



FITTING OF A TRANSFORMATION GEOID MODEL TO THE GRAVIMETRIC-GEOMETRIC GEOID MODEL OF BENIN CITY

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ABSTRACT

The application of the transformation geoid model in Benin City has necessitated its fitting to the existing gravimetric-geometric geoid model of the study area. The transformation geoid model was determined using the Kotsakis (2008) model for the transformation of global geoid heights to local geoidal undulations. To obtain its accuracy, the root mean square error (RMSE) index was applied. The computed accuracy is 2.0172 m. To apply the determined geoid model in the study area, as well as improving on the computed accuracy, the model was fitted to the gravimetric-geometric geoid model of the study area. The fitting result shows that geoid heights can be computed using the determined geoid model with an accuracy of 1.1041 m in the study area.

Keyword: fitting, gravimetric-geometric, geoid, model, transformation

INTRODUCTION

The geoid is an equipotential sea surface that is extended through the land. It is a surface adopted as a reference for the vertical coordinate system. The local geoid model of Nigeria has not been determined. As a result, different local geoid models have been established in various parts of the country. The local geoid model of Benin City was determined with the gravimetric-geometric method, which involves the combination of gravimetrically and geometrically obtained data. The transformation of the global geoid model to local has been proposed by Okeke and Nnam (2017) for the determination of the local geoid model of Nigeria. The method has to do with the transformation of the geoid heights from the global geopotential model such as EGM 08 to local geoid heights using the transformation model given by Kotsakis (2008). The proposed method has been tested by Okeke and Nnam (2017) using the geoid heights from EGM 08 in part of the Federal Capital Territory, Abuja and accuracy of 0.14 cm was obtained. To verify the consistency of the accuracy of the proposed method in other parts of the country, it was applied and compared with the local gravimetric-geometric geoid model of Benin City. The accuracy obtained was 2.0172 m. It is inconsistent with the

one obtained by Okeke and Nnam (2017). To apply the proposed method geoid model in the study area for the computation of geoidal undulations, the accuracy of the model needs to be improved. This can be done by fitting the transformation geoid model to the local geoid model of the study area. As a result, this paper presents the fitting of a transformation geoid model to the gravimetric-geometric geoid model of Benin City.

The Study Area

According to Oduyebo *et al.* (2019), Benin City is the capital of Edo State in Southern Nigeria. It is a City approximately 40 kilometres north of the Benin River. The City is also linked by roads to Asaba, Sapele, Siluko, Okene, and Ubiaja and is served by air and the Niger River delta ports of Koko and Sapele (Oduyebo *et al.*, 2019). The City is made up of three Local Government Areas, Oredo LGA, Ikpoba Okha LGA and Egor LGA. It has a total population of 1,782,000 according to the 2021 NPC projection. It covers a total area of about 1,204 km². Benin City is bounded by UTM zone 31 coordinates 660000 mN and 712500 mN, and 770000 mE and 815000 mE (Oduyebo *et al.*, 2019). Figures 1 and 2 show the maps of the study area.

Properties of the WGS 84 and Clarke 1880 Ellipsoids

The equatorial radius (a) and the flattening (f) of the WGS 84 and the Clarke 1880 ellipsoids are respectively 6378137 m and 1/298.257223563, and 6378249.145 m and 1/293.465 (Eteje *et al.*, 2019).

Global Earth Gravimetric Models

According to Idrizi (2013), in the absence of local/state gravimetric networks, geodetic practice often uses global Earth Gravimetric Models, which includes data for the entire area of the world. Idrizi (2013) further stated that, to date, the National Geospatial-Intelligence Agency (NGA) has established three global Earth gravimetric models in the years 1984, 1996 and 2008, recognized as EGM84, EGM96 and EGM08. The Geosciences Division in the Office of Geomatics at NGA is responsible for collecting, processing, and evaluating gravity data (free-air and Bouguer gravity anomalies). These data are then used to compute gravimetric quantities such as mean gravity anomalies, geoid heights, deflections of the vertical, and gravity disturbances. All of these quantities are used in World Geodetic System 1984 support, navigation systems, mapping projects, and different types of surveys (Idrizi, 2013).

Earth's Gravitational Model (EGM) derived geoidal undulations, N_{GGM} (Long and Medium Wavelength) is mathematically expressed as (Odumusu *et al.*, 2016):

$$N_{GGM} = \frac{GM}{R^2} \sum_{n=0}^{\infty} \left(\frac{a}{r}\right)^n \sum_{m=2}^n [S_{nm} \sin m\lambda + C_{nm} \cos m\lambda] P_{nm} \cos \varphi \quad (12)$$

Where,

GM = Gravity Mass Constant of GEM

γ = Normal Gravity

a = Equatorial Scale Factor of GEM

r, θ , = Geocentric Radius, Spherical Co-latitude and Longitude of Computation Point.

P_{nm} = Fully – Normalized Legendre Function

C_{nm}, S_{nm} = Fully – Normalised Coefficients of GEM

Computation of Root Mean Square Error (RMSE)

The fitting of a new local geoid model to an existing local geoid model requires the computation of the RMSE of the model to obtain its accuracy. The computation of the RMSE of the fitted geoid model is done by comparing the geoid heights of points from the existing and the new geoid models to obtain the residuals. The computed residuals and the total number of points are used for the computation of the RMSE of the fitted geoid model. The model for the computation of RMSE as given by Oduyebo *et al.* (2019) is

$$RMSE = \pm \sqrt{\frac{V^T V}{n}} \quad (13)$$

Where,

$$V = N_{Existing} - N_{New} \text{ (Residual)}$$

$N_{Existing}$ = Geoid Height from Existing Geoid Model

N_{New} = Geoid Height from New Geoid Model

n = Number of Points

Gravimetric-Geometric Geoid Model of Benin City

The gravimetric-geometric geoid model of Benin City was determined using the combination of the gravimetrically and geometrically obtained data. The realization, as well as the determination of the model, is detailed in Oduyebo *et al.* (2019). In Oduyebo *et al.* (2019), three gravimetric-geometric geoid models were determined and the one with the highest accuracy (third-degree gravimetric-geometric geoid model with RMSE of 0.6746 m among the three models was recommended for application in Benin City.

METHODOLOGY

Data Acquisition

The transformation local geoid model was determined with a total of 49 points (See Figure 3). The positions and the ellipsoidal heights of the points were obtained by carrying out a GNSS observation in relative mode (See Figures 4 and 5). The observation was done using CHC 900 dual-frequency GNSS receivers. The observed points were the same as the ones used when the local gravimetric-geometric geoid model of the study area was determined.

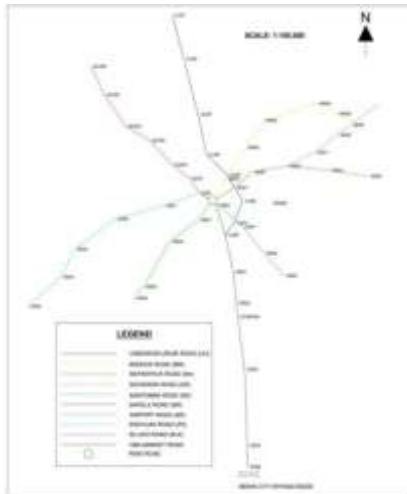


Figure 3: GNSS Observation Points Figure 4: Base Receiver at Control Station XSU92 Figure 5: Rover Receiver at One of the Selected Points (RR01) at Ring Road

Data Processing

The GNSS observations were respectively downloaded and processed with HcLoader and Compass post-processing software (See Figure 6). Since the adopted method involved the transformation of a global dataset to local, the geographic coordinates of the points were processed in the WGS 84 datum. The global positions of the points were used to obtain the global geoid heights of the points from EGM 08 using the GeoidEval online software, as well as equation (12) (See Figure 7).

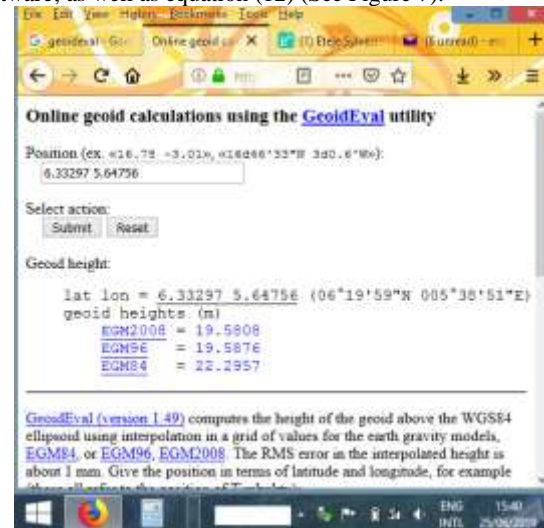
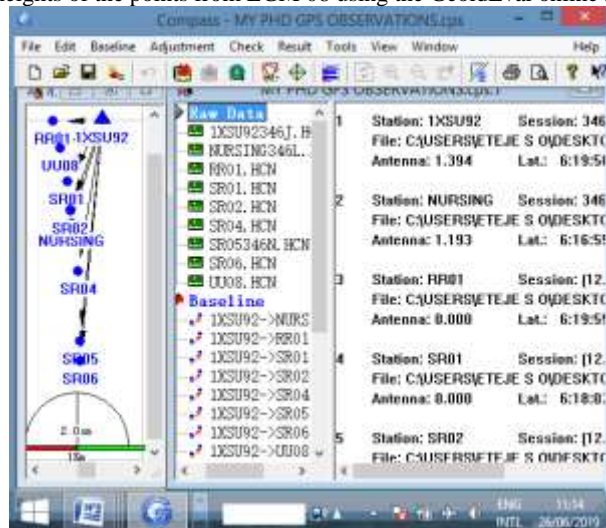


Figure 6: Processing of the GNSS Data Using Compass Software Figure 7: Computation of the Global Geoid Heights of the Points Using GeoidEval Software

The local geoid heights of the points were computed with the properties of the WGS 84 and the Clarke 1880 ellipsoids, the transformation parameters, the global geographic coordinates and the EGM 08 geoid heights of the points using the transformation method, as well as equation (1). The computation was done using a Microsoft Excel program developed in the study (See Figure 8).

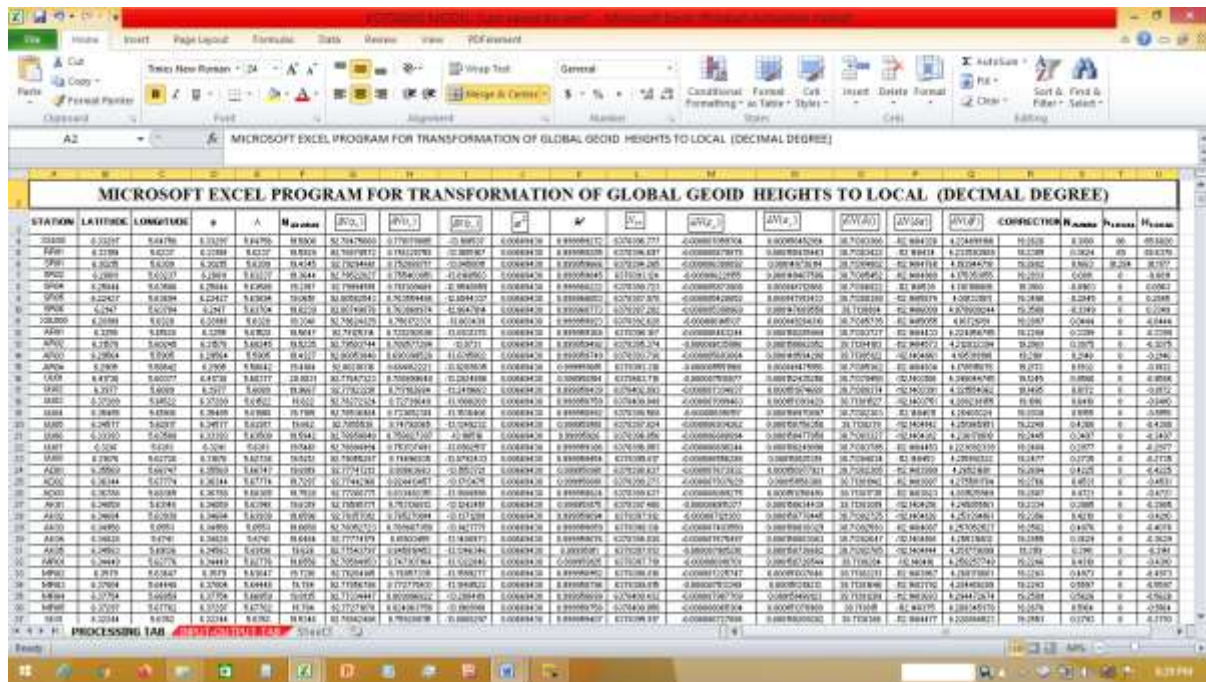


Figure 8: Computation of Geoid Heights of the Points Using the Transformation Method
The RMSE, as well as the accuracy of the fitted local geoid model, was computed using equation (13).

RESULTS PRESENTATION AND ANALYSIS
Comparison of the Gravimetric-Geometric and the Transformation Method Geoid Heights

Table 1 presents the mean of the differences between the local gravimetric-geometric and the transformation method models geoid heights, the RMS errors, as well as the accuracy of the transformation geoid model, the existing gravimetric-geometric geoid model and that of the transformation geoid model when it was applied in part of the Federal Capital Territory (FCT), Abuja. It was done to present the accuracy of the transformation geoid model in the study area and when it was applied in part of the FCT, Abuja. Also, to present the accuracy of the gravimetric-geometric geoid model and the mean deviation of the transformation method local geoid model from the existing gravimetric-geometric geoid model of the study area. From Table 1, it can be seen that the mean of the differences between the geoid heights from the transformation geoid model and the gravimetric-geometric geoid model is 1.6882 m. This implies that the transformation

method geoid model of the study area deviated with an average value of 1.6882 m from the existing local gravimetric-geometric geoid model of the study area. Also from Table 1, the RMS error, as well as the accuracy of the transformation geoid model is 2.0172 m which implies that geoid heights can be computed with an accuracy of 2.0172 m applying the model in the study area. Again from table 1, the RMS errors of the transformation geoid model as obtained by Okeke and Nnam (2017) and that of the gravimetric-geometric geoid models are respectively 0.0014 m and 0.675 m. These also show the high accuracy of the two geoid models. Comparing the obtained accuracy of the transformation geoid model in the study area (2.0172 m) with that obtained by Okeke and Nnam (2017) (0.0014 m) when the model was applied in part of the FCT, Abuja, shows the inconsistency of the accuracy of the method. It implies that the method cannot be used for the determination of the local geoid model of Nigeria.

Table 1: Mean of Differences and Computed RMS Errors

MEAN OF DIFFERENCES =	1.6882 m
RMS Error of Transformation Model = SQRT of Average =	2.0172 m
RMS Error of Gravimetric-Geometric Model =	0.6746 m
RMS Error of Transformation Model as obtained by Okeke and Nnam (2017) =	0.0014 m

Fitting of the Transformation Geoid Model to the Gravimetric-Geometric Geoid Model

Table 2 presents the fitting of the transformation geoid model to the gravimetric-geometric geoid model. It was done to improve on the accuracy of the transformation geoid model of the study area. The improvement of the accuracy of the transformation geoid model was necessary to enable its application in the study area. The fitting was carried out by finding the mean (1.6882 m) of the differences between the gravimetric-geometric model geoid heights of the points

given in Table 2 and their respective geoid heights from the transformation geoid model of the study area. Subsequently, the mean was added to the transformation model geoid heights to fit the transformation geoid model to the gravimetric-geometric geoid model. Having fitted the transformation geoid model to the gravimetric-geometric geoid model, the two models geoid heights were compared to obtain the improved accuracy, as well as the RMS error of the transformation geoid model as given in Table 2. From Table 2, it can be seen that the improved accuracy of the transformation geoid model of the study area is 1.1041 m.

This implies that geoid heights can be obtained in the study area using the fitted geoid model with an accuracy of 1.1041 m. Comparing this accuracy of the transformation geoid model with its accuracy before fitting given in Table 1 (2.0172 m) shows an improvement in the accuracy of the model. It can also be seen from Table 2 that the mean of the

differences between the transformation and the gravimetric-geometric models' geoid heights is 1.6882 m. It shows that to achieve the accuracy of 1.1041 m for the transformation geoid model, 1.6882 m will be added to any geoid height computed with the transformation geoid model.

Table 2: Fitting of the Transformation Geoid Model to the Gravimetric-geometric Geoid Model

Point	Gravimetric-Geometric Geoid Heights (A) (m)	Transformed EGM08 Geoid Heights (B) (m)	Difference between A & B (m)	Difference between A & B Squared (P) (m ²)
XSU92	2.086	2.0062	0.0798	0.0064
RR01	2.420	2.0506	0.3694	0.1365
SR01	1.588	1.8545	-0.2665	0.0710
SR02	1.978	1.7693	0.2087	0.0436
SR04	2.520	1.5919	0.9281	0.8613
SR05	2.802	1.4037	1.3983	1.9553
SR06	3.266	1.3533	1.9127	3.6585
XSU100	2.098	1.7326	0.3654	0.1335
AR01	0.685	2.0281	-1.3431	1.8039
AR02	0.720	1.9957	-1.2757	1.6274
AR03	1.436	1.9028	-0.4668	0.2179
AR04	1.439	1.8814	-0.4424	0.1957
UU01	4.658	2.6468	2.0112	4.0448
UU02	3.498	2.5054	0.9926	0.9853
UU03	1.981	2.3292	-0.3482	0.1212
UU04	1.276	2.2037	-0.9277	0.8607
UU05	1.346	2.1262	-0.7802	0.6087
UU06	1.489	2.0379	-0.5489	0.3013
UU07	1.329	1.9859	-0.6569	0.4315
UU08	1.263	1.9617	-0.6987	0.4882
AD01	2.986	2.1107	0.8753	0.7662
AD02	4.019	2.1413	1.8777	3.5256
AD03	4.420	2.1603	2.2597	5.1062
AK01	1.376	2.0867	-0.7107	0.5051
AK02	1.473	2.1092	-0.6362	0.4048
AK03	2.252	2.0958	0.1562	0.0244
AK04	3.101	2.0511	1.0499	1.1022
AK05	3.954	2.0023	1.9517	3.8090
MR01	1.300	2.1192	-0.8192	0.6711
MR02	1.488	2.1855	-0.6975	0.4864
MR03	1.614	2.2479	-0.6339	0.4018
MR04	4.037	2.2508	1.7862	3.1905
MR05	4.313	2.2046	2.1084	4.4455
SK01	1.500	1.9675	-0.4675	0.2185
SK02	2.035	1.8748	0.1602	0.0257
SK03	2.379	1.7966	0.5824	0.3392

EKS	0.665	2.1024	-1.4374	2.0662
SLK0	0.668	2.1528	-1.4848	2.2046
SLK01	0.781	2.2080	-1.4270	2.0364
SLK02	1.326	2.3009	-0.9749	0.9504
SLK03	1.736	2.3646	-0.6286	0.3951
SLK04	2.688	2.4766	0.2114	0.0447
SLK05	3.357	2.5701	0.7869	0.6192
EK01	0.078	2.1041	-2.0261	4.1050
EK02	0.983	2.1171	-1.1341	1.2861
EK03	1.729	2.0743	-0.3453	0.1192
EK04	2.001	2.0143	-0.0133	0.0002
EK05	2.516	1.9720	0.5440	0.2959
AIRPORT	0.578	2.0042	-1.4262	2.0339
RMS ERROR = SQRT OF AVERAGE (P) =				1.1041 m
MEAN OF THE DIFFERENCES =				1.6882 m

CONCLUSION

In conclusion, the study has fitted a transformation geoid model to the gravimetric-geometric geoid model of Benin City. The obtained results showed that the transformation geoid model deviated from the gravimetric-geometric geoid model of the study area with a mean value of 1.6882 m. The study also obtained the accuracy of the transformation geoid model before fitting as 2.0172 m. The computed results of the fitted geoid model showed that geoid heights can be obtained using the model with an accuracy of 1.1041 m in the study area.

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