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BOUNDARY LAYER ANALYSIS FLOW PAST A POROUS ROTATING DISK WITH DUFOUR AND SORET EFFECTS

*Abdulhakeem Yusuf., Abdulrahman Bima, Nasiru Omeiza Salihu

Department of Mathematics, Federal University of Technology, Minna, Niger State, Nigeria

*Corresponding authors' email: yusuf.abdulhakeem@futminna.edu.ng

ABSTRACT

Analysis of a boundary layer flow past a porous rotating disk with Dufour and Soret effects are carried out mathematically with the formulated problem presented in its rectangular form. These problems correspond to the continuity, momentum, energy and concentration equations. The formulated equations (PDE) are contracted to a reduced nonlinear coupled equation (ODE) via some variables. The ODEs were solve using the decomposition method and the results obtained agreed with those in the literatures. The physical quantities that occur are presented and varied graphically on the fluid radial, tangential, axial velocity, temperature and concentration profiles. It seens that both ferro hydro dynamic (FHD) interactive parameter and rotational parameter enhances the radial, axial, temperature and concentration while they lead to decrease in tangential velocity of the fluid.

Keywords: Adomian decomposition, Boundary layer, Dufour effect, Ferro hydro dynamics (FHD), Soret effect, Rotating disk

INTRODUCTION

The importance of the study of boundary layer flow in a rotating disk in fluid dynamics can never be over emphasize due to it wide application, practically and theoretically. The viscous forces are balance by the Coriolis forces when the fluid rotate in the disk as against the inertia forces. Seeing from the rotational reference frame, the forces are defined as apparent deflection of an object in motion.

Omokhuale and Jabaka (2022) investigated the effects of heat sink and radiation on unsteady hydromagnetic convective Couette flow of a viscous, electrically conducting and incompressible fluid. The formulated problems were solved numerically and the fluid momentum were found to increase with radiation. Considering heat source and viscosity effects, flow of unsteady viscous fluid via horizontal linear stretching plate was carried out by Sharma (2012). The description of the impact of viscous dissipation and Brownian motion on slip flow over curved surface with Jefferey nanomaterial was depicted recently by Khan *et al.* (2021). The significant of system of rotating disk with heat transfer can never be over emphasize in engineering applications such as in computer disk drive and gas turbine cooling (Owen et al. 1989; Herrero et al. 1994). Devi and Ganga (2009), considered heat transfer MHD flow over linear stretching sheet with viscous dissipation effects. The analysis of the implication of viscous dissipation over a radiative heat transfer was carried out by Chen (2010). Turkyilmazoglu (2014) discussed Joule heating effects and viscosity on MHD flow over a moving that is radially stretched with time dependent variable. Maleque (2010) studied time-dependent electrically conducting fluid flow in a rotating disk with variable viscousity. Al-Hadhrami *et al.* (2003) developed a model in a porous media with viscous dissipation that is adequate for practical reasons.

Several Researchers such as Suleiman and Yusuf (2020), Yusuf *et al.* (2021a), Yusuf *et al.* (2021b), Bolarin *et al.* (2019), Yusuf *et al.* (2019) have all demonstrated in their work the reliability of the method of Adomian in solving boundary layer problems.

The analysis of a boundary layer flow past a porous rotating disk with Dufour and Soret effects using the Adomian decomposition method is a new advancement that will further improve the literature.

PROBLEM FORMULATION

Considering a flow of a laminar flow fluid, steady case, in a moving circular disk with concentration $c = c_w$ and $c \to c_\infty$ at $z = 0$ and $z \to \infty$ respectively. Following the formulation of Sharma et al. (2021) by incorporating thermo-diffusion and diffusion-thermo. The model formulation is given as:

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{\mu_0}{\rho}|M|\frac{\partial}{\partial r}|H|+v\left[\frac{\partial^2 u}{\partial r^2}+\frac{\partial}{\partial r}\left(\frac{u}{r}\right)+\frac{\partial^2 u}{\partial z^2}\right]+2\Omega v-\frac{\mu_\infty}{\rho K}u=u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r} \quad (2)$$

$$v\left[\frac{\partial^2 v}{\partial r^2}+\frac{\partial}{\partial r}\left(\frac{v}{r}\right)+\frac{\partial^2 v}{\partial z^2}\right]-2\Omega u-\frac{\mu_\infty}{\rho K}v=u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}+\frac{uv}{r} \quad (3)$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{\mu_0}{\rho}|M|\frac{\partial}{\partial z}|H|+v\left[\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right]-\frac{\mu_\infty}{\rho K}w=u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z} \quad (4)$$

$$\rho c_p\left(u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}\right)=k\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right)+\mu\left[\left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2\right]+\frac{D_m K_T}{c_p}\frac{\partial^2 c}{\partial z^2} \quad (5)$$

$$u\frac{\partial c}{\partial r}+w\frac{\partial c}{\partial z}=D_B\frac{\partial^2 c}{\partial z^2}+\frac{D_m K_T}{T_m}\frac{\partial^2 T}{\partial z^2} \quad (6)$$

$$\begin{aligned}&(u,v,w)=\left(0,r\Omega,\frac{\upsilon}{\delta}\right),\ \ p=p_0,\ \ \mathrm{T}=T_w,\ c=c_w\ \text{ as } z=0\\&(u,v,w)\to(0,0,0),\ \ p\to p_\infty,\ \ T\to T_\infty,\ c\to c_\infty\ \text{ as } z\to\infty\end{aligned} \quad (7)$$

Where u,v,w are velocity component, density stand for ρ, fluid pressure is p, M stands for magnetism, H stands for magnetic field, μ viscousity, K permeability (Darcy), c_p heat capacity, T fluid temperature, k thermal conductivity, Ω angular velocity, T_M mean fluid temperature, K_T diffusion ratio, D_M mean diffusion.
The similarity transformation is given as:

$$\left.\begin{aligned}&u=r\Omega\frac{\partial U}{\partial\eta},\ \ v=r\Omega V,\ \ w=\frac{\upsilon}{\delta}W,\ \ \ p=\frac{\rho\upsilon^2}{\delta^2}P\\&T-T_\infty=(T_w-T_\infty)\theta,\ \ \mathrm{c}-c_\infty=(c_w-c_\infty)\varphi,\ \ \eta=\frac{z}{\delta}\end{aligned}\right\} \quad (8)$$

According to Sharma (2021) $\frac{1}{\rho}\frac{\partial p}{\partial r}=r\Omega^2,\ \ |H|=\frac{m}{2\pi r^2},\ \ |M|=k_1(T_c-T)$

Where $\eta,U,V,W,\mathrm{P},\theta$ and φ corresponds to fluid dimensionless distance, dimensionless radial, tangential, and axial velocity, fluid Pressure, temperature and fluid concentration respectively. δ stands for scalar factor, k_1 is pyro magnetic coefficient, T_c stands for curie temperature.
Introducing the similarity transformation in (8) and the linearized equation above into equation (1) to (7), the equations simplify to nonlinear coupled ordinary differential below:

$$\left.\begin{aligned}
&\frac{\partial^3 U}{\partial \eta^3}-R\left\{\left(\frac{\partial U}{\partial \eta}\right)^2-V^2-2V+1-2U\frac{\partial^2 U}{\partial \eta^2}+\beta\frac{\partial U}{\partial \eta}+\frac{2B}{\mathrm{Re}^2}\right\}=0\\
&\frac{\partial^2 V}{\partial \eta^2}-2R\left\{\left(\frac{\partial U}{\partial \eta}V-U\frac{\partial V}{\partial \eta}\right)-\frac{\partial U}{\partial \eta}+\frac{\beta}{2}V\right\}=0\\
&\frac{\partial^2 W}{\partial \eta^2}-W\frac{\partial W}{\partial \eta}-R\beta W-\frac{\partial P}{\partial \eta}=0\\
&\frac{\partial^2 \theta}{\partial \eta^2}+2\Pr RU\frac{\partial \theta}{\partial \eta}+\Pr Ec\left[\left(\frac{\partial^2 U}{\partial \eta^2}\right)^2+\left(\frac{\partial V}{\partial \eta}\right)^2\right]+Du\frac{\partial^2 \varphi}{\partial \eta^2}=0\\
&\frac{\partial^2 \varphi}{\partial \eta^2}+2RLeU\frac{\partial \varphi}{\partial \eta}+LeSr\frac{\partial^2 \theta}{\partial \eta^2}=0\\
&\left.\frac{\partial U}{\partial \eta}\right|_{\eta=0}=0,\ V(0)=1,\ \mathrm{W}(0)=1,\ \theta(0)=1,\ \ \varphi(0)=1,\ \ \eta=0\\
&\left.\frac{\partial U}{\partial \eta}\right|_{\eta=\infty}=V(\infty)=W(\infty)=\theta(\infty)=\varphi(\infty)=0, \qquad \eta\to\infty
\end{aligned}\right\} \tag{9}$$

Where $R=\frac{\Omega\delta^2}{\upsilon}$, $\beta=\frac{\upsilon\Omega}{K}$, $\mathrm{Re}=\frac{\Omega r^2}{\upsilon}$, $B=\frac{m\mu_\infty k_1\left(T_c-T\right)\rho}{2\pi\mu_\infty^2}$, $\Pr=\frac{\upsilon\rho c_p}{k}$, $Ec=\frac{r^2\Omega^2}{\left(T_w-T_\infty\right)c_p}$,

$Du=\frac{D_m K_T\left(c_w-c_\infty\right)}{\upsilon c_p\left(T_w-T_\infty\right)}$, $Le=\frac{\upsilon}{D_B}$, $Sr=\frac{D_m K_T\left(T_w-T_\infty\right)}{T_m\left(c_w-c_\infty\right)\upsilon}$.

Are rotational parameter, Darcy number, Renolds number, FHD interactive parameter, Prandtl number, Eckert number, Dufour number, Lewis number, and Soret number respectively.

METHODOLOGY

In order to solve the problem in equation (9), the method of Decomposition was introduced by introducing the following ADM polynomials in to (9):

$$\left.\begin{aligned}
&\sum_{n=0}^{\infty}U_n=L_1^{-1}\left[R\left\{\sum_{n=0}^{\infty}A_n-\sum_{n=0}^{\infty}B_n-2V+1-2\sum_{n=0}^{\infty}C_n+\beta\frac{\partial U}{\partial \eta}\right\}\right]\\
&\sum_{n=0}^{\infty}V_n=L_2^{-1}\left[2R\left\{\left(\sum_{n=0}^{\infty}D_n-\sum_{n=0}^{\infty}E_n\right)-\frac{\partial U}{\partial \eta}+\frac{\beta}{2}V\right\}\right]\\
&\sum_{n=0}^{\infty}W_n=L_2^{-1}\left[\sum_{n=0}^{\infty}F_n+R\beta W\right]\\
&\sum_{n=0}^{\infty}\theta_n=L_2^{-1}\left[-2\Pr R\sum_{n=0}^{\infty}G_n-\Pr Ec\left[\sum_{n=0}^{\infty}H_n+\sum_{n=0}^{\infty}I_n\right]-Du\frac{\partial^2 \varphi}{\partial \eta^2}\right]\\
&\sum_{n=0}^{\infty}\varphi_n=L_2^{-1}\left[-2RLe\sum_{n=0}^{\infty}J_n-LeSr\frac{\partial^2 \theta}{\partial \eta^2}\right]
\end{aligned}\right\} \tag{10}$$

where

$$\left.\begin{aligned} &A_n=\sum_{k=0}^{n}U_{n-k}U_k, B_n=\sum_{k=0}^{n}V_{n-k}V_k, C_n=\sum_{k=0}^{n}U^{//}_{n-k}U_k, D_n=\sum_{k=0}^{n}U^{/}_{n-k}V_k, E_n=\sum_{k=0}^{n}V^{/}_{n-k}U_k, F_n=\sum_{k=0}^{n}W^{/}_{n-k}W_k,\\ &G_n=\sum_{k=0}^{n}U^{/}_{n-k}\theta^{/}_k, \mathrm{H}_n=\sum_{k=0}^{n}U^{/}_{n-k}U^{/}_k, \mathrm{I}_n=\sum_{k=0}^{n}V^{/}_{n-k}V^{/}_k, J_n=\sum_{k=0}^{n}\varphi^{/}_{n-k}U_k \end{aligned}\right\} \quad (11)$$

Are the polynomials.
Decomposing equation (11) to:

$$\left.\begin{aligned} &U_n=L_1^{-1}\left[R\left\{\sum_{k=0}^{n}U_{n-k}U_k-\sum_{k=0}^{n}V_{n-k}V_k-2V+1-2\sum_{k=0}^{n}U^{//}_{n-k}U_k+\beta\frac{\partial U}{\partial\eta}\right\}\right]\\ &V_n=L_2^{-1}\left[2R\left\{\left(\sum_{k=0}^{n}U^{/}_{n-k}V_k-\sum_{k=0}^{n}V^{/}_{n-k}U_k\right)-\frac{\partial U}{\partial\eta}+\frac{\beta}{2}V\right\}\right]\\ &W_n=L_2^{-1}\left[\sum_{k=0}^{n}W^{/}_{n-k}W_k+R\beta W\right]\\ &\theta_n=L_2^{-1}\left[-2\Pr R\sum_{k=0}^{n}U^{/}_{n-k}\theta^{/}_k-\Pr Ec\left[\sum_{k=0}^{n}U^{/}_{n-k}U^{/}_k+\sum_{k=0}^{n}V^{/}_{n-k}V^{/}_k\right]-Du\frac{\partial^2\varphi}{\partial\eta^2}\right]\\ &\varphi_n=L_2^{-1}\left[-2RLe\sum_{k=0}^{n}\varphi^{/}_{n-k}U_k-LeSr\frac{\partial^2\theta}{\partial\eta^2}\right] \end{aligned}\right\}$$

(12)

Maple 17 software was used to carry out the integrals in equation (12) and the initial guesses are assumed to be:

$$\left.\begin{aligned} &U_0(\eta)=\frac{a_1\eta^2}{2}-\iiint\left(1+\frac{2B}{\mathrm{Re}^2}\right)d\eta d\eta d\eta\\ &V_0(\eta)=1+a_2\eta\\ &W_0(\eta)=1+a_3\eta\\ &\theta_0(\eta)=1+a_4\eta\\ &\varphi_0(\eta)=1+a_5\eta \end{aligned}\right\} \quad (13)$$

The final solutions are given as

$$\left.\begin{aligned} &U(\eta)=\sum_{n=0}^{3}U_n\\ &V(\eta)=\sum_{n=0}^{3}V_n\\ &W(\eta)=\sum_{n=0}^{3}W_n\\ &\theta(\eta)=\sum_{n=0}^{3}\theta_n\\ &\varphi(\eta)=\sum_{n=0}^{3}\varphi_n \end{aligned}\right\} \quad (14)$$

RESULTS AND DISCUSSION

The nonlinear coupled ordinary differential equations with corresponding boundary conditions in equation (9) are highly nonlinear. ADM is employed to obtain the solution. The results obtained are compared with the existing literature and a good agreement are observed as seen in Table 1 below.

Figures 1 to 5 present the variation of FHD interactive parameter on the velocity, temperature and concentration profiles respectively. It was observed that increase in the parameter leads to thicknesses in the radial and axial velocity but force the tangential velocity to drop. The parameter also has influence on the fluid temperature and slightly on the concentration profiles.

Figures 6 to 10 is the graphical presentation of the effects of rotational parameter on radial, tangential, axial velocity, temperature and concentration profiles. The rotational parameter is an increasing agent of radial and axial velocity and a reduction agent of the tangential velocity. It also boosts the fluid temperature and concentration.

Table 1: Comparison of $U_{\eta\eta}(0)$ with β when $B = R = \mathrm{Re} = 1$

β	Sharma eta al.(2021)	resent results
0.5	-0.605825	-0.630321599
1.5	-0.59867	-0.597171257
2	-0.592452	-0.585886645
3	-0.577199	-0.56640511

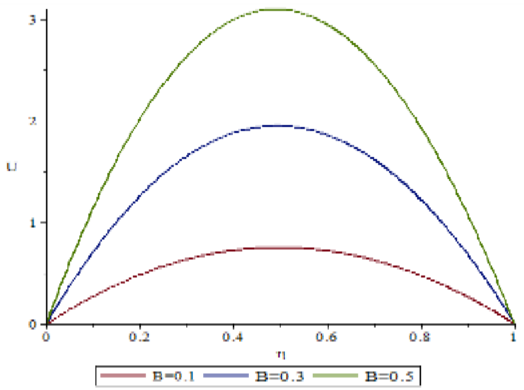

Figure 1: Variation of B on U

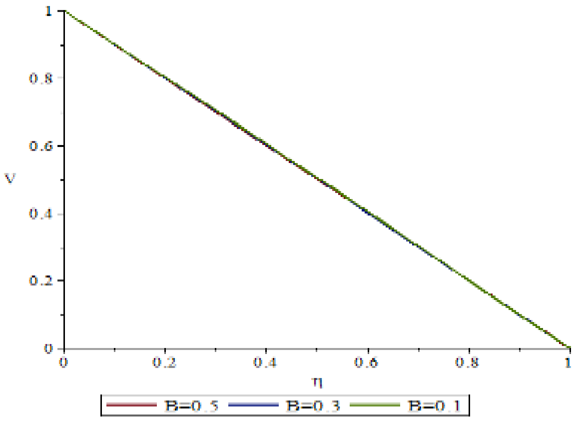

Figure 2: Variation of B on V

Figures 11 to 15 depicts the influence of Darcy number on the fluid velocity Temperature and concentration profiles respectively. As the Darcy number thickens, radial velocity also thickens while the tangential and axial velocity thin. This causes gain in momentum in the radial velocity and loss in momentum in the case of tangential and axial velocity which is as a result of Kevin force from the magnetic field. The fluid temperature and concentration are slightly enhanced.

Figures 16 to 18 displayed the variation of Renolds number on the radial, tangential and axial velocity respectively. The Renolds number causes the radial velocity to drop while tangential and axial velocity are enhanced as the Renolds number increases.

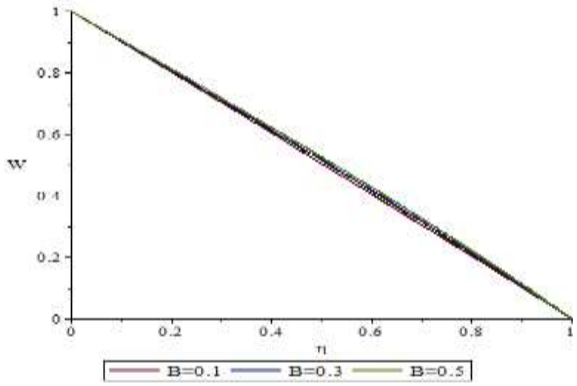

Figure 3: Variation of B on W

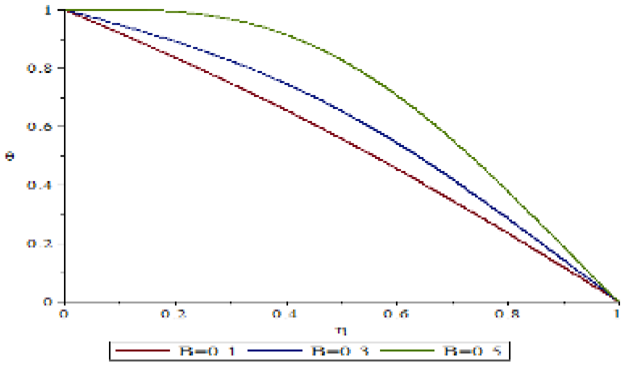

Figure 4: Variation of B on θ

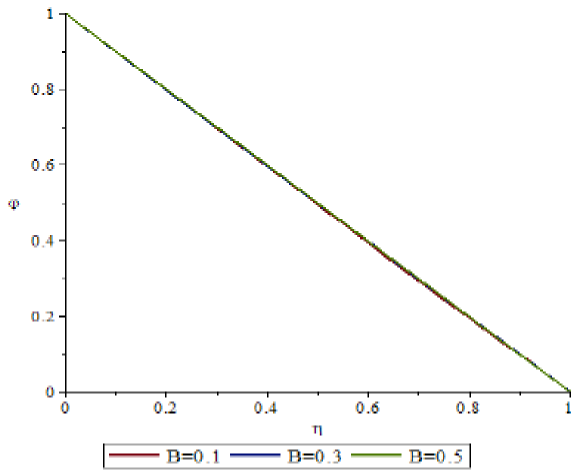

Figure 5: Variation of B on θ

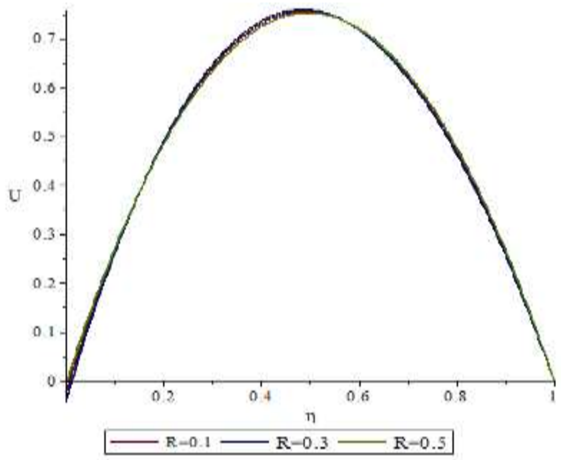

Figure 6: Variation of R on U

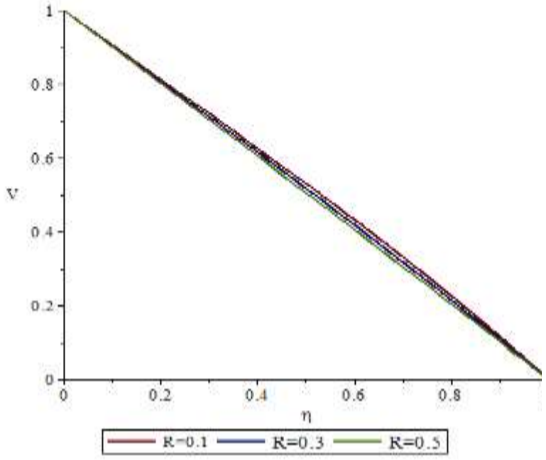

Figure 7: Variation of R on V

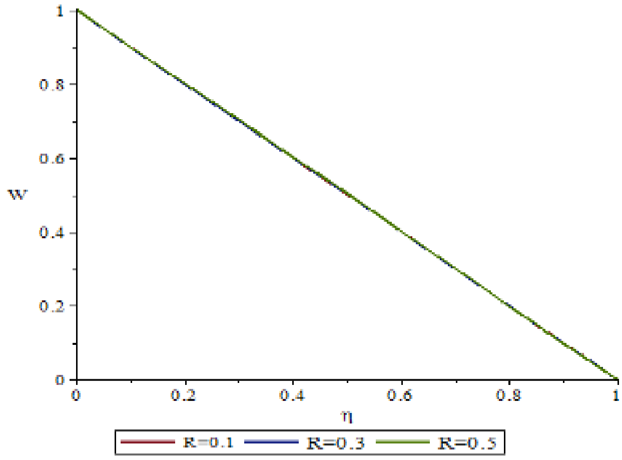

Figure 8: Variation of R on W

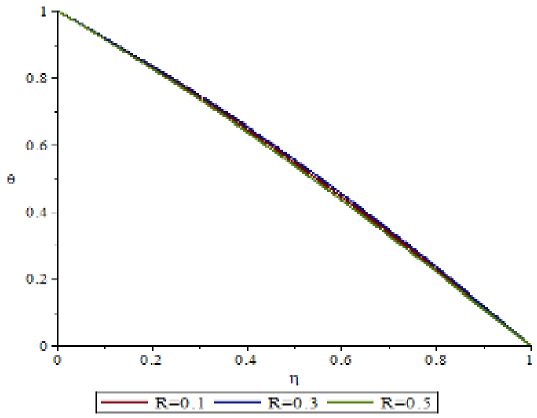

Figure 9: Variation of R on θ

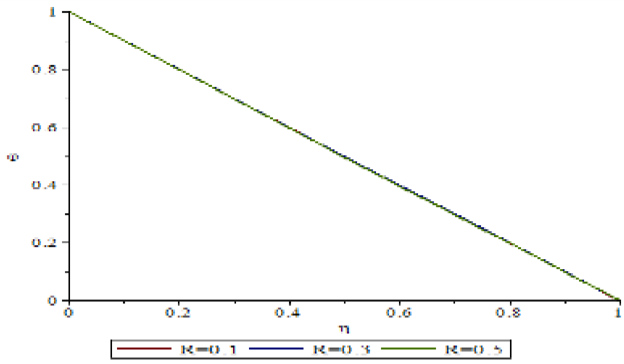

Figure 10: Variation of R on φ

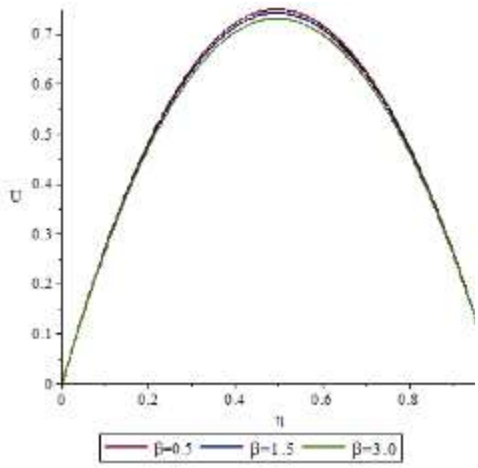

Figure 11: Variation of β on U

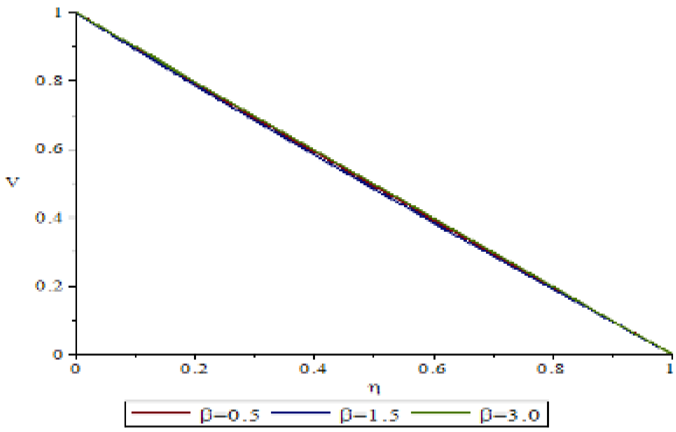

Figure 12: Variation of β on V

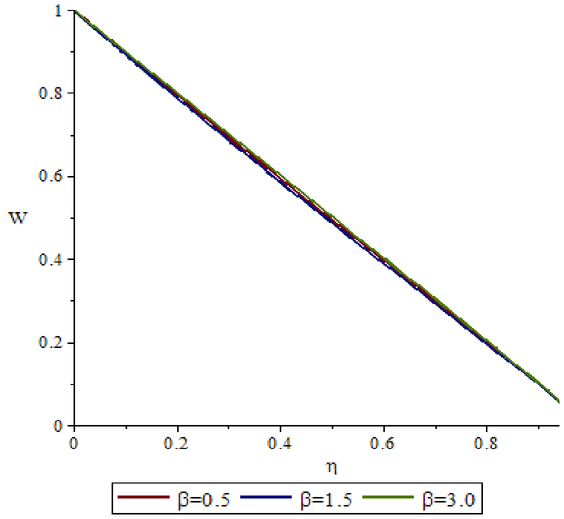

Figure 13: Variation of β on W

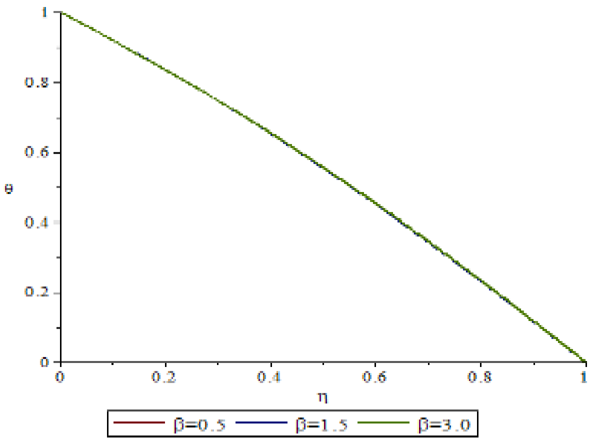

Figure 14: Variation of β on θ

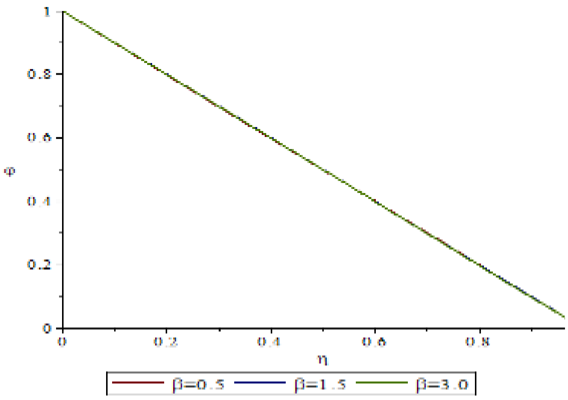

Figure 15: Variation of β on φ

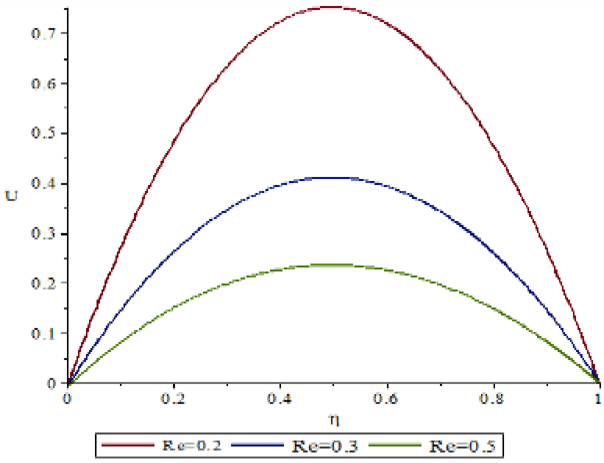

Figure 16: Variation of Re on U

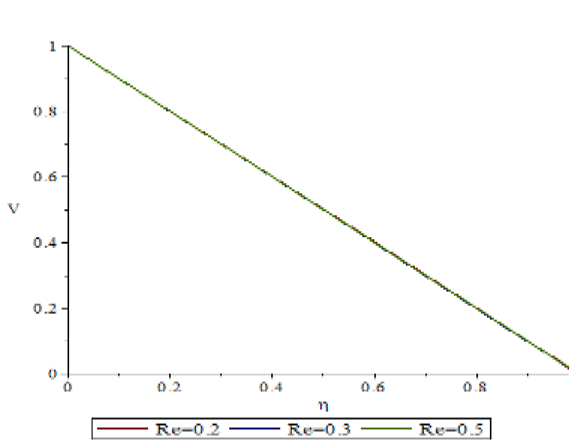

Figure 17: Variation of Re on V

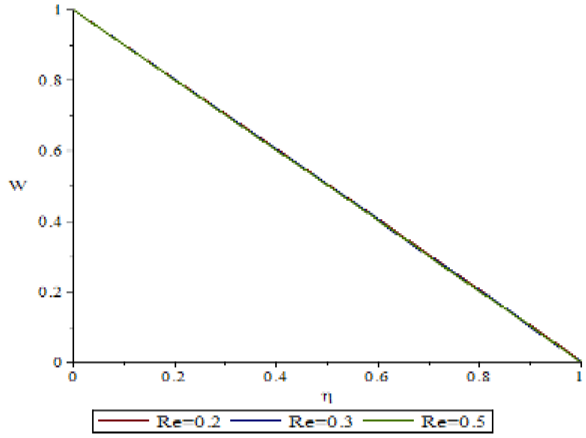

Figure 18: Variation of Re on W

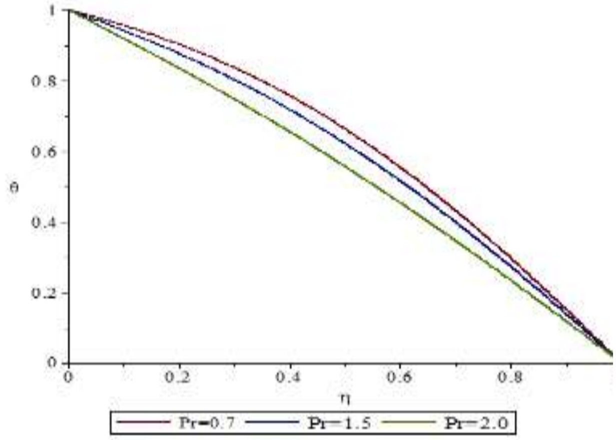

Figure 19: Variation of Pr on θ

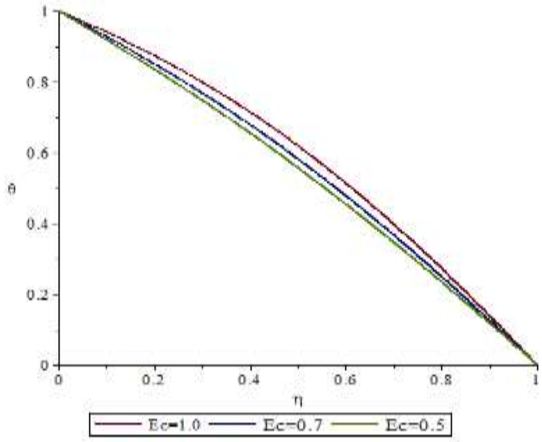

Figure 20: Variation of Ec on θ

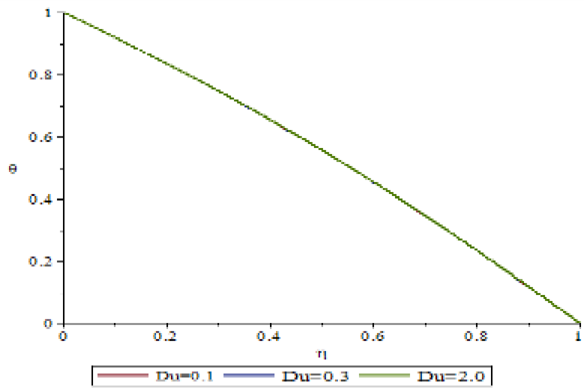

Figure 21: Variation of Du on θ

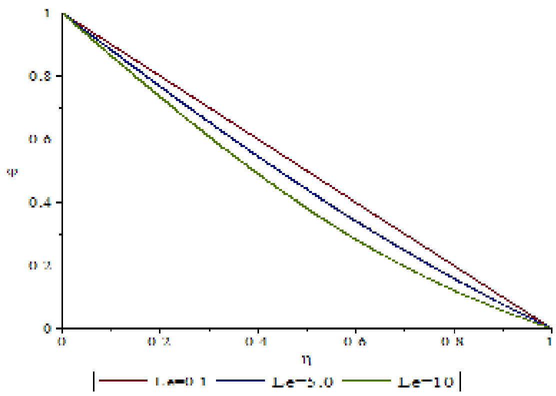

Figure 22: Variation of Le on φ

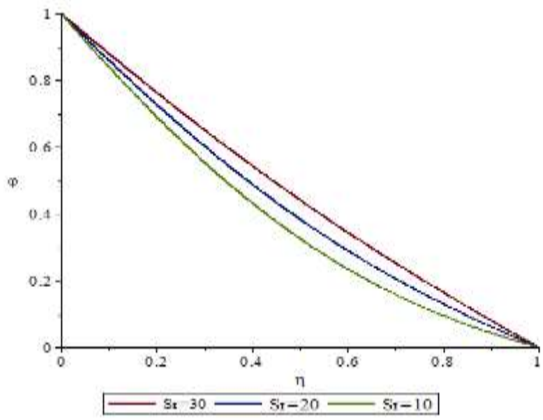

Figure 23: Variation of Sr on φ

Figures 19 to 20 shows the effect of Prandtl number and Eckert number on the fluid temperature respectively. As Prandtl number increases, the fluid temperature is observed to lose momentum which result to reduction in temperature of the fluid. Eckert number is found to be an increasing agent of the fluid temperature which is due to viscousity. When temperature rises, the fluid viscousity also rise.
Figure 21 shows the variation of Dufour number on the temperature profile and it is seen that the fluid temperature is enhance with Dufour number.
Figure 22 display the effect of Lewis number on the fluid concentration and it is observed that increase in Lewis number leads to decrease in the fluid concentration.
Figure 23 is the variation of Soret number on the fluid concentration profile. Soret number is found to be an increasing agent of the concentration profile.

CONCLUSION

This present work extended the work of Sharma et al. (2021) by introducing an additional model to study the fluid concentration and also introduced the thermo-diffusion term (Dufour) and diffusion-thermo (Soret) to the flow model. The formulated problems were solved using decomposition method and the results obtained were compared with literature as presented in Table 1. These results show an agreement between the present work and the literature. The FHD interactive parameter and rotational parameter have same effects on the fluid velocity, temperature and concentration profiles. The Dufour number slightly increase the temperature profile while Soret number increases the fluid concentration significantly. The graphs presented in this work all satisfy both the initials and the boundary conditions which depict that the problem is well poss. All other parameters were kept constant while a parameter is varied.in conclusion, the radial, tangential, axial velocity, temperature and concentration all decayed to zero at free stream.

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