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MULTI-FUZZY SET RELATIONAL STRUCTURES

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ABSTRACT
In this paper, the concepts of multi-fuzzy set relational structures were introduced and some of their properties were presented. Relations were established on both the class of multi-fuzzy sets and on the power multi-fuzzy set of a given multi-fuzzy set. It is demonstrated that as relation on the latter turn out to be a linear order, relations on the former were only partial orders. Some related results were also presented.

Keywords: Relations, Sets, power multi-fuzzy, partial order

INTRODUCTION

Set theory introduced in 1873 by Georg Cantor appears to be one of the greatest achievements of modern mathematics, as almost all mathematical concepts, methods and results could be represented within axiomatic set theory. However, many contemporary Mathematicians expressed their concern over the Cantor's insistence on the distinctness and definiteness of an element in a set as repetition and vagueness are obvious in real life situations. These prompt a second thought on the theory which brings about the theories of fuzzy set, rough set, multiset, soft set, soft multiset, etc., which were studied in (Zadeh, 1965; Moldtsov, 1999; Blizard, 1989; Pawlak, 1982; Isah, 2019). Admitting repetition of an element in a set gave rise to multiset theory, which was studied by various scholars such as (Blizard, 1991; Yager, 1986; Singh and Isah, 2016). Moreover, in order to comprehend the notion of loose concepts like classes with vague boundaries, etc., Zadeh (1965) introduced fuzzy set theory. It is a mathematical theory introduced to model vagueness and other loose concepts. A fuzzy set is a generalized set of objects occurring with a continuum of degrees (grades) of membership. Moreover, fuzzy multisets model the case where otherwise indistinguishable objects possess a particular property to a certain degree. Thus, by fuzzifying the number of occurrences of each object, we get what we call a multi-fuzzy set. Many researchers such as (Syropoulos, 2006; Singh et al., 2014; Yager, 1986; Singh et al., 2015) have contributed to the developments of both the theories.

The idea of relational structures were presented and studied by various scholars, such as (Grefen and de By, 1994; Girish and Sunil, 2009; Singh and Isah, 2014). In this paper, the notions of multi-fuzzy set relational structures were introduced and some of their properties presented.

PRELIMINARIES

Definition 1 [Zadeh, 1965; Klir and Yuan, 1995]: A fuzzy set (class) $\check{A}$ in X is characterized by a membership function $\mu_{\check{A}}(x)$ which associates with each point x in X, a real number $\mu_{\check{A}}(x)$ in the interval $[0,1]$. The value of $\mu_{\check{A}}(x)$ represents the grade of membership of x in $\check{A}$.

Let $\check{A}$ and $\check{B}$ be two fuzzy sets, then

(a) $\check{A} \subseteq \check{B}$ if and only if $\mu_{\check{A}}(x) \le \mu_{\check{B}}(x), \quad \forall x \in X$.

(b) $\check{A} \cup \check{B} = \check{C}$ such that

$$\mu_{\check{C}}(x) = \max[\mu_{\check{A}}(x), \mu_{\check{B}}(x)], \forall x \in X.$$

(c) $\check{A} \cap \check{B} = \check{C}$ such that

$$\mu_{\check{C}}(x) = \min[\mu_{\check{A}}(x), \mu_{\check{B}}(x)], \forall x \in X.$$

Definition 2 [Blizard, 1991; Singh et al., 2007]: Let $X = \{x_1, x_2, x_3, \cdots, x_j, \cdots\}$ be set. A multiset or mset A over X is a *cardinal-valued* function i.e., $A: X \to N = \{0,1,2,\cdots\}$ such that for $x \in Dom(A)$ implies $A(x)$ is a cardinal and $A(x) = m_A(x) > 0$, where $m_A(x)$ denotes the number of times an object *x* occurs in A.

Let A be a multiset containing one occurrence of a, two occurrences of b, and three occurrences of c, then A can be represented as $A = [[a,b,b,c,c,c]] = [a,b,b,c,c,c] = [a,b,c]_{1,2,3} = [a,2b,3c] = [a.1,b.2,c.3] = [1/a,2/b,3/c] = [a^1,b^2,c^3] = [a^1b^2c^3]$. For convenience, the curly brackets are also used in place of the square brackets.

Definition 3 [Blizard, 1991; Singh et al., 2007]: Let A and B be two msets over a given domain set X. Then

(a) $A \subseteq B$ if $m_A(x) \le m_B(x), \forall x \in X$.
(b) $A = B$ if $m_A(x) = m_B(x), \forall x \in X$.

(c) $A \cup B = \max[m_A(x), m_B(x)], \forall x \in X$.
(d) $A \cap B = \min[m_A(x), m_B(x)], \forall x \in X$.
(e) $A + B = A \uplus B = m_A(x) + m_B(x), \forall x \in X$.
(f) $A - B = \max[m_A(x) - m_B(x), 0], \forall x \in X$.

Definition 4 [Blizard, 1991; Singh et al., 2007]: Let A be a multiset, then the power multiset of A, denoted by $P(A)$, is the multiset of all sub multiset of A.

Example 1 [Singh et al. 2007]: Let $A = [a, a, b]$, then

$$P(A) = [\emptyset, \{a\}, \{a\}, \{a, a\}, \{b\}, \{a, b\}, \{a, b\}, \{a, a, b\}].$$

However, as been observed by some scholars in the field of multiset such as (Hickman, 1980) and (Blizard, 1989), there is no good reason for admitting repeated elements into a power multiset. Thus, a power multiset may be called a power set. For example, in the multiset above, $P(A) = [\emptyset, \{a\}, \{a, a\}, \{b\}, \{a, b\}, \{a, a, b\}]$ and is called the power set of A.

Definition 5 [Syropoulos, 2006]: Let X be a (fixed) universe, then a multi-fuzzy set A is a function $A: X \to N_0 \times I$, where N_0 is the set of all positive integers including zero and $I = [0, 1]$. Thus $A(x) = (n, i)$ denotes that the degree to which x occurs n times in the multi-fuzzy set is equal to i.

Let A be a multi-fuzzy set, then
(a) the multiplicity function is defined as $A_m(x): X \to N_0$ and
(b) the membership function is defined as $\mu_A(x): X \to I$.
Obviously, if $A(x) = (n, i)$, then $A_m(x) = n$ and $\mu_A(x) = i$.

Definition 6 [Syropoulos, 2006]: Let A and B be two multi-fuzzy sets over X, then
Their Union is

$$(A \cup B)(x) = (\max\{A_m(x), B_m(x)\}, \max\{\mu_A(x), \mu_B(x)\}), \forall x \in X.$$

Their Intersection is

$$(A \cap B)(x) = (\min\{A_m(x), B_m(x)\}, \min\{\mu_A(x), \mu_B(x)\}), \forall x \in X.$$

Their Sum is

$$(A \uplus B)(x) = (A_m(x) + B_m(x), (\mu_A + \mu_B)(x)), \forall x \in X,$$

where $(\mu_A + \mu_B)(x) = \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x)$.

Their Difference is

$$A - B = (\max\{A_m(x) - B_m(x), 0\}, \max\{\mu_A(x) - \mu_B(x), 0\}), \forall x \in X.$$

Example 2: Let $A = \{(3,0.5)/x, (5,0.7)/y, (2,0.4)/z\}$ and $B = \{(2,0.2)/x, (4,0.8)/y, (5,0.3)/z\}$ be two Multi-fuzzy sets over $X = \{x, y, z\}$. Then

$$A \cup B = \{(3,0.5)/x, (5,0.8)/y, (5,0.4)/z\}$$
$$A \cap B = \{(2,0.2)/x, (4,0.7)/y, (2,0.3)/z\}$$
$$A \uplus B = \{(5,0.6)/x, (9,0.94)/y, (7,0.58)/z\}.$$

In the following section, the concepts of Multi-fuzzy Set Relational Structures together with their properties were presented which is the main contribution of the paper.

MULTI-FUZZY SET RELATIONAL STRUCTURES

Definition 7: Let A and B be two multi-fuzzy sets over X, then
(i) A is a Sub multi-fuzzy set of B written $A \subseteq B$
if $A_m(x) \le B_m(x)$ and $\mu_A(x) \le \mu_B(x), \forall x \in X$.
(ii) A is equal to B written $A = B$

if $A_m(x) = A_m(x)$ and $\mu_A(x) = \mu_A(x), \forall x \in X$.

Proposition 1: Let A, B and C be multi-fuzzy sets over X.

i. If $A \subseteq C$ and $B \subseteq C$, then $A \cup B \subseteq C$ and $A \cap B \subseteq C$.
ii. If $C \subseteq A$ and $C \subseteq B$, then $C \subseteq A \cup B$ and $C \subseteq A \cap B$.
iii. If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

Proof

i. If $A \subseteq C$ and $B \subseteq C \Longrightarrow A_m(x) \le C_m(x),\ \mu_A(x) \le \mu_C(x)$ and $B_m(x) \le C_m(x),\ \mu_B(x) \le \mu_C(x), \forall x \in X$
$\Longrightarrow \max\{A_m(x), B_m(x)\} \le C_m(x)$ and $\max\{\mu_A(x),\ \mu_B(x)\} \le \mu_C(x)$
$$\Longrightarrow A \cup B \subseteq C$$
Now, since $\min\{A_m(x), B_m(x)\} \le \max\{A_m(x), B_m(x)\}$ and $\min\{\mu_A(x),\ \mu_B(x)\} \le \max\{\mu_A(x),\ \mu_B(x)\}, \forall x \in X$, it follows that $A \cap B \subseteq C$.

ii. If $C \subseteq A$ and $C \subseteq B \Longrightarrow C_m(x) \le A_m(x),\ \mu_C(x) \le \mu_A(x)$ and $C_m(x) \le B_m(x),\ \mu_C(x) \le \mu_B(x), \forall x \in X$.
$\Longrightarrow C_m(x) \le \min\{A_m(x), B_m(x)\}$ and $\mu_C(x) \le \min\{\mu_A(x),\ \mu_B(x)\}$
$$\Longrightarrow C \subseteq A \cap B$$
Moreover, since $\min\{A_m(x), B_m(x)\} \le \max\{A_m(x), B_m(x)\}$ and $\min\{\mu_A(x),\ \mu_B(x)\} \le \max\{\mu_A(x), \mu_B(x)\}, \forall x \in X$, it follows that $C \subseteq A \cup B$.

iii. Since $A \subseteq B$ and $B \subseteq C \Longrightarrow A_m(x) \le B_m(x),\ \mu_A(x) \le \mu_B(x)$ and $B_m(x) \le C_m(x),\ \mu_B(x) \le \mu_C(x), \forall x \in X$.
$\Longrightarrow A_m(x) \le C_m(x)$ and $\mu_A(x) \le \mu_C(x), \forall x \in X$.
$$\Longrightarrow A \subseteq C.$$

Definition 8: Let $F(M)$ be the collection of all multi-fuzzy sets over X. Let $A, B \in F(M)$ and R_1 be a relation on $F(M)$ such that AR_1B if $A_m(x) = B_m(x)$ and $\mu_A(\text{x}) \le \mu_B(x), \forall x \in X$. Then

i. R_1 is reflexive: $\forall A \in F(M)$ and $\forall x \in X, A_m(x) = A_m(x)$ and $\mu_A(\text{x}) \le \mu_A(x)$.
ii. R_1 is antisymmetry: $\forall A, B \in F(M)$ and $\forall x \in X$, if $A_m(x) = B_m(x), \mu_A(\text{x}) \le \mu_B(x)$ and $\mu_B(\text{x}) \le \mu_A(x), \Longrightarrow A = B$.
iii. R_1 is transitive: $\forall A, B, C \in F(M)$ and $\forall x \in X$, if $A_m(x) = B_m(x) = C_m(x), \mu_A(\text{x}) \le \mu_B(x)$ and $\mu_B(\text{x}) \le \mu_C(x)$, $\Longrightarrow \mu_A(\text{x}) \le \mu_C(x)$.

Thus, R_1 is a partial order.

However, symmetry property fails in R_1, since $\forall A, B \in F(M)$ and $\forall x \in X$, if $A_m(x) = B_m(x), \mu_A(\text{x}) \le \mu_B(x) \nRightarrow \mu_B(\text{x}) \le \mu_A(x)$.

Remark 1: R_1 is not complete.
Counter Example
Let $A = \{(3,0.5)/x, (5,0.7)/y, (8,0.4)/z\}$ and $B = \{(4,0.2)/x, (5,0.8)/y, (8,0.5)/z\}$. Then neither AR_1B nor BR_1A.

Definition 9: Let $F(M)$ be the collection of all multi-fuzzy sets over X. Let $A, B \in F(M)$ and R_2 be a relation on $F(M)$ such that AR_2B if $A_m(x) \le B_m(x)$ and $\mu_A(\text{x}) = \mu_B(x), \forall x \in X$. Then

i. R_2 is reflexive: $\forall A \in F(M)$ and $\forall x \in X, A_m(x) \le A_m(x)$ and $\mu_A(\text{x}) = \mu_A(x)$.
ii. R_2 is antisymmetry: $\forall A, B \in F(M)$ and $\forall x \in X$, if $\mu_B(\text{x}) = \mu_A(x),\ A_m(x) \le B_m(x)$ and$B_m(x) \le A_m(x) \Longrightarrow A = B$.
iii. R_2 is transitive: $\forall A, B, C \in F(M)$ and $\forall x \in X$, if $\mu_A(\text{x}) = \mu_B(\text{x}) = \mu_C,\ \ A_m(x) \le B_m(x)$ and $B_m(x) \le C_m(x), \Longrightarrow A_m(x) \le C_m(x)$ i.e., AR_2C.

Thus, R_2 is a partial order.

However, R_2 is not symmetry since $\forall A, B \in F(M)$ and $\forall x \in X$, if $\mu_A(\text{x}) = \mu_B(x),\ A_m(x) \le B_m(x) \nRightarrow B_m(x) \le A_m(x)$.

Remark 2: R_2 is not complete.
Counter Example
Let $A = \{(3,0.5)/x, (5,0.7)/y, (8,0.4)/z\}$ and $B = \{(4,0.2)/x, (5,0.8)/y, (8,0.5)/z\}$. Then neither AR_1B nor BR_2A.

Definition 10: Let $F(M)$ be the collection of all multi-fuzzy sets over X. Let $A, B \in F(M)$ and R_3 be a relation on $F(M)$ such that AR_3B if $A_m(x) \leq B_m(x)$ and $\mu_A(\text{x}) \leq \mu_B(x), \forall x \in X$. Then

i. R_3 is reflexive: $\forall A \in F(M)$ and $\forall x \in X, A_m(x) \leq A_m(x)$ and $\mu_A(\text{x}) \leq \mu_A(x)$.

ii. R_3 is antisymmetry: $\forall A, B \in F(M)$ and $\forall x \in X$, if $\mu_A(\text{x}) \leq \mu_B(x)$, $A_m(x) \leq B_m(x)$ and $B_m(x) \leq A_m(x)$, $\mu_B(\text{x}) \leq \mu_A(x) \Longrightarrow \mu_A(\text{x}) = \mu_B(x)$, $A_m(x) = B_m(x)$ i.e., $A = B$.

iii. R_3 is transitive: $\forall A, B, C \in F(M)$ and $\forall x \in X$, if $A_m(x) \leq B_m(x)$, $\mu_A(\text{x}) \leq \mu_B(\text{x})$ and $B_m(x) \leq C_m(x)$, $\mu_B(\text{x}) \leq \mu_C(\text{x})$, $\Longrightarrow A_m(x) \leq C_m(x)$, $\mu_A(\text{x}) \leq \mu_C(\text{x})$ i.e., AR_3C.

Thus, R_3 is a partial order.

However, R_3 is not symmetry since $\forall A, B \in F(M)$ and $\forall x \in X$, if $\mu_A(\text{x}) \leq \mu_B(x)$, $A_m(x) \leq B_m(x) \nRightarrow B_m(x) \leq A_m(x)$, $\mu_B(\text{x}) \leq \mu_A(x)$.

Remark 3: R_3 is not complete.

Counter Example

Let $A = \{(4{,}0.7)/x, (5{,}0.1)/y, (2{,}0.4)/z\}$ and $B = \{(6{,}0.2)/x, (5{,}0.3)/y, (8{,}0.9)/z\}$. Then neither AR_3B nor BR_3A.

Definition 11: Let A be a multi-fuzzy set over X, then the power multi-fuzzy set of A, denoted by $P(A)$is the set of all possible submulti-fuzzy set of A.

Example 3: Let $A = \{(3{,}0.1)/x, (2{,}0.9)/y\}$, then

$$P(A) = \{\{(1{,}0.1)/x, (1{,}0.9)/y\}, \{(1{,}0.1)/x, (2{,}0.9)/y\}, \{(2{,}0.1)/x, (1{,}0.9)/y\}, \{(2{,}0.1)/x, (2{,}0.9)/y\}, \{(3{,}0.1)/x, (1{,}0.9)/y\}, \{(3{,}0.1)/x, (2{,}0.9)/y\}\}.$$

Definition 12: Let $A, B \in P(A)$ and R_4 be a relation on $P(A)$ such that AR_4B if $A_m(x) \leq B_m(x)$ and $\mu_A(\text{x}) \leq \mu_B(x), \forall x \in X$. Then

i. R_4 is reflexive: $\forall A \in P(A)$ and $\forall x \in X, A_m(x) \leq A_m(x)$ and $\mu_A(\text{x}) \leq \mu_A(x)$.

ii. R_4 is complete: $\forall A \neq B \in P(A)$ either AR_4B or BR_4A.

iii. R_4 is antisymmetry: $\forall A, B \in P(A)$ and $\forall x \in X$, if $\mu_A(\text{x}) \leq \mu_B(x), A_m(x) \leq B_m(x)$ and $B_m(x) \leq A_m(x)$, $\mu_B(\text{x}) \leq \mu_A(x) \Longrightarrow \mu_A(\text{x}) = \mu_B(x)$, $A_m(x) = B_m(x)$ i.e., $A = B$.

iv. R_4 is transitive: $\forall A, B, C \in P(A)$ and $\forall x \in X$, if $A_m(x) \leq B_m(x)$, $\mu_A(\text{x}) \leq \mu_B(\text{x})$ and $B_m(x) \leq C_m(x)$, $\mu_B(\text{x}) \leq \mu_C(\text{x})$, $\Longrightarrow A_m(x) \leq C_m(x)$, $\mu_A(\text{x}) \leq \mu_C(\text{x})$ i.e., AR_4C.

Moreover, R_4 is a linear order.

However, R_4 is not symmetry since $\forall A, B \in P(A)$ and $\forall x \in X$, if $\mu_A(\text{x}) \leq \mu_B(x)$, $A_m(x) \leq B_m(x) \nRightarrow B_m(x) \leq A_m(x)$, $\mu_B(\text{x}) \leq \mu_A(x)$.

The binary relation R_4 and power multi-fuzzy set of A gives rise to a multi-fuzzy set relational structure $(P(A), R_4)$; refer to (Grefen and de By, 1994; Singh and Isah, 2014) for related descriptions.

Definition 13: Let $Q = (P(A), R_4)$; where $P(A)$ consists of all Sub multi-fuzzy sets of A and R_4 its binary relation Then,

A_i is a maximal element of Q iff $\forall A_j \in P(A)$, $A_jR_4A_i$.

A_i is a minimal element of Q iff $\forall A_j \in P(A)$, $A_iR_4A_j$.

A_i is an isolated element of Q iff A_i is both a maximal and minimal elementof Q.

Example 4: Let $A = \{(2,0.7)/x, (4,0.3)/y\}$, then

$$P(A) = \{\{(1,0.7)/x, (1,0.3)/y\}, \{(1,0.7)/x, (2,0.3)/y\}, \{(1,0.7)/x, (3,0.3)/y\}, \{(1,0.7)/x, (4,0.3)/y\}, \{(2,0.7)/x, (1,0.3)/y\}, \{(2,0.7)/x, (2,0.3)/y\}, \{(2,0.7)/x, (3,0.3)/y\}, \{(2,0.7)/x, (4,0.3)/y\}\}.$$

Now, $\{(1,0.7)/x, (1,0.3)/y\}$ is the minimal element and $\{(2,0.7)/x, (4,0.3)/y\}$ is the maximal element. However, there is no isolated element.

Moreover, suppose $A = \{(1,0.8)/x\}$, then $P(A) = \{(1,0.8)/x\}$ and $\{(1,0.8)/x\}$ is the isolated element.
Note that, Q will have an isolated element iff A is of singleton element with multiplicity 1.

Proposition 2: $Q = (P(A), R_4)$ always has a unique maximal element.

Proof
Let $A(x)/x \neq B(x)/x \in P(A)$ and let $A(x)/x$, $B(x)/x$ be both maximal elements of Q. Since $A(x)/x$ is a maximal element $\Longrightarrow$ $B(x)/x \ R_4 \ A(x)/x$.
Also, since $B(x)/x$ is a maximal element $\Longrightarrow A(x)/x \ R_4 \ B(x)/x$.
Thus, by antisymmetry of these relations we have $A(x)/x = B(x)/x$.
This contradicts our assumption that $A(x)/x \neq B(x)/x$.
Hence Q has a unique maximal element.

Proposition 3: $Q = (P(A), R_4)$ always has a unique minimal element

Proof
Let $A(x)/x \neq B(x)/x \in P(A)$ and let $A(x)/x$, $B(x)/x$ be both minimal elements of Q. Since $A(x)/x$ is a minimal element we have $A(x)/x \ R_4 \ B(x)/x$.
Also, as $B(x)/x$ is a minimal element, $B(x)/x \ R_4 \ A(x)/x$.
Now, by antisymmetry of these relations, $A(x)/x = B(x)/x$.
This contradicts the assumption that $A(x)/x \neq B(x)/x$.
Therefore, Q has a unique minimal element.

CONCLUSION
In this work, considering the applications of relational structures in mathematics, computer science and other sciences etc., multi-fuzzy set relational structures were introduced. Moreover, the reflexivity, antisymmetry, transitivity, completeness etc., and otherwise of these relations were also established.

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