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AN ESTIMATOR OF MOMENT OF INERTIA OF MAGIC SQUARE ARRAY OF MASSES

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ABSTRACT

The classical definition of magic square of order N is a square array of consecutive natural numbers from 1 to N^2 such that the raw, column and diagonal sum add up to the same number. When the magic square entries are considered as an array of masses of a rigid body, it is established that its moment of inertia is a function of N. This work consider a more general magic array of masses of a rigid body to establish its inertial moment as a function of the magic sum and the central entry of the array. The paper further discusses the advantage of this development.

Keywords: Magic square, moment of inertia, kinetic energy, axis

INTRODUCTION

Magic square is a square matrix array of numbers consisting of a distinct positive consecutive integers 1,2,3,…… N^2, arranged such that the sum of the numbers in any horizontal, vertical,or main diagonal line is always the same number, (Martin, 2014). Magic squares are categorised in to five major types: Normal magic square, Even/Odd Order magic square, Doubly Even magic squares, Singly even magic squares, Pandiagonal magic squares, Regular magic squares (Pickover, 2002).

4	9	2
3	5	7
8	1	6

Figure 1: A magic square array of consecutive natural numbers with magic sum 15.

Magic squares of order N having entries $1, 2, \dots, N^2$ such that the sum of all entries along the raws, columns and main diagonals are equal to the magic constant of the square, where studied as an array of masses by Loly, (2004). This communication extends the work of Loly, (2004) to consider a more general magic squares (Ward, 1980) of order 3 with entries from a set of real numbers that is $a_{ij} \epsilon R$. The littereture on magic square array of real numbers were mostly develop to establish a vector space for a general collection of magic square arrays (Cross, 1966; Eperson, 1962; Holmes, 1970; Mayoral, 1996; Ward, 1980). Moment of inertia was instrumentally incorporated in mathematical derivation of Rayleigh beam with damping coefficient in the work of Usman et al., (2020).

Consequently, the magic array was interpreted in this work to have entries as masses of a rigid body proportional to real numbers that form a magic square array. Thus, we derived the moment of inertia about the centre of the magic square which is the axis of rotation of the magic array of masses as function of the magic sum and the central entry of the magic array. The scalar moment of inertia I_s is found by summing mr^2 for each entry of the real number in the magic square where m is the mass of the real number and r is the distance of the real number from the axis of rotation (centre of the magic square). The result of this research work provides a great example of the properties of the inertia tensor in the field of classical mechanics and extend the real life application of magic square other than its conventional recreational purposes.

METHOD AND MATERIALS

In this section, we introduce the principal concepts that are necessary in the derivation of the result presented and more elaborate analysis on the consequence of the result.

Kinetic Energy (K.E.)

An object's kinetic energy is the energy it possesses as a result of its motion(Jain, 2012). In this case the work required to accelerate a given mass body from rest to its stated velocity. Unless the body's speed changes, it retains its kinetic energy after gaining it during acceleration.

Moment of Inertia of Rotational Kinetic Energy

Moment of inertia refers to a measure of a body's propensity to resist angular acceleration is the sum of the products of the masses of all of its constituent particles and their squares of distance from the axis of rotation (Marion & Thornton, 1995). Consider a rigid body rotating about a fixed axis O and made up of masses $m_1, m_2, \dots, m_n$ each having distances $r_1, r_2, \dots, r_n$ respectively from axis of rotation (Fig. 2.) (Kashimbila, 2003).

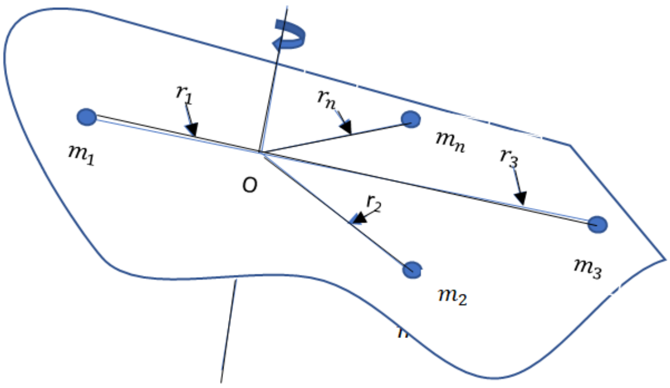

Figure 2: Rigid body rotating about an axis O

The kinetic energy (K.E) for each mass is respectively given by

$$KE_1 = \frac{1}{2}m_1 {v_1}^2,$$

$$KE_2 = \frac{1}{2}m_2 {v_2}^2,$$

.

.

.,

$$KE_n = \frac{1}{2}m_n {v_n}^2$$

where, $m_1, m_2, \ldots, m_n$ are masses that made up the rigid body and $v_1, v_2, \ldots, v_n$ as their linear velocities respectively.

The total kinetic energy KE_T of the rigid body is given by

$$KE_T = KE_1 + KE_2 + \ldots + KE_n$$

$$= \frac{1}{2}m_1 {v_1}^2 + \frac{1}{2}m_2 {v_2}^2 + \ldots + \frac{1}{2}m_n {v_n}^2$$

But, $v = r\omega$, where, v is the linear velocity and ω is angular velocity. So,

$$KE_T = \frac{1}{2}m_1(r_1\omega)^2 + \frac{1}{2}m_2(r_2\omega)^2 + \ldots + \frac{1}{2}m_n(r_n\omega)^2$$

$$= \frac{1}{2}m_1{r_1}^2\omega^2 + \frac{1}{2}m_2{r_2}^2\omega^2 + \cdots \frac{1}{2}m_n{r_n}^2\omega^2$$

$$= \frac{1}{2}\omega^2(m_1{r_1}^2 + m_2{r_2}^2 +, \ldots + m_n{r_n}^2)$$

$$= \frac{1}{2}\omega^2 \sum_{i=1}^{n} m_i{r_i}^2$$

$$= \frac{1}{2}\omega^2 I$$

where I is the moment of inertia, with

$$I = \sum_{i=1}^{n} m_i{r_i}^2 \tag{2}$$

Magic Square

A classic definition of magic square of order N as a square array of numbers consisting of the distinct positive integers (1,2,3, …. N^2) arranged such that the sum of the 'N' numbers in any horizontal, vertical, main diagonal and anti-diagonal line is always the same number (Martin, 2014). In a more elaborate mathematical form, magic square in its perfect form refers to a square array of N^2 boxes (cells) filled with consecutive numbers from 1 to N^2 such that the row sums, column sums and diagonal sums of the entries are all equal to the same number called magic sum S.

4	9	2
3	5	7
8	1	6

A

m_{11}	m_{12}	m_{13}
m_{21}	m_{22}	m_{23}
m_{31}	m_{32}	m_{33}

B

Figure 3: A) Magic square with natural number entries and B) A magic square array of masses as entries.

RESULTS AND ANALYSIS

Moment of Inertia of Magic Square

The moment of inertia of a magic square of order 3 with magic sum S and entries $m_{i,j} \in R$, $(i,j = 1,2,3)$ can be described using the illustration in fig2. The axis of rotation is cantered at the central cell of the magic square with mass m_{22} of 0 unit radius away from the centre. The perpendicular cells with masses m_{12}, m_{21}, m_{23} and m_{32} respectively positioned at the centre of their corresponding cells were 1unit radii away from the centre of the magic square. The diagonal cells with masses m_{11}, m_{13}, m_{31} and m_{33} which are respectively positioned at the centre of the cells were all √2units radii away from the centre of the magic square. The 1unit radii r_p were represented with straight lines of fig.3 while √2units radii r_d were represented with broken lines of fig.3. The diagonal distance of √2units is arrived at using a Pythagoras theorem that relates the blue and the red lines.

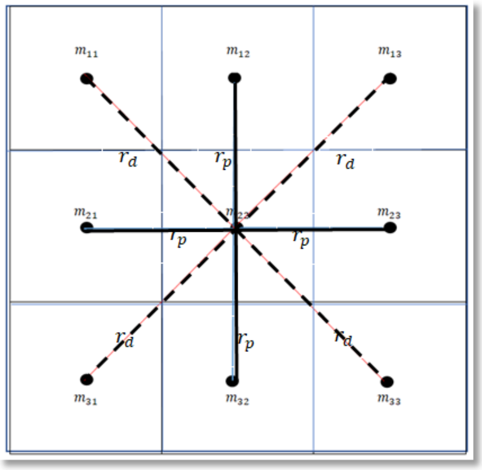

Figure 4: Description of magic square array of masses, m,$_{ij}$, *i,j=1,2,3* and respective distances away from center.

Now, if we rotate the magic square array of masses around its axis of rotation (centre of the magic square), then the moment of inertia of a magic square is given by

Theorem 1.0: *Let the entries of order 3 magic square be an array of masses m_{ij} with their respective distances as describe in Figure 4. Then the moment of inertia of order 3 magic square array of masses is given by $I_s = 5S - 3m_{22}$, where S is the magic sum and m_{22} is the central mass in the magic array of masses.*

Proof:

Relating the array in Figure 4 and derivation of general moment of inertia in

Equation (1), we can define moment of inertia of the array as follows.

$$I_s = (m_{12}r_p^2 + m_{21}r_p^2 + m_{23}r_p^2 + m_{32}r_p^2) + (m_{11}r_d^2 + m_{13}r_d^2 + m_{32}r_d^2 + m_{33}r_d^2) + m_{22}(0^2)$$

$$= (m_{12} + m_{21} + m_{23} + m_{32})r_p^2 + (m_{11} + m_{13} + m_{31} + m_{33})r_d^2$$

$$= (m_{12} + m_{21} + m_{23} + m_{32})1^2 + (m_{11} + m_{13} + m_{31} + m_{33})\sqrt{2}^2$$

$$= (m_{12} + m_{21} + m_{23} + m_{32}) + 2(m_{11} + m_{13} + m_{31} + m_{33})$$

$$= (m_{21} + m_{23}) + (m_{11} + m_{12} + m_{13}) + (m_{31} + m_{32} + m_{33}) + (m_{11} + m_{13}) + (m_{31} + m_{33})$$

$$= (m_{21} + m_{23}) + (m_{11} + m_{12} + m_{13}) + (m_{31} + m_{32} + m_{33}) + (m_{11} + m_{33}) + (m_{13} + m_{31})$$

$$= (S - m_{22}) + S + S + (S - m_{22}) + (S - m_{22})$$

$$= 5S - 3m_{22}$$

Hence, the moment of inertia of order three magic square array of masses is five times the magic sum S minus 3 times the central mass m_{22}. This is mathematically given by.

$$I_s = 5S - 3m_{22} \tag{2}$$

Discussion of Result

The result in Theorem 1.0 simplifies the computational requirement of estimating a moment of inertia of a magic array of masses from computational requirement of the series estimator in Equation (1) to a simplified version of estimator in our result of equation (2) which require a linear combination of magic sum S and the central entry of the array. To this note, we can as well observe the following real-life application of the magic square array of masses and the proposed estimator of its inertial moment.

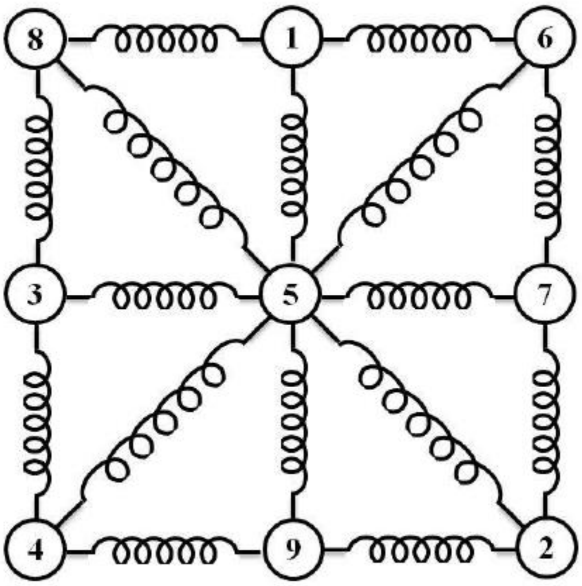

Figure 5: Particles and springs system of a 3 x 3 magic square array of masses

Application in a System of Particles and Springs.

Suppose we have n^2 discrete array of masses and $2(n^2 - 1)$ identical springs vibrating system which construct a system of particles of magic squares array of masses. In particular, for the system in Figure 5, we have 9 masses which connect to each other by 16 springs for the first step. The initially conditions for these oscillators can be describe as follows,

(i) The two-dimensional coordinates.

(ii) Masses form an array mass of magic squares.

(iii) The given springs constant are K.

(iv) Center of mass along with geometrical center of the system coincide and are constant.

(v) The viscous or frictional forces effects or external gravitational fields are negligible.

If the system is magic square array of order 3, we then have a system with 3 constants of the motion in minimum, namely, the vertical and horizontal linear momentums and also the angular momentum. In this case the three of the frequencies vanish from 18 modes of oscillation. To illustrate, these three zeroes result from symmetry on shift and transference in two directions at the system's plane and symmetry on rotation about the plane's perpendicular axis. The inertial moment of the system can be derived from the magic sum of 15 units and central mass of 5 units using Theorem 1.0 of our result giving by

$$I_s = 5S + m_{22} = 5 \times 15 + 5 = 80$$

However, a specific physical application of a magic square array described above can be seen in a clutching system such that the clutch plate (disc) which is arranged in such a way that the springs form an array of objects positioned at equal distances from one another. The clutch serves as a mechanical link between the engine and transmission, disconnecting or separating the engine from the transmission system for a brief period of time. When the clutch pedal is depressed, the drive wheels are disconnected, allowing the driver to change gears smoothly. In a torque controlled drill, for example, one shaft is driven by a motor and the other by a drill chuck. A clutch connects the two shafts, allowing them to be locked together and spin at the same speed (engaged), locked together but spinning at different speeds (slipping), or unlocked and spinning at different speeds (disengaged).

CONCLUSION

In this paper an expression of moment of inertia for a magic square array of masses was derived. We first consider transforming non-unit entities of numbers to masses in a 3 x 3 magic square. Then the magic array of masses was assumed to be that of a rigid body rotating about a fixed axis. Consequently, the derived expression has simplified the computational requirement of finding the moment of inertia of a rigid body with magic array of masses by simply using the magic sum S and the central mass m_{22} located at the centre of the body. The formulation has provided a great example of the properties of the inertia tensor in the field of classical mechanics and extend the real life application of magic square other than its conventional recreational purposes. Generalization of the formulation for order n magic square array of masses of a rigid body will be an interesting future research to consider.

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