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DETERMINATION OF AN UNKNOWN DIFFUSION COEFFICIENT IN A PARABOLIC INVERSE PROBLEM

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ABSTRACT
In this paper we studied the finite difference scheme for the solution of one dimensional parabolic inverse problem of finding the function $\phi(x,t)$ and the unknown positive coefficient $b(t)$. The Backward time centered space (BTCS) which is unconditionally stable is studied and its convergent is proved using application of discrete maximum principle. Error estimates for $\phi(x,t)$ and $b(t)$ is studied and to give clear overview of the methodology several model problems are solved numerically. According to the experimental numerical results and the concluding remark are presented.

Keywords: finite difference methods, parabolic inverse problem, convergence, Error estimates, maximum principle.

INTRODUCTION
The problem of solving two unknown functions $\phi(x,t)$ and the diffusion coefficient $b(t)$ in the parabolic inverse problem is considered:

$\phi_t = b(t)\phi_{xx} \quad in \ \Omega^T \qquad (1)$

$\phi(x,0) = \psi(x), \quad 0 \le x \le 1 \qquad (2)$

$\phi(0,t) = H_1(t), \quad 0 \le t \le T \qquad (3)$

$\phi(1,t) = H_2(t), \quad 0 \le t \le T \qquad (4)$

Where $\Omega^T = \{(x,t): x \in (0,1),\ t \in (0,T)\}, T > 0, and\ \psi, H_1, H_2$are well-known function, while $\phi(x,t)$ and $b(t)$ are unknown it is clear that with the data mentioned above this problem is under-determined, so to solve the inverse problem we most introduce a supplementary boundary condition such that the one and only solution of $\phi(x,t)$ and $b(t)$ are obtained. In particular, this may take form of the heat flux $q(t)$ at a given point $x^* = 0\ or\ 1$, that is,

$-b(t)\phi_x(x^*,t) = q(t), \quad 0 \le t \le T. \qquad (5)$

As a matter of choice, one may recommend other function, say

$\phi(x^*,t) = q(t), \quad 0 \le t \le T, \qquad (6)$

Where $x^* \in (0,1)$ thus a resumption of the function $b(t)$ together with the solution $\phi(x,t)$ can be formed.

The problem of restoring a time dependent coefficient in a parabolic inverse problem has drawn so many interests and considered by many scientists. In the past decenary a countless covenant of attentiveness has been focused towards the resolve of unknown diffusion coefficients in partial differential equation. One of the motivations behind this paper is to regulate the unknown variables in a section by quantifying only the data on the boundary and specific consideration has been concentrated on coefficients that denote the physical quantities, such as, the conductivity of a medium. The approaches used depend toughly on the nature of the equations and variables on which the unknown quantity is projected a priori to depend. A significant but challenging situation is when the new conductivity builds upon the dependent variable of the solution $\phi(x,t)$.

For a heat energy challenging, this has a physical clarification in which the temperature reliant on conductivity. The spatial transformation of the function $\phi(x,t)$ is insignificant in association with the variation in time, and then a rational estimate to this state of actions may be to consider the coefficient to be the function only of the time variable. The mathematical solution of the problem $(1)-(4)$ has been talk over by numerous authors [1, 2, 3]. For parabolic inverse problem of discovering$b(t)$ Azari [1] studied$\phi_t = b(t)\phi_{xx}$ with the respect to initial-boundary and over quantified condition to regulate the time reliant on coefficient and then converted the inverse problem to neoclassical equation. The maximum principle was then applied to this problem and global existence clarification to these problems were achieved from the continuity techniques. In [2] the numerical solution of $(1)-(6)$ are also debated using Explicit, Implicit and Crank Nicolson numerical schemes and higher order was recommended to determine the function ϕ and the unknown time reliant on coefficient $b(t)$, in which so many numerical investigations were obtained to examine the effectiveness and accuracy of the numerical consequence, error approximation and numerical solution of $\phi(x,t)$ and $b(t)$ were developed. In [3] Pseudospectral Legendre scheme is engaged to solve problem $(1)-(4)$ where the Errors of $\phi(x,t)$ and $b(t)$ are acquired by using Explicit, implicit, Crank Nicolson, Saulyev's first and second kind. In [4] the author discussed over the problem of determining concurrent time reliant on thermal diffusivity and the temperature circulation in one dimensional parabolic equation in nonlocal boundary and integral over resolve conditions, the uniqueness and existence condition of classical clarification of the problem were also discussed. In [5] finite difference estimate to an inverse problem $(1)-(6)$ were also deliberated, the Implicit Euler scheme is considered and is shown that the scheme is stable using maximum norm and convergence are proved using discrete maximum principle. The error estimation and numerical investigation of $\phi(x,t)$ and $b(t)$, and some newly projected procedures are presented. Author in[6] also researched on the problem $(1)-(4)$, but the numerical results of the investigation are far-off from tolerable. In [7] Cannon and Jones studied $\phi_t = b(t)\phi_{xx}$ subject to time reliant on boundary conditions. The foremost target of the research is to decrease the problematic case to nonlinear integral equation for the quantity$b(t)$. This suggestion, which depends on the explicit arrangement of elementary solution of the heat equation, does not simply lead to the separation of m space variable for $m \geq 2$. In [8] Cannon and William verified the fortitude of a time reliant on conductivity for potential arbitrary field in $\mathbb{R}^n$, their technique can be labeled as a "lenient" amendment of the methodology of Jones, and depends on the compactness and maximum principle of a convinced smoothing to produce a desire effect by sequential estimates.

FINITE DIFFERENCE TECHNIQUE

The numerical methods of one dimensional parabolic inverse problem is advanced to solve (1)-(4) the finite difference consequent from substituting the space and the time derivative. The parabolic space domain $[0,1]\times[0,T]$ is derived in to mesh of $M \times N$ with the spatial step size $h=\frac{1}{M}$ and the time step size$k=\frac{1}{N}$.

Now we can design the grid points (x_i, t_i) by

$$x_i = i \times h, \qquad i = 0,1,2, \ldots \ldots M$$
$$t_j = j \times k, \qquad j = 0,1,2,, \ldots, N$$

Where M and N are any integers, the notation ϕ_i^j , b_j are used to designate the finite difference estimates of

$$\phi(i \times h, j \times k) \quad and \quad b(j \times k).$$

Transformation of the Inverse Problem

Taking the derivative of equation(6) with the respect to t, we obtained

$$q'(t) = \phi_t(x^*, t). \qquad (2)$$

Substituting equation (2) in eqution (1.1) we have

$$q'(t) = b(t)\phi_{xx}(x^*, t), \qquad (3)$$

this yields to

$$b(t) = \frac{q'(t)}{\phi_{xx}(x^*,t)}, \qquad t \in [0,T] \qquad (4)$$

provided that $\phi_{xx}(x^*, t) \neq 0$;

Now equations $(1)-(6)$ change to the resulting problem

$$\phi_t = \frac{q'(t)}{\phi_{xx}(x^*,t)}\phi_{xx} \quad in \ \Omega^T \qquad (5)$$
$$\phi(x,0) = \psi(x), \qquad 0 \leq x \leq 1, \qquad (6)$$
$$\phi(0,t) = \mathrm{H}_1(t), \qquad 0 \leq t \leq T, \qquad (7)$$
$$\phi(1,t) = \mathrm{H}_1(t), \qquad 0 \leq t \leq T. \qquad (8)$$

Our process is based on the following alteration, by setting

$$w(x,t) = \phi_{xx}(x,t), (9)$$

taking derivative of (9) with respect to t we have

$w_t(x,t) = \phi_{xxt}(x,t),$

$$= \frac{\partial}{\partial x^2}\left(\frac{q'(t)}{\phi_{xx}(x^*,t)}\phi_{xx}(x,t)\right),$$
$$w_t(x,t) = \frac{\partial}{\partial x^2}\left(\frac{q'(t)}{w(x^*,t)}w(x,t)\right).$$

Therefore

$$w_t(x,t) = \frac{q'(t)}{w(x^*,t)}w_{xx}(x,t).$$

For initial condition at $t=0$,

$$\phi(x,0) = \psi(x).$$

Second derivative of $\phi(x,0)$ with the respect to x, yield to

$\phi_{xx}(x,0) = \psi_{xx}(x)$, Then

$$w(x,0) = \psi_{xx}(x). \qquad (10)$$

Transformation of the left and right boundary condition at $x=0$ and $x=1$, respectively, we know

FUDMA Journal of Sciences (FJS) Vol. 5 No. 4, December, 2021, pp 291 - 299

$$b(t) = \frac{q'(t)}{\phi_{xx}(x^*, t)}.$$

That is

$$b(t) = \frac{q'(t)}{w(x^*, t)},$$

for the left boundary condition

$$\phi(0,t) = \mathrm{H}_1(t), \qquad \text{then}$$

$$\phi_{xx}(0,t) = w(0,t) = \frac{1}{b(t)}\phi_t(0,t) = \frac{1}{b(t)}\mathrm{H}_1'(t).$$

It implies that

$$w(0,t) = \frac{\mathrm{H}_1'(t)}{q'(t)} w(x^*, t). \qquad (11)$$

Similarly, for the right boundary condition$x = 1$,

$$\phi(1,t) = \mathrm{H}_2(t) \text{ then}$$

$$\phi_{xx}(1,t) = w(1,t) = \frac{\phi_t(1,t)}{b(t)} = \frac{1}{b(t)}\mathrm{H}_2'(t).$$

Then it implies that

$$w(1,t) = \frac{\mathrm{H}_2'(t)}{q'(t)} w(x^*, t). \qquad (12)$$

Where$w(x,t)$is the solution of the following problem

$$w_t = \frac{q'(t)}{w(x^*, t)} w_{xx} \quad in \ \Omega^T \qquad (13)$$

$$w(x,0) = \psi_{xx}(x), \ \ 0 \le x \le 1 \qquad (14)$$

$$w(0,t) = \frac{\mathrm{H}_1{}'(t)}{q'(t)} w(x^*, t), \quad 0 \le t \le T, \ (15)$$

$$w(1,t) = \frac{\mathrm{H}_2{}'(t)}{q'(t)} w(x^*, t), 0 \le t \le T. (2.16)$$

We make some assumptions that holds throughout this paper:

P(1)Let$\psi(x) \in C^{4+\partial}[0,1]$ and $\psi_{xx}(x) > 0, \psi_{xx}(x^*) = \frac{\varepsilon}{2} > 0$ and $\psi_{xxxx}(x) > 0$on[0,1].

P(2) Let $\mathrm{H}_1(t), \mathrm{H}_2(t)$ and $q(t) \in C^{l+\frac{\partial}{2}}[0,T]$.

Furthermore, $q'(t) > 0$ on $[0,T]$,

$$0 < \frac{\mathrm{H}_1(t)}{q'(t)} < 1, 0 < \frac{\mathrm{H}_2(t)}{q'(t)} < 1, and$$

$$\left(\frac{\mathrm{H}_1(t)}{q'(t)}\right)' > 0, \left(\frac{\mathrm{H}_2(t)}{q'(t)}\right)' > 0, \qquad t \in [0,T].$$

Backward Time Centered Space (BTCS)

Backward time centered space can be defined using the forward derivative approximation for the time derivative ϕ_t and second order approximation for the spatial derivative ϕ_{xx} defined at the point(x_i, t_{j+1}). Then the overall approximation is called Backward Time Centered space or Backward Euler scheme.

Lemma 2.1 [6] suppose that $f(x) \in C^2[0,1]$ and there exist i_0 such that $x^* \in [x_{i_0}, x_{i_0+1})$.

$$f(x^*) = \frac{h - \varepsilon_x}{h} f(x_{i_0}) + \frac{\varepsilon_x}{h} f(x_{i_{0+1}}) + O(h^2), \qquad (18)$$

where

$$\varepsilon_x = x^* - x_{i_0}.$$

Now the backward time centered space (see Figure 1) can now be defined by

$$\frac{w_i^{j+1} - w_i^j}{k} = \frac{B^{j+1}}{w_*^j} \frac{w_{i+1}^{j+1} + w_{i-1}^{j+1} - 2w_i^{j+1}}{h^2}, \qquad i = 0,1,..M-1, j \ge 0, \qquad (19)$$

$$w_i^0 = \psi_{xxi} i = 0,1, \dots M, \qquad (20)$$

$$w_i^{j+1} = Q_i^{j+1} w_*^j, i = 0 \ \text{or} M, \quad j \ge 1 \qquad (21)$$

where

$$w_*^j = \phi_{xx}(x^*, t^j) = \frac{1}{h^2}\left[\phi(x^* + h, t^j) - 2\phi(x^*, t^j) + \phi(x^* - h, t^j)\right]$$

Equivalently

$$w_*^{j+1} = \frac{h - \varepsilon_x}{h} w_{i_0}^{j+1} + \frac{\varepsilon_x}{h} w_{i_0+1}^{j+1} \qquad (22)$$

FUDMA Journal of Sciences (FJS) Vol. 5 No. 4, December, 2021, pp 291 – 299

And $B^{j+1} = q'(t_j)$,
$Q_0^{j+1} = \left(\frac{\mathrm{H}_1(t_{j+1})}{q(t)}\right)'$ *and* $Q_M^{j+1} = \left(\frac{\mathrm{H}_2(t_{j+1})}{q(t)}\right)'$.

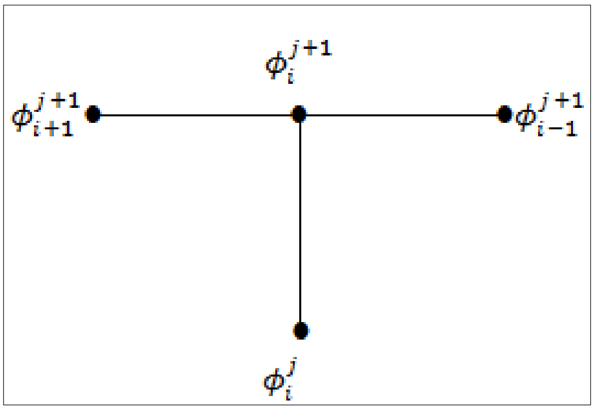

Figure 1. Computational molecule for BTCS

It is easy to see that $(19) - (22)$ is a semi$-$ implicit scheme because $w(x^*, t)$ is approximated using values at the previous time level. The scheme (19) result in a truncation error $O(h^2 + k)$, which is the same as the standard backward finite difference scheme for parabolic equations. It can easily be seen that any standard numerical solver for parabolic equations can be used to solve$(19) - (21)$.
Let us define

$$\nabla^+ w_i = \frac{w_{i+1} - w_i}{h} \;\; \nabla^- w_i = \frac{w_i - w_{i-1}}{h}. \quad (23)$$

Therefore, we have

$$\nabla^2 w_i = \nabla^- \nabla^+ w_i = \frac{w_{i+1} - 2w_i + w_{i-1}}{h^2} \quad (24)$$

And for some $\{d_i\}_{i=1}^{p}$ it follows

$$\nabla^2(d_i w_i) = d_i \nabla^2 w_i + w_{i+1} \nabla^2 d_{i+1} + \nabla^+ d_{i-1} \nabla^+ w_i + \nabla^+ d_{i-1} \nabla^+ w_{i-1} \qquad (25)$$

CONVERGENCE

Convergent Estimate Theorem

Theorem 3.2 [1, 7]: Suppose that$\phi \in C^{4,2}(\Omega)$. Then there exist $h_0 > 0$ and $k_0 > 0$, dependent upon the data $\mathrm{H}_1, \mathrm{H}_2, q, s = \max\{x^*, 1 - x^*\}$, and $T > 0$, such that $h \in (0, h_0)$ and$k \in (0, k_0)$, $\forall C > 0$, which is depending on s, T and$C^{4,2}$ norm of w, such that

$$\max_{i,j} \left| w(x_i, t_{j+1}) - w_i^{j+1} \right| \leq C(h^2 + k) \; (26)$$

Error Estimate for$\boldsymbol{\phi(x,t)}$ and $\boldsymbol{b(t)}$

It is easy to observe that the numerical solution of w_i^{j+1} is not the solution of initial inverse value problem. So to solve the original problem, we must recover $\phi(x, t)$ from $w(x, t)$. We need to resolve the resulting boundary value problem by dealing with time t as a parameter form

$$\phi_{xx}(x, t) = w(x, t) x \in (0,1), (27)$$
$$\phi(0, t) = \mathrm{H}_1(t),$$
$$\phi(1, t) = \mathrm{H}_2(t).$$

Our goal is to obtain the function $\phi(x, t)$. The differentiability of $\phi(x, t)$ with respect to t is also obvious. By using maximum principle [7],we obtained

$$|\phi(x, t)| \leq 2 \max_{0 \leq x \leq 1, 0 \leq t \leq 1} \left(|\mathrm{H}_1|, |\mathrm{H}_2|, \frac{1}{m} |w(x, t)| \right), (28)$$

Where $m = \min_{0 \leq x 1} (\tau^2 e^{-\tau x})$, $\qquad \tau > 0. (3.35)$

Now the finite difference solution of ϕ_i^{j+1} fromw_i^{j+1} is define by (3.33)

$$w_i^{j+1} = \frac{\phi_{i+1}^{j+1} - 2\phi_i^{j+1} + \phi_{i-1}^{j+1}}{h^2},$$
$$1 \le i \le M-1, j \ge 0, \quad (29)$$
$$\phi_0^{j+1} = \mathrm{H}_1^{j+1} \quad j \ge 0,$$
$$\phi_i^{j+1} = \mathrm{H}_2^{j+1} \quad j \ge 0,$$

Observe that w_i^{j+1} is the solution of $(5)-(9)$. by applying discrete maximum principle for two-point boundary problem, we have the following approximation to $\phi(x_i, t_j)$:

$$|\phi_i^{j+1}| \le 2 \max_{0\le i\le M, 0\le j\le N} \left(|\mathrm{H}_1|, |\mathrm{H}_2|, \frac{1}{\eta}|\phi_i^{j+1}|\right) \quad (30)$$

where

$\eta = \min_{0\le i\le M} (\theta e^{-\theta i h}), \quad \theta > 0.$

From (3.1) and equation, (3.36) and , (3.37) we obtained

$|\phi(x_i, t_j) - \phi_i^{j+1}| = O(h^2 + k), \quad i = 0,1,,..M, \ \ j = 0,1,N, \quad (31)$

For every h and k sufficiently small.

NUMERICAL RESULTS AND DISCUSSION

Now we will present the numerical experiment for solving two model problems by using the numerical procedures discussed above in order to give clear overview of the approaches. Each model problem we used various values of h with the fixed value of k and the point $x^* = 0.25$ choose as an interior point of the domain Ω^T for the two model problems. In order to verify the accuracy of $b(t)$ and $\phi(x,t)$ using proposed finite difference schemes the following error calculation are used

$$e_\phi = |\phi(x_i, t_j) - \phi_j^i|,$$

And

$$e_\phi = \|\phi(x_i, t_j) - \phi_j^i\|_\infty.$$

Similarly

$$e_b = \|b(t_j) - b_j^i\|_\infty,$$

And

$$e_b = |b(t_j) - b_j^i|.$$

Problem 1

Consider the problem $(1.1)-(1.5)$ with
Subject to the given initial condition

$$\psi(x) = 2e^x, \qquad 0 \le x \le 1$$

and boundary conditions

$$\mathrm{H}_1(t) = 1 + \frac{1+2t^3}{1+t^3}, \qquad 0 \le t \le 1$$
$$\mathrm{H}_2(t) = e^1 + \frac{e^1(1+2t^3)}{1+t^3}, \qquad 0 \le t \le 1$$

with fixed point $x^* = \frac{1}{4}$,

$$q(t)\ 1.28403 + \frac{1.28403(1+2t^3)}{1+t^3}, \quad 0 \le t \le 1,$$

For which the exact solution is

$$\phi(x,t) = e^x + \frac{e^x(1+2t^3)}{1+t^3}, \quad 0 \le t \le 1,$$
$$b(t) = \frac{3t^2}{2+5t^3+3t^6}, \qquad 0 \le t \le 1.$$

FUDMA Journal of Sciences (FJS) Vol. 5 No. 4, December, 2021, pp 291 - 299

Table 1: Exact and Approximate values of ϕ with $\Delta t = 0.00025$, and $T = 1$

x	Exact $\phi(x,t)$	$\Delta x = 0.1$	$\Delta x = 0.01$	$\Delta x = 0.001$
0.1	2.628177740940	2.6304302555089	2.6304947084293	2.636700615141
0.2	2.762927295189	2.7652952986495	2.7653630561419	2.772134284105
0.3	2.904585606820	2.9070750204148	2.9071462519080	2.914277838170
0.4	3.053506895400	3.0561239439588	3.0561988275688	3.063696543677
0.5	3.210063541719	3.2128147692266	3.2128934922013	3.220775637080
0.6	3.374647018940	3.3775393048979	3.3776220640859	3.385908334890
0.7	3.547668871483	3.5507094481131	3.5507964504554	3.559507539422
0.8	3.729561744103	3.7327582144306	3.7328496774784	3.742006304184
0.9	3.920780463725	3.9241408205911	3.9242369730496	3.933833087411
1.0	4.121803176750	4.1253358227967	4.1254369050972	4.134955543870

Table 2: Exact and Approximate values for $b(t)$ with $\Delta t = 0.000025$ at $T = 1$

T	Exact b(t)	$\Delta x = 0.001$	$\Delta x = 0.01$	$\Delta x = 0.1$
0.1	0.0037488284	0.0019213333	0.0010988100	0.0015511716
0.2	0.0149625711	0.0131291666	0.0122128633	0.0094266548
0.3	0.0334670499	0.0316632234	0.0307696470	0.0280435443
0.4	0.0588179936	0.0570175453	0.0534136161	0.0534167675
0.5	0.0901937757	0.0883597778	0.0874453459	0.0846108763
0.6	0.1263342890	0.1243966550	0.1234332450	0.1205477702
0.7	0.1655487586	0.1638590106	0.1626984449	0.1598373434
0.8	0.2058064870	0.2039553148	0.2420569876	0.2394062424
0.9	0.2449067167	0.2431393330	0.2422563455	0.2396507841
1.0	0.2807017544	0.2789248712	0.2786433109	0.2753688249

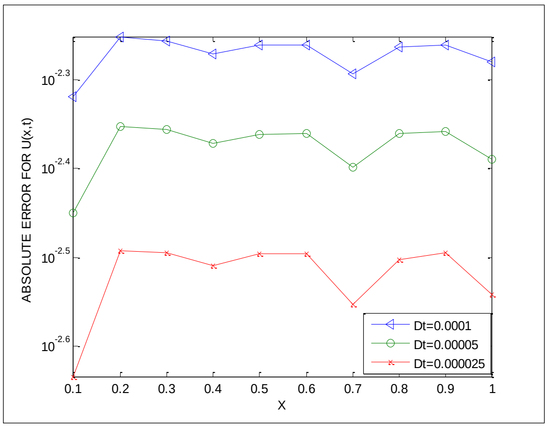

Figure 2: Absolute error for $\phi(x,t)$ and $\Delta x = 0.01, T = 1$

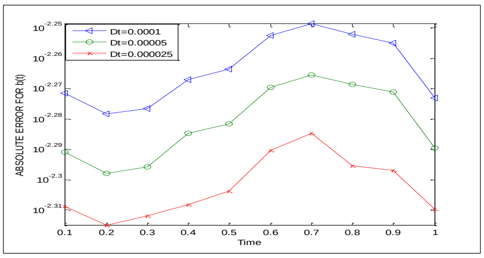

Figure 3: Absolute error for $b(t)$ and $\Delta x = 0.01$ at each time level

Problem 2

Consider the problem $(1)-(5)$ with
$\phi_t = b(t)\phi_{xx}$,inΩ^T
subject to the initial condition

$$\psi(x) = e^{\frac{x}{2}}, x \in [0,1],$$

and the boundary conditions

$$\mathrm{H}_1(t) = \frac{1+2t^3}{1+t^3} + \sin\left(\frac{t}{2}\right), \qquad t \in [0,1],$$

$$\mathrm{H}_2(t) = \sqrt{e^1}\left[\frac{1+2t^3}{1+t^3} + \sin\left(\frac{t}{2}\right)\right]. \qquad t \in [0,1]$$

With fixed point $x^* = 0.25$,

$$q(t) = \frac{1.13315(1+2t^3)}{1+t^3} + 1.13315\sin\left(\frac{t}{2}\right),$$

for which the exact solution is

$$\phi(x,t) = e^{\frac{x}{2}}\left[\frac{1+2t^3}{1+t^3} + \sin\left(\frac{t}{2}\right)\right],$$

$$b(t) = \frac{2\left[6t^2 + (1+t^3)^2\cos\left(\frac{t}{2}\right)\right]}{(1+t)^3\left[1+2t^3+(1+t^3)\sin\left(\frac{t}{2}\right)\right]}.$$

Table 3: Exact and approximate values of ϕ with $\Delta x = 0.01, x^* = 0.25$ and $T = 1$

x	Exact ϕ	$\Delta t = 0.0001$	$\Delta t = 0.00005$	$\Delta t = 0.000025$
0.1	2.080912856	2.0799698300	2.0793100351	2.0798145576
0.2	2.187603540	2.1865573436	2.0793100351	2.1863920237
0.3	2.299764373	2.2981636342	2.2979172032	2.2983145576
0.4	2.417675813	2.4159052213	2.4148335668	2.4162854439
0.5	2.541632703	2.5395335842	2.5396375432	2.5403305592
0.6	2.671944999	2.6701207451	2.6700438147	2.6707356278
0.7	2.808938548	2.8067963407	2.8087237854	2.8078141151
0.8	2.952955907	2.9509682421	2.9516299007	2.2947399658
0.9	3.104357192	3.1026559118	3.1060514115	3.1026144625
1.0	3.263520991	3.2614316231	3.2654559224	3.2617257632

Table 4: Exact and approximate values of $b(t)$ for $\Delta x = 0.01$ at $T = 1$

T	Exact b	$\Delta t = 0.0001$	$\Delta t = 0.00005$	$\Delta t = 0.000025$
0.1	1.979634405	1.976432650	1.977233816	1.973481614
0.2	2.014562188	2.011267253	2.012114123	2.011564574
0.3	2.098282728	2.094776455	2.094786440	2.095352100
0.4	2.222861861	2.219497253	2.221629098	2.220161956
0.5	2.378381379	2.374881765	2.376814069	2.371243057
0.6	2.552887320	2.569284432	2.559333432	2.549777645
0.7	2.732893051	2.729096564	2.729422650	2.729689346
0.8	2.904389231	2.901266201	2.905432431	2.901565492
0.9	3.054179413	3.050364431	3.046972533	3.051177134
1.0	3.171252165	3.167971771	3.167856549	3.168307690

FUDMA Journal of Sciences (FJS) Vol. 5 No. 4, December, 2021, pp 291 - 299

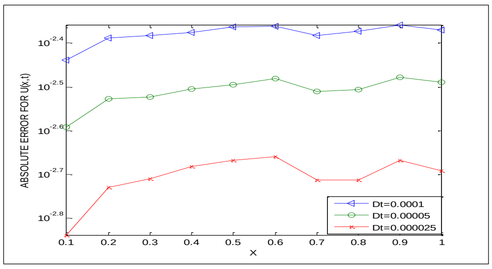

Figure 4: Absolute errors for $\phi(x,t)$ with $\Delta x = 0.01$at $T = 1$

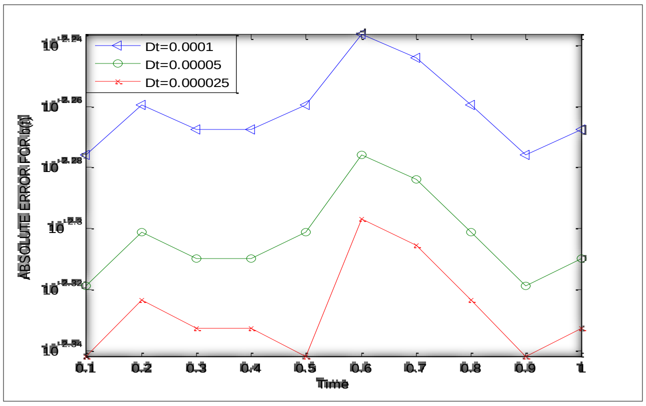

Figure 5: Absolute errors for $b(t)$ for $\Delta x = 0.01$ at each time level

CONCLUSION

The following conclusion can be drawn from the two presented numerical problems**:**

We used backward time centered space to compute the numerical solution of $\phi(x,t)$ at $T = 1$, with the various value of Δt and Δx. The errors plotted in Figure.2, and Figure.4, respectively, we can easily observed that the errors decreases rapidly when Δt is decreases with the fixed value of $\Delta x = 0.01$. It is obvious to see that on the both side of the boundary of Figure 2. The errors is nearly zero because of the existing of boundary conditions on both side, Therefore the error at the boundary points are sufficiently small.

The numerical errors for the diffusion coefficient $b(t)$ at different time level that are plotted In Figure 3 and Figure 5, respectively it was observed that if Δx is fixed as $\Delta x = 0.01$, the error of $b(t)$ decreases rapidly when Δt decreases. Likewise, when Δt is fixed the as $\Delta t = 0.000025$, the error for $b(t)$ decrease rapidly when Δx is decreasing. It was also observed that the error is nearly zero when $(t \to 0)$. This is sensible because the initial condition is logically existing so therefore when $t = 0$ the errors disappear.

CONCLUSION AND FUTURE WORK

In this paper backward time centered space of the finite difference scheme were applied for recovering time dependent diffusion coefficient of one-dimensional parabolic inverse problem. The suggested numerical approaches for solving these two model problems are very reasonable and these test experiment backed our theoretical expectation. Using the backward time centered space formula for the one –dimensional diffusion problem with an additional measurement defined our model well. Several of issues can be ten dents as subject for future examinations in this field. We can mentioned some of them in the following: We can extend this research to two or three dimensional problems, Employing Crank Nicolson finite difference techniques to solve the current problems, we can also extend to higher-order accurate finite difference methods, we can also apply on explicit formula which is conditionally stable, dealing with the more difficult extra measurements, using new numerical measures for solving Backward time centered space problems by using the described methods for simplifying the present problem with the Neumann's boundary condition.

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