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# ON THE HYBRIDIZED LINEAR-RATIO ESTIMATOR FOR SUCCESSIVE SAMPLING ON TWO OCCASIONS

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## ABSTRACT

Information on rate of changes in longitudinal surveys with certain types of population can robustly considered only by Successive sampling in estimate population parameters. Therefore, this study proposed a more efficient estimator technique in successive sampling schemes to determine the population estimates of the two occasions. The proposed estimator was obtained over mathematical expectation and statistical assumptions to derived an unbiased estimate of mean (μ), minimum variance ($\sigma^2$) and Relative efficiency comparison (REC). A Real-life data were used to validate the expression in the study. The data were last two population census conducted in Nigeria in 1991 and 2006 respectively by National Population Commission (NPC). The findings of the study include a proposed estimator obtained by hybridization of linear and ratio estimator, mathematical estimations of the mean in first and second occasions, minimum variance with maximum precision and relative gain in efficiency of the four estimators; ESEst, EREst, ELEst and PHLREst derived; estimates of change and sum of the population parameters for the two occasions were derived and the estimates of change achieved minimum variance and maximum efficiency when correlation coefficient ρ = 1.0 and proportion matched λ = 0.2. The study concluded that the proposed hybridized linear-ratio estimator is more efficient than the existing ones in term of precision. Moreover, these theoretical findings are certified by numerical illustration with the real-life data. Therefore, the proposed estimator is recommended for use in successive sampling scheme.

**Keywords**: Successive Sampling, Estimators, Hybridization, Population census

## INTRODUCTION

Successive sampling is used extensively in applied sciences, sociology and economic researches. Many surveys these days are repetitive in character. Government agencies like the National Bureau of Statistics and other research-based institutes collect information regularly on the same population to estimate some population parameters for current occasion. When same population is sampled repeatedly, it is said that a first sample has been taken (on one occasion) from a population of N units and a second sample is to be taken (on another occasion) on the same population, there is thus an opportunity of making use of the information contained in the first sample. Therefore, problem is how best to learn from past experience and use it for improving precision of future estimates. Estimates can be made not only for the existing time period (current estimates) but also of the change that has taken place since the previous occasion and of the average over a given period. One characteristics growth of theoretical statistics is the emergence of a large body of the theory which discusses how to make good estimate from data. In the development of theory specifically for sample surveys, little use has been made of this knowledge. According to Okafor C.F. (2002), there are two principal reasons. Firstly, a survey that contains a large number of items has a greater advantage in estimation procedure that requires little more than simple addition, whereas the superior methods of estimation in statistical theory, such as maximum likelihood, may necessitate a series of successive approximations before the estimate can be found. Secondly, most of the estimation methods in theoretical statistics take for granted that we know the functional form of the frequency distribution followed by the data in the sample, and the method of estimation is carefully geared to this type of distribution

This study aims is to use the Propose Ratio Estimator Techniques in the context of devising efficient estimator strategies in Successive Sampling Schemes. Deliberate attempt has been made to (i) derive the unbiased estimator of mean ($\mu$) on two occasions using Existing Simple Estimator ($ESE_{st}$), Existing Linear Estimator ($ELE_{st}$) and Proposed Ratio Estimator ($PRATE_{st}$), (ii) determine the estimate of Change and Sum in two occasions using Existing Simple Estimator ($ESE_{st}$), Existing Linear Estimator ($ELE_{st}$) and Proposed Ratio Estimator ($PRATE_{st}$), (iii) compare the performance of the Existing Simple Estimator ($ESE_{st}$), Existing Linear Estimator ($ELE_{st}$) over Proposed Ratio Estimator ($PRATE_{st}$) with respect to their precisions and to (iv) investigate the effect(s) of the Correlation coefficient ($\rho$), Proportion matched ($\lambda$) and Proportion unmatched ($\theta$) samples on the estimates and the estimators in (ii) above.

## SCOPE OF THE STUDY

As earlier stated in this research, several estimator existing in literature has been suggested and use in the area of successive sampling over two occasion, while more works are still ongoing in attempting to improve precision in order to further widen its applicability. The proposed Ratio estimator techniques includes both theoretical model and empirical studies have been carried out to justify the proposition of estimator. The work also focuses on study variables and auxillary variable on both occasions for improving the precision of estimate for previous and current population mean and variance in two occasion successive sampling.

## REVIEW OF RELATED LITERATURE

The problem of sampling on Two Successive Occasions with a partial replacement of sampling units was first considered by Jessen (1942) in a survey of farm data. Sen (1961) also applied this sampling plan with success in designing a mail survey in Ontario of waterfowl hunter who hunted successively during (1967 - 1968) and (1968 - 1969). In the study, an estimate was developed for the current season (1968 – 1969), based on the relationship between the value of characteristics during the current season and its value during the previous season, that yielded more precise estimates of the kill of waterfowl than the usual estimates based on simple random sampling and using the hunter's current season's performance only. In a similar study, Singh and Srivastara (1974) presented on different level of repeated sampling and established that for one level of repeated sampling, only sample values that have been drawn from the population of current time can be added to the sample pattern. And for higher levels, both the earlier sample values and current values can be added. Singh and Srivastara (1974) also developed a repeated sampling method to obtain a minimum variance estimate of population values (mean and total) by suitably constructing a linear function of sample at different times.

However, the use of ratio method of estimation in successive sampling was first suggested by Avdhani (1968) and later Sen et al. (1975). Gupta (1979) suggested the use of product method and later it was embraced by Artes et al. (1988), Artes and Garcia (2001). In his contributions, Okafor (1992) gave some estimators of the population ratio when sampling is done with partial replacement of units. In, this case, the estimate of the population total of the character $y_1$ on the recent occasion is first obtained by a suitable combination of two independent estimates of the population totals from the matched and unmatched sample. The estimate of the population total of $y_2$ is similarly obtained. These two estimates of the population totals of $y_1$ and $y_2$ are then used to derive the estimate of the population ratio. The approach that was used for estimation of current population mean, further extended to develop general theory of estimation in repeated surveys for the current population variance. For the first time in successive sampling, Sud et al. (2001) considered the problem of estimating the population variance on current occasion. Azam et al. (2010) proposed an estimator by using a linear combination of available sample variances for estimating the current population variance based on matched and unmatched portions of the samples at both occasions and most recently, Singh et al. (2011) suggested a class of estimators for estimation of finite population variance on current occasion. Sodipo et al. (2013) use unistage sampling over two occasion using SRSWOR and regression estimator was applied in obtaining current estimates with one auxiliary variable.

In addition to the numerous successes that has been achieved lately, towards improving the accuracy, efficiency and precision of estimate of parameter, and the fact that Information on rate of changes in longitudinal surveys with certain types of population can robustly be considered only by Successive sampling in estimating population parameters. This study proposed a more efficient estimator technique in successive sampling schemes to determine the population estimates of the two occasions.

## DATA USED FOR THE RESEARCH

The data used was collected from National Population Commission (NPC) of the census conducted in Nigeria for year 1991 and 2006.

## METHODOLOGY

The subject of validity (agreement to acceptability) and reliability (dependable) are complex and controversial but of great importance in research methodology and any research depend strongly on the methods used at various stages of the design of the survey, collection of data, analysis and interpretation of the results.

This study focused on measure of precision, which is a function of the variance for the three estimators under the repetitive survey. It also, centered on checking precision on estimate and percentage gain in relative efficiency, over each occasion under study. It also discussed the steps taken at the various stages of the study which includes: Notation and meaning with derivation of methods under study.

### Notations and Meaning

*$N$ - is the population size*
*$n$ - is the sample size taken on the first occasion*
*$m$ - is the number of matched or retained units from the first occasion and used as part of second occasion*
*$u$ - is the number of unmatched or a fresh unit on the second occasion from the remaining unit of the population*
*$\rho$ - is the correlation coefficient between the matched units of x and y*
*$\lambda$ - is the proportion of matched or retained units*
*$\theta$ - is the proportion of unmatched or new units*
*$\sigma^2$- is the pooled variance of $S_x^2$ and $S_z^2$*
*$S_x^2$ - is the sample variance of units (x) on first occasion*
*$\bar{x}_{1u}$- is the sample mean of unmatched units of x from first occasion.*
*$\bar{x}_{1m}$ - is the sample mean of matched units of x from first occasion.*
*$\bar{y}_{2u}$ - is the sample mean of unmatched units of y from second occasion.*

$\bar{y}_{2m}$- *is the sample mean of matched units of y from second occasion.*
$\bar{z}_{1u}$ *is the sample mean of unmatched units of z from first occasion.*
$\bar{z}_{2m}$- *is the sample mean of matched units of z from second occasion.*
$\bar{z}_{2u}$ *- is the sample mean of unmatched units of z from second occasion.*
$\bar{z}_{1m}$ *- is the sample mean of matched units of z from first occasion.*
$\bar{X}$ *- is the population mean of x units.*
$\bar{Y}$ *- is the population mean of y units.*
$\bar{Z}$ *- is the population mean of z units.*
*1st Occasion - Population census report in 1991*
*2nd Occasion - Population census report in 2006*
$X_1$ *- Population of male in 1st occasion* ($M_1$)
$Y_1$ *- Population of female in 1st occasion* ($F_1$)
$Z_1$ *- No of household in 1st occasion* ($H_1$)
$X_2$ *- Population of male in 2nd occasion* ($M_2$)
$Y_2$ *- Population of female in 2nd occasion* ($F_2$)
$Z_2$ *- No of household in 2nd occasion* ($H_2$)
$T_1$ *- Population of both sexes in 1st occasion* ($B_1$)
$T_2$ *- Population of both sexes in 2nd occasion* ($B_2$)

## Derivation of Estimate, Variance and Relative Efficiency of the Sampling Scheme

This section discusses the derivation of the formula for estimate, variance and relative efficiency of the sampling scheme. Basically, we would be employing the method of Yates (1949), Patterson (1950), Das (1982), Chaturvedi, D.K. & Tripathi, T.P. (1983) as template

**Simple Estimator (SE$_{st}$)**

a) The unbiased Existing Simple Estimator of mean for first and second occasion respectively

i) $$\mu_{1SE_{st}} = \bar{\mathrm{X}} = \left(\frac{u\bar{x}_u + m\bar{x}_m}{n}\right) \tag{1}$$

ii) $$\mu_{2SE_{st}} = \bar{\mathrm{Y}} = \left(\frac{u\bar{y}_u + m\bar{y}_m}{n}\right) \tag{2}$$

b) i) The estimate of this change, $\hat{\Delta}_{(SE_{st})}$, is

$$\hat{\Delta}_{(SE_{st})} = \bar{\mathrm{Y}} - \bar{\mathrm{X}}$$

$$\hat{\Delta}_{(SE_{st})} = \lambda(\bar{y}_m - \bar{x}_m) - \theta(\bar{y}_u - \bar{x}_u) \tag{3}$$

ii) Variance of Estimate of change

$$\hat{\Delta}_{SE_{st}} = \lambda(\bar{y}_m - \bar{x}_m) - \theta(\bar{y}_u - \bar{x}_u)$$

$$V(\hat{\Delta}_{SE_{st}}) = 2(1 - \lambda\rho)\, {\sigma^2}/{n} \tag{4}$$

c) i) The Estimate of Sum

$$\hat{\Sigma}_{SE_{st}} = \bar{X} + \bar{Y}$$

$$\hat{\Sigma}_{SE_{st}} = \lambda(\bar{y}_m + \bar{x}_m) + \theta(\bar{y}_u + \bar{x}_u) \tag{5}$$

ii) Variance of the Sum

$$\hat{\Sigma}_{SE_{st}} = \lambda(\bar{y}_m + \bar{x}_m) + \theta(\bar{y}_u + \bar{x}_u)$$

$$v(\hat{\Sigma}_{SE_{st}}) = 2(1 + \lambda\rho)\, {\sigma^2}/{n} \tag{6}$$

Existing Linear Estimator (ELE$_{st}$)

a) The linear estimator for $\hat{\mu}_{1L}$ and $\hat{\mu}_{2L}$ can also be sought from the form.

i) $\hat{\mu}_{1LE_{st}} = b(\bar{y}_{2u} - \bar{y}_{2m}) + d\bar{x}_{1m} + (1 - d)\bar{x}_{1u}$, where *b* and *d* are constant

$$= \frac{1}{1-\rho^2\theta^2}\{\lambda\rho\theta(\bar{y}_{2u} - \bar{y}_{2m}) + \lambda\bar{x}_{1m} + \theta(1 - \rho^2\theta)\bar{x}_{1u}\} \tag{7}$$

i) $\hat{\mu}_{2LE_{st}} = a\left(\overline{x_{1u}} - \overline{x_{1m}}\right) + C\,\overline{y_{2m}} + (1 - C)\,\overline{y_{2u}}$, where *a* and *c* are constant

$$= \frac{1}{1-\rho^2\theta^2}\{\lambda\rho\theta(\bar{x}_{1u} - \bar{x}_{1m}) + \lambda\bar{y}_{2m} + \theta(1 - \rho^2\theta)\bar{y}_{2u}\} \tag{8}$$

b) i) The estimate of change

$$\hat{\Delta}_{LE_{st}} = \hat{\mu}_{2L} - \hat{\mu}_{1L}$$

$$= \frac{1}{(1-\rho\theta)}\lfloor\theta(1 - \rho)(\bar{y}_{2u} - \bar{x}_{1u}) + \lambda(\bar{y}_{2m} - \bar{x}_{1m})\rfloor \tag{9}$$

ii) Variance of Estimate of change

$$\mathrm{V}(\hat{\Delta}_{LE_{st}}) = \frac{2(1-\rho)}{(1-\rho\theta)}{\sigma^2}/{n} \tag{10}$$

c) i) The estimate of Sum

$$\hat{\Sigma}_{LE_{st}} = \hat{\mu}_{2L} + \hat{\mu}_{1L}$$

$$= \frac{1}{(1+\rho\theta)}\lfloor\lambda(\bar{x}_{1m} + \bar{y}_{2m}) + \theta(1 + \rho)(\bar{x}_{1u} + \bar{y}_{2u})\rfloor \tag{11}$$

ii) Variance of Sum

$$\mathrm{V}(\hat{\Sigma}_{TSO_L}) = \frac{2(1+\rho)}{(1+\rho\theta)}{\sigma^2}/{n} \tag{12}$$

**Properties of the Proposed Ratio Estimator**

1. Bias
2. Mean square Error (MSE)
3. Minimum Mean Square Errors (MMSE)
4. Efficiency comparison

Consider a population containing of N unit. Let the character under study in the first and second occasion be denoted by *x and y*. It is assumed that the information on auxiliary variable z is available on the first as well as on the second occasion. We consider the population to be large enough and the sample size is constant on each occasion. Using sample random without replacement (SRSWOR), we select a sample of size n on the first occasion of these n units, a sub-sample of size $m = n\lambda$ is retained on the second occasion. This sub sample is supplemented by selecting of $u = (n - m) = n\mu$ units afresh from the units that were not selected on the first occasion.

Following Yates (1949), Petterson (1950) and Singh (2005) methods, we use

$$\hat{\mu}_{1LE_{st}} = b(\bar{y}_{2u} - \bar{y}_{2m}) + d\bar{x}_{1m} + (1 - d)\bar{x}_{1u} \qquad (13)$$

$$\mu_{2_{LE_{st}}} = a(\bar{x}_{1u} - \bar{x}_{1m}) + c\bar{y}_{2m} + (1 - c)\bar{y}_{2u} \qquad (14)$$

We Proposed a ratio estimator $\bar{z}$ on the both occasion which is based on a sample of size m common to both the occasion and is given by

$$\mu_{1_{PRATE_{st}}} = b\left(\frac{\bar{y}_{2u}}{\bar{Z}_{2u}}\bar{Z} - \frac{\bar{y}_{2m}}{\bar{z}_{2m}}\bar{Z}\right) + d\,\frac{\bar{x}_{1m}}{\bar{z}_{1m}}\bar{Z} + (1 - d)\,\frac{\bar{x}_{1u}}{\bar{z}_{1u}}\bar{Z} \qquad (15)$$

$$\mu_{2_{PRATE_{st}}} = a\left(\frac{\bar{x}_{1u}}{\bar{Z}_{1u}}\bar{Z} - \frac{\bar{x}_{1m}}{\bar{z}_{1m}}\bar{Z}\right) + c\,\frac{\bar{y}_{2m}}{\bar{z}_{2m}}\bar{Z} + (1 - c)\,\frac{\bar{y}_{2u}}{\bar{z}_{2u}}\bar{Z} \qquad (16)$$

Where a, b, c and d are constants

- ✓ To determine the value of constants a, b, c and d.
- ✓ Find the variance of $\mu_{1_{PRATE_{st}}}$ and $\mu_{2_{PRATE_{st}}}$ then take the derivatives with respect to constants a, b, c and d, therefore equate the resulting equations to zero to obtain a, b, c and d. Hence, the proposed ratio estimator of the means is as given in section 3.3.1 below.

**The Proposed Ratio Estimator of the means are:**

i) $$\mu_{1_{PRATE_{st}}} = \frac{cH-M}{G}\left(\frac{\bar{y}_{2u}}{\bar{Z}_{2u}}\bar{Z} - \frac{\bar{y}_{2m}}{\bar{z}_{2m}}\bar{Z}\right) + \frac{GF-ME}{GD-HE}\frac{\bar{x}_{1m}}{\bar{z}_{1m}}\bar{Z} + \left(\frac{GD+ME-HE-GF}{GD-HE}\right)\frac{\bar{x}_{1u}}{\bar{z}_{1u}}\bar{Z}$$

ii) $$\mu_{2_{PRATE_{st}}} = \frac{CH-M}{G}\left(\frac{\bar{x}_{1u}}{\bar{Z}_{1u}}\bar{Z} - \frac{\bar{x}_{1m}}{\bar{z}_{1m}}\bar{Z}\right) + \frac{GF-ME}{GD-HE}\frac{\bar{y}_{2m}}{\bar{z}_{2m}}\bar{Z} + \left(\frac{GD+ME-HE-GF}{GD-HE}\right)\frac{\bar{y}_{2u}}{\bar{z}_{2u}}\bar{Z}$$

Where,

$$C = \frac{GF-ME}{GD-HE}, \qquad D = \frac{A_3A_4}{\theta\lambda} - \frac{A_0k^2B_3}{n}, \quad E = \frac{A_0k^2B_2}{n} - \frac{A_7A_8}{\theta\lambda}, \quad F = \frac{A_5A_6}{\theta\lambda}$$

$$G = \frac{A_1A_2}{\theta\lambda} - \frac{A_0k^2B_1}{n}, \quad H = \frac{A_0k^2B_2}{n} - \frac{A_7A_8}{\theta\lambda}, \quad M = \frac{A_9A_{10}}{\theta\lambda}$$

Where also,

$$A_0 = \frac{1}{\lambda^2} - \frac{1}{\theta\lambda} + \frac{1}{\theta^2}, \quad A_1 = 4 - {R_1}^{-2} - 2{R_1}^{-1}\rho, \quad A_2 = 1 - 2{R_1}^{-2} - 2{R_1}^{-1}\rho,$$

$$A_3 = 1 + 2{R_2}^{-2} - 3{R_2}^{-1}\rho, \quad A_4 = 1 - 2{R_2}^{-2}, \quad A_5 = 1 + 3{R_2}^{-2} - 3{R_2}^{-1}\rho,$$

$$A_6 = {R_2}^{-2} + {R_2}^{-1}\rho, \quad A_7 = {R_1}^{-1}{R_2}^{-1} - {R_2}^{-1}\rho - \rho, \quad A_8 = 2{R_1}^{-1}{R_2}^{-1} - 2{R_2}^{-1}\rho - {R_1}^{-1}\rho$$

$$A_9 = 2{R_2}^{-2} - 2{R_2}^{-1}\rho - {R_1}^{-1}\rho + \rho, \quad A_{10} = {R_2}^{-2} - {R_2}^{-1}\rho$$

And that,

$$B_1 = {R_1}^{-2} - {R_1}^{-1}\rho + \rho^2, \quad B_2 = {R_1}^{-1}\rho - {R_1}^{-1}{R_2}^{-1} + {R_2}^{-1}\rho - \rho^2, \quad B_3 = {R_2}^{-2} - 2{R_2}^{-1}\rho + \rho^2$$

Where,

$$R_1 = \frac{\bar{z}}{\bar{x}}, \quad R_2 = \frac{\bar{z}}{\bar{y}}, \quad k^2 = \frac{S^2x}{\bar{z}^2}, \text{ and } \rho = \rho xy = \rho xz = \rho yz$$

**Estimate of change:**

i) The estimate required $\hat{\Delta}_{PRATE_{st}} = \mu_{2_{PRATE_{st}}} - \mu_{1_{PRATE_{st}}}$

That is,

$$\Delta_{PRATE_{st}} = \frac{1}{G(GD-HE)}\left\{\left[(G^2F + GFH - GME - MGD)\left(\frac{\bar{y}_{2m}}{\bar{z}_{2m}}\bar{Z} - \frac{\bar{x}_{1m}}{\bar{z}_{1m}}\bar{Z}\right)\right] + \left[\left((G^2D - G^2F + GME + MGD - GHE - GFH)\right)\left(\frac{\bar{y}_{2u}}{\bar{Z}_{2u}}\bar{Z} - \frac{\bar{x}_{1u}}{\bar{z}_{1u}}\bar{Z}\right)\right]\right\} \qquad (17)$$

ii) Variance of Estimate of change

$$V\left(\Delta_{PRATE_{st}}\right) = \frac{2}{G(GD-HE)}\left\{(G^2D - GHE) - (G^2F + GFH - GME - MGD)\frac{\rho}{\lambda}\right\}\frac{\sigma^2}{n} \qquad (18)$$

**Estimate of Sum:**

i) The estimate required $\Sigma_{PRATE_{st}} = \hat{\mu}_{2PRATE_{st}} + \hat{\mu}_{1PRATE_{st}}$

$$\Sigma_{PRATE_{st}} = \frac{1}{G(GD-HE)}\left\{\left[(G^2F + MGD - GME - GFH)\left(\frac{\bar{y}_{2m}}{\bar{z}_{2m}}\bar{Z} + \frac{\bar{x}_{1m}}{\bar{z}_{1m}}\bar{Z}\right)\right] + \left[(G^2D - G^2F + GFH + GME - MED - GHE)\left(\frac{\bar{x}_{1u}}{\bar{z}_{1u}}\bar{Z} + \frac{\bar{y}_{2u}}{\bar{Z}_{2u}}\bar{Z}\right)\right]\right\} \quad (19)$$

ii) Variance of Estimate of Sum

$$V\left(\Sigma_{PRATE_{st}}\right) = \frac{2}{G(GD-HE)}\left\{(G^2D - MED) - (G^2F - GME)\frac{\rho}{\lambda}\right\}\frac{\sigma^2}{n} \quad (20)$$

**Relative Gain in Precision**

a) Relative gain in precision of change in Linear Estimator (LE$_{st}$) over Simple Estimator (SE$_{st}$)

$$R\left(\frac{\Delta_{SE_{st}}}{\hat{\Delta}_{LE_{st}}}\right) = \frac{v\left(\hat{\Delta}_{SE_{st}}\right) - v\left(\hat{\Delta}_{LE_{st}}\right)}{v\left(\hat{\Delta}_{LE_{st}}\right)}$$
$$= \frac{\rho^2\lambda\theta}{1-\rho} \quad (21)$$

b) Relative gain in precision of sum in Linear Estimator (LE$_{st}$) over Simple Estimator (SE$_{st}$)

$$R\left(\frac{\hat{\Sigma}_{SE_{st}}}{\hat{\Sigma}_{LE\,st}}\right) = \frac{v\left(\hat{\Sigma}_{SE_{st}}\right) - v\left(\hat{\Sigma}_{SE\,st}\right)}{v\left(\hat{\Sigma}_{LE\,ST}\right)}$$
$$= \frac{\rho^2\lambda\theta}{1+\rho} \quad (22)$$

c) Relative gain in precision of change in Proposed Ratio Estimator (PRATE$_{st}$) over Simple Estimator (SE$_{st}$)

$$R\left(\frac{\Delta_{SE_{st}}}{\Delta_{PRATE_{st}}}\right) = \frac{V(\Delta_{SE_{st}}) - V\left(\Delta_{PRATE_{st}}\right)}{v(\Delta_{PRATE_{st}})} \times 100\%$$
$$= \frac{G(1-\lambda\rho)(GD-HE)}{(G^2D-GHE)-(G^2F+GFH-GME-MGD)}\left(\frac{\rho}{\lambda}\right) \quad (23)$$

d) Relative gain in precision of sum in Proposed Ratio Estimator (PRATE$_{st}$) over Simple Estimator (SE$_{st}$)

$$R\left(\frac{\Sigma_{SE_{st}}}{\Sigma_{PRATE_{st}}}\right) = \frac{v\left(\Sigma_{SE_{st}}\right) - v\left(\Sigma_{PRATE_{st}}\right)}{v\left(\Sigma_{PRATE_{st}}\right)} \times 100\%$$
$$= \frac{G(1+\lambda\rho)(GD-HE)}{(G^2D-MED)-(G^2F-GMF)}\left(\frac{\rho}{\lambda}\right) \quad (24)$$

## RESULT OF THE ANALYSIS

### Empirical Study I

The data from census conducted in Nigeria in 1991 (1st occasion) and 2006 (2nd occasion) was considered. We define the variables X and Y as the population of males and female in each state and Z is defined as the auxiliary variable which is the total number of households in each state and T is the total number of both sex (see section 3.1 for more details)

**Table 1: Selection Procedure**

| | | Parameters | | | | | |
|---|---|---|---|---|---|---|---|
| **S/N** | All States | N | n | U | m | Ө | $\lambda$ |
| 1 | 36 states & FCT | 37 | 25 | 13 | 12 | 0.52 | 0.48 |

**Table 2: Estimate of Mean by Country**

| | | | Previous Estimate | | | | Current Estimate | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| S/n | Country | Estimator | $H_1$ | $M_1$ | $F_1$ | $B_1$ | $H_2$ | $M_2$ | $F_2$ | $B_2$ |
| 1. | Nigeria | SEst | 514,569.24 | 1,290,049.76 | 1,284,269.56 | 2,573,927.5 | 689,922 | 1,731,831 | 1,695,244.44 | 3,433,199.16 |
| | | Lest | 537,66.21 | 1,765,011.32 | 1,659,34.5 | 2,957,811.6 | 765,325 | 2,361869.2 | 2,791,349.5 | 3,980,981.1 |
| | | PRATEst | 685,231.11 | 2,310,789.6 | 2,012,176.7 | 3,557,918.6 | 1,067,387.6 | 2,989,120.3 | 2,621,520.7 | 4,759,240.5 |

**Table 3: Estimate of Change and its Variance**

| | | | Change | | | | Variances | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| S/n | Country | Estimator | H | M | F | B | H | M | F | B |
| 1. | Nigeria | SEst | 175,352.8 | 441,781.24 | 410,974.9 | 859,271.64 | 186,321.2 | 165,321.3 | 432,32.5 | 876,532.5 |
| | | LE$_{st}$ | 227,658.8 | 396,825.6 | 532,028 | 1,023,169.5 | 165,321.3 | 452,123.5 | 420,165.7 | 786.625.2 |
| | | PRATE$_{st}$ | 382,156.5 | 678,321.7 | 609,344 | 1,201,321.8 | 95,321.4 | 311,211.7 | 300,184.6 | 605,321.9 |

**Table 4: Estimate of Sum and its Variance**

| | | | Sum | | | | Variances | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| S/n | Country | Estimator | H | M | F | B | H | M | F | B |
| 1. | Nigeria | SE$_{st}$ | 1,204,491.24 | 3,021,880.76 | 2,979,514 | 6,007,126.7 | 382,625.6 | 941,652.6 | 894,644 | 1,753,069 |
| | | LE$_{st}$ | 1,302,991.2 | 4,126,880.5 | 3,640,671 | 6,938,792.7 | 330,648.8 | 924,267.8 | 851,323.4 | 1,584,415.6 |
| | | PRATE$_{st}$ | 1,752,618.7 | 5,299,918.9 | 5,001,297 | 8,317,159.2 | 191,642.8 | 621,423.4 | 600,469.5 | 1,221,645.9 |

**Table 5: Relative Gain in Precision (%)**

| | | | Change | | | | Sum | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| S/n | Country | Estimator | H | M | F | B | H | M | F | B |
| 1. | Nigeria | LE$_{st}$ | 12.7 | 9.5 | 2.9 | 11.4 | 15.7 | 1.9 | 5.1 | 10.6 |
| | | PRATE$_{st}$ | 95.5 | 49.5 | 44.0 | 44.80 | 99.7 | 51.5 | 49.0 | 43.5 |

**Table 6: Estimate of Change (Δ) with varying** $\lambda$’s and $\rho$’s, ($N = 37$ $n = 25$)

| $\rho$ | $\lambda = 0.2$ | | | | $\lambda = 0.5$ | | | | $\lambda = 0.8$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | SE$_{st}$ | RE$_{st}$ | LE$_{st}$ | PHLRE$_{st}$ | SE$_{st}$ | RE$_{st}$ | LE$_{st}$ | PHLRE$_{st}$ | SE$_{st}$ | RE$_{st}$ | LE$_{st}$ | PHLRE$_{st}$ |
| 0.05 | 159346.7 | 166112.3 | 177811.7 | 182321.6 | 186723.6 | 193321.1 | 265391.2 | 478321.7 | 196321.0 | 211321.6 | 345391.6 | 503316.0 |
| 0.15 | 135111.7 | 141211.8 | 156212.5 | 166378.5 | 175352.8 | 178521.5 | 227658.8 | 382156.5 | 186371.6 | 191211.9 | 285345.7 | 485368.8 |
| 0.25 | 121321.8 | 139321.1 | 149316.7 | 158379.6 | 169321.9 | 172321.4 | 216381.9 | 365491.6 | 179321.5 | 186841.9 | 281496.7 | 467496.3 |
| 0.35 | 120116.8 | 132233.7 | 138321.6 | 149891.7 | 163867.1 | 169341.6 | 202341.7 | 342369.9 | 172411.5 | 179382.1 | 278327.9 | 455351.8 |
| 0.45 | 119321.5 | 129321.1 | 133678.9 | 138321.9 | 159321.1 | 162381.7 | 199381.8 | 335419.7 | 166321.7 | 175841.6 | 269329.4 | 441867.2 |
| 0.55 | 117381.6 | 126481.8 | 130781.6 | 136891.7 | 154311.8 | 161321.6 | 190384.6 | 321321.6 | 159311.9 | 171321.9 | 256794.5 | 436321.4 |
| 0.65 | 115678.6 | 123321.7 | 128321.9 | 132799.6 | 151869.1 | 159785.8 | 186321.5 | 302311.5 | 156281.6 | 169847.6 | 248991.2 | 429321.8 |
| 0.75 | 110321.6 | 120785.6 | 125321.6 | 130311.7 | 149321.9 | 152314.6 | 179311.6 | 295314.6 | 147321.9 | 163321.5 | 236321.6 | 427321.1 |
| 0.85 | 108567.1 | 111321.8 | 121321.9 | 128869.5 | 138891.2 | 149385.7 | 170311.6 | 281345.7 | 138319.6 | 160876.9 | 229316.9 | 420691.7 |
| 0.95 | 102321.7 | 109887.8 | 112369.8 | 120221.6 | 132776.8 | 142328.9 | 168321.7 | 276388.8 | 136311.5 | 156331.7 | 220321.6 | 401321.9 |
| 1.00 | 97321.6 | 102321.6 | 109369.8 | 112321.5 | 129321.0 | 138326.4 | 156321.9 | 251321.9 | 130816.6 | 149321.0 | 199371.9 | 398321.6 |

**Table 7: Estimate of Variance of Change (Δ) with varying** $\lambda$’s and $\rho$’s, ($N = 37$ $n = 25$)

| $\rho$ | $\lambda = 0.2$ | | | | $\lambda = 0.5$ | | | | $\lambda = 0.8$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | SE$_{st}$ | RE$_{st}$ | LE$_{st}$ | PHLRE$_{st}$ | SE$_{st}$ | RE$_{st}$ | LE$_{st}$ | PHLRE$_{st}$ | SE$_{st}$ | RE$_{st}$ | LE$_{st}$ | PHLRE$_{st}$ |
| 0.05 | 197491.1 | 176491.2 | 171991.7 | 107419.6 | 204311.8 | 188794.5 | 185321.9 | 110341.9 | 216317.9 | 199381.7 | 190421.4 | 120416.1 |
| 0.15 | 186321.5 | 165321.9 | 160381.6 | 95411.9 | 197345.3 | 176471.9 | 171691.4 | 107381.9 | 201341.6 | 192321.9 | 185461.2 | 110341.1 |
| 0.25 | 162321.5 | 159321.1 | 146321.1 | 80317.6 | 186821.3 | 165321.3 | 160111.3 | 95321.4 | 196321.6 | 185411.7 | 171321.9 | 107321.6 |
| 0.35 | 179334.6 | 162,99.7 | 142311.8 | 76915.4 | 184321.4 | 163661.2 | 165998.4 | 97481.4 | 191411.7 | 179497.4 | 169391.6 | 101321.1 |
| 0.45 | 177637.1 | 159.441.7 | 140917.9 | 74321.5 | 179841.6 | 162591.2 | 162111.9 | 92441.9 | 186321.5 | 168559.7 | 164321.9 | 97381.7 |
| 0.55 | 174.391.4 | 157816.9 | 138416.7 | 72491.6 | 177736.1 | 159331.6 | 158321.7 | 85441.7 | 180416.7 | 164321.7 | 160321.7 | 92321.1 |
| 0.65 | 171321.5 | 150881.9 | 132781.6 | 70321.6 | 174934.6 | 157778.4 | 153811.7 | 79991.9 | 176161.8 | 162871.8 | 158389.8 | 85321.9 |
| 0.75 | 163311.9 | 147991.9 | 130617.9 | 67911.8 | 171896.7 | 150311.9 | 149661.8 | 64331.9 | 170321.7 | 159321.6 | 153894.5 | 79891.6 |
| 0.85 | 159341.6 | 143678.8 | 126321.5 | 63511.6 | 163481.9 | 147921.7 | 145811.9 | 60349.8 | 165497.9 | 155321.7 | 149381.7 | 64321.8 |
| 0.95 | 152319.7 | 138391.6 | 121341.9 | 61348.9 | 159221.7 | 143621.1 | 140321.1 | 54995.5 | 160321.5 | 150001.1 | 145321.1 | 60321.7 |
| 1.00 | 149321.0 | 130117.9 | 119345.5 | 59811.6 | 152378.3 | 138321.9 | 135321.9 | 51321.7 | 154949.5 | 149321.9 | 140783.1 | 54321.1 |

**Table 8: Relative Gain in precision of Change (Δ) with varying** $\lambda$’s and $\rho$’s, $(N = 37\ n = 25)$

| $\rho$ | $\lambda = 0.2$ | | | $\lambda = 0.5$ | | | $\lambda = 0.8$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | RE$_{st}$ | LE$_{st}$ | PHLRE$_{st}$ | RE$_{st}$ | LE$_{st}$ | PHLRE$_{st}$ | RE$_{st}$ | LE$_{st}$ | PHLRE$_{st}$ |
| 0.05 | 11.9 | 24.8 | 83.9 | 8.2 | 10.2 | 52.1 | 5.5 | 8.6 | 39.8 |
| 0.15 | 21.7 | 34.2 | 95.3 | 11.8 | 24.9 | 63.7 | 6.7 | 9.7 | 42.5 |
| 0.25 | 30.9 | 43.8 | 102.1 | 12.0 | 36.4 | 75.5 | 7.9 | 10.6 | 44.9 |
| 0.35 | 40.0 | 53.5 | 133.2 | 14.6 | 41.0 | 89.1 | 8.6 | 11.0 | 58.9 |
| 0.45 | 50.6 | 62.9 | 135.4 | 16.6 | 50.9 | 94.5 | 9.5 | 12.4 | 61.3 |
| 0.55 | 60.5 | 71.8 | 140.6 | 19.6 | 62.3 | 108.0 | 10.8 | 13.5 | 75.4 |
| 0.65 | 70.6 | 80.7 | 143.0 | 20.9 | 73.7 | 119.0 | 11.2 | 14.2 | 96.5 |
| 0.75 | 80.4 | 90.2 | 147.0 | 24.4 | 84.9 | 127.0 | 12.9 | 15.7 | 103.2 |
| 0.85 | 91.2 | 101.8 | 150.9 | 30.5 | 92.1 | 130.9 | 13.8 | 16.8 | 117.3 |
| 0.95 | 93.4 | 108.5 | 158.1 | 40.8 | 98.5 | 149.5 | 16.9 | 18.3 | 125.7 |
| 1.00 | 100.5 | 128.2 | 169.6 | 50.1 | 102.6 | 156.0 | 18.8 | 20.1 | 130.4 |

**Table 9: Estimate of Sum (Σ) with varying** $\lambda$’s and $\rho$’s, $(N = 37\ n = 25)$

| $\rho$ | $\lambda = 0.2$ | | | | $\lambda = 0.5$ | | | | $\lambda = 0.8$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | SE$_{st}$ | RE$_{st}$ | LE$_{st}$ | PHLRE$_{st}$ | SE$_{st}$ | RE$_{st}$ | LE$_{st}$ | PHLRE$_{st}$ | SE$_{st}$ | RE$_{st}$ | LE$_{st}$ | PHLRE$_{st}$ |
| 0.05 | 1711611.5 | 1511341.6 | 1301321.9 | 1138417.8 | 1045321.9 | 1034167.5 | 1145917.8 | 996851.0 | 851321.0 | 765811.0 | 653211.0 | 541321.0 |
| 0.15 | 1811911.6 | 1617811.2 | 1491112.7 | 1236321.5 | 1165186.5 | 1141321.5 | 1291314.8 | 1001351.7 | 985321.0 | 854341.0 | 789411.0 | 658321.0 |
| 0.25 | 1965914.2 | 1732651.6 | 1514164.2 | 1381341.6 | 1204491.2 | 1219188.5 | 1302991.2 | 1216314.8 | 1012314.0 | 996841.0 | 876321.0 | 745321.0 |
| 0.35 | 2001111.6 | 1813711.7 | 1621311.9 | 1411416.7 | 1311341.8 | 1301144.8 | 1411414.3 | 1311614.9 | 1121321.4 | 1001112.4 | 954321.0 | 895811.0 |
| 0.45 | 2114334.6 | 1932165.6 | 1731211.1 | 1586316.9 | 1421617.5 | 1411321.6 | 1564321.6 | 1432116.8 | 1224381.5 | 1134114.5 | 1003321.4 | 955873.0 |
| 0.55 | 2214314.6 | 2016321.7 | 1821441.1 | 1669319.4 | 1531811.6 | 1501481.9 | 1665891.7 | 1531716.5 | 1321341.7 | 1241681.5 | 1112321.5 | 1012321.5 |
| 0.65 | 2319314.5 | 2116381.9 | 1911321.6 | 1781375.9 | 1658321.7 | 1624881.5 | 1795321.6 | 1632381.9 | 1416312.8 | 1321471.6 | 1321395.6 | 1112381.7 |
| 0.75 | 2418314.6 | 2217381.6 | 2033391.7 | 1896321.2 | 1765321.6 | 1741660.3 | 1865811.8 | 1711816.5 | 1567817.9 | 1443211.8 | 1313496.8 | 1212391.8 |
| 0.85 | 2521816.7 | 2318321.9 | 2144391.8 | 1997321.9 | 1845321.8 | 1832117.9 | 1965327.7 | 1811178.8 | 1651321.6 | 1511614.9 | 1461321.5 | 1321416.2 |
| 0.95 | 2616317.7 | 2417416.8 | 2279316.7 | 2011319.8 | 1985445.7 | 1932441.4 | 2014321.9 | 1961821.5 | 1781414.9 | 1653416.7 | 1532416.7 | 1432119.6 |
| 1.00 | 2717318.6 | 2597845.0 | 2345381.9 | 2179341.6 | 2001416.5 | 2000116.8 | 2001216.7 | 2000121.4 | 1814321.7 | 1791341.6 | 1652321.9 | 1515117.8 |

**Table 10: Estimate of Variance of Sum (Σ) with varying** $\lambda$’s and $\rho$’s, $(N = 37\ n = 25)$

| $\rho$ | $\lambda = 0.2$ | | | | $\lambda = 0.5$ | | | | $\lambda = 0.8$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | SE$_{st}$ | RE$_{st}$ | LE$_{st}$ | PHLRE$_{st}$ | SE$_{st}$ | RE$_{st}$ | LE$_{st}$ | PHLRE$_{st}$ | SE$_{st}$ | RE$_{st}$ | LE$_{st}$ | PHLRE$_{st}$ |
| 0.05 | 461321.3 | 365361.8 | 296791.7 | 165321.8 | 365321.8 | 311414.6 | 295821.6 | 171321.9 | 354351.8 | 305311.8 | 299861.8 | 159341.9 |
| 0.15 | 470321.3 | 377311.7 | 305311.6 | 174811.4 | 378311.6 | 320116.9 | 305178.5 | 181179.6 | 365517.3 | 316411.6 | 306179.8 | 164311.8 |
| 0.25 | 481341.6 | 387321.6 | 318771.9 | 186791.4 | 382625.6 | 330648.8 | 310569.6 | 191642.8 | 376519.6 | 321411.5 | 317411.9 | 175821.6 |
| 0.35 | 492481.8 | 397321.0 | 324111.6 | 198321.5 | 394411.8 | 341117.9 | 324511.1 | 200119.4 | 386317.7 | 331481.7 | 328817.6 | 183411.9 |
| 0.45 | 513118.9 | 401831.6 | 331416.5 | 200116.7 | 400114.8 | 352311.4 | 335891.6 | 210219.5 | 395321.8 | 345381.6 | 337661.6 | 194399.8 |
| 0.55 | 521321.3 | 412871.8 | 342117.8 | 211321.8 | 412321.9 | 366516.6 | 341417.9 | 221321.7 | 400119.9 | 358916.5 | 348118.7 | 204311.9 |
| 0.65 | 532417.6 | 423314.9 | 354161.2 | 221416.9 | 422391.6 | 379811.7 | 356819.9 | 235311.8 | 410121.6 | 366811.8 | 357311.8 | 217821.9 |
| 0.75 | 544311.6 | 434.816.8 | 365381.6 | 234314.6 | 439381.7 | 381656.9 | 366314.5 | 246321.8 | 420321.7 | 377411.3 | 366381.6 | 225119.6 |
| 0.85 | 552311.8 | 445317.1 | 378114.8 | 245981.7 | 447821.6 | 398778.1 | 375391.7 | 257451.9 | 436311.8 | 389111.4 | 375781.7 | 238124.5 |
| 0.95 | 563217.9 | 456811.7 | 388818.9 | 255817.9 | 458011.7 | 405811.8 | 384951.6 | 267811.6 | 447381.9 | 398118.5 | 384891.6 | 249321.6 |
| 1.00 | 571321.8 | 476812.8 | 391811.8 | 268111.8 | 469812.9 | 410911.9 | 391652.8 | 278618.7 | 458312.0 | 400321.7 | 39816.7 | 251321.8 |

**Table 11: Relative Gain in precision of Sum (Σ) with varying** $\lambda$'s and $\rho$'s, ($N = 37\ n = 25$)

| $\rho$ | $\lambda = 0.2$ | | | $\lambda = 0.5$ | | | $\lambda = 0.8$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | $RE_{st}$ | $LE_{st}$ | $PHLRE_{st}$ | $RE_{st}$ | $LE_{st}$ | $PHLRE_{st}$ | $RE_{st}$ | $LE_{st}$ | $PHLRE_{st}$ |
| 0.05 | 26.3 | 55.4 | 179.4 | 47.3 | 83.5 | 193.2 | 56.1 | 118.2 | 222.4 |
| 0.15 | 25.1 | 54.8 | 165.6 | 46.8 | 82.7 | 181.3 | 55.1 | 117.8 | 214.7 |
| 0.25 | 24.8 | 53.3 | 155.6 | 45.2 | 81.6 | 176.4 | 54.3 | 116.9 | 201.6 |
| 0.35 | 23.4 | 52.9 | 146.7 | 44.9 | 80.4 | 165.4 | 53.7 | 115.3 | 199.6 |
| 0.45 | 22.1 | 51.7 | 137.8 | 43.2 | 79.6 | 155.8 | 52.9 | 114.6 | 186.8 |
| 0.55 | 21.9 | 50.2 | 126.7 | 42.6 | 78.5 | 145.6 | 51.2 | 113.2 | 175.9 |
| 0.65 | 20.6 | 49.6 | 116.8 | 41.8 | 77.6 | 136.7 | 50.1 | 112.6 | 165.1 |
| 0.75 | 19.1 | 47.6 | 107.6 | 40.1 | 76.5 | 126.7 | 49.6 | 103.7 | 155.8 |
| 0.85 | 18.9 | 46.3 | 98.6 | 39.8 | 75.5 | 116.8 | 48.9 | 98.4 | 146.5 |
| 0.95 | 17.6 | 45.9 | 88.6 | 38.2 | 74.9 | 98.7 | 47.2 | 88.5 | 136.7 |
| 1.00 | 16.8 | 44.9 | 76.5 | 37.6 | 73.4 | 86.8 | 46.5 | 76.7 | 128.9 |

## DISCUSSION OF RESULTS

The result of analysis presented in Tables 1 – 11 can be summarized as follow:

Generally we have undertaken an investigation covering a ratio estimator techniques in the context of devising efficient sampling strategies for successive sampling schemes with a view to derive the unbiased estimator of mean ($\mu$) on two occasion (previous and current) using Existing Simple Estimator ($ESE_{st}$), Existing Linear Estimator ($ELE_{st}$) and Propose Ratio Estimator (PRATEst) and also to examine the Estimates of change (Δ) and Sum (⍰) respectively and establish minimum variance of the three estimators and to achieve the relative gain in precision of the estimators. This study considered the census figure of the 36 states and FCT of Nigeria in 1996 and 2006 and the necessary parameters of populations for computing the estimators are given in Table 1. (using Random Number table as a selection procedure).

Table 2 reveals that both previous and current Estimate sustained the expected population mean under the variables of interest ($M_{1_{\alpha2}}, F_{1_{\alpha2}}\ and\ B_{1_{\alpha2}}$) and auxiliary variables ($H_{1_{\alpha2}}\ and\ T_{1_{\alpha2}}$) using Existing Simple Estimator ($ESE_{st}$), Existing Linear Estimator ($ELE_{st}$) and Proposed Ratio Estimator (PRATEst) compare to actual population mean.

According to Table 3, the Estimate of change (Δ) in variables of interest (M, F, and B) and auxiliary variables (H and T) over the time interval of the census are unique and consistent considered the actual population value and expected population value. In the same vein it was observed that Proposed Ratio Estimator (PRATEst) has least variance than other two estimators.

It could be noted, in Table 4, that Estimate of sum (⍰) in both variables of interest (M, F and B) and auxiliary variables (H and T) over a given time interval 1999 – 2006 were also unique and justifiable considering the actual value and expected value among the three Estimators. However, Table 5 revealed that gain in efficiency exist by using Proposed Ratio Estimator (PRATEst) than usual Existing Linear Estimator ($ELE_{st}$) both at Estimate of change (Δ) and sum (⍰) most especially for auxiliary variable (H and T) with high reasonable gain in precision.

In the same vein, it was observed, from Table 6, that the correlation coefficient (p) increases as the estimate of change (Δ) in estimator decreases and proportion matched (λ) increase as the estimate of change (Δ) in estimator increasing with different varying $\lambda$'s and $\rho$'s.

We learned (Table 7) that for varying $\lambda$'s and $\rho$'s minimum variance and maximum precision is achieved as $\rho \rightarrow 1$ as well as $\lambda \rightarrow 0$. This implies that there is perfect positive relationship between first and second samplings occasions. Similarly, for varying $\lambda$'s and $\rho$'s minimum variance and maximum precision is achieved as $\rho's \rightarrow 0\ and\ \lambda's \rightarrow 1$ under the estimate of sum of variance (Table 10).

Table 8 shows that the relative efficiency or gain in precision was achieved when $\rho's \rightarrow 1\ and\ \lambda's \rightarrow 0$ under estimate of change with substantial gain in $PHLRE_{st}$. It is equally showed that, when $\rho's \rightarrow 0\ and\ \lambda's \rightarrow 1$, the relative efficiency or gain in precision was achieved under estimate of sum, with substantial gain in $PHLRE_{st}$. (Table 11)

Finally, From Table 9, it can be seen that the correlation coefficient ($\rho$) increases as each estimator increases and proportion matched ($\lambda$) increases as the estimators decreases with different $\lambda$'s and $\rho$'s under the estimate of the sum(Σ).

## CONCLUSION

In view of the above result, we concluded that our proposed Estimator has not only certified all the properties of an estimator, it has equally given rise to a more robust estimates which, when compared with estimate from the other existing estimator, possessed the following qualities:

- Least variance
- Higher gain in Precision (independent of sample size)
- Higher efficiency (when estimating average over time)

## RECOMMENDATIONS

To enhance quality of estimates, we recommend that:

- the replacement of part of the sample on the current occasion should be ensured, in order to achieve efficiency improvement in the required estimate
- attempt should be made to retain a greater portion of the samples on the current occasion to enhance the estimate of change.
- Fresh sample should be taken on each occasion to facilities efficiency when estimating average over time.

To enhance availability of quality data, we recommend that Federal Government should:

- give priority to population census and ensure it timely conduct, as stipulated in the constitution, for every interval of 10 years.
- allow population census to be an administrative and technical exercise devoid of politics of numbers, region or ethnicity.

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