*FUDMA Journal of Sciences (FJS)*
*ISSN online: 2616-1370*
*ISSN print: 2645 - 2944*
*Vol. 5 No. 4,December, 2021, pp 167 - 173*
*DOI: https://doi.org/10.33003/fjs-2021-0504-803*

# COMPARATIVE STUDY ON CHARGED AND NEUTRAL PION-NUCLEON COUPLING CONSTANTS USING YUKAWA POTENTIAL MODEL

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**ABSTRACT**

The meson field theory based on the Yukawa model for nuclear forces is estimated using a simple relationship between the charged and neutral pion- nucleon coupling constant. This signifies that the charged coupling constant is larger compared to the value obtains for neutral pions. Therefore since np interaction is stronger than the pp interaction, we look at the value obtained for charged pion – nucleon constant which gives good agreement with one of the recent experimental values, and then the splitting between the charged and neutral pion – nucleon coupling constants is practically the same as that between charged and neutral pion mass. In this case, the mass difference between the charged and neutral pion is also calculated to assess the amount of charge dependence of the Neutron – Neutron scattering length.

PACS numbers: 13.75.Cs, 13.75.Gx, 14.20.Dh, 14.40.Be, 25.40.Cm, 25.40.Dn, 03.70.+k
**Keywords:** Nucleaar forces, *pp* interaction, splitting, pion-nucleon

## INTRODUCTION

The investigation of nucleon-nucleon and pion–nucleon interaction Alarcon *et al*. (2013) plays an important role in the understanding of the pion–nucleon coupling constants which are said to have fundamental characteristics in nuclear force with a great application in the field of theoretical physics. Their qualitative undertakes the knowledge of their exact instrumental values; this gives us the understanding of a broad variety of hadron and nuclear physics phenomena (Arndt *et al.*, 2006). The of study the pion – nucleon coupling constants was investigate on nucleon – nucleon scattering length to assess the level of charge dependence.

The charge independence of the pion- nucleon coupling constant which is likely to be violated or arise as a result of between a pion – nucleon coupling constant on a neutral and charged pions, is the core problem that had attracted much attention of many researchers for decade. These differences also yielded different values of the charge pion nucleon constant $g^2_{\pi^\pm}$ rendering it of most importance as in the result of the recent experiment estimated a value of $g^2_{\pi^\pm}$ which varies between 13.54 and 13.49 (Arndt *et al.,* 1995) and (Arriola *et al.*, 2016). The result of the neutral pion – nucleon constant$g^2_{\pi^0}$, gives an estimated value of 13.6 / 13.6 (Alarcon *et al.*, 2013).

Thus, the estimated measured values of the charged pion constant $g^2_{\pi^\pm}$ as in Arndt *et al.* (2004) are slightly close to that of the neutral – pion constant $g^2_{\pi^0}$ which falls within the range of the charged independence based on the pion nucleon coupling. Similarly, the results of the measurement of Alarcon *et al.*(2013), yield large significant values of $g^2_{\pi^\pm}$ than that of $g^2_{\pi^0}$ . With this, the results of the charge independence on the pion nucleon constant is said to be the open fundamental problem that calls for more studies with further experimental values, and with theoretical computation (Babenko *et al.*,2015).

From the investigation of pion nucleon coupling constant based on the Yukawa model (Yukawa 1938) for nucleon-nucleon interaction and with the help of contemporary experimental data on low-energy parameters of nucleon-nucleon scattering. It is known that the nucleon-nucleon system cannot obtained from the Yukawa potential is not accurately as soon as its parameters are defined and selected using fundamental quantities of the field theory. The coupling constants and then pion –masses, by selecting of the parameter value of the Yukawa potential, we tend to rely on the measured low energy parameters of nucleon-nucleon scattering in the effective range theory.

The Yukawa model framework is accepted as the means of analyzing the pion nucleon coupling constant since the Yukawa model is said to correspond to one pion exchange which is corresponding to its dominant mechanism at the lowest collision energies on the nucleon interaction. This signifies long-range interactions. The heavier $\ell -$ and $w -$meson exchange are the two pion exchanges which are power at mediums and small distances with quark-gluon degrees of freedom gain remarkably at the smallest interaction ranges. In a nutshell, determining the pion-nucleon coupling constant is said to be characterized by pion-nucleon interaction, which may likely depend on the long-range – data ( peripheral) nucleon-nucleon interaction which is one of the pion exchange are been dominant Yukawa (1938).

In quantum field theory, Pseudo-scalar (PS) or a pseudo-vector (PV), lagrangian formulae are described in the interaction of pion nucleon as Yukawa (1938).

$$\mathcal{L}^{PV}_{\pi\mathrm{N}} = g\pi\sqrt{4\pi}(i\bar{\psi}\gamma 5\psi)\phi \qquad (1)$$

$$\mathcal{L}_{\pi \mathrm{N}}^{PV} = \frac{f\pi}{M_s}\sqrt{4\pi}(\psi\gamma_\mu\gamma_5\psi)\delta^\mu\phi \quad (2)$$

$$\frac{g_\pi}{M_1+M_2} = \frac{f\pi}{M_{\pi^\pm}} \quad (3)$$

In this case $M_1\ and\ M_2$ are the masses of interacting nucleons. Therefore the $\pi^0$ and $\pi^\pm$ pseudo-scalar coupling constants, $g\pi^0$ and$g\pi^\pm$, where $f\pi^0$ and $f\pi^\pm$are the pseudo-vector constants, which be interpolate through (Bergervo *et al.,* 1990).

$$g\pi^0 = \frac{2M_p}{M_{\pi^\pm}}f\pi^0 \quad (4)$$

$$g\pi^\pm = \frac{M_p+M_n}{M_{\pi^\pm}}f\pi^\pm \quad (5)$$

Where $M_p$ is the mass of proton and $M_n$ is the mass of neutron respectively

**Deduces of some major equations between charged and neutral pion – nucleon coupling constants in the Yukawa model**

According to the Meson-field theory, there is a strong interaction between two nucleons at their lowest energies which is dominated by the exchange of virtual pions, this helps in determining the long-range nucleon-nucleon interaction. The nucleon-nucleon interaction by meson field theory explains the classical one – pion – exchange potential, which is referred to as Yukawa potential, for the pure singlet $^1S_0$ state is expressed in simpler and well-known forms as (Yukawa H, 1938).

$$V_Y(r) = -\frac{V_0e^{-\mu r}}{\mu r} \quad (6)$$

Whereby r and $\mu$ are the distance between the nucleon and the pion mass $m(\pi)$.

$$\mu = \frac{m_\pi C}{\hbar} \quad (7)$$

Where C and $\hbar$ are the speed of light with Planck constant. According to (7) gives us the nuclear – force range $R \equiv {}^1/_\mu$ which is inversely proportional to the pion mass $m_\pi$ and has a small value of $R{\sim}1.4$. Then the depth of the Yukawa potential ($V_0$) is expressed through pseudo-vector pion-nucleon coupling constant $f\pi$ (Blomgren *et al.,* 2000).

$$V_0 = m_\pi C^2 f_\pi^2 \quad (8)$$

This is the direct consequence of the lagrangian presented by the meson–field theory. In the case of interaction between two charged protons mediated by the exchange of the parameters of the Yukawa potential $V_0^{pp}$ and a neutral $\pi^0$ meson. The $V_{pp}$ is the expression of $\pi^0$ mass $m_{\pi^0}$ and the coupling constant $f\pi^0$ action involves the exchange of both the exchanged $\pi^\pm$ mesons and neutral meson$\pi^0$. The estimated $\mu_{np}$ and $V_0^{np}$ parameters of the potential (6) involve the substitutions of the average pion – mass (Bohr, 1969).

$$\bar{m}_\pi = \frac{1}{3}(m_{\pi^0} + 2m_{\pi^\pm}) \quad (9)$$

Pion – nucleon coupling constant average is given as (Bugg, 2004).

$$\bar{F}_\pi^2 = \frac{1}{3}(f_{\pi^0}^2 + 2f_{\pi^\pm}^2) \quad (10)$$

Unfortunately, there is no accurate quantitative representation of the nucleon-nucleon system which can be deduced from the Yukawa potential as its parameters are defined and also selected from the fundamental quantities of the field theory for both of the pion masses and coupling constants. As the matter of fact in this study, the parameters of the Yukawa potential are frequently assigned values with the nucleon-nucleon scattering in the effective range theory at low energy (Bugg *et al.*, 1973). The measured purely nuclear proton-proton scattering length and effective range one can use to estimate the "effective" mass $M_{\pi^0}^Y$ and pion – nucleon coupling constant $\left(f_{\pi^0}^Y\right)^2$ for a neutral $\pi^0$ meson. For instance, the Yukawa forms for the proton-proton potential. This the effective mass $M_{\pi^0}^Y$ and the pion – nucleon coupling constant $\left(f_{\pi^0}^Y\right)^2$gives values prove to exceeds the directly measured values de Swart *et al.* (1998) this yield the given expression.

$$M_{\pi^0}^Y = C_1 m_{\pi^0}, \left(f_{\pi^0}^Y\right)^2 = C_2 f_{\pi^0}^2 \quad (11)$$

Whereby $C_1$ and $C_2$ are computed numerically Dumbrajs *et al.*(1998), it said to be quite natural to assume that equation as analog which can hold for the masses and pion – nucleon coupling constants of charged $\pi^\pm$ mesons, and let the average pion – mass and the average pion nucleon coupling constant.

The relationship between neutron-proton Yukawa potential $\mu_{np}$ and $V_0^{np}$ is associated with the analog parameter of proton-proton potential, $\mu_{pp}$ and $V_0^{pp}$(Yukawa, 1938).

$$\mu_{np} = \frac{\bar{m}_\pi}{m_{\pi^0}} \mu_{pp} \tag{12}$$

$$V_0^{np} = \frac{\bar{m}_\pi}{m_{\pi^0}} \frac{\bar{f}_\pi^2}{f_{\pi^0}^2} V_0^{pp} \tag{13}$$

Equation (10) and (13) implies the relationship between the pseudo-vector pion-nucleon coupling constants for the charged and neutral pion (Ebel *et al.,* 1971).

$$f_{\pi^\pm}^2 = C_f^2 f_{\pi^0}^2 \tag{14}$$

Where the factor $C_f^2$ is expresses as (Yukawa, 1938).

$$C_f^2 = \frac{1}{2}\left(3\frac{m_{\pi^0}}{m_\pi}\frac{V_0^{np}}{V_0^{pp}} - 1\right) \tag{15}$$

Equations (4), (5), and (14) we obtain that the pseudo-scalar charged and neutral pion nucleon coupling constants are interrelated as (Dumbrajs *et al.,* 1983)?

$$g_{\pi^\pm}^2 = C_g^2 g_{\pi^0}^2 \tag{16}$$

Where

$$C_g^2 = \left(\frac{M_p + M_n}{2M_p}\right) C_f^{2^{\,2}} \tag{17}$$

Equation (15) is the measured pion – masses. In the considered model based on meson field theory, it proportion between the neutral pion – nucleon coupling constant and the charged is said to be fully estimated between the depth of neutron-proton and proton Yukawa potentials, $V_0^{np} / V_0^{np}$. similarly, we illustrate that the neutron-proton potentials are appreciably deeper than the proton-proton one $V_0^{np} > \frac{\bar{m}_\pi}{m_{\pi^0}} V_0^{pp}$. The factor in equation (15) is more than two, therefore we considered formalizing the charged pion nucleon coupling constants said to be larger than the neutral ones (Ebel *et al*., 1971).

$$f_{\pi^\pm}^2 > f_{\pi^0}^2,\ g_{\pi^\pm}^2 > g_{\pi^0}^2 \tag{18}$$

In this case, we can observe that the charged pion-nucleon constant is greater than the neutral one. We also considered a scheme of a strong $np$ and $pp$ interaction in the spin-singlet $^1S_0$ state, which is a reliably established phenomenon. It was observed to be a single length of $np$ scattering which is larger than the purely nuclear $pp$ scattering length of Ebel G *et, al*.,(1971).

$$|a_{np}| > |a_{pp}| \tag{19}$$

About literature review of experimental works of (Ebel *et al*., 1971).gives the values of pion - nucleon coupling constants said to obey the inequalities (Ericson *et al.*,1995). Similarly, the results of other experiments (Baru *et al.,* 2011) indicate a constant charge indep0endence of the pion nucleon coupling constant. That is the approximate equalities are given by $f_{\pi^\pm}^2 \cong f_{\pi^0}^2$ and $g_{\pi^\pm}^2 \cong g_{\pi^0}^2$ holds therein within the experimental uncertainties. In our proposed model, the violation of change independent of the pion – nucleon coupling constant reveals itself which is associated with charge dependence of nuclear forces.

**Estimation of charge independence breaking of the pion – nucleon coupling constant**

The Radius $R_{NN} = 1/\mu_{NN}$ and the depth $V_0^{NN}$ of the Yukawa potential based on nucleon-nucleon can be deduced from measured low energies parameters of the expansion in the effective range. Employing the unknown values of the pure nuclear low–energy parameters on nucleon-nucleon scattering (Arriola.*et al.,* 1957).

$$a_{pp} = -17.3,\ r_{pp} = 2.85,\ a_{np} = -23.72 \tag{20}$$

The $np$ and $pp$ interaction we obtain from the Yukawa – potential parameters. We then apply the phase variable approach (Arndt.*et al.,* 2004).

$$V_0^{pp} = 44.83\ MeV,\ \mu_{pp} = 0.8392 \tag{21}$$

$$V_0^{np} = 48.01\, MeV,\ \mu_{np} = 0.8583 \quad (22)$$

Therefore the neutron-proton interaction was obtained using equation (12) and later on we substitute for the scattering length as (20) the results of the $np$ scattering effective range of the Yukawa potential is given as (Arndt *et al*., 2006).

$$r_{np} = 2.69 \quad (23)$$

These results said to be in agreement with the measured value (Babenko *et al.*, 2016).

$$r_{np} = 2.70 \quad (24)$$

We then have (Arndt *et al.*, 2000).

$$V_0^{np} > \frac{\bar{m}_\pi}{m_{\pi^0}} V_0^{pp} = 45.843\, MeV \quad (25)$$

This satisfied the aforementioned condition leading to inequalities. We then substitute the derived value (21) and (22) of the $np$ and $pp$ depth in the potentials with the measured pion and nucleon masses Arriola *et al.* (2000).as in Ericson *et al.*(1995) and Bugg *et al.* (1957), for the neutral pion – nucleon coupling constant and the factors is related to charged pion.

$$C_f^2 = 1.073,\ C_g^2 = 1.074 \quad (26)$$

The values of $C_g^2$ and $C_f^2$ are likely to be the same or similar as it was expected to be obtained from the proximity of the neutron masses band the proton (Arriola *et al.* 2000).

In contrast with the charged pion – nucleon coupling constant $g_{\pi^\pm}^2$ there is no controversy to the reliability of the measured neutral constant $g_{\pi^0}^2$ from some literature$g_{\pi^0}^2 = 13.52$, said to be the best measurement which is fully agreed with the recent experimental values of $g_{\pi^0}^2 = 13.55$ and $g_{\pi^0}^2$=13.61 quoted in Babenko *et al.* (2016) by substituting equation (16) the latest experimental values of the pseudo-scalar and neutral constant.

$$g_{\pi^0}^2 = 13.53 \quad (27)$$

Where by $C_g^2$ value for the pseudo-scalar charged pion-nucleon coupling constant we find

$$g_{\pi^\pm}^2 = 14.53 \quad (28)$$

By employing equation (Babenko *et al.*, 2016) the pion – nucleon coupling constants of the pseudo-vector are determined as.

$$f_{\pi^0}^2 = 0.0748 \quad (29)$$

$$f_{\pi^\pm}^2 = 0.0803 \quad (30)$$

The value of $g_{\pi^\pm}^2$ in (28) derived by using the Yukawa – model with the recent experimental values

$$g_{\pi^\pm}^2 = 14.53 \quad (31)$$

**THE UPPSALA NEUTRON RESEARCH GROUP**

The reported experimental result of Uppsala Research Group gives the measurement value as it is in equation (31) which is similar to the measurement of the same group as, $g_{\pi^\pm}^2 = 14.62$ and $g_{\pi^\pm}^2 = 14.74$ and to the obtained values of $g_{\pi^\pm}^2 = 14.28$ earlier obtained in (Alarcon *et al.*, 2013). Closely, the charged pion – nucleon coupling constant reported by Nijmegen Group as$g_{\pi^\pm}^2 = 13.54$, these coincide with neutral$\pi^0$.

The result obtains shows the change independence breaking of the pion-nucleon coupling has the difference between the charged and neutral pion–nucleon coupling constants (Alarcon *et al.*, 2013).

$$\Delta f_{CIB}^2 \equiv f_{\pi^\pm}^2 - f_{\pi^0}^2,\ \Delta g_{CIB}^2 \equiv g_{\pi^\pm}^2 - g_{\pi^0}^2 \quad (32)$$

Equation (14) and (16) follows the explicit expressions for the quantities

$$\Delta f_{CIB}^2 = (C_f^2 - 1) f_{\pi^0}^2 \quad (33)$$

$$\Delta g_{CIB}^2 = (C_f^2 - 1) g_{\pi^0}^2 \quad (34)$$

By substituting the numerical value Babenko *et al*.(2015) for the factor $C_g^2$, in this both the pseudo-scalar charged and neutral pion – nucleon coupling constants, as well as the reliably measured value (27) for the neutral constant $g_{\pi^0}^2$. In this case, there is an absolute breaking of charge independence of pion nucleon coupling constant in the Yukawa potential model is given as (Alarcon *et al.*, 2013).

$$\Delta g_{CIB}^2 = 1.006 \quad (35)$$

In its relative units, the charge independence breaking of the pion – nucleon coupling constants is given as follows (Alarcon *et al.*, 2013).

$$\frac{\Delta f_{CIB}^2}{f_{\pi^0}^2} = \left(C_f^2 - 1\right) = 0.0729 \quad (36)$$

$$\frac{\Delta g_{CIB}^2}{g_{\pi^0}^2} = \left(C_f^2 - 1\right) = 0.0744 \quad (37)$$

From this, we concluded that the violation of charge independence of the pion – nucleon coupling constant is approximately about 7.4%.

**4.0 The connection between charge splitting of the pion nucleon coupling constant and the pion – mass** as in equation (14) and (26) the neutral pseudo-vector and the charged pion – nucleon coupling constants are in the ratio of (Alarcon *et al.*, 2013).

$$\frac{f_{\pi^\pm}}{f_{\pi^0}} = C_f = 1.036 \quad (38)$$

This approximately is the same with the ratio between measured masses of the charged and neutral pion (Babenko *et al*., 2015)

$$\frac{m_{\pi^\pm}}{m_{\pi^0}} = 1.034 \quad (39)$$

We have a high precise as

$$\frac{f_{\pi^\pm}}{f_{\pi^0}} \cong \frac{m_{\pi^\pm}}{m_{\pi^0}} \quad (40)$$

$$f_{\pi^\pm}^2 = 0.0799 \quad (41)$$

Therefore for the pseudo-scalar coupling constant as it is in equation (5) and (41) yield a values of

$$g_{\pi^\pm}^2 = 14.478 \quad (42)$$

These values were computed and formulae (28) and (16). Then, from (27) and (42), the pion – nucleon coupling constant break the charge independence which arises from the difference in mass between the $\pi^\pm$ and $\pi^0$ mesons ($\Delta m_\pi = 4.51 MeV$) which is estimated as follows.

$$\Delta g_{CIB}^2 = 0.956 \quad (43)$$

With a relative units approximately 7% while the pseudo-scalar coupling constants obey the approximately equation

$$\frac{m_{\pi^\pm}}{m_{\pi^0}} \cong \frac{g_{\pi^\pm}}{g_{\pi^0}} \quad (44)$$

Which is empirically estimated to equation (40) for the pseudo-vector coupling constants for formulae (44) which attracted the attention of researchers as in Babenko *et al*.(2015) then it was analyzed the low–energy $pp$ $and$ $np$

The Scattering parameters by using the traditional classical Yukawa model by relying on measured values (Alarcon *et al.*, 2013) and (Babenko *et al.*, 2015).

Equation (40) can be rewritten as

$$f_{\pi^\pm} R_{\pi^\pm} \cong f_{\pi^0} R_{\pi^0} \quad (45)$$

Where the meson exchange potential at a radius $R_{\pi^0}$ for the $\pi^0$ exchange

$$R_{\pi^0} \equiv \frac{\hbar}{m_{\pi^0} c} = 1.462 \quad (46)$$

This is very larger than the radius $R_{\pi^{\pm}}$ with reference to the $\pi^{\pm}$ exchange

$$R_{\pi^{\pm}} \equiv \frac{\hbar}{m_{\pi^{\pm}}C} = 1.414 \quad (47)$$

The pion – nucleon coupling constant $f_{\pi}$ and the radius of the meson exchange potential$R_{\pi}$. Due to the mass splitting between the $\pi^{\pm}$ and $\pi^{0}$ meson which is proved to be charge-dependent quantities. The obtained result shows that the $\pi^{\pm}$ mesons are heavier than the $\pi^{0}$ meson these effectively raises the charge pion constant $f_{\pi^{\pm}}$ for the neutral – pion constant $f_{\pi^{0}}$and then reduces the effect of $\pi^{\pm}$ exchange radius $R_{\pi^{\pm}}$ concerning the $\pi^{0}$ exchange radius $R_{\pi^{0}}$. Our result shows that the pion nucleon constant $f_{\pi}$ and the $\pi$- exchange radius $R_{\pi}$ given as (Alarcon *et al.*, 2013)

$$f_{\pi} R_{\pi} = B \quad (48)$$

Let substitute the measured value of the neutral pion-nucleon constant

$$F_{\pi^{0}} = 0.294 \quad (49)$$

The value of (46) for the $\pi^{0}$ exchange radius, where by B is estimated as

$$B = 0.399 \quad (50)$$

Thus, we developed a relationship to correlate the pion- nucleon coupling constant $f_{\pi}\ and\ \pi$ – exchange radius $R_{\pi}$ through the given relation

$$f_{\pi} \cong \frac{B}{R\pi} \quad (51)$$

**NUCLEON – NUCLEON SCATTERING LENGTH CHANGE DEPENDENCE**

$$\Delta a_{CIB}^{expt} \equiv a_{pp} - a_{np} \quad (52)$$

According to the (19) and (20) the difference in the experimental values are given as

$$\Delta a_{CIB}^{expt} = 6.423 \quad (53)$$

With relative units of$\sim 30\%$, that the difference is negligible beyond the experimental prediction which signifies that the hypothesis of the charge independence of the nuclear forces is violated at low energies (Babenko *et al.*, 2015). The recorded charge dependence of the nuclear forces is often influenced by the difference between the charged and the neutral pions Koch R., *et, al*.,(1980). Whereby the half difference $\Delta a_{CIB}^{expt}$ has been shown recorded due to the mass difference between the $\pi^{\pm}$ and $\pi^{0}$ meson. The singlet $np$- scattering length value where computed, we assume that the inequality Alarcon *et al.* (2013) is exact.

$$a_{np} = -22.90 \quad (54)$$

This differ in the scattering length of proton – proton as

$$a_{pp} = -17.34 \quad (55)$$

Where by the scattering length of neutron – proton scattering length is

$$\Delta a_{CIB}^{\pi} = 5.59 \quad (56)$$

This is consistent with the experimental values (Arriola *et al.*, 2000). Thus, in our model framework, the charge independence of the nuclear forces is fully violated due to the difference in mass between the charge and the neutral pions. It's also look at the scattering length of proton-proton and neutron-proton by predicting their differences. $\Delta a_{CIB}^{\pi}$, which is approximately 90% of its corresponding experimental values as$\Delta a_{CIB}^{expt}$. Similarly to this, the $\Delta a_{CIB}^{\pi}$ values deduced in the previous analyses reached approximately 50% of $\Delta a_{CIB}^{expt}$ (Alarcon *et al.*, 2013).

**CONCLUSION**

In this study, the developed physical consistent formalization of the nucleon-nucleon interaction by using Yukawa potential based on the meson field model. The parameters are $np$ and $pp$ systems in the spin-singlet state $^{1}S_{0}$ are determined by empirical relation to some characteristic of the pion – nucleon interaction. The model makes use of the pion masses $m_{\pi}$ and the pion–nucleon constant as the framework of the study. The employed formulated relationship from equation (14 – 17) between the charged and neutral pion – nucleon coupling constants.

The result obtained shows that the neutral constant $f_{\pi^{0}}^{2}$ is less than the charged pion – nucleon coupling constant $f_{\pi^{\pm}}^{2}$. So that the pseudo-vector and the pseudo-scalar pion – nucleon coupling constant, $f_\pi^2$ and $g_\pi^2$ said to violate the charge independence of the nuclear forces.

The result of the empirical relation for pseudo-scalar charged constantly to yield a value of $g_{\pi^\pm}^2 = 14.536$, which is similar with the experimental value reported by Uppsala Neutron Research Group (6) as $g_{\pi^\pm}^2$=14.52.

The result also shows, difference between the charged and neutral pseudo-scalar pion- nucleon coupling constants, $\Delta f_{CIB} \equiv f_{\pi^\pm} - f_{\pi^0}$, which is deduced as $\Delta f_{CIB} = 0.00929$. While its relative units, that is the ratio $\frac{\Delta f_{CIB}}{f_{\pi^0}}$= 3.60% is shown to be closely the same as $\frac{\Delta m_\pi}{m_{\pi^0}}$=3.39%. Therefore, the relative units of the pion-nucleon coupling constant splitting seem to be the same as that of the pion – masses.

The illustration for both the pion – nucleon coupling constant $f_\pi$ and the $\pi$ – meson exchange radius $R_\pi$ are all charged dependent to a high precision with a product of $f_\pi R_\pi$ with a charge independent quantity. Their relative unit is given as the difference between their products for the charged pion and neutral pion said to be less than 0.19%.

In the Yukawa potential model, we approximate like 89.9% of the difference between the experimental values of the $pp$ and $np$ scattering length. Whereby the $\pi^\pm$ and $\pi^0$ mesons is the mass difference between them as follows $\Delta m_\pi = 4.548 MeV$.

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