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NEW CALIBRATION OF FINITE POPULATION MEAN OF COMBINED RATIO ESTIMATORS IN STRATIFIED RANDOM SAMPLING

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ABSTRACT

This study deals with using calibration estimation approaches to modified the combined ratio estimator in stratified random sampling. Calibration distance measures with their associate constraints were used to modify combined ratio estimator. In stratified random sampling, new sets of optimum calibration weights are created and used to obtain new calibration estimators of population mean. Empirical study through simulation was conducted to look into the efficiency of the suggested estimators obtained. The suggested calibration estimators are more efficient than other existing estimators investigated in the study, according to the findings.

Keywords: Weights for calibration, Combined Ratio, Estimators, Mean Squared Error, Stratified Sampling

INTRODUCTION

Estimation of calibration by incorporating the known population characteristics of auxiliary variables, the original design weights can be adjusted. In stratified random sampling, a calibration approach is used to determine the optimum stratum weights for enhancing the precision of survey estimates of population parameters. Deville and Sarndal established the approach of estimate by calibration in survey sampling in 1992. The ideal is to use auxiliary information to improve parameter estimates for the population of interest. Following Deville and Sarndal (1992), many researchers have investigated survey sample design calibration estimation utilizing various calibration constraints on auxiliary variables. Singh *et al.* (1998) was the first researcher that extended calibration approach to stratified sampling design. In stratified sampling, Tracy *et al.* (2003), Singh (2003), Kim *et al.* (2007), Clement and Enang (2015) and Ozgul (2018) used calibration estimation in ratio-type estimators. Rao *et al.* (2012) used different distance measures with two auxiliary variables in stratified sampling to construct a multivariate calibration estimate for the population mean. Based on distinct calibration constraints of auxiliary information, Koyuncu and Kadilar (2016) proposed calibration estimators for estimating the population mean in stratified sampling. The existing calibration estimators such as Singh *et al.* (1998), Tracy *et al.* (2003), Kim *et al.* (2007), Rao *et al.* (2012), Koyuncu and Kadilar (2013), Clement and Enang (2015), Koyuncu and Kadilar (2016), Clement (2017) and Ozgul (2018) adjusted the stratum weight of unbiased estimators utilizing auxiliary variables in stratified sampling to increase efficiency, favoring and more accurately representing strata with large sizes. To overcome this challenge, the strata sample means of the study variable are modified using ratio function of population and sample means of auxiliary variable within the strata using calibration techniques to produce more efficient estimate. Muili, *et al.* (2019).

Notations and Review of Existing Estimators

Take a look at a finite population ψ N elements, $\psi=\{\psi_1,\psi_2,\psi_3,...,\psi_N\}$ consists of L strata with N_h units in the h th stratum from which a simple random of size n_h can be generated from the population using SRSWOR. Total Population size $N=\sum_{h=1}^{L}N_h$, sample size $n=\sum_{h=1}^{L}n_h$ $y_{hi}, i=1,2,...,N_{hi}$ and $x_{hi}, i=1,2,...,N_{hi}$ of study variable y and x auxiliary variable. Let $W_h=N_h/N$ the weights of the strata, $\bar{Y}_h=N^{-1}\sum_{i=1}^{n_h}y_{hi}$ and $\bar{y}_h=n^{-1}\sum_{i=1}^{n_h}y_{hi}$ are the population and sample means respectively for the study variables.

The classic estimator of population mean in stratified sampling, according to Cochran (1977), is: $\bar{y}_{st}=\sum_{h=1}^{L}W_h\bar{y}_h$ (2.0)

$$V(\bar{y}_{st})=\sum_{h=1}^{L}W_h^2\left(\frac{1-f_h}{n_h}\right)s_{hy}^2 \quad (2.1)$$

where $s_{hy}^2=(n_h-1)^{-1}\sum_{h=1}^{n_h}(y_{hi}-\bar{y}_h)^2,\ f_h=n_h/N_h$

A combined ratio estimator was proposed by Hansen *et al*. (1946).

$$\bar{y}_{st}^{RC} = \frac{\bar{y}_{st}}{\bar{x}_{st}} \bar{X} \tag{2.2}$$

where $\bar{x}_{st} = \sum_{i=1}^{n_h} W_h \bar{x}_h \ and \ \bar{y}_{st} = \sum_{i=1}^{n_h} W_h \bar{y}_h$

The combined ratio estimator's variance is

$$V\left(\bar{y}_{st}^{RC}\right) = \sum_{h=1}^{L} W_h^2 \gamma_h \left(S_{yh}^2 + R^2 S_{xh}^2 - 2RS_{yxh}^2\right) \tag{2.3}$$

where $R = \frac{\bar{Y}}{\bar{X}}$

Lata *et al*. (2017) used a distance function of the sum of weighted squared deviation of calibrated and strata weights to build a calibration estimator for estimating population mean under stratified random sampling:

$$\bar{y}_{st}^{LA} = \sum_{h=1}^{L} \Omega_h^{LA} \bar{y}_h \tag{2.4}$$

$$Z = \sum_{h=1}^{L} S_{hx}^2 \left(Q_h\right)^{-1} \left(\Omega_h^{LA} - W_h\right)^2 \tag{2.5}$$

where Ω_h^{LA} are the calibrated weights and S_{hx}^2 is the auxiliary variable's variance in the hth stratum, subject to the calibration constraint.

$$\sum_{h=1}^{L} \Omega_h^{LA} \bar{x}_{1h} = \bar{X} \tag{2.6}$$

The calibrated weights and the estimator of (2.4) are obtained as show in (2.7) and (2.8) respectively.

$$\Omega_h^{LA} = W_h + \frac{\bar{x}_h Q_h \left(S_{hx}^2\right)^{-1}}{\sum_{h=1}^{L} \bar{x}_h^2 Q_h \left(S_{hx}^2\right)^{-1}} \left(\bar{X} - \sum_{h=1}^{L} W_h \bar{x}_h\right) \tag{2.7}$$

$$\bar{y}_{st}^{LA} = \sum_{h=1}^{L} W_h \bar{y}_h + \hat{\beta}\left(\bar{X} - \sum_{h=1}^{L} W_h \bar{x}_h\right) \tag{2.8}$$

where $\hat{\beta} = \frac{\sum_{h=1}^{L} \bar{x}_h \bar{y}_h Q_h \left(S_{hx}^2\right)^{-1}}{\sum_{h=1}^{L} \bar{x}_h^2 Q_h \left(S_{hx}^2\right)^{-1}}$

The calibrated estimator's variance is given by

$$V\left(\bar{y}_{st}^{LA}\right) = \sum_{h=1}^{L} \Omega_h^{LA^2} \frac{\left(1 - f_h\right)}{n_h} s_{eh}^2 \tag{2.9}$$

where $s_{eh}^2 = \left(n_h - 1\right)^{-1} \sum_{i=1}^{nh} e_{hi}^2$ is the mean square of the hth stratum sample and $e_{hi} = \left(y_{hi} - \bar{y}_h\right) - \hat{\beta}\left(x_{hi} - \bar{x}_h\right)$.

MATERIALS AND METHODS

Suggested Estimator

Having studied Lata *et al.* (2017) calibration estimator, and motivated by his work. A new set of calibration estimators are suggested as:

$$\bar{y}_{st}^{MJ} = \sum_{h=1}^{L} \Omega_h^{MJ} \bar{y}_h \tag{3.0}$$

$$Z^* = \sum_{h=1}^{L} S_{hx}^2 \left(Q_h\right)^{-1} \left(\Omega_h^{MJ} - W_h^*\right)^2 \tag{3.1}$$

where $W_h^* = W_h \bar{X} / \sum_{h=1}^{L} W_h \bar{x}_h$, Ω_h^{MJ} are the new calibrated weights and S_{hx}^2 is the variance of auxiliary variable in hth stratum. The form of the suggested estimators is determined by Q_h , which is a selected weight.
Subject to the calibration constraint

$$\sum_{h=1}^{L} \Omega_h^{MJ} \bar{x}_h = \sum_{h=1}^{L} W_h^* \bar{X}_h \tag{3.2}$$

The Lagrange Multiplier function L of $\bar{y}_{st}^{MJ}$ is defined as follows to get new calibrated weights $\left(\Omega_h^{MJ}\right)$ for the suggested estimator $\left(\bar{y}_{st}^{MJ}\right)$:

$$L = \sum_{h=1}^{L} S_{xh}^2 \left(Q_h\right)^{-1} \left(\Omega_h^{MJ} - W_h^*\right)^2 - 2\lambda \left(\sum_{h=1}^{L} \Omega_h^{MJ} \bar{x}_h - \sum_{h=1}^{L} W_h^* \bar{X}_h \right) \tag{3.3}$$

where λ is Lagrange's multiplier, differentiate (3.3) in relation to Ω_h^{MJ} , λ and are equal to zero to obtained (3.4), and (3.5) as follows:

$$\Omega_h^{MJ} = W_h^* + \lambda Q_h \bar{x}_h \left(S_{xh}^2\right)^{-1} \tag{3.4}$$

$$\sum_{h=1}^{L} \Omega_h^{MJ} \bar{x}_h - \sum_{h=1}^{L} W_h^* \bar{X}_h = 0 \tag{3.5}$$

Substitute (3.4) in (3.5), the results are obtained as:

$$\lambda \sum_{h=1}^{L} Q_h \bar{x}_h^2 \left(S_{xh}^2\right)^{-1} = \sum_{h=1}^{L} W_h^* \bar{X}_h - \sum_{h=1}^{L} W_h^* \bar{x}_h \tag{3.6}$$

Make λ the subject of formula, obtained as:

$$\lambda = \frac{\sum_{h=1}^{L} W_h^* \bar{X}_h - \sum_{h=1}^{L} W_h^* \bar{x}_h}{\sum_{h=1}^{L} Q_h \bar{x}_h^2 \left(S_{xh}^2\right)^{-1}} \tag{3.7}$$

When (3.7) is substituted for (3.4), the calibrated weights become:

$$\Omega_h^{MJ} = W_h^* + \left(Q_h \bar{x}_h \left(S_{xh}^2\right)^{-1} \right) \left(\frac{\sum_{h=1}^{L} W_h^* \bar{X}_h - \sum_{h=1}^{L} W_h^* \bar{x}_h}{\sum_{h=1}^{L} Q_h \bar{x}_h^2 \left(S_{xh}^2\right)^{-1}} \right) \tag{3.8}$$

Substituting (3.8) in (3.0), obtain the new combined calibration estimator $\left(\bar{y}_{st}^{MJ}\right)$ as:

$$\bar{y}_{st}^{MJ} = \sum_{h=1}^{L} W_h^* \bar{y} + \hat{\beta}_1 \sum_{h=1}^{L} W_h^* \left(\bar{X}_h - \bar{x}_h \right) \tag{3.9}$$

Substituting $W_h^* = W_h \bar{X} / \sum_{h=1}^{L} W_h \bar{x}_h$ in (3.9), gives

$$\bar{y}_{st}^{MJ} = \bar{X} \left(\sum_{h=1}^{L} W_h \bar{y} \right) \left(\sum_{h=1}^{L} W_h \bar{x}_h \right)^{-1} + \hat{\beta}_1 \bar{X} \left(\sum_{h=1}^{L} W_h \right) \left(\sum_{h=1}^{L} W_h \bar{x}_h \right)^{-1} \left(\bar{X}_h - \bar{x}_h \right) \tag{3.11}$$

where $\hat{\beta}_1 = \dfrac{\sum_{h=1}^{L} Q_h \bar{x}_h \bar{y} \left(S_{xh}^2\right)^{-1}}{\sum_{h=1}^{L} Q_h \bar{x}_h^2 \left(S_{xh}^2\right)^{-1}}$

Setting $Q_h = 1$, $Q_h = \bar{x}_h^{-1}$, and $Q_h = \left(S_{xh}^2\right)^{-1}$, we have the following new set of calibration combined ratio estimations respectively:

$$\bar{y}_{st11}^{MJ} = \bar{X}\left(\sum_{h=1}^{L} W_h \bar{y}\right)\left(\sum_{h=1}^{L} W_h \bar{x}_h\right)^{-1} + \hat{\beta}_{11}\bar{X}\left(\sum_{h=1}^{L} W_h\right)\left(\sum_{h=1}^{L} W_h \bar{x}_h\right)^{-1}\left(\bar{X}_h - \bar{x}_h\right) \quad (3.12)$$

$$\bar{y}_{st12}^{MJ} = \bar{X}\left(\sum_{h=1}^{L} W_h \bar{y}\right)\left(\sum_{h=1}^{L} W_h \bar{x}_h\right)^{-1} + \hat{\beta}_{12}\bar{X}\left(\sum_{h=1}^{L} W_h\right)\left(\sum_{h=1}^{L} W_h \bar{x}_h\right)^{-1}\left(\bar{X}_h - \bar{x}_h\right) \quad (3.13)$$

$$\bar{y}_{st13}^{MJ} = \bar{X}\left(\sum_{h=1}^{L} W_h \bar{y}\right)\left(\sum_{h=1}^{L} W_h \bar{x}_h\right)^{-1} + \hat{\beta}_{13}\bar{X}\left(\sum_{h=1}^{L} W_h\right)\left(\sum_{h=1}^{L} W_h \bar{x}_h\right)^{-1}\left(\bar{X}_h - \bar{x}_h\right) \quad (3.14)$$

where $\hat{\beta}_{11} = \dfrac{\sum_{h=1}^{L} \bar{x}_h \bar{y}\left(S_{xh}^2\right)^{-1}}{\sum_{h=1}^{L} \bar{x}_h^2\left(S_{xh}^2\right)^{-1}}$, $\hat{\beta}_{12} = \dfrac{\sum_{h=1}^{L} \bar{y}\left(S_{xh}^2\right)^{-1}}{\sum_{h=1}^{L} \bar{x}_h\left(S_{xh}^2\right)^{-1}}$ and $\hat{\beta}_{13} = \dfrac{\sum_{h=1}^{L} \bar{x}_h \bar{y}\left(S_{xh}^4\right)^{-1}}{\sum_{h=1}^{L} \bar{x}_h^2\left(S_{xh}^4\right)^{-1}}$

Table 1: Populations Involved in the Empirical Research

Population	Auxiliary variable x	Study variable y
I	$x_h \approx \exp\left(\theta_h\right), \theta_1 = 5, \theta_2 = 6,$ $\theta_3 = 4, h = 1,2,3$	
II	$x_h \approx gamma\left(\theta_h, \eta_h\right), \theta_1 = 3, \eta_1 = 2,$ $\theta_2 = 3, \eta_2 = 1, \theta_3 = 3, \eta_3 = 3,$	$y_{hi} = \alpha_h x_{hi}^j + \xi_{hi}$, $\alpha_{1h} = E\left(x_h\right),$ $\alpha = 1.0,\ \xi_h \approx N\left(0,1\right), h = 1,2,3.\ j = 2,3,4$
III	$x_h \approx \text{chisq}\left(\theta_h\right), \theta_1 = 5, \theta_2 = 6,$ $\theta_3 = 4, h = 1,2,3$	

Table 2: MSE and PRE of Some Existing and Suggested Estimators Using Population I

Estimator	$y_{hi} = \alpha_h x_{hi}^2 + \xi_{hi}$	
	MSE	**PRE**
$\bar{y}_{st}$	4.400796	100
Combined ratio $\bar{y}_{st}^{RC}$	1.685452	261.1048
Lata *et al*. (2017) $\bar{y}_{st11}^{LA}$	1.93655	227.2493
Lata *et al*. (2017) $\bar{y}_{st12}^{LA}$	2.028274	216.9725
Lata *et al*. (2017) $\bar{y}_{st13}^{LA}$	2.149103	204.7736
Suggested Estimator $\bar{y}_{st11}^{MJ}$	**0.8508302**	**517.2355**
Suggested Estimator $\bar{y}_{st12}^{MJ}$	**0.8646045**	**508.9953**
Suggested Estimator $\bar{y}_{st13}^{MJ}$	**0.8871836**	**496.0412**

Table 3: MSE and PRE of Some Existing and Suggested Estimators Using Population I

Estimator	$y_{hi} = \alpha_h x_{hi}^3 + \xi_{hi}$	
	MSE	**PRE**
$\bar{y}_{st}$	556.1768	100
Combined ratio $\bar{y}_{st}^{RC}$	354.32	156.9702
Lata *et al*. (2017) $\bar{y}_{st11}^{LA}$	398.3078	139.6349
Lata *et al*. (2017) $\bar{y}_{st12}^{LA}$	412.5612	134.8107
Lata *et al*. (2017) $\bar{y}_{st13}^{LA}$	431.4772	128.9006
Suggested Estimator $\bar{y}_{st11}^{MJ}$	**271.1696**	**205.1029**
Suggested Estimator $\bar{y}_{st12}^{MJ}$	**277.8351**	**200.1823**
Suggested Estimator $\bar{y}_{st13}^{MJ}$	**287.1083**	**193.7167**

Table 4: MSE and PRE of Some Existing and Suggested Estimators Using Population I

Estimator	$y_{hi} = \alpha_h x_{hi}^4 + \xi_{hi}$	
	MSE	**PRE**
$\bar{y}_{st}$	94055.7	100
Combined ratio $\bar{y}_{st}^{RC}$	71867.57	130.8736
Lata *et al*. (2017) $\bar{y}_{st11}^{LA}$	79068.55	118.9546
Lata *et al*. (2017) $\bar{y}_{st12}^{LA}$	81246.93	115.7652
Lata *et al*. (2017) $\bar{y}_{st13}^{LA}$	84021.24	111.9428
Suggested Estimator $\bar{y}_{st11}^{MJ}$	**63159.03**	**148.9188**
Suggested Estimator $\bar{y}_{st12}^{MJ}$	**64357.91**	**146.1447**
Suggested Estimator $\bar{y}_{st13}^{MJ}$	**65918.68**	**142.6844**

Table 5: MSE and PRE of Some Existing and Suggested Estimators Using Population II

Estimator	$y_{hi} = \alpha_h x_{hi}^2 + \xi_{hi}$	
	MSE	**PRE**
$\bar{y}_{st}$	0.7243471	100
Combined ratio $\bar{y}_{st}^{RC}$	0.3669404	197.4018
Lata *et al*. (2017) $\bar{y}_{st11}^{LA}$	0.4105255	176.4439
Lata *et al*. (2017) $\bar{y}_{st12}^{LA}$	0.4556056	158.9855
Lata *et al*. (2017) $\bar{y}_{st13}^{LA}$	0.4856501	149.15
Suggested Estimator $\bar{y}_{st11}^{MJ}$	**0.2231978**	**324.5314**
Suggested Estimator $\bar{y}_{st12}^{MJ}$	**0.239531**	**302.4022**
Suggested Estimator $\bar{y}_{st13}^{MJ}$	**0.2519612**	**287.4836**

Table 6: MSE and PRE of Some Existing and Suggested Estimators Using Population II

Estimator	$y_{hi}=\alpha_h x_{hi}^3+\xi_{hi}$	
	MSE	**PRE**
$\bar{y}_{st}$	37.77247	100
Combined ratio $\bar{y}_{st}^{RC}$	26.38607	143.1531
Lata *et al*. (2017) $\bar{y}_{st11}^{LA}$	29.33956	128.7424
Lata *et al*. (2017) $\bar{y}_{st12}^{LA}$	31.85461	118.5777
Lata *et al*. (2017) $\bar{y}_{st13}^{LA}$	33.45616	112.9014
Suggested Estimator $\bar{y}_{st11}^{MJ}$	**20.8789**	**180.9122**
Suggested Estimator $\bar{y}_{st12}^{MJ}$	**22.42852**	**168.4127**
Suggested Estimator $\bar{y}_{st13}^{MJ}$	**23.44996**	**161.0769**

Table 7: MSE and PRE of Some Existing and Suggested Estimators Using Population II

Estimator	$y_{hi}=\alpha_h x_{hi}^4+\xi_{hi}$	
	MSE	**PRE**
$\bar{y}_{st}$	2601.545	100
Combined ratio $\bar{y}_{st}^{RC}$	2093.923	124.2426
Lata *et al*. (2017) $\bar{y}_{st11}^{LA}$	2274.021	114.4028
Lata *et al*. (2017) $\bar{y}_{st12}^{LA}$	2408	108.0376
Lata *et al*. (2017) $\bar{y}_{st13}^{LA}$	2490.562	104.4562
Suggested Estimator $\bar{y}_{st11}^{MJ}$	**1863.71**	**139.5896**
Suggested Estimator $\bar{y}_{st12}^{MJ}$	**1955.269**	**133.053**
Suggested Estimator $\bar{y}_{st13}^{MJ}$	**2012.79**	**129.2507**

Table 8: MSE and PRE of Some Existing and Suggested Estimators Using Population III

Estimator	$y_{hi}=\alpha_h x_{hi}^2+\xi_{hi}$	
	MSE	**PRE**
$\bar{y}_{st}$	3.505272	100
Combined ratio $\bar{y}_{st}^{RC}$	1.267897	276.4634
Lata *et al*. (2017) $\bar{y}_{st11}^{LA}$	1.404499	249.5745
Lata *et al*. (2017) $\bar{y}_{st12}^{LA}$	1.596074	219.6185
Lata *et al*. (2017) $\bar{y}_{st13}^{LA}$	1.674268	209.3615
Suggested Estimator $\bar{y}_{st11}^{MJ}$	**0.636964**	**550.3093**
Suggested Estimator $\bar{y}_{st12}^{MJ}$	**0.6503848**	**538.9537**
Suggested Estimator $\bar{y}_{st13}^{MJ}$	**0.6660098**	**526.3094**

Table 9: MSE and PRE of Some Existing and Suggested Estimators Using Population III

Estimator	$y_{hi} = \alpha_h x_{hi}^3 + \xi_{hi}$	
	MSE	**PRE**
$\bar{y}_{st}$	342.7701	100
Combined ratio $\bar{y}_{st}^{RC}$	210.0028	163.2217
Lata *et al*. (2017) $\bar{y}_{st11}^{LA}$	225.9159	151.7246
Lata *et al*. (2017) $\bar{y}_{st12}^{LA}$	245.4806	139.6323
Lata *et al*. (2017) $\bar{y}_{st13}^{LA}$	253.9479	134.9765
Suggested Estimator $\bar{y}_{st11}^{MJ}$	**151.2407**	**226.6388**
Suggested Estimator $\bar{y}_{st12}^{MJ}$	**159.5417**	**214.8467**
Suggested Estimator $\bar{y}_{st13}^{MJ}$	**163.5477**	**209.5842**

Table 10: MSE and PRE of Some Existing and Suggested Estimators Using Population III

Estimator	$y_{hi} = \alpha_h x_{hi}^4 + \xi_{hi}$	
	MSE	**PRE**
$\bar{y}_{st}$	44896.74	100
Combined ratio $\bar{y}_{st}^{RC}$	34022.95	131.9602
Lata *et al*. (2017) $\bar{y}_{st11}^{LA}$	35868.74	125.1695
Lata *et al*. (2017) $\bar{y}_{st12}^{LA}$	37950.3	118.304
Lata *et al*. (2017) $\bar{y}_{st13}^{LA}$	38854.48	115.551
Suggested Estimator $\bar{y}_{st11}^{MJ}$	**28877.71**	**155.4719**
Suggested Estimator $\bar{y}_{st12}^{MJ}$	**29974.4**	**149.7836**
Suggested Estimator $\bar{y}_{st13}^{MJ}$	**30470.31**	**147.3459**

RESULTS AND DISCUSSION

Tables 2 – 10 show the Mean Square Error (MSE) and Percentage Relative Efficiency (PRE) of the suggested combined calibration estimators and other estimators using simulated data. The results revealed that the suggested combined calibration estimators have minimum MSE compared to the traditional estimators and other estimators considered under stratified random sampling.

CONCLUSION

The new combined stratified calibration estimators for estimating population mean is suggested with a set of constraint which minimized a given chi-squared distance measure. New calibration weights are developed. The simulation analysis reveals that the suggested calibration estimators outperform other estimators in the study (with lower MSE and higher PRE). As a result, we conclude that the suggested calibration estimators are more efficient than the traditional estimator, combined ratio estimator, and Lata *et al*. (2017) estimators in estimating the population mean of the study variable.

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