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LINEAR PROGRAMMING MODEL OF PRODUCTION, INVENTORY AND DISTRIBUTION PROBLEM BASED ON RANDOM SAMPLING

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ABSTRACT

A linear programming model using the random sampling technique was used for the optimization of the products in the production line of Nigerian Bottling Company (NBC), Lagos State, Nigeria, which has many production facilities and multi-products systems. Products from the company are distributed to a number of depots across the country of which the demand for each product is known. The problem of interest involves determining what product/s should be made, how much of each product should be produced and where production should take place. The objectives of the company are to minimize the total cost as well as maximize the total sales revenue based on the set of decisions including demands, capacity restriction and budget constraints. Linear Programming software solver (LiPs) was used to solve large linear programming problems based on the mathematical model developed. The model improved the profit of the company under study by 5% and reduced the costs of production, inventory and distribution (PID). This improvement on the profit realized provides the need for using Linear Programming in solving production planning problem in the Nigerian Bottling Company (NBC).

Keywords: Optimization, Decision variables, Constraints, Linear programming

INTRODUCTION

Operations Research is designed to provide quantitative tools to decision-making processes. It comprises a set of mathematical optimization and simulation methods and models, such as Linear Programming, Non-linear Programming, Theory of Queues, Dynamic Programming, Theory of Decisions, etc. (Erinle-Ibrahim et al., 2020).Company managers are often faced with decisions relating to the use of limited resources. These resources may include materials, money and men. Implementing optimized solutions by linear programming has reduced costs significantly in many middle to large scale companies in several industrialized countries. Linear programming has demonstrated to be an alternative solution to plan replacing the traditional solutions based on trial and error (Cardoen, et al., 2010, Balogun et al., 2012).

Problems involving the production, Inventory and Distribution of goods and services can either be a transportation model or inventory models and variations of both types of models can be found in the literature (Stephen and Akpan, 2019; Erinle-Ibrahim et al., 2020). The transportation model proposed in this paper will be used to determine the quantity of the NBC products that should be transported through each route (at a minimum total cost) of a transportation problem which has 'i' plants (as supply centers) and 'k' depots as consumption centers(David *et al*.,2017).

Similarly, inventory models can either be a deterministic or stochastic model. Inventories are stocks of items in finished, semi-finished, unfinished or raw form that are stored in depots, stores or at suitable places for future use or sales (Igwe and Onyeweaku, 2013). According to Lucey, (1996), inventory will help NBC to reduce the variations in demand and production of their products by deposing goods at each finite time intervals to cushion the effects of excess demand which could be taken care of by initial quantity of goods in the depot. The optimal quantity that should be ordered and which will also minimize the total inventory cost depends on the type of inventory model under consideration (Balogun*et al*., 2012). While inventory models can be broadly classified into two major groups, deterministic and stochastic transportation model is entirely a deterministic model (Agbadudu, 1996; Akpan and Iwoke, 2016). The seasonal demand for beverages is creating a chain of intricate and far reaching effects that could not be responded so adequately. As a result, the company's profit was being seriously eroded by steeply escalating production, inventory and distribution costs. The web of interacting influences which spanned the company's principal activities - Production, inventory and distribution, required an integrated computer-based planning system to uncover the appropriate decisions, hence the basis for this research work.

Aim and Objectives

The overall aim of the study is to optimize the production, inventory and distribution of the Nigerian Bottling Company products so as to maximize profit. The specific objectives of the study include the following

1. To carry out an assessment of the NBC production, inventory and distribution systems.
2. To fit a linear programming model relating to the production, inventory and distribution of the NBC products.
3. To identify production activities that will achieve high efficiency in the production processes of the NBC.
4. To identify the decision variables, parameters and constraints necessary for formulating a model of the NBC production, inventory and distribution operations.
5. To be able to evaluate the profit of using optimization approach in the production planning of the NBC.

METHODOLOGY

Linear Programming Model Formulation

The Linear Programming model was formulated based on three basic components:

1. Decision variables to be determined.
2. Objective (goal or aim) to optimize.
3. Constraints that need to be satisfied.

The parameters, variables and constraints were determined according to the notations thus:

Index
j = no of products.
i = no of plants.
k = no of depots.
t = no of periods.
Parameters
Sjk = unit selling price of product, j at depot, k.
Hjk= unit-holding cost of product, j at depot, k.
Vjik = unit distribution cost of product, j from plant, i to depot,
Ijit = inventory of product, j at plant, i in period, t.
Ai = maximum storage capacity at plant, i.
Wji = unit production time (cycle time) of product, j at plant, i.
Ti = maximum available production time at plant, i.
Bu = upper budget limit.
BL = lower budget limit.
Cjit = unit production cost of product, j at plant, i in period, t.
Djkt =demand for product, j at depot, k in period, t.

Variables
Qjit = quantity of product, j produced at plant, i in period, t.
Xjikt = quantity of product, j from plant, i supplied to depot, k in period, t.
INVjkt= inventory of product, j at depot, k in period, t.

Assumptions
For simplicity of the problem, some assumptions are proposed as follows:
1. Ready market for all products.
2. All plant locations have same standard operating process.
3. Enough personnel (skill and knowledge) exist.
4. Effective transport system is in place.
5. Demand estimation can be made.

Constraints
1. Capacity restriction on production in every period.
2. Inventory balance at production facility.
3. Inventory balance at depot.
4. Total demand for each item at customer location.
5. Total budget.

Model Development
Component formulation
Profit = Sales Revenue – Total cost
Sales Revenue
$$=\sum_{j=1}^{m}\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{t=1}^{m} Sjikt\ Xjikt \quad (1)$$

Total Cost
= production cost + inventory cost + distribution cost.
Production cost
$$=\sum_{j=1}^{m}\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{t=1}^{m} Cjikt\ Qjikt \quad (2)$$
Inventory cost = average sum of starting inventory and ending inventory cost
$$= 0.5hjk\sum_{j=1}^{m}\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{t=1}^{m} INVjkt - 1 + INVjkt \quad (3)$$

Distribution cost
$$=\sum_{j=1}^{m}\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{t=1}^{m} Vjikt\ Xjikt \quad (4)$$

Objective function
Maximize:
Profit=Sales Revenue – Total cost
Profit = Sales Revenue – (Production cost + Inventory cost + Distribution cost)
Profit
$$= \sum_{j=1}^{m}\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{t=1}^{m} Sjikt\ Xjikt - \sum_{j=1}^{m}\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{t=1}^{m} Cjikt\ Qjikt - 0.5hjk\sum_{j=1}^{m}\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{t=1}^{m}(INVjkt - 1 + INVjkt) - \sum_{j=1}^{m}\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{t=1}^{m} Vjikt\ Xjikt \quad (5)$$

Constraints
Subject to:
Period capacity restriction on production
$$\sum_{j=1}^{m}\sum_{i=1}^{m}\sum_{t=1}^{m} Wjit\ Qjit \le Ti \quad (6)$$

Production facility inventory balance
$$\sum_{j=1}^{m}\sum_{i=1}^{m}\sum_{t=1}^{m}(Ijit + Qjit - Xjikt) \le Ai \quad (7)$$

Depot inventory balance =
$$\sum_{j=1}^{m}\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{t=1}^{m} Xjikt - \begin{pmatrix}\sum_{j=1}^{m}\sum_{k=1}^{m}\sum_{t=1}^{m} INVjkt - \\ \sum_{j=1}^{m}\sum_{k=1}^{m}\sum_{t=1}^{m} INVjkt - 1\end{pmatrix} \quad (8)$$
$$= \sum_{j=1}^{m}\sum_{k=1}^{m}\sum_{t=1}^{m} Djkt \quad (9)$$

Production-demand balance
$$\sum_{j=1}^{m}\sum_{i=1}^{m}\sum_{t=1}^{m} Qjit + \sum_{j=1}^{m}\sum_{i=1}^{m}\sum_{t=1}^{m} Ijit \le \sum_{j=1}^{m}\sum_{k=1}^{m}\sum_{t=1}^{m} Djkt \quad (10)$$
Budgetary constraint
$$\sum_{j=1}^{m}\sum_{i=1}^{m}\sum_{t=1}^{m} Cjit\ Qjit - \sum_{j=1}^{m}\sum_{k=1}^{m}\sum_{t=1}^{m} 0.5hjkt - (INVjkt - 1 + INVjkt) - \sum_{j=1}^{m}\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{t=1}^{m} Vikt\ Xjikt \le \text{BU} \quad (11)$$

$$\sum_{j=1}^{m}\sum_{i=1}^{m}\sum_{t=1}^{m} Cjit\ Qjit - \sum_{j=1}^{m}\sum_{k=1}^{m}\sum_{t=1}^{m} 0.5hjkt - (INVjkt - 1 + INVjkt) - \sum_{j=1}^{m}\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{t=1}^{m} Vikt\ Xjikt \ge \text{BL} \quad (12)$$

For ease of identification and simplification, the variables used are redefined as shown in table 1.

Table 1: Quantity of product 'j' produced at plant 'i' in period't' in year 2021.

Qjit	j	i	T
Q1	1	1	1
Q2	2	1	1
Q3	1	2	1

where,
Q1 is quantity of product '1' made at plant '1' in period '1',
Q2 is quantity of product '2' made at plant '1' in period '1',
Q3 is quantity of product '1' made at plant '2' in period '1'.
Product '1' is **'**Coke**'** while Product '2' is **'**Fanta**'**.

Xjit is quantity of product j produced at plant I distributed to depot k in period t e.g
X1 is quantity of product '1' produced at plant '1 distributed to depot 1 in period 1.
Ijit is inventory of product 'j' at depot 'k' in period 't', e.g. I1 is inventory of product '1' at depot '1' in period '1'

Table 2: Demand for product ‘j’ at depot ‘k’ in period‘t’ (quarters), Djkt (unit) in year 2021

PERIOD, t (Quarter)	DEPOT, k	PRODUCT, j 1	PRODUCT, j 2
	1	75500	250000
1	2	125500	400000
	3	199000	640000
Total		400000	1290000
	1	50000	205000
2	2	100000	300000
	3	140000	435000
Total		340000	1040000
	1	40000	105000
3	2	75000	260000
	3	125000	425000
Total		290000	890000
	1	100000	385000
4	2	180000	560000
	3	260000	805000
Total		440000	1650000

N.B: Starting inventory of product ‘j’ at plant ‘i’ in period‘t’ = 1, assumed to be zero,

Similarly,
Starting inventory of product ‘j’ at depot ‘k’ in period‘t’ = 1, assumed to be zero.

Table 2 shows the various quantities of products 1 and 2 that were demanded at depots 1, 2, 3 in time periods 1, 2, 3 and 4 with their respective totals.

Table 3: Unit production time, wji(second, s) of product in year 2021

PLANT, i	PRODUCT, j	
	1	**2**
1	4.32	4.32
2	2.4	2.4

Maximum available time at each plant:
Plant 1 = 20hrs
Therefore, the maximum available time (in seconds) in each period or quarter (1 period = a quarter = 3 months) = 60mins/1hr x 60secs/1min x 20hrs/1day x 20days/1mnth x 3mnths/1 period.
= 4,320,000 seconds/quarter

From table 3, it could be deduced that 4.32 seconds were used to produce products 1 and 2 in plant 1 while 2.4 seconds were used to produce products 1 and 2 in plant 2.
Similarly,
Plant 2 = 20hrs
Therefore, the maximum available time (in seconds) in each period or quarter (1 period = a quarter = 3 months = 60mins/1hr x 60secs/1min x 20hrs/1day x 20days/1mnth x 3mnths/1 period
= 4,320,000 seconds/quarter

Table 4: Total unit cost of producing product ‘j’ at plant ‘i’, (₦/unit) in year 2021 **DEPOT, ‘k’**

PLANT, ‘i’	PRODUCTION COST OF ‘j’(₦/UNIT)	
	1	**2**
1	830	830
2	830	830

Table 4 shows that ₦830 was the total unit production cost of products 1 and 2 in plants 1 and 2. 17 11 9

Table 5: Selling price per unit of product ‘j’ at depot ‘k’, (₦/unit) in year 2021.

PRODUCT, ‘j’	DEPOT, ‘k’		
	1	**2**	**3**
1	980	980	980
2	980	980	980

Table 5 shows that the unit selling price of products 1 and 2 at depots 1, 2 and 3 were ₦980.
Selling price per unit product (crate) = ₦2,200 (50cl)
Selling price per bottle (liquid content only) = ₦120 (50cl)
Selling price per unit product (crate) = ₦1300 (35cl)
Selling price per bottle (liquid content only) = ₦90 (35cl)

Table 6: Distribution cost per unit of product, (₦/unit) in year 2021

PLANT, ‘i’	DEPOT, ‘k’		
	1	**2**	**3**
1	7	9.5	11
2	11	9	6

From table 6, it could be deduced that the unit distribution costs of product 'j' at plant 1 and in depots 1, 2 and 3 were ₦7, ₦9.5k and ₦11 respectively while that at plant 2 and at depots 1, 2 and 3 were ₦11, ₦9 and ₦6 respectively.

Table 7: Inventory holding cost at each depot, ₦/unit (assumed constant over a long range) in year 2021

DEPOT, 'k'	PRODUCT, 'j'	
	1	2
1	13.5	15
2	14.5	14
3	15	13

Maximum storage capacity at warehouse:
Plant 1 = 100,000 units
Plant 2 = 150,000 units
From table 7, it could be inferred that the inventory holding costs of product 1 at depots 1, 2 and 3 were ₦13.5k; ₦14.5k and ₦15 respectively while that of product 2 at depots 1, 2 and 3 were ₦15, ₦14 and ₦13 respectively.

Objective Function to Optimize:
Profit = Sales Revenue – (Production cost + Inventory cost + Distribution cost)
Maximize
973X1 + 970.5X2 + 969X3 + 969X4 + 971X5 + 974X6 + 973X7 + 970.5X8 + 969X9 + 969X10 + 971X11 + 974X12 + 973X13 + 970.5X14 + 969X15 + 969X16 + 971X17 + 974X18 + 973X19 + 970.5X20 + 969X21 + 969X22 + 971X23 + 974X24 + 973X25 + 970.5X26 + 969X27 + 969X28 + 971X29 + 974X30 + 973 X31 + 970.5X32 + 969X33 + 969X34 + 971X35 + 974X36 + 973X37 + 970.5X38 + 969X39 + 969X40 + 971X41 + 974X42 + 973X43 + 970.5X44 + 969X45 + 969X46 + 971X47 + 974X48 – 830Q1 – 830Q2 – 830Q3 – 830Q4 – 830Q5 – 830Q6 – 830Q7 – 830Q8 – 830Q9 – 830Q10 – 830Q11 – 830Q12 – 830Q13 – 830Q14 – 830Q15 – 830Q16 – 13.5INV1 – 14.5INV2 – 15INV3 – 15INV4 – 14INV5 – 13INV6 – 13.5INV7 – 14.5INV8 – 15INV9 – 15INV10 – 14INV11 – 13INV12 – 13.5INV13 – 14.5INV14 – 15INV15 – 15INV16 – 14INV17 – 13INV18 – 13.5INV19 – 14.5INV20 – 15INV21 – 15INV22 – 14INV23 – 13INV24
forXjikt : Quantity of product 'j' from plant 'i' supplied to depot 'k' in period 't'

Maximise Profit:
973X1 + 970.5X2 + 969X3 + 969X4 + 971X5 + 974X6
Subject To:
-X1 – X2 – X3 ≤ 100000
- X4 – X5 – X6 ≤ 150000
X1 + X4 = 75500
X2 + X5 =125500
X3 + X6 =199000
-X1 – X2 – X3 – X4 – X5 – X6 ≤290000
-X1 – X2 – X3 – X4 – X5 – X6 ≤ 240000
-X1 – X2 – X3 – X4 – X5 – X6 ≤ 540000
X1, X2, X3, X4, X5, X6 ≥ 0
For Qjit: Quantity of product 'j' produced at plant 'i' in period't'

Maximise Profit :
980Q1 + 980Q2 + 980Q3 + 980Q4
Subject To:
4.32Q1 + 4.32Q2 ≤6264000
2.4Q3 + 2.4Q4 ≤6264000
Q1 + Q2 ≤ 100000
Q3 + Q4 ≤150000
Q1 + Q ≤ 400000
Q2 + Q4 ≤ 1290000
Q1 + Q3 ≤ 290000
Q2 + Q4 ≤ 940000
Q1 + Q3≤ 240000
Q2 + Q4 ≤ 790000
Q1 + Q3 ≤ 540000
Q2 + Q4 ≤ 1750000
Q1, Q2, Q3, Q4 ≤ 0

For INVjkt: Inventory of product 'j' at depot 'k' in period't'

Maximize Profit:
13.5INV1 + 14.5INV2 + 15INV3 + 15INV4 + 14INV5 + 13INV6
Subject To:
0.5INV1 = 75500
0.5INV2 =125500
0.5INV3 = 199000
0.5INV4 = 250000
0.5INV5 = 400000
0.5INV6 = 640000
INV1, INV2, INV3, INV4, INV5, INV6 ≥ 0

RESULTS AND DISCUSSION

Profit Analysis

Profit analysis of year 2021 without optimization
Selling Price per unit product (35cl) = ₦980
Average Sales Volume range per day = 24,000 cases per day
Sales Revenue =₦980 X 24,000 X 365 = ₦8,584,800,000
Unit Production time of Products = 8hrs per day
Filler Capacity Cases (35cl) = 1,083 cases per hour
Unit Production Cost per Case (35cl) = ₦830
Production Cost = ₦830 X 1,083 X 8 X 365 = ₦2,624,758,800
Maximum Storage Capacity at the warehouse = 150,000 units
Inventory holding Cost at the warehouse per unit product = ₦15
Inventory Cost= ₦15 X 150,000 X 365 = ₦821,250,000
Sales Volume Range per day = 24,000 cases per day
Distribution Cost per unit case (35cl) = ₦11
Distribution Cost = ₦11 X 24,000 X 365 = ₦96,360,000
Profit = Sales Revenue – Total Cost
Where Total Cost = Production Cost + Inventory Cost + Distribution Cost
Profit = Sales Revenue – Production Cost – Inventory Cost – Distribution Cost
Profit = ₦8,584,800,000 – ₦2,624,758,800 – ₦821,250,000 – ₦96,360,000
Profit = ₦5,042,431,200.00K

Profit analysis of year 2021 with optimization
Profit = Sales Revenue – Total Cost
where
Total Cost = Production Cost + Inventory Cost + Distribution Cost
Profit = Sales Revenue – Production Cost – Inventory Cost – Distribution Cost
Profit = ₦389,148,000 + ₦1,255,010,000 + ₦282,110,000 + ₦914,455,000 + ₦233,495,000+ ₦768,575,000 +

₦525,320,000 + ₦1,702,440,000 – ₦207,500,000 – ₦207,500,000 – ₦207,500,000 – ₦207,500,000 – ₦46,988,000 – ₦34,310,000 – ₦28,485,000 – ₦63,880,000
Profit= ₦5,066,890,000.00K

Where
389148000, 1255010000, 282110000, 914455000, 233495000, 768575000, 525320000, 1702440000, 207500000, 207500000, 207500000, 207500000, 46988000, 34310000, 28485000, 63880000, are the values of theoptimum solutions generated from the Linear Programming Solver Software.

Optimum solutions
The Objective function gives a maximum profit value of ₦5,066,890,000.00K (target year – 2021) and the inventory level is kept at minimum as against the existing profit margin of ₦5,042,431,000.00K without Optimization principle.
Hence, the Optimization method employed in the profit analysis of year 2021 has yielded a significant extra 10% profit increment of ₦24,458,800.00K as against the profit margin recorded in year 2021without Optimization principle.

Sensitivity analysis
The Optimal Solution of a Linear Programming is mostly based on a snapshot of the conditions that prevail at the time the model is formulated and solved. In reality, the decision environments rarely remain static. Hence, it is essential to determine how the optimal solution changes when the models parameters are changed. Sensitivity analysis provides efficient computational techniques to study the dynamic behavior of the optimal solution resulting from making changes in the parameters of the model.
The transportation model proposed in this research work was used to determine the quantity of NBC products that should be transported through each route at a minimum total cost of a transportation problem which has ‘i’ plants as supply centers and ‘k’ depots as consumption centers (David *et al*., 2017).
Similarly, inventory models can either be a deterministic or stochastic model. Inventories are stocks of items in finished, semi-finished, unfinished or raw form that are stored in depots, stores or at suitable places for future use or sales (Ogunrinde, and Ishola, 2020). The inventory model will help NBC to reduce the variations in demand and production of their products by disposing goods at each finite time intervals to cushion the effect of excess demand which could be taken care of by an initial quantity of goods in the depot.
Furthermore, the Optimization method employed in the profit analysis of year 2021 has yielded an extra profit increment of ₦24,458,800.00K which represents a 10% increment as against the profit margin recorded in year 2021 without Optimization principle.
The latest operational report for the twelve-month period ended December 31, 2021 showed an increase in gross profit from ₦5,042,431,200.00K (without optimization) to ₦5,066,890,000.00K (with optimization). This represents a significant profit increase of about 10% with the optimization principle and cost is at minimum. The profit before tax rose from ₦5.59 billion by December, 2020 to ₦5.79 billion in December, 2021 (without Optimization) and to ₦5.82 billion in December, 2021 (with Optimization). Similarly, profit after tax also leapt from ₦5.24 billion by December, 2020 to ₦5.42 billion in December, 2021 (without Optimization) and to ₦5.66 billion by December, 2021 (with Optimization). Higher margins pushed profit before tax up by 17.5% while profit after tax rose by 20.3%.

Finally, the model has helped NBC to measure the efficiency of their production, inventory and distribution activities by capturing: “the right product to be produced in the right place at the right period and supplied to the right de pot from the right plant at the right time”.

CONCLUSION
The Production, inventory and distribution systems of NBC has been assessed and the potentials for using Linear Programming model in managing large scale Production, inventory and distribution problems subsequently identified. The decision variables, parameters and constraints necessary for formulating a model of the company’s Production, inventory and distribution operations so identified have been solved using the linear Programming Solver, LiPs. A linear Programming model which consists of eighty-eight (88) variables and fifty-four (54) constraints has been fitted and subsequently formulated. The Optimal Production, inventory and distribution of NBC products has been analyzed and the model improved the profit of the company under study by 10% and also enhanced the Production, inventory and distribution (PID) strategy used by the company. The model developed resulted in significant profit increase of about 5% with the Optimization principle and cost is at minimal (a profit margin of ₦5,403,890,000.00k - year 2021) in contrast with the existing profit of ₦5,141,682,000.00k without the Optimization principle in year 2021 and a profit increment of ₦262,208,000.00K with the Optimization principle in year 2020. Hence, the Process of formulating the model, testing the model, and analyzing the model’s results has provided valuable benefits to NBC.

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