FUDMA Journal of Sciences (FJS)
ISSN online: 2616-1370
ISSN print: 2645 - 2944
Vol. 6 No. 4, August, 2022, pp 60 - 67
DOI: https://doi.org/10.33003/fjs-2022-0604-989

NUMERICAL SOLUTIONS OF COVID-19 SIRD MODEL IN NIGERIA

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ABSTRACT

A mathematical model of the Susceptible, Infectious, Recovery and Death (SIRD) for the spread of the COVID-19 disease in Nigeria was considered in this paper. The model values and parameters were obtained from Nigeria Centre for Disease Control Coronavirus COVID-19. The model was solved using Heun's method and Runge-Kutta's method of orders four and five to obtain an approximated solution for the model with the help of the Matlab Program. The result obtained was illustrated in plots to show the progression of the disease in the various classes. The comparisons of the three numerical methods used correspond well with each other by showing the same behavior pattern.

Keywords: COVID-19, Nigeria, SIRD, Heun's method, Fourth-order Runge-Kutta method, Fifth-order Runge-Kutta method, Matlab

INTRODUCTION

Nigeria the most populous country in Africa reported her first index case of Covid-19 confirmed case on 27th February 2020, when an Italian citizen in Lagos tested positive for the disease (First case of COVID-19) (NCDC, 2020). Since the first index case, Nigeria has since then experienced a rapid progression of the COVID-19 disease in all the 36 states in the country. COVID-19 which is known as Coronavirus disease 2019 is highly transmittable and its virus is responsible for Severe Acute Respiratory Syndrome (SARS). COVID-19 symptoms range from mild to severe symptoms and can also lead to dead when treatment is delayed (Jamilu *et al*., 2020). To be able to understand the progression of the spread, different tools has be developed, one of which is a mathematical model. The development of the mathematical model has become an imperative tool for understanding infectious disease dynamics such as COVID 19, Ebola Virus, HIV/AIDs, Malaria, Lassa fever, and lots more. The use of these tools for predicting and analyzing the spread and control of disease is useful for decision-making policies to reduce the mortality rate and as well protect the population. The fundamental procedure in modeling the spread of disease is through the use of compartmental models. Compartmental models are described using a set of mathematical equations to understand how individuals in various "compartments" in a population interact with one another (Towers, 2013).

The most common compartmental models are deterministic models which can be divided into ordinary differential equations (ODEs) or partial differential equations (PDEs). The SIR model is the basic compartment model widely used to study epidemic diseases. This model is divided into three compartments that describe the movement of an individual between the Susceptible, Infective I, and Recovery R compartments, respectively. Several modifications have been made to the SIR model, such as SIS, SEIS, SEIR models, etc. The SIR model can be modified also to include Death compartment D. The modified SIR which is called the SIRD model is described as Susceptible(S), Infected (I), Recovery (R), and Death (D). SIRD model has been used by (Al-Raeei, 2020; Calafiore and Fracastoro, 2022; Fernandez-Villaverde and Jones, 2022; Ferrari *et al., 2021;* Martinez, 2021; Mohamed and Al-Raeei, 2021; Norah *et al.*, 2020; Sooppy Nisar *et al*., 2020). Most mathematical models for infectious disease dynamics are generally written as a system of nonlinear differential Equations, in which their exact solution is difficult to solve analytically. To compute the solutions of these models, approximate or perturbation methods are often employed (He, 2004). One of the numerical techniques is the Runge-Kutta method.

The Runge-Kutta method was developed by Runge and Kutta (Kutta 1901; Runge 1895) to find the solution to the differential equation in the field of atomic spectra. This method has since then been the most popular numerical method for finding numerical approximate for nonlinear differential equations because of its accuracy, stability, and ease of programming, it is particularly suitable when the compilation of higher derivatives is complicated and at the same time decreases the error (Islam, 2015).

In this work, we aimed at analyzing the progression of the COVID-19 pandemic in Nigeria from February 27th, 2020 to May 10th, 2022 (NCDC, 2022) using numerical methods. Numerical methods such as Heun's and Runge-Kutta methods of orders four and five (RK4 and RK5) would be used to determine an approximated solution for the COVID-19 SIRD model as proposed by (Martinez, 2021).

MATERIALS AND METHODS.

COVID-19 SIRD MODEL

SIRD model is a compartment model which is divided into four epidemiological classes to determine how each of these classes would be affected by the diseases. The SIRD model is also a deterministic model formulated in terms of ordinary differential equations (ODEs) and these equations are divided into a system of four differential equations. The COVID-19 SIRD model proposed by (Martinez, 2021) was used in this paper and is given below

$$\frac{dS}{dt} = -\beta S(t)I(t) \tag{1}$$

$$\frac{dI}{dt} = \beta S(t)I(t) - \alpha I(t) - \gamma I(t) \tag{2}$$

$$\frac{dR}{dt} = \alpha I(t) \quad (3)$$
$$\frac{dD}{dt} = \gamma I(t) \quad (4)$$

The initial conditions and parameters that are used are cumulative valves for the various classes, which are taken from the Nigeria Centre for Disease Control Coronavirus COVID-19 Microsite (NCDC, 2022). The initial conditions and parameters are described in the table below.

Table 1: Description of Parameter. The values and parameters used in the model are from March 27th, 2020 to May 10th, 2022.

Symbol	Description	Source and Values
S	Individual are susceptible to the disease	5114703 (NCDC, 2022)
I	Individual per unit of time who are infected with the disease	255,802 (NCDC, 2022)
R	Individual per unit of time who recovered from the disease	249,936 (NCDC, 2022)
D	Individual who dead of the disease.	3143 (Nigeria Covid-19 Tracker, 2022)
α	Recovery rate per unit of time	0.97716 (Nigeria Covid-19 Tracker, 2022)
β	Infected rate per unit of time	4.69385*10^(-7) Estimated
γ	Death rate per unit of time	0.01229 (Nigeria Covid-19 Tracker, 2022

NUMERICAL METHODS

DEFINITION OF HEUN'S AND RUNGE-KUTTA METHODS OF FOURTH AND FIFTH ORDER:

(A). Heun's method

Heun's method is named after a German mathematician Karl Heun and it is a stage two Runge-Kutta method that is also known as the improved Euler method (Ender and David, 2003). Heun's method is a numerical procedure for solving ordinary differential equations with given initial conditions. We consider the given problem $y' = f(t, y)$ with initial condition $y(t_0) = t_0$

Computing for the approximate solution y_{i+1} we have

$$y_{i+1}^{p} = y_i + hf(t_i,\ y_i) \quad (5)$$

$$y_{i+1} = y_i + h/2[f(t_i, y_i) + f(t_{i+1} + y_{i+1}^{p})] \quad (6)$$

y' represent the derivative function of f, y_{i+1}^{p} is the initial intermediate value, y_{i+1} is the approximate solution, $h = (t_{i+1} - t_i)/n \ for \ t_0 \leq t \leq t_i \ and \ i = 1,2,3,\dots,n.$ t is the independent variable, y is the dependent variable, h is the fixed step size and n is the number of points for both methods.

(B). Runge-kutta methods of fourth and fifth order:

Runge-Kutta fourth-order method (RK4) is also known as the classical Runge-Kutta method, this method is a four-stage method that requires four function evaluations to advance the numerical solution in a one-time step (Roslan and KihCman, 2013). Runge-Kutta fifth-order method (RK5) is a six-stage method that requires six function evaluations to advance the numerical solution in one time step and it has two more function evaluations than RK4 (Gopa *et al*., 2006).

The Runge-Kutta method of order four and five are defined below. Consider the given problem

$$\begin{cases} y' = f(t, y) \\ y(t_0) = y_0 \end{cases} \quad (7)$$

Where y' represent the derivative function of f, t is an independable variable and y is a dependable variable.

Computing for the approximate solution for both methods, we have

For the Fourth Order Method (RK4)

$$y_{i+1} = y_i + \frac{1}{6}h(k_1 + 2k_2 + 2k_3 + k_4) \quad (8)$$

$$where \ \ k_1 = f(t_i, y_i)$$

$$k_2 = f(t_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h)$$

$$k_3 = f(t_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2 h)$$

$$k_4 = f(t_i + h, y_i + k_3 h)$$

and for Fifth Order Method(RK5)

$$y_{i+1} = y_i + \frac{1}{90}h(7k_1 + 32k_3 + 12k_4 + 32k_5 + 7k_6) \quad (9)$$

$$where\ k_1 = f(t_i, y_i)$$

$$k_2 = f(t_i + \tfrac{1}{4}h, y_i + \tfrac{1}{4}k_1 h)$$

$$k_3 = f(t_i + \tfrac{1}{4}h, y_i + \tfrac{1}{8}k_1 h + \tfrac{1}{8}k_2 h)$$

$$k_4 = f(t_i + \tfrac{1}{2}h, y_i - \tfrac{1}{2}k_2 h + k_3 h)$$

$$k_5 = f(t_i + \tfrac{3}{4}h, y_i + \tfrac{3}{16}k_1 h + \tfrac{9}{10}k_4 h)$$

$$k_6 = f(t_i + h, y_i - \tfrac{3}{7}k_1 h + \tfrac{2}{7}k_2 h + \tfrac{12}{7} + k_3 h - \tfrac{12}{7} + k_4 h + \tfrac{8}{7}k_5 h)$$

Where
$h = (t_{i+1} - t_i)/n\ for\ t_0 \le t \le t_i\ and\ i = 1,2,3,\dots,n.$ $t$is the independent variable, y is the dependent variable, h is the fixed step size and n is the number of points for both methods.

TRANSFORMATION OF RUNGE-KUTTA METHODS FOR SIRD MODEL

The general form of equation (1-4) of the SIRD COVID-19 Model can be solved by using Heun's method, fourth and fifth order Runge-Kutta (RK4 & RK5) methods with the given initial conditions S_0 , I_0 , R_0, D_0 and parameters as described in Table 1.

The Heun's iterative formulas for $S(t), I(t), R(t)\ and\ D(t)$ of SIRD model can be written out as

$$S_{i+1}^p = S_i + hf_S(t_i, S_i, I_i, R_i, D_i) \quad (10)$$
$$I_{i+1}^p = I_i + hf_I(t_i, S_i, I_i, R_i, D_i) \quad (11)$$
$$R_{i+1}^p = R_i + hf_R(t_i, S_i, I_i, R_i, D_i) \quad (12)$$
$$= D_i + hf_D(t_i, S_i, I_i, R_i, D_i) \quad (13)$$

$$where \quad \frac{dS}{dt} = f_S, \frac{dI}{dt} = f_I, \quad \frac{dR}{dt} = f_R, \frac{dD}{dt} = f_D \text{ and}$$

$$S_{i+1} = S_i + h/2[f_S(t_i, S_i, I_i, R_i, D_i) + f_S(t_{i+1} + S_{i+1}^p), f_I(t_i, S_i, I_i, R_i, D_i) + f_I(t_{i+1} + I_{i+1}^p), f_R(t_i, S_i, I_i, R_i, D_i) + f_R(t_{i+1} + R_{i+1}^p), f_D(t_i, S_i, I_i, R_i, D_i) + f_D(t_{i+1} + D_{i+1}^p)] \quad (14)$$

$$I_{i+1} = I_i + h/2[f_S(t_i, S_i, I_i, R_i, D_i) + f_S(t_{i+1} + S_{i+1}^p), f_I(t_i, S_i, I_i, R_i, D_i) + f_I(t_{i+1} + I_{i+1}^p), f_R(t_i, S_i, I_i, R_i, D_i) + f_R(t_{i+1} + R_{i+1}^p), f_D(t_i, S_i, I_i, R_i, D_i) + f_D(t_{i+1} + D_{i+1}^p)] \quad (15)$$

$$R_{i+1} = R_i + h/2[f_S(t_i, S_i, I_i, R_i, D_i) + f_S(t_{i+1} + S_{i+1}^p), f_I(t_i, S_i, I_i, R_i, D_i) + f_I(t_{i+1} + I_{i+1}^p), f_R(t_i, S_i, I_i, R_i, D_i) + f_R(t_{i+1} + R_{i+1}^p), f_D(t_i, S_i, I_i, R_i, D_i) + f_D(t_{i+1} + D_{i+1}^p)] \quad (16)$$

$$D_{i+1} = D_i + h/2[f_S(t_i, S_i, I_i, R_i, D_i) + f_S(t_{i+1} + S_{i+1}^p), f_I(t_i, S_i, I_i, R_i, D_i) + f_I(t_{i+1} + I_{i+1}^p), f_R(t_i, S_i, I_i, R_i, D_i) + f_R(t_{i+1} + R_{i+1}^p), f_D(t_i, S_i, I_i, R_i, D_i) + f_D(t_{i+1} + D_{i+1}^p)] \quad (17)$$

The RK4 iterative formulas for $S(t), I(t), R(t)\ and\ D(t)$ of SIRD model can be written out as

$$S_{i+1} = S_i + \frac{1}{6}h(kS_1 + 2kS_2 + 2kS_3 + kS_4) \quad (18)$$
$$I_{i+1} = I_i + \frac{1}{6}h(kI_1 + kI_2 + 2kI_3 + kI_4) \quad (19)$$
$$R_{i+1} = R_i + \frac{1}{6}h(kR_1 + 2kR_2 + 2kR_3 + kR_4) \quad (20)$$
$$D_{i+1} = D_i + \frac{1}{6}h(kD_1 + 2kD_2 + 2kD_3 + kD_4) \quad (21)$$

$$where \quad \frac{dS}{dt} = f_S, \frac{dI}{dt} = f_I, \quad \frac{dR}{dt} = f_R, \frac{dD}{dt} = f_D \text{ and}$$

$$kS_1 = f(t_i, S_i, I_i, R_i, D_i)$$
$$kI_1 = f_S(t_i, S_i, I_i, R_i, D_i)$$
$$kR_1 = f_R(t_i, S_i, I_i, R_i, D_i)$$

$$kD_1 = f_D(t_i, S_i, I_i, R_i, D_i)$$

$$kS_2 = f_S\left(t_i + \frac{h}{2}, S_i + \frac{kS_1}{2}, I_i + \frac{kI_1}{2}, R_i + \frac{kR_1}{2}, D_i + \frac{kD_1}{2}\right)$$
$$kI_2 = f_I\left(t_i + \frac{h}{2}, S_i + \frac{kS_1}{2}, I_i + \frac{kI_1}{2}, R_i + \frac{kR_1}{2}, D_i + \frac{kD_1}{2}\right)$$
$$kR_2 = f_R\left(t_i + \frac{h}{2}, S_i + \frac{kS_1}{2}, I_i + \frac{kI_1}{2}, R_i + \frac{kR_1}{2}, D_i + \frac{kD_1}{2}\right)$$
$$kD_2 = f_D\left(t_i + \frac{h}{2}, S_i + \frac{kS_1}{2}, I_i + \frac{kI_1}{2}, R_i + \frac{kR_1}{2}, D_i + \frac{kD_1}{2}\right)$$
$$kS_3 = f_S\left(t_i + \frac{h}{2}, S_i + \frac{kS_2}{2}, I_i + \frac{kI_2}{2}, R_i + \frac{kR_2}{2}, D_i + \frac{kD_2}{2}\right)$$
$$kI_3 = f_I\left(t_i + \frac{h}{2}, S_i + \frac{kS_2}{2}, I_i + \frac{kI_2}{2}, R_i + \frac{kR_2}{2}, D_i + \frac{kD_2}{2}\right)$$
$$kR_3 = f_R\left(t_i + \frac{h}{2}, S_i + \frac{kS_2}{2}, I_i + \frac{kI_2}{2}, R_i + \frac{kR_2}{2}, D_i + \frac{kD_2}{2}\right)$$
$$kD_3 = f_D\left(t_i + \frac{h}{2}, S_i + \frac{kS_2}{2}, I_i + \frac{kI_2}{2}, R_i + \frac{kR_2}{2}, D_i + \frac{kD_2}{2}\right)$$

$$kS_4 = f_S(t_i + h, S_i + kS_3, I_i + kI_3, R_i + kR_3, D_i + kD_3)$$
$$kI_4 = f_I(t_i + h, S_i + kS_3, I_i + kI_3, R_i + kR_3, D_i + kD_3)$$
$$kR_4 = f_R(t_i + h, S_i + kS_3, I_i + kI_3, R_i + kR_3, D_i + kD_3)$$
$$kD_4 = f_D(t_i + h, S_i + kS_3, I_i + kI_3, R_i + kR_3, D_i + kD_3)$$

For the Fifth Order Method:

The RK5 iterative formulas for $S(t), I(t), R(t)$ *and* $D(t)$ of SIRD model can be written out as

$$S_{i+1} = S_i + \frac{1}{90}h(7kS_1 + 32kS_3 + 12kS_4 + 32kS_5 + 7kS_6) \quad (22)$$
$$I_{i+1} = I_i + \frac{1}{90}h(7kI_1 + 32kI_3 + 12kI_4 + 32kI_5 + 7kI_6) \quad (23)$$
$$R_{i+1} = R_i + \frac{1}{90}h(7kR_1 + 32kR_3 + 12kR_4 + 32kR_5 + 7kI_6) \quad (24)$$
$$D_{i+1} = D_i + \frac{1}{90}h(7kD_1 + 32kD_3 + 12kD_4 + 32kD_5 + 7kD_6) \quad (25)$$

$$kS_1 = f_S(t_i, S_i, I_i, R_i, D_i)$$
$$kI_1 = f_I(t_i, S_i, I_i, R_i, D_i)$$
$$kR_1 = f_R(t_i, S_i, I_i, R_i, D_i)$$
$$kD_1 = f_D(t_i, S_i, I_i, R_i, D_i)$$

$$kS_2 = f_S(t_i + \tfrac{1}{4}h, S_i + \tfrac{1}{4}kS_1h, I_i + \tfrac{1}{4}kI_1h, R_i + \tfrac{1}{4}kR_1h, D_i + \tfrac{1}{4}kD_1h)$$
$$kI_2 = f_I(t_i + \tfrac{1}{4}h, S_i + \tfrac{1}{4}kS_1h, I_i + \tfrac{1}{4}kI_1h, R_i + \tfrac{1}{4}kR_1h, D_i + \tfrac{1}{4}kD_1h)$$
$$kR_2 = f_R(t_i + \tfrac{1}{4}h, S_i + \tfrac{1}{4}kS_1h, I_i + \tfrac{1}{4}kI_1h, R_i + \tfrac{1}{4}kR_1h, D_i + \tfrac{1}{4}kD_1h)$$
$$kD_2 = f_D(t_i + \tfrac{1}{4}h, S_i + \tfrac{1}{4}kS_1h, I_i + \tfrac{1}{4}kI_1h, R_i + \tfrac{1}{4}kR_1h, D_i + \tfrac{1}{4}kD_1h)$$

$$kS_3 = f_S(t_i + \tfrac{1}{4}h, S_i + \tfrac{1}{8}kS_1h + \tfrac{1}{8}kS_2h, I_i + \tfrac{1}{8}kI_1h + \tfrac{1}{8}kI_2h, R_i + \tfrac{1}{8}kR_1h + \tfrac{1}{8}kR_2h, D_i + \tfrac{1}{8}kD_1h + \tfrac{1}{8}kD_2h)$$
$$kI_3 = f_I(t_i + \tfrac{1}{4}h, S_i + \tfrac{1}{8}kS_1h + \tfrac{1}{8}kS_2h, I_i + \tfrac{1}{8}kI_1h + \tfrac{1}{8}kI_2h, R_i + \tfrac{1}{8}kR_1h + \tfrac{1}{8}kR_2h, D_i + \tfrac{1}{8}kD_1h + \tfrac{1}{8}kD_2h)$$
$$kR_3 = f_R(t_i + \tfrac{1}{4}h, S_i + \tfrac{1}{8}kS_1h + \tfrac{1}{8}kS_2h, I_i + \tfrac{1}{8}kI_1h + \tfrac{1}{8}kI_2h, R_i + \tfrac{1}{8}kR_1h + \tfrac{1}{8}kR_2h, D_i + \tfrac{1}{8}kD_1h + \tfrac{1}{8}kD_2h)$$
$$kD_3 = f_D(t_i + \tfrac{1}{4}h, S_i + \tfrac{1}{8}kS_1h + \tfrac{1}{8}kS_2h, I_i + \tfrac{1}{8}kI_1h + \tfrac{1}{8}kI_2h, R_i + \tfrac{1}{8}kR_1h + \tfrac{1}{8}kR_2h, D_i + \tfrac{1}{8}kD_1h + \tfrac{1}{8}kD_2h)$$

$$kS_4 = f_S(t_i + \tfrac{1}{2}h, S_i - \tfrac{1}{2}kS_2h + kS_3h, I_i - \tfrac{1}{2}kI_2h + kI_3h, R_i - \tfrac{1}{2}kR_2h + kR_3h, D_i - \tfrac{1}{2}kD_2h + kD_3h)$$
$$kI_4 = f_I(t_i + \tfrac{1}{2}h, S_i - \tfrac{1}{2}kS_2h + kS_3h, I_i - \tfrac{1}{2}kI_2h + kI_3h, R_i - \tfrac{1}{2}kR_2h + kR_3h, D_i - \tfrac{1}{2}kD_2h + kD_3h)$$
$$kR_4 = f_R(t_i + \tfrac{1}{2}h, S_i - \tfrac{1}{2}kS_2h + kS_3h, I_i - \tfrac{1}{2}kI_2h + kI_3h, R_i - \tfrac{1}{2}kR_2h + kR_3h, D_i - \tfrac{1}{2}kD_2h + kD_3h)$$
$$kD_4 = f_D(t_i + \tfrac{1}{2}h, S_i - \tfrac{1}{2}kS_2h + kS_3h, I_i - \tfrac{1}{2}kI_2h + kI_3h, R_i - \tfrac{1}{2}kR_2h + kR_3h, D_i - \tfrac{1}{2}kD_2h + kD_3h)$$

$$kS_5 = f(t_i + \tfrac{3}{4}h, S_i + \tfrac{3}{16}kS_1h + \tfrac{9}{10}kS_4h, I_i + \tfrac{3}{16}kI_1h + \tfrac{9}{10}kI_4h, R_i + \tfrac{3}{16}kR_1h + \tfrac{9}{10}kR_4h, D_i + \tfrac{3}{16}kD_1h + \tfrac{9}{10}kD_4h)$$

$$kI_5 = f_S(t_i + \tfrac{3}{4}h, S_i + \tfrac{3}{16}kS_1h + \tfrac{9}{10}kS_4h, I_i + \tfrac{3}{16}kI_1h + \tfrac{9}{10}kI_4h, R_i + \tfrac{3}{16}kR_1h + \tfrac{9}{10}kR_4h, D_i + \tfrac{3}{16}kD_1h + \tfrac{9}{10}kD_4h)$$

$$kR_5 = f_I(t_i + \frac{3}{4}h, S_i + \frac{3}{16}kS_1h + \frac{9}{10}kS_4h, I_i + \frac{3}{16}kI_1h + \frac{9}{10}kI_4h, R_i + \frac{3}{16}kR_1h + \frac{9}{10}kR_4h, D_i + \frac{3}{16}kD_1h + \frac{9}{10}kD_4h)$$

$$kD_5 = f_R(t_i + \frac{3}{4}h, S_i + \frac{3}{16}kS_1h + \frac{9}{10}kS_4h, I_i + \frac{3}{16}kI_1h + \frac{9}{10}kI_4h, R_i + \frac{3}{16}kR_1h + \frac{9}{10}kR_4h, D_i + \frac{3}{16}kD_1h + \frac{9}{10}kD_4h)$$

$$k_6 = f_D(t_i + h, y_i - \frac{3}{7}k_1h + \frac{2}{7}k_2h + \frac{12}{7} + k_3h - \frac{12}{7} + k_4h + \frac{8}{7}k_5h)$$

Where
$S(t_0) = S_0\ , I(t_0) = I_0\ , R(t_0) = R_0\ \ and\ D(t_0) = D_0\ are\ initial\ conditions,\ h = (t_{i+1} - t_i)/n\ \ and\ for\ t_0 \leq t \leq t_i,$ t is the independent variable, S, I, R, D are dependent variables, h is the fixed step size and n is the number of points for the above methods.

SIMULATION RESULTS AND DISCUSSIONS

We used Heun's method, fourth and fifth-order Runge-Kutta to solve the COVID-19 SIRD model and the numerical result were obtained with the implementation of the MATLAB program. The initial conditions and parameter values in Table 1 were used to solve the model. Also, we considered a duration of 26 months, 11 days (t) starting from 27th February 2020 to 10th May 2022 and we take the fixed time step as, h=0.1

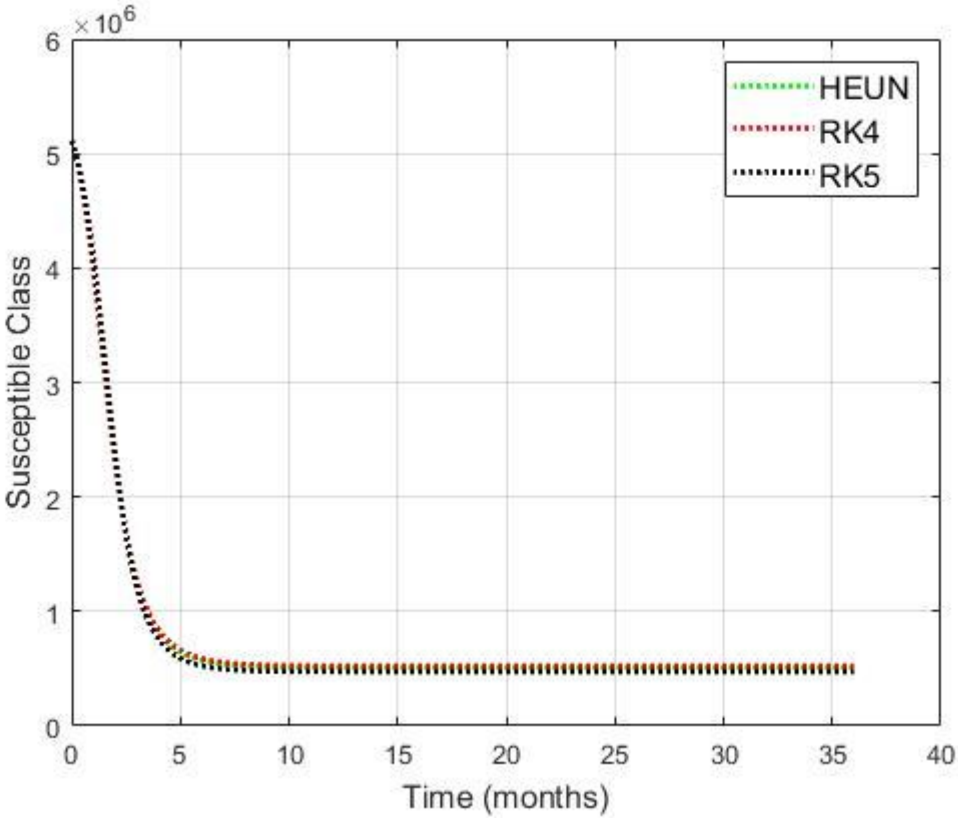

Figure 1: Graphical representation of susceptible class against time for Heun's, Rk4 and Rk5

It was observed from Figure1, that the three methods show the same graphical result. Also, as the number of month's increases, the progression of the susceptible class decreases as a result of migration to the infected class, also the susceptible class becomes linear, this linearity was due to the various control measures and vaccination put in place by Nigeria's government during the duration.

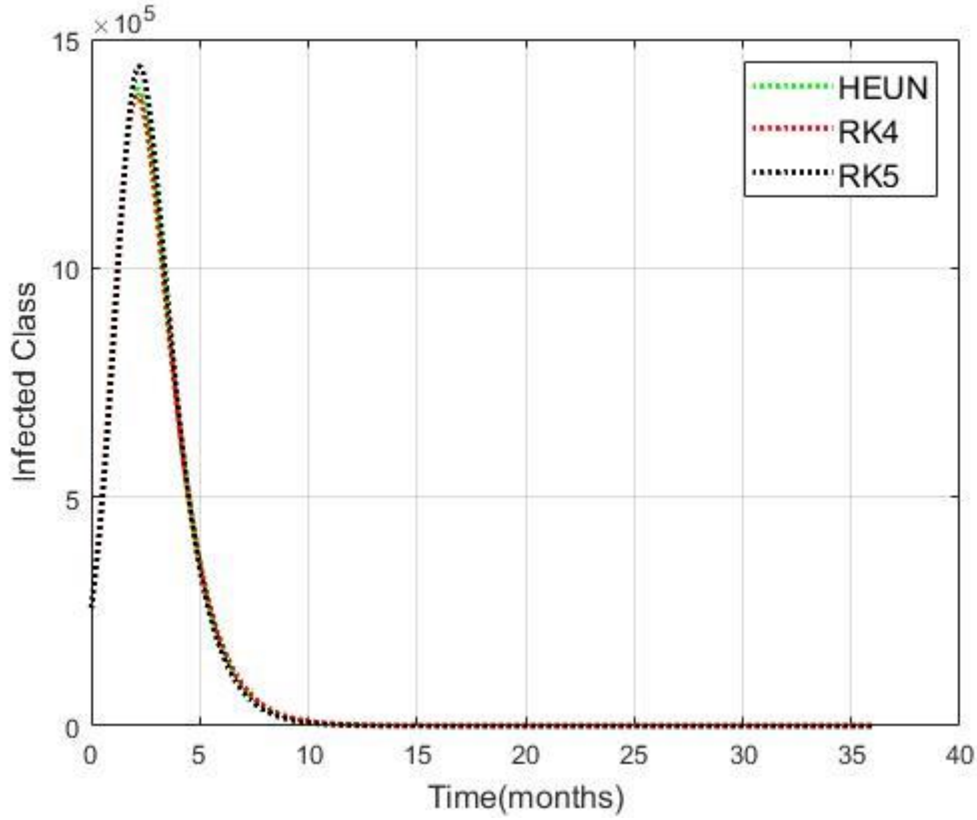

Figure 2: graphical representation of infected class against time for Heun, Rk4 and Rk5

As the Susceptible Class was been exposed to infection, a rapid increase in the Infected Class was observed in Figure2. This rise was due to a lack of awareness of the disease thereby leading to a drastic increase in the Death Class. The simulation result shows that the RK5 method is more stable and gives a better peak of the infection than the Heun's and the RK4 methods.

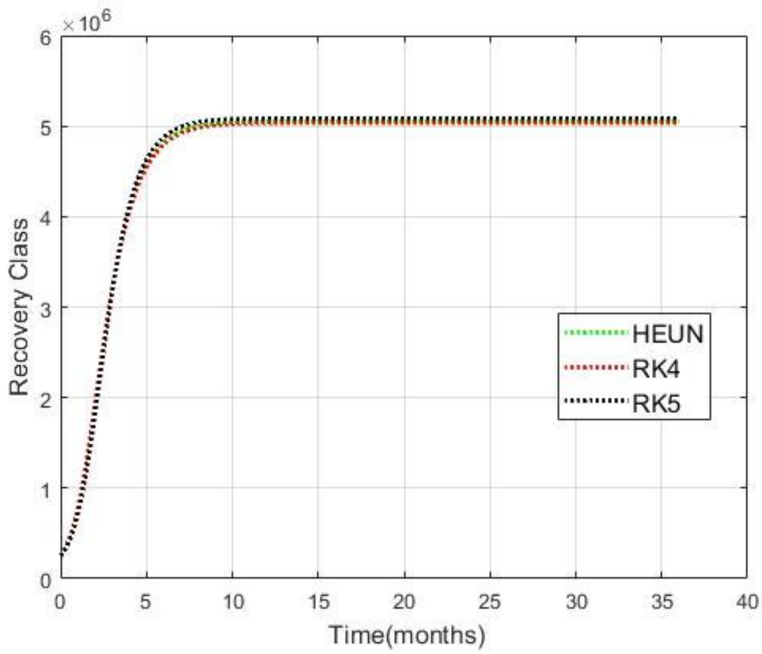

Figure 3: Graphical representation of recovery class against time for Heuns, Rk4 and Rk 5

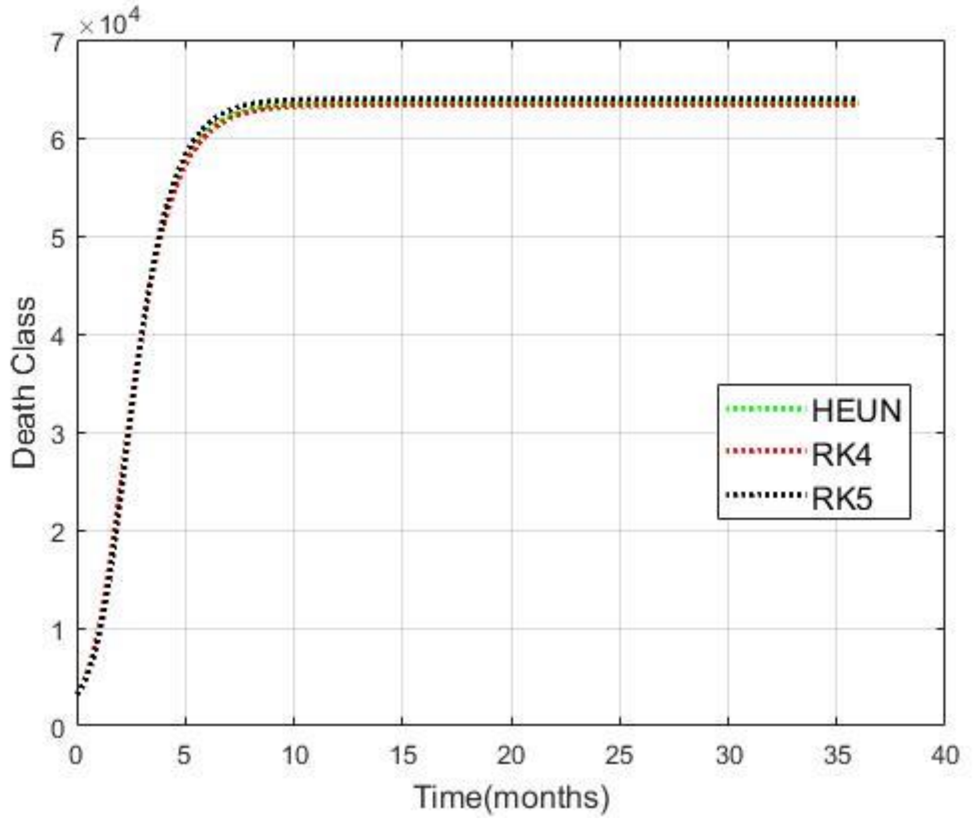

Figure 4: Graphical representation of death class against time for Heuns Rk4 and Rk 5

We observed from figure 3 and 4 that the three methods aligned with each other. The graphs show that the Recovery class and the Death Class rise rapidly and gradually converge to stability, this was due to the migration from the Infected as various control measures are been put in place, as well as an increase in awareness policy

CONCLUSION

In this paper, the SIRD model of the COVID-19 disease in Nigeria was analyzed to determine the progression of the COVID-19 disease in Nigeria. The accumulated result of the Susceptible Class, Infected Class, Recovery Class, and Death Class as of May 10th, 2022 was used as the initial conditions. We used Heun's method, Runge-Kutta methods of order four and five numerical methods for the analysis and the Matlab program was used to obtain the simulation result. The graphs from the methods corresponded well with each other and show the same behavior pattern. From our research, we can see that COVID-19 disease in Nigeria is progressing towards total elimination through the use of the numerical method.

ACKNOWLEGDEMENT

The authors are grateful to the Management of Sheda Science and Technology for the opportunity given to us to carry out this research in their organization

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