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UNSTEADY MHD FLOW OF NON -NEWTONIAN FLUID WITH MASS TRANSFER IN A HORIZONTAL CHANNEL

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ABSTRACT

This study analyzes the unsteady MHD flow of non-Newtonian fluid in horizontal channel. The upper plate is oscillating and moving while the bottom plate is stationary. Solutions for momentum, energy and concentration equation are obtained by the He-Laplace scheme. The effect of various flow parameters controlling the physical situation is discussed with the aid of graphs. Significant results from this study, shows that velocity and concentric fields decrease with the increase in chemical reaction parameter.

Keywords: Unsteady, MHD, Chemical reaction, Non-Newtonian fluid, He-Laplace

INTRODUCTION

Non-Newtonian fluid is a type that is capable of describing shear thinning and shear thickening effects (examples are ketchup, blood, paint, cream, nail polish, etc).

Over time researches on non – Newtonian fluids, have resulted into the discovery of many empirical and semi-empirical non-Newtonian models or constitutive equations have been proposed. Rehan *et al.* (2010) considered the steady flow of a fourth grade fluid, between two parallel plates. They analyzed four types of flows: Couette flow, plug flow, Poiseuille flow and generalized Couette flow. The nonlinear differential equation describing the velocity field was solved by optimal homotopy asymptotic method (OHAM). They observed that the OHAM was more efficient and flexible than the perturbation and Homotopy analyses method. Islam *et al.* (2011) considered the steady flow of a non-Newtonian fluid with slippage between the plate and the fluid. The constitutive equations of the fluids were modelled for fourth-grade non-Newtonian fluid with partial slip. They employed homotopy perturbation and optimal homotopy asymptotic methods to solve the non-linear differential equation (Islam et al., 2011). Shehzad *et al.* (2018) reported the electro-osmotic Couette-Poiseuille flow of power law Al2O3- PVC nanofluid through a channel, in which upper wall is moving with constant velocity. The influences of magnetic field, mixed convection, joule heating, and viscous dissipation were also incorporated. The flow was generated because of constant pressure gradient in axial direction. The resulting flow problem was coupled nonlinear ordinary differential equations, which were at first modeled and then transform into dimensionless form through appropriate transformation. Analytical solution of the governing equation was carried out.

Khan *et al.* (2018) discussed the unsteady flow of non-Newtonian fluid with the properties of heat/sink in the presence of thermal radiation through a binary mixture embedded in a porous. Santhosha *et al.* (2017) studied the radiation and chemical combined effects on MHD free convective heat and mass transfer flow of viscous, incompressible, conducting elastic fluid through porous medium finite by a porous plate within the presence of heat generation. The momentum, energy and mass diffusion equation were coupled non-linear partial differential equations. They employed two term perturbation method.

Taza *et al.* (2016) studied the unsteady thin film flow of a fourth grade fluid over a moving and oscillating vertical belt. They employed adomian decomposition method (ADM) and optimal homotopy asymptotic method (OHAM) to find the solution of the non- linear differential equations that governed the flow. Hayat *et al.* (2007) presented the exact solution four four types of flows between two parallel plates, viz. Couette flow, plug flow, Poiseuille flow and generalized Couette flow. The nonlinear second-order differential equation for the velocity field was solved exactly in each case. The nonlinear differential equation describing the velocity field was solved by optimal homotopy asymptotic method (OHAM). They observed that the OHAM is more efficient and flexible than the perturbation and Homotopy analyses method. Arifuzzaman *et al.* (2018) analysed heat and mass transfer characteristics of naturally corrective hydro-magnetic flows of fourth grade radiative fluid resulting from vertical porous plate. They considered non-linear order chemical reaction and heat generation with thermal diffusion. The complete fundamental equations were transformed into dimensionless equations by implementing finite difference scheme explicitly.

Idowu and Sani (2019) carried out an analysis for unsteady magnetohydrodynamic (MHD) flow of a generalized third grade fluid between two parallel plates. The fluid flow was as a result of the plate oscillating, moving and pressure gradient. Three flow problems were investigated, namely: Couette, Poiseuille and Couette-Poiseuille flows and a number of nonlinear partial differential equations were obtained which were solved using the He-Laplace method. Expressions for the velocity field, temperature and concentration fields were given for each case and finally, effects of physical parameters on the fluid motion, temperature and concentration were plotted and discussed. They found that an increase in the thermal radiation parameter increases the temperature of the fluid and hence reduces the viscosity of the fluid while the concentration of the fluid reduces as the chemical reaction parameter increases.

Joseph *et al.* (2021) investigated the unsteady MHD flow of fourth-grade fluid in horizontal parallel plates channel. The upper plate was oscillating and moving while the bottom plate remained stationary. Solutions for momentum, energy and concentration equation were obtained by the He-Laplace scheme. The effect of various flow parameters controlling the physical situation is discussed with the aid of graphs. Significant results from this study, showed that velocity and temperature fields increase with the increase in thermal radiation parameter, while the velocity and concentric fields decrease with increase in chemical reaction parameter.

Furthermore, velocity, temperature and concentric fields decrease with the increase in suction parameter.

MATERIALS AND METHOD

Formulation of the Problem

We consider the unsteady flow of an electrically conducting incompressible fourth grade fluid between two horizontal parallel plates channel as shown in figure 1. The fluid is subjected to a uniform transverse magnetic field. We assumed the bottom plate is fixed (stationary) and the top plates is moving with constant velocity. The nomenclature used is expressed in table1.

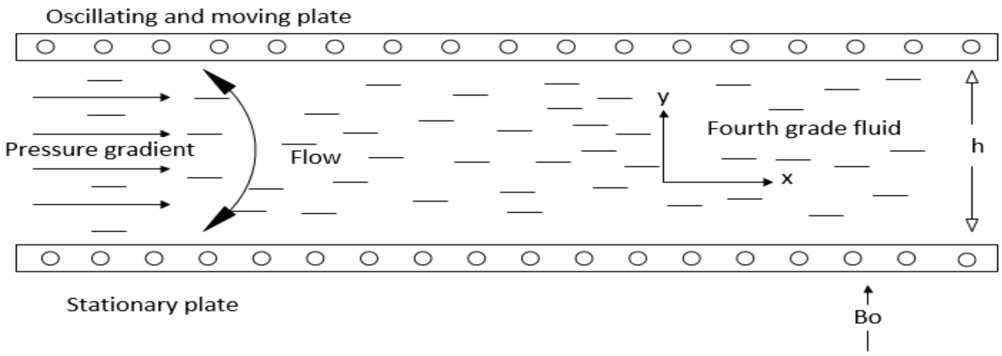

Figure 1: Physical Schematic of the Flow Configuration

Table 1: **Nomenclature**

B_0	External magnetic field.
C	Species concentration
u	Fluid velocity
S	Suction parameter
Ha	Hartmann number
K_r	Chemical reaction parameter
S_c	Schmidt number
C_w	Concentration at the surface
C_∞	Concentration as $y \to \infty$
x, y	Cartesian coordinates
Greek Symbols	
μ	Coefficient of shear viscosity
α	Second grade parameter
β_a, β_b	Third grade parameters
γ_a, γ_b	Fourth grade parameters
β_c	Concentration expansion coefficient
	Stefan – Boltzmann constant
ρ	Density of the fluid
ν	Kinematic viscosity

The chemically reactive flow is heading x – direction along infinite porous plate with heat generation. Here, U_0 is the uniform velocity and C_∞ is the species concentration.

Under the above consideration, the equations that described the physical circumstances are

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + v\frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1 \nu}{\rho}\frac{\partial^3 u}{\partial y^2 \partial t} + \frac{\beta_1 \nu^2}{\rho}\frac{\partial^4 u}{\partial y^2 \partial t^2} + \frac{6(\beta_2+\beta_3)}{\rho}\left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^2 u}{\partial y^2} + \frac{\gamma_1 \nu^3}{\rho}\frac{\partial^5 u}{\partial y^2 \partial t^3} + \frac{2\nu(3\gamma_2+\gamma_3+\gamma_4+\gamma_5+\gamma_7+\gamma_8)}{\rho C_p}\left[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t \partial y} + \left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^3 u}{\partial y^2 \partial t}\right] - \frac{\sigma B_0^2}{\rho C_p}u + g\beta_C(C - C_\infty) - \frac{\nu}{k}u \tag{2}$$

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - K_c(C - C_\infty) \tag{3}$$

The initial and boundary conditions are

$$\left.\begin{array}{l} u = U_0 e^{-yh}, C = C_0 + (C_w - C_\infty)e^{-yh} \ \ at\ t = 0\ for\ \ 0 \le y \ge h \\ u(y,t) = U,, C(y,t) = C_w\ at\ y = h\ for\ t \ge 0 \\ u(y,t) \to \infty, C(y,t) \to \infty\ as\ y \to \infty\ for\ t > 0 \end{array}\right\} \tag{4}$$

Where u is the fluid velocity and C is the species concentration equation, ρ is the density of the fluid, C_p is the heat capacity, B_0 is the external magnetic field.

In order to transform equations (1) – (4), we use the following dimensionless parameters

$u^*=\frac{u}{U_0},\ p^*=\frac{p}{\mu U_0^2},t^*=\frac{tU_0^2}{\nu},\ G_c=\frac{g\beta_C(C_w-C_\infty)\nu}{U_0^3},Ha^2=\frac{\sigma B_0^2\nu}{\rho U_0^2},Da=\frac{KU_0^2}{h^2},S_c=\frac{D}{\nu}\ ,\ y^*=\frac{yU_0}{\nu},x^*=\frac{x}{h},h=\frac{U_0}{\nu}\ ,S=\frac{\mathrm{v}_0}{U_0},\mathrm{v}=\frac{\mathrm{v}}{U_0},C^*=\frac{C-C_0}{C_w-C_\infty},\alpha=\frac{\alpha_1U_0^2}{\rho\nu^2},\beta_a=\frac{\beta_1U_0^4}{\rho\nu^3},\beta_b=\frac{(\beta_2+\beta_3)U_0^4}{\rho\nu^3},\gamma_a=\frac{\gamma_1U_0^6}{\rho\nu^3},\gamma_b=\frac{2(3\gamma_2+\gamma_3+\gamma_4+\gamma_5+3\gamma_7+\gamma_8)U_0^6}{\rho\nu^4},Da=\frac{kU_0^2}{\nu^2},K_r=\frac{K_c\nu}{U_0^2}$

(5)

Substituting equation (5) into equations (1) – (4) and by dropping the asterisks, we have the following:

$$\frac{\partial v}{\partial y}=0\Rightarrow \mathrm{v}=-\mathrm{v}_0 \qquad (6)$$

$$\frac{\partial u}{\partial t}-S\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{\partial^2u}{\partial y^2}+\alpha\frac{\partial^3u}{\partial y^2\partial t}+\beta_a\frac{\partial^4u}{\partial y^2\partial t^2}+\beta_b\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2u}{\partial y^2}+\gamma_a\frac{\partial^5u}{\partial y^2\partial t^3}+\gamma_b\left[2\frac{\partial u}{\partial y}\frac{\partial^2u}{\partial y^2}\frac{\partial^2u}{\partial t\partial y}+\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3u}{\partial y^2\partial t}\right]-\left(Ha+\frac{1}{Da}\right)u+G_cC \qquad (7)$$

$$\frac{\partial C}{\partial t}-S\frac{\partial C}{\partial y}=\frac{1}{S_c}\frac{\partial^2C}{\partial y^2}-K_rC \qquad (8)$$

And the initial and boundary conditions becomes

$$\left.\begin{array}{c} u(y,t)=e^{-y},C(y,t)=e^{-y}\ at\ t=0\ for\ 0\le y\ge 1\\ u(y,t)=1,C(y,t)=1\ at\ y=1\ for\ t\ge 0\\ u(y,t)\to\infty,C(y,t)\to\infty\ as\ y\to\infty\ for\ t>0\end{array}\right\} \qquad (9)$$

Solution of the Problem

Here, we employed the He – Laplace scheme to solve equations (6) to (8) subjects to the initial and boundary conditions (9). Since equation (7) is a coupled non – linear partial differential equation, we have to solve equation (8) first.

Now applying Laplace transform on equation (8), we have;

$$L\left\{\frac{\partial C}{\partial t}\right\}-SL\left\{\frac{\partial C}{\partial y}\right\}=\frac{1}{S_c}L\left\{\frac{\partial^2C}{\partial y^2}\right\}-L\{K_rC\} \qquad (10)$$

Applying the initial condition and dividing through by s and rearranging, we obtain;

$$L\{C(y,t)\}=\frac{e^{-y}}{s}+\frac{1}{s}\left\{\frac{1}{S_c}L\left\{\frac{\partial^2C}{\partial y^2}\right\}+SL\left\{\frac{\partial C}{\partial y}\right\}-L\{K_rC\}\right\} \qquad (11)$$

Taking the inverse Laplace transform of both sides of equation (21), gives;

$$C(y,t)=e^{-y}+L^{-1}\left[\frac{1}{s}\left\{\frac{1}{S_c}L\left\{\frac{\partial^2C}{\partial y^2}\right\}+SL\left\{\frac{\partial C}{\partial y}\right\}-L\{K_rC\}\right\}\right] \qquad (12)$$

Applying the Homotopy perturbation technique, equation (12) yields

$$\sum_{n=0}^{\infty}P^nC_n(y,t)=e^{-y}+P\left[L^{-1}\left\{\frac{1}{s}\left\{\frac{1}{S_c}L\left\{\frac{\partial^2C}{\partial y^2}\right\}-L\{K_rC\}\right\}\right\}\right] \qquad (13)$$

Comparing the coefficients of the like powers of $'P'$, the following approximations were obtained;

$$P^0:C_0(y,t)=e^{-y} \qquad (14)$$

$$P^1:C_1(y,t)=L^{-1}\left[\frac{1}{s}\left\{\frac{1}{S_c}L\left\{\frac{\partial^2C_0}{\partial y^2}\right\}+SL\left\{\frac{\partial C_0}{\partial y}\right\}-L\{K_rC_0\}\right\}\right]=L^{-1}\left\{\frac{1}{S_c}\left(\frac{e^{-y}}{s^2}\right)-S\left(\frac{e^{-y}}{s^2}\right)-K_r\left(\frac{e^{-y}}{s^2}\right)\right\}$$

$$=\left(\frac{e^{-y}}{S_c}-Se^{-y}-K_re^{-y}\right)t \qquad (15)$$

$$P^2:C_2(y,t)=L^{-1}\left[\frac{1}{s}\left\{\frac{1}{S_c}L\left\{\frac{\partial^2C_1}{\partial y^2}\right\}+SL\left\{\frac{\partial C_1}{\partial y}\right\}-L\{K_rC_1\}\right\}\right]$$

$$=L^{-1}\left[\frac{1}{s}\left\{\frac{1}{S_c}L\left\{\left(\frac{e^{-y}}{S_c}-Se^{-y}-K_re^{-y}\right)t\right\}+SL\left\{\left(Se^{-y}-\frac{e^{-y}}{S_c}+K_re^{-y}\right)t\right\}-L\left\{K_r\left(\frac{e^{-y}}{S_c}-Se^{-y}-K_re^{-y}\right)t\right\}\right\}\right]$$

$$=\left(\frac{e^{-y}}{S_c^2}-\frac{2Se^{-y}}{S_c}-\frac{2K_re^{-y}}{S_c}++2K_rSe^{-y}+S^2e^{-y}+K_r^2e^{-y}\right)\frac{t^2}{2!} \qquad (16)$$

Therefore, in view of equations (14), (15), (16), the solution is,

$$C(y,t)=C_0(y,t)+C_1(y,t)+C_2(y,t)+C_3(y,t)\cdots$$

$$C(y,t)=e^{-y}+\left(\frac{e^{-y}}{S_c}-Se^{-y}-K_re^{-y}\right)t+\left(\frac{e^{-y}}{S_c^2}-\frac{2Se^{-y}}{S_c}-\frac{2K_re^{-y}}{S_c}++2K_rSe^{-y}+S^2e^{-y}+K_r^2e^{-y}\right)\frac{t^2}{2!} \qquad (17)$$

Finally, we now solve equation (7), which is rearranged to give

$$\frac{\partial u}{\partial t}-S\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{\partial^2u}{\partial y^2}+\alpha\frac{\partial^3u}{\partial y^2\partial t}+\beta_a\frac{\partial^4u}{\partial y^2\partial t^2}+\beta_b\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2u}{\partial y^2}+\gamma_a\frac{\partial^5u}{\partial y^2\partial t^3}+\gamma_b\left[2\frac{\partial u}{\partial y}\frac{\partial^2u}{\partial y^2}\frac{\partial^2u}{\partial t\partial y}+\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3u}{\partial y^2\partial t}\right]-l_2u+G_cC$$

where, $Ha+\frac{1}{Da}=l_2$ (17i)

Applying the Laplace transform on both sides of equation (17i) gives

$$L\left\{\frac{\partial u}{\partial t}\right\}-L\left\{S\frac{\partial u}{\partial y}\right\}=L\left\{-\frac{\partial p}{\partial x}+\frac{\partial^2u}{\partial y^2}+\alpha\frac{\partial^3u}{\partial y^2\partial t}+\beta_a\frac{\partial^4u}{\partial y^2\partial t^2}+\beta_b\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2u}{\partial y^2}+\gamma_a\frac{\partial^5u}{\partial y^2\partial t^3}+\gamma_b\left[2\frac{\partial u}{\partial y}\frac{\partial^2u}{\partial y^2}\frac{\partial^2u}{\partial t\partial y}+\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3u}{\partial y^2\partial t}\right]-l_2u+G_cC\right\} \qquad (18)$$

$$L\{u(y,t)\}=\frac{u(y,0)}{s}+\frac{1}{s}L\left\{-\frac{\partial p}{\partial x}+S\frac{\partial u}{\partial y}+\frac{\partial^2u}{\partial y^2}+\alpha\frac{\partial^3u}{\partial y^2\partial t}+\beta_a\frac{\partial^4u}{\partial y^2\partial t^2}+\beta_b\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2u}{\partial y^2}+\gamma_a\frac{\partial^5u}{\partial y^2\partial t^3}+\gamma_b\left[2\frac{\partial u}{\partial y}\frac{\partial^2u}{\partial y^2}\frac{\partial^2u}{\partial t\partial y}+\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3u}{\partial y^2\partial t}\right]-l_2u+G_cC\right\} \qquad (19)$$

Taking the inverse Laplace transform of both sides of equation (19), we have;

$$L^{-1}\{L\{u(y,t)\}\} = L^{-1}\Bigl\{\frac{u(y,0)}{s} - \frac{\partial p}{\partial x} + \frac{1}{s}L\Bigl\{+S\frac{\partial u}{\partial y} + \frac{\partial^2 u}{\partial y^2} + \alpha\frac{\partial^3 u}{\partial y^2 \partial t} + \beta_a\frac{\partial^4 u}{\partial y^2 \partial t^2} + \beta_b\Bigl(\frac{\partial u}{\partial y}\Bigr)^2\frac{\partial^2 u}{\partial y^2} + \gamma_a\frac{\partial^5 u}{\partial y^2 \partial t^3} + \gamma_b\Bigl[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t \partial y} + \Bigl(\frac{\partial u}{\partial y}\Bigr)^2\frac{\partial^3 u}{\partial y^2 \partial t}\Bigr] - l_2 u + \frac{G_c}{s}\Bigl(e^{-y} + \Bigl(\frac{e^{-y}}{S_c} - Se^{-y} - K_r e^{-y}\Bigr)t + \Bigl(\frac{e^{-y}}{S_c^2} - \frac{2Se^{-y}}{S_c} - \frac{2K_r e^{-y}}{S_c} + +2K_r Se^{-y} + S^2 e^{-y} + K_r^2 e^{-y}\Bigr)\frac{t^2}{2!}\Bigr)\Bigr\}\Bigr\} \tag{20}$$

Or,

$$u(y,t) = \lambda + e^{-y} + (G_c e^{-y})t + \Bigl(l_1 e^{-y} + \frac{e^{-y}}{S_c} - K_r e^{-y}\Bigr)\frac{t^2}{2!} + \Bigl(\frac{e^{-y}}{S_c^2} - \frac{2Se^{-y}}{S_c} - \frac{2K_r e^{-y}}{S_c} + 2K_r Se^{-y} + K_r^2 e^{-y}\Bigr)\frac{t^3}{3!} + +L^{-1}\Bigl\{\frac{1}{s}L\Bigl\{+S\frac{\partial u}{\partial y} + \frac{\partial^2 u}{\partial y^2} + \alpha\frac{\partial^3 u}{\partial y^2 \partial t} + \beta_a\frac{\partial^4 u}{\partial y^2 \partial t^2} + \beta_b\Bigl(\frac{\partial u}{\partial y}\Bigr)^2\frac{\partial^2 u}{\partial y^2} + \gamma_a\frac{\partial^5 u}{\partial y^2 \partial t^3} + \gamma_b\Bigl[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t \partial y} + \Bigl(\frac{\partial u}{\partial y}\Bigr)^2\frac{\partial^3 u}{\partial y^2 \partial t}\Bigr] - l_2 u\Bigr\}\Bigr\} \tag{21}$$

Applying the Homotopy perturbation method to equation (21), gives

$$\sum_{n=0}^{\infty} P^n u_n(y,t) = \lambda + e^{-y} + (G_c e^{-y})t + \Bigl(l_1 e^{-y} + \frac{e^{-y}}{S_c} - K_r e^{-y}\Bigr)\frac{t^2}{2!} + \Bigl(l_1^2 e^{-y} + \frac{e^{-y}}{S_c^2} - \frac{2Se^{-y}}{S_c} - \frac{2K_r e^{-y}}{S_c} + 2K_r Se^{-y} + K_r^2 e^{-y}\Bigr)\frac{t^3}{3!} + +P\Bigl(L^{-1}\Bigl\{\frac{1}{s}L\Bigl\{S\frac{\partial u}{\partial y} + \frac{\partial^2 u}{\partial y^2} + \alpha\frac{\partial^3 u}{\partial y^2 \partial t} + \beta_a\frac{\partial^4 u}{\partial y^2 \partial t^2} + \beta_b H_a(u_n) + \gamma_1\frac{\partial^5 u}{\partial y^2 \partial t^3} + \gamma_b\Bigl[2H_b(u_n) + H_c(u_n)\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t \partial y} + \Bigl(\frac{\partial u}{\partial y}\Bigr)^2\frac{\partial^3 u}{\partial y^2 \partial t}\Bigr] - l_2 u\Bigr\}\Bigr\}\Bigr) \tag{22}$$

Where, $H_a(u_n), H_b(u_n)$ and $H_c(u_n)$ are the He's polynomials for $\left(\frac{\partial u}{\partial y}\right)^2, \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t \partial y}$ and $\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3 u}{\partial y^2 \partial t}$ respectively.
Now, comparing the like powers of "P" in equation (44) and equating their coefficients gives

$$P^0; u_0(y,t) = \lambda + e^{-y} + (G_c e^{-y})t + \Bigl(l_1 e^{-y} + \frac{e^{-y}}{S_c} - K_r e^{-y}\Bigr)\frac{t^2}{2!} + \Bigl(l_1^2 e^{-y} + \frac{e^{-y}}{S_c^2} - \frac{2Se^{-y}}{S_c} - \frac{2K_r e^{-y}}{S_c} + 2K_r Se^{-y} + K_r^2 e^{-y}\Bigr)\frac{t^3}{3!} \tag{23}$$

$$P^1; u_1(y,t) = L^{-1}\Bigl\{\frac{1}{s}L\Bigl\{S\frac{\partial u_0}{\partial y} + \frac{\partial^2 u_0}{\partial y^2} + \alpha\frac{\partial^3 u_0}{\partial y^2 \partial t} + \beta_a\frac{\partial^4 u_0}{\partial y^2 \partial t^2} + \beta_b(u_0')^2(u_0'') + \gamma_a\frac{\partial^5 u_0}{\partial y^2 \partial t^3} + \gamma_b[2u_0'''u''_{0t} + (u_0')^2(u_0''u_{0t}')] - l_2 u_0\Bigr\}\Bigr\} \tag{24}$$

Or,

$$u_1(y,t) = \Bigl(e^{-y} - Se^{-y} + \alpha G_c e^{-y} + \frac{\beta_a e^{-y}}{S_c} - \beta_a K_r e^{-y} - \beta_b e^{-y} + \frac{\gamma_a e^{-y}}{S_c^3} - \frac{2\gamma_a Se^{-y}}{S_c} - \frac{2\gamma_a K_r e^{-y}}{S_c} + 2\gamma_a K_r Se^{-y} + \gamma_a K_r^2 e^{-y} + \gamma_a l_1^2 e^{-y} - 2\gamma_a G_c e^{-2y} + \gamma_b G_c e^{-3y} - l_2 e^{-y} - \lambda l_2\Bigr)t + \Bigl(G_c e^{-y} - G_c Se^{-y} + \frac{\alpha e^{-y}}{S_c} - 2\alpha Se^{-y} + \alpha l_1 e^{-y} + \frac{\beta_a e^{-y}}{S_c^2} - \frac{2\beta_a Se^{-y}}{S_c} - \frac{2\beta_a K_r e^{-y}}{S_c} + 2\beta_a K_r Se^{-y} + \beta_a K_r^2 e^{-y} + \beta_a l_1^2 e^{-y} + 3\beta_b G_c e^{-3y} + \frac{\gamma_a e^{-y}}{S_c^3} - \frac{3\gamma_a e^{-y}}{S_c^2} - \frac{6\gamma_a K_r Se^{-y}}{S_c} + \frac{6\gamma_a S^2 e^{-y}}{S_c} - 6\gamma_a K_r S^2 e^{-y} - 3\gamma_a K_r^2 Se^{-y} - \gamma_a K_r^3 e^{-y} - \gamma_a l_1^2 Se^{-y} - \gamma_a l_1 S^2 e^{-y} + \gamma_a l_1^3 e^{-y} - 2\gamma_b G_c^2 e^{-3y} - 3\gamma_b G_r^2 e^{-3y} - 4\gamma_b G_C G_r e^{-3y} - l_2 G_c e^{-y} - l_2 G_r e^{-y}\Bigr)\frac{t^2}{2!} + \Bigl(\frac{e^{-y}}{S_c} - K_r e^{-y} + \frac{e^{-y}}{P_r} + l_1 e^{-y} - \frac{Se^{-y}}{S_c} + K_r Se^{-y} - \frac{Se^{-y}}{P_r} + Sl_1 e^{-y} + \frac{\alpha e^{-y}}{S_c^2} - \frac{2\alpha Se^{-y}}{S_c} - \frac{2\alpha K_r e^{-y}}{S_c} + 2\alpha K_r Se^{-y} + \alpha K_r^2 e^{-y} + \frac{\alpha e^{-y}}{P_r^2} + \frac{2\alpha l_1 e^{-y}}{P_r} + \alpha l_1^2 e^{-y} + \frac{\beta_a e^{-y}}{S_c^3} - \frac{3\beta_a Se^{-y}}{S_c^2} - \frac{2\beta_a K_r e^{-y}}{S_c^2} + \frac{6\beta_a K_r Se^{-y}}{S_c} + \frac{3\beta_a S^2 e^{-y}}{S_c} + \frac{3\beta_a K_r^2 e^{-y}}{S_c} - \frac{\beta_a K_r e^{-y}}{S_c} - 3\beta_a K_r S^2 e^{-y} - 3\beta_a K_r^2 Se^{-y} - \beta_a K_r^3 e^{-y} - \beta_a l_1^2 Se^{-y} - \beta_a l_1 S^2 e^{-y} + \beta_a l_1^3 e^{-y} + \beta_a S^3 e^{-y} - 6\beta_b G_c^2 e^{-3y} - \frac{l_1 e^{-y}}{S_c} + l_2 K_r e^{-y} + 2\gamma_b G_c^3 e^{-3y} - l_1 l_2 e^{-y}\Bigr)\frac{t^3}{3!} \tag{25}$$

Therefore, the solution to equation (7) is;

$$u(y,t) = u_0(y,t) + u_1(y,t) + \cdots \tag{26}$$

Where, $u_0(y,t)$ and $u_1(y,t)$ are defined in equations (23) and (25) respectively.

RESULTS AND DISCUSSION

Theoretical work on unsteady MHD flow of non-Newtonian fluid in horizontal channel with mass transfer has been analyzed. The impact of thermal radiation, chemical reaction, suction, third and fourth-grade parameters along with other pertinent flow parameters are plotted graphically on different flow fields. The default values for the pertinent flow parameters are taken as (Arifuzzaman *et al.*, 2018)), $\lambda = 0.30, \alpha = 0.20, \beta_a = 0.005, \beta_b = 0.005,\ \gamma_a = 0.05, \gamma_b = 0.0.05,\ S_c = 0.50,\ G_c = 3, P_r = 0.71, Ha = 0.50, Da = 1.00, K_r = 0.50$.

The effect of chemical reaction parameter (K_r) on velocity and concentration profiles are depicted in figs. 2 and 3 respectively. Increase in chemical reaction parameter (K_r), the velocity field and the concentration field both decreases.

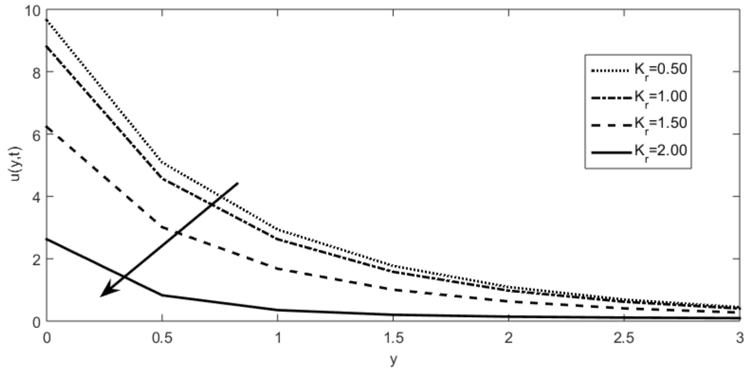

Figure 2. Effect of K_r on Velocity profile u

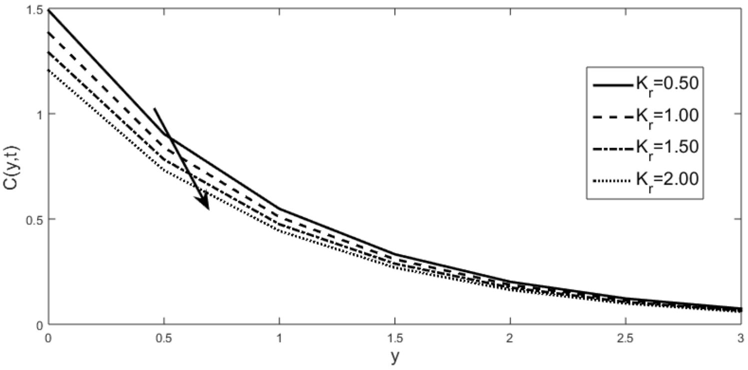

Figure 3. Effect of K_r on Concentration distribution Type equation here.

Figure 4 illustrates the drag force effect on fluid flow. The velocity profile decreases with the increment of Hartmann number.

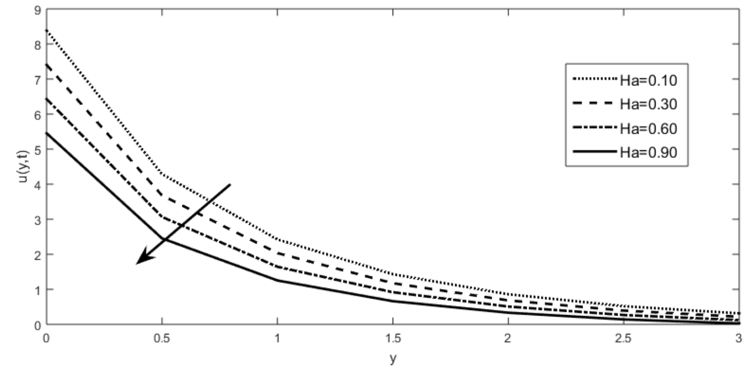

Figure 4. Effect of Ha on Velocity profile u

The impact of suction parameter S on velocity and concentration profiles are depicted in figs. 5 and 6 respectively. It is clearly seen that velocity and concentration profiles diminish with the increase of S. This is due to the porosity of plates.

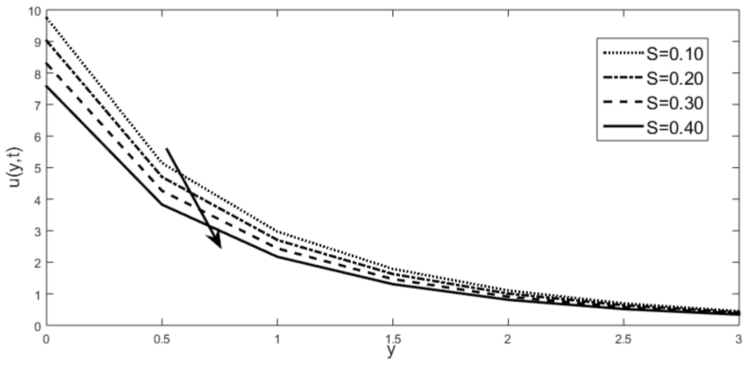

Figure 5. Effect of S on Velocity profile u

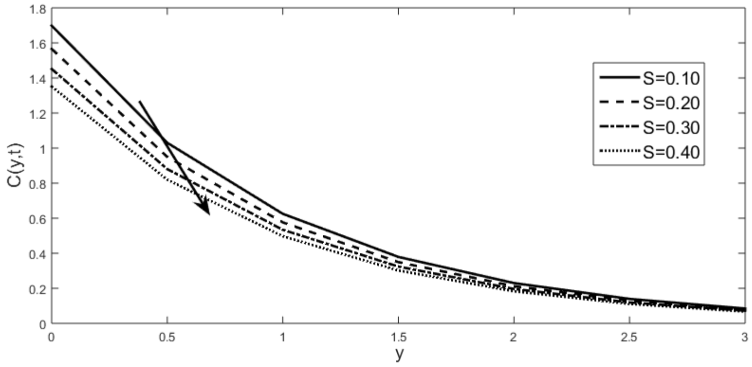

Figure 6. Effect of S on Concentration distribution θ

CONCLUSION

Unsteady MHD flow of non- Newtonian fluid has been analyzed. The solution for the nonlinear partial differential equations are obtained by He-Laplace scheme. The effects of flow parameters on velocity and concentration profiles are depicted in figures and discussed. From the results obtained, the findings are:

(i) For increasing values of chemical reaction, velocity and concentration fields diminish.
(ii) Velocity and concentration fields diminish due to the increment of suction parameter.
(iii) Velocity and skin friction fields decline due to the increment of magnetic parameter.

REFERENCES

Arifuzzaman S.M., Shakhaoth Khan Md., Al-Mamum A., Reza-E-Rabbi S.K., Biswas P., and Karim I.,(2018) , *Hydrodynamic Stability and Heat and Mass Transfer Flow Analysis of MHD Radiative Fourth-Grade Fluid Through Porous Plate with Chemical Reaction*. Journal of King Saud University, **31** (4) pp1388 – 1398.

Hayat T., Ellahi R. and Mahomed F.M., (2007) , *Exact solutions for Couette and Poiseuille flows for Fourth grade Fluids*, Acta Mechanica, **188** pp69 – 78.

Idowu A.S. and Sani U. (2019): *Thermal radiation and chemical reaction effects on unsteady magnetohydrodynamic third-grade fluid flow between stationary and oscillating plates*. – International Journal of Applied Mechanics and Engineering, vol.24, pp.269 – 293.

Islam S., Bano Z., Siddique I., and Siddiqui A.M., (2011), *The Optimal Solutions for the Flow of a Fourth-Grade Fluid with Partial Slip*, Computer and Mathematics with Application,**6** pp1507 – 1516.

Joseph K.M., Ayankop-Andi, E. and Mohammed, S.U.,(2021): *Unsteady MHD Plane Couette-Poiseuille Flow of Fourth-Grade Fluid with Thermal Radiation, Chemical Reaction and Suction Effects*. International Journal of Applied Mechanics and Engineering, 26 (4): pp77-98.

Rehan A.S., Islam S. and Siddiqui A.M. (2010): *Couette and poiseuille flows for fourth-grade fluids using optimal homotopy asymptotic method.* – World Applied Science Journal, vol.9, No.1, pp.1228-1236.

Santhosa B., Younus S., Kamala G., Ramana Murthy MV.,(2017), *Radiation and Chemical Effects on MHD Free Convective Heat and Mass Transfer Flow of a Viscoelastic Fluid Past a Porous Plate with Heat Generation/Absorption*. International Journal of Chemical Sci., **15** (3): 170

Shehzad N., Zeeshan A., and Ellahi R., (2018), *Electroosmotic Flow of MHD Power Law Al2O3-PVC Nanofluid in a Horizontal Channel: Couette-Poiseuille Flow Model.* Communication Theoretical Physics **69** (6) pp655–666.

Taza G., Fazle G., Islam S., Shah R.A., Khan I., Nasir S. and Sharidan, (2016) ,*Unsteady Thin Film Flow of a Fourth-Grade Fluid over a Vertical Moving and Oscillating Belt*, Propulsion and Power Research,**5** (3) pp223 -235.

Zeeshan Khan, Ilyas Khan , Murad Ullah, Tlili I., (2018), *Effect of Thermal Radiation and Chemical Reaction on non-Newtonian Fluid through a Vertically Stretching Porous Plate with Suction.* Results in Physics, **9** pp1086 – 1095.