FUDMA Journal of Sciences (FJS)
ISSN online: 2616-1370
ISSN print: 2645 - 2944
Vol. 4 No. 1, March, 2020, pp 528 – 538

OPTIMAL INVESTMENT STRATEGY AND CAPITAL MANAGEMENT IN A BANK UNDER STOCHASTIC INTEREST RATE AND STOCHASTIC VOLATILITY

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ABSTRACT

In this research work, we have looked at how a financial institution can optimally allocate its wealth among three assets namely: treasury, security and loan, and also manage it assets in stochastic interest rate and stochastic volatility setting. We derived the optimal investment policy through the application of dynamic programming principle for the case of constant relative risk aversion (CRRA) utility function. Furthermore, we derived the Stochastic Differential Equation (SDE) for the capital adequacy ratio under Basel Accord, the SDE for the Total Risk – Weighted Assets (TRWA), SDE for the capital required to maintain the capital adequacy ratio under Basel II and Central Bank of Nigeria (CBN) standards and solve the SDEs numerically to study how the financial institution can manage its assets. We also presented numerical examples to illustrate the dynamics of the optimal investment policy, TRWA SDE and SDE of the capital required to maintain the capital adequacy ratio under Basel II and Nigeria CBN standards.

Keywords: Financial Institution, Investment Strategy, Stochastic Optimization Theory, Stochastic Interest Rate, Stochastic Volatility.

INTRODUCTION

Management of a financial institution's assets majorly involves achieving profit maximization through high returns on loans and securities, reducing risk and meeting the liquidity needs of the financial institution. Specifically, financial institutions attempt to manage their assets in the following ways: they try to grant loans to customers who are likely to pay high interest rates and have a low tendency to default. Secondly, they tend to purchase securities with high returns and low risk. Also, in attempt to manage their assets, financial institutions try to lower risk by diversifying their investment portfolio through investment in different types of assets (Mukuddem – Petersen and Petersen, 2008). Financial institutions must also manage their assets so that it can satisfy the reserve – requirements without incurring high cost.

The need for financial institutions to invest in assets with an acceptable level of risk and high returns is very important. For instance, if the return on a particular loan turns out to be very high at the end of the loan contract period, the financial institution might regret not having allocated a large enough portion of its capital to such loan. Therefore, optimal asset allocation is very crucial in a financial institution management. Numerous studies have been done on how to optimally allocate a financial institution's wealth among its assets. In particular, Dangl and Lehar (2004) and Decamps *et al.* (2004), constructed a continuous – time models which solved the optimal control problems of a bank in the context of portfolios selection and capital requirements. Peter *et al*., (2011) studied an optimal assets allocation problem with stochastic interest rates which takes into account specific features of bank. Their goal is to present a numerical aspect of the derived Hamilton Jacob Bellman (HJB) equation and to focus on the optimal assets allocation model results from a practical viewpoint. Fouche *et al* (2006) also considered assets allocation problem. In their work, they illustrated that it is possible to use an analytic approach to optimize assets allocation strategies for banks. They formulated an optimal bank valuation problem through optimal choices of loan rate and demand which leads to maximal deposits, provisions for deposits withdrawals and bank profitability subject to cash flow, loan demand, financing and balance sheet constraints.

Several studies have also investigated the assets allocation problems using stochastic control theory developed by Merton (1969 and 1971) in discrete and continuous time setting (Wachter, 2002; Munk *et al.*, 2004). The approach solved nonlinear partial differential Hamilton – Jacobi – Bellman (HJB) equation to find the closed form solution for the value function.

Also, failures spark risk management strategies and regulatory prescripts to mitigate this risk. One of these prescriptions is the Basel Accord on capital adequacy requirements, which states that all major international financial institution e.g. banks should hold capital in proportion to their perceived risks (Peter *et al*., 2011). Although, an internal model may be used by the financial institution to make an assessment of their portfolio risk and determine the capital requirement. Capital management deals with the decision about the amount of capital the financial institution should hold and how it should be accessed (Diamond and Rajan, 2000). A double burden seems to be associated with the financial institution capital management because it benefits the owners as it reduces the institution failure likelihood but is costly since the higher the level of capital, the lower the return on equity for a prescribed return on assets. Hence, when considering the amount of capital a financial institution should hold, the owners must decide how much is the benefits that results from the higher capital they are willing to trade – off for the lower return on equity that comes from the associated higher capital.

The global economic crisis provided an opportunity for fundamental changes of the approach to risk and regulation in financial sector. An agreement on reforms to strengthen global capital and liquidity rules with the goal of promoting a stronger or more resilient banking sector, which being referred to Basel Accords came into being in 1988. The purpose of the Basel Accords is to ensure that internationally active banks hold enough capital to meet obligations and to absorb unexpected losses (Von - Thadden, 2004). Therefore, the Basel committee on banking supervision (BCBS) administers the regulation and supervision of the international banking industry by imposing the minimum capital requirements and other measures on the banking industry.

Under Basel I Accord, banks are to maintain Total Capital (calculated as the sum of Tier 1 and Tier 2 Capital) equal to at least 8% of its total – risk – weighted assets which is referred to as capital adequacy ratio (CAR) (BCBS, 2004). However, Basel I Accord was based on simplified calculations and classification which have led to its disappearance. As a result, the BCBS issued the Basel II Accord and further agreements as symbol of continuous refinement of risk and capital requirement (Investopedia, 2019). Basel III Accord is the third global, voluntary regulatory standard on bank capital adequacy, stress testing and market liquidity risk. It is a set of reform measures introduced in response to the 2007 – 2008 financial crises. The Accord which was issued in 2010 (Debajyoti *et al.*, 2013), aimed at improving the regulation, supervision and risk management within the banking sector. It also shows the continuous effort made by BCBS to improve the banking regulatory framework. It also important to note that CAR for banks in Nigeria currently stands at 10% and 15% for national/regional banks and banks with international license respectively (Ugo, 2014).

Therefore, many mathematical models have been formulated over the past years to explore the dynamics of asset allocation and capital management problem in financial institutions in stochastic interest rate setting. In our contribution, we attempt to explore the dynamics of a financial institution asset allocation and capital management problem in a stochastic interest rate and stochastic volatility framework. Our goal is to maximize an expected utility of the assets at a future time, derive the SDE for the capital adequacy ratio under Basel Accord, derive the SDE for the total risk – weighted assets and SDE for the capital required to maintain the capital adequacy ratio under Basel II and Nigeria CBN standards.

The Mathematical Model Formulation

We consider a financial institution that dynamically allocates its wealth among three assets namely: treasury, loan and security. The assets prices satisfy the geometric Brownian motion, assets can be bought and sold without incurring any transaction costs or restriction on short sales and the interest rate is described by Affine model. The risk preference of the investor satisfies CRRA utility function.

The Financial Market

We consider a complete and frictionless financial market which is continuously open over a fixed time interval $[0,T]$ and Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where $\mathcal{F} = \{\mathcal{F}_t\}_{t\geq 0}$ is the filtration generated by the Brownian motions, $\mathbb{P}$ is the real world probability. The first asset in the financial market is a riskless treasury and its price at time t can be denoted by $S_0(t)$. It evolves according to the following stochastic differential equation

$$\frac{dS_0(t)}{S_0(t)} = r(t)dt, \qquad S(0) = S_0 \tag{1}$$

The dynamics of the short rate process, $r(t)$, is given by the stochastic differential equation

$$dr(t) = \big(a - br(t)\big)dt - \sigma_r\sqrt{r(t)}dw_r(t), \quad r(t) = r_0 \tag{2}$$

Where a, b and $\sigma_r = \sqrt{k_1}$ are constants.

The second asset is a loan to be amortized over a period $[0,T]$ whose price at time $t \geq 0$ is denoted by $L(t)$. Its dynamics can be described by the stochastic differential equation.

$$\frac{dL(t)}{L(t)} = \big(r(t) + b_1\lambda_r k_1 r(t)\big)dt + b_1\sigma_r\sqrt{r(t)}dw_r(t) \tag{3}$$

where b_1, λ_r and k_1 are constants. The loan return has a risk premium $b_1\lambda_r r(t)$ that changes with t both implicitly through the dependence on $r(t)$ and explicitly through the dependence on b_1.

The third asset in the financial market is a risky security whose price is denoted by $S(t), t \geq 0$. Its dynamics can be described by the equation:

$$\frac{dS(t)}{S(t)} = \big(r(t) + \upsilon\eta(t) + \sigma_s\lambda_r k_1 r(t)\big)dt + \sigma_s\sigma_r\sqrt{r(t)}dw_r(t) + \sqrt{\eta(t)}dw_s(t) \quad (4)$$

Here, we assumed that the volatility $\eta(t)$ satisfies the Heston model:

$$d\eta(t) = \alpha\big(\delta - \eta(t)\big)dt + \sigma_\eta\sqrt{\eta(t)}dw_r(t) \quad (5)$$

where α, δ andσ_η are positive constant and satisfied the condition $2\alpha\delta > \sigma_\eta^2$ and it ensures $\eta(t) > 0\ \forall\ t \in [0, T]$.

Here we assume that there is no correlation between $w_s(t)$ and $w_r(t)$, and between $w_\eta(t)$ and $w_r(t)$. The correlation between $w_s(t)$ and $w_\eta(t)$ is ρ.

The Asset Portfolio of the Financial Institution

Let $X(t)$ denotes the value of the financial institution assets portfolio at time $t \in [0, T], \pi_s(t)$ and $\pi_l(t)$ denote the amount invested in the security and loan respectively. Therefore, $\pi_0(t) = X(t) - \pi_s(t) - \pi_l(t)$ denotes the amount invested in the riskless asset. The dynamics of the assets portfolio is given by

$$\begin{aligned} dX(t) &= \big(X(t) - \pi_s(t) - \pi_l(t)\big)\frac{dS_0(t)}{S_0(t)} + \pi_s(t)\frac{dS(t)}{S(t)} + \pi_l(t)\frac{dL(t)}{L(t)} \\ &= [X(t)r(t) + \pi_s(t)\upsilon\eta(t) + \pi_s(t)\sigma_s\lambda_r k_1 r(t) + \pi_l(t)b_1\lambda_r k_1 r(t)]dt \\ &+ \left[\pi_s(t)\sigma_s\sigma_r\sqrt{r(t)} + \pi_l(t)b_1\sigma_r\sqrt{r(t)}\right]dw_r(t) + \pi_s(t)\sqrt{\eta(t)}dw_s(t) \end{aligned} \quad (6)$$

Definition (Admissible Strategy)

An investment strategy $\pi(t) = (\pi_s(t), \pi_l(t))$ is said to be admissible if the following conditions are satisfied.

i. $\pi_s(t)$ and $\pi_l(t)$ are all $f_t -$ measurable.

ii. $E\left(\int_0^T (\pi_s^2(t)\eta(t) + [\pi_s(t)\sigma_s\sigma_r + \pi_l(t)b_1\sigma_r]^2 r(t))dt\right) < \infty$.

iii. The stochastic differential equation (6) has a unique solution $\forall\ \pi(t) = (\pi_s(t), \pi_l(t))$.

2.4 The Portfolio Optimization Problem

Let the set of all admissible strategy be denoted by Π. Under the asset portfolio (6), the financial institution looks for an optimal strategy $\pi_s{}^*(t)$ and $\pi_l{}^*(t)$ which maximizes the expected utility of the terminal wealth. i.e.:

$$\max_{\pi(t)\in\Pi} E[U(X(T))] \quad (7)$$

Based on the classical tools of stochastic optimal control, we state the problem as follows:

Maximize $E[U(X(T))]$

Subject to:

$$dr(t) = \big(a - br(t)\big)dt - \sigma_r\sqrt{r(t)}dw_r(t)$$

$$d\eta(t) = \alpha\big(\delta - \eta(t)\big)dt + \sigma_\eta\sqrt{\eta(t)}dw_r(t)$$

$$\begin{aligned} dX(t) &= [X(t)r(t) + \pi_s(t)\upsilon\eta(t) + \pi_s(t)\sigma_s\lambda_r k_1 r(t) + \pi_l(t)b_1\lambda_r k_1 r(t)]dt \\ &+ \left[\pi_s(t)\sigma_s\sigma_r\sqrt{r(t)} + \pi_l(t)b_1\sigma_r\sqrt{r(t)}\right]dw_r(t) + \pi_s(t)\sqrt{\eta(t)}dw_s(t) \end{aligned}$$

$$X(0) = x_0, \quad r(0) = r_0, \quad \eta(0) = \eta_0, 0 \le t \le T$$

The objective is to maximize the expected utility of the financial institution's portfolio at a future date $T > 0$. That is, find the optimal value function

$$H(t, r, \eta, x) = \max_{\pi(t)\in\Pi} E\big[U\big(X(T)\big)\big|r(t) = r, \eta(t) = \eta, X(t) = x\ \big] \quad (8)$$

and the optimal strategy is $\pi^*(t) = (\pi_s{}^*(t), \pi_l{}^*(t))$ such that

$$\underset{\pi^*(t)}{\mathrm{H}}(t, r, \eta, x) = H(t, r, \eta, x) \quad (9)$$

The Derivation of the Hamilton – Jacobi – Bellman Equation Associated with the Portfolio Optimization Problem

The Hamilton – Jacobi – Bellman equation associated with the portfolio optimization problem is:

$$\begin{aligned} &\max_{\pi(t)\in\Pi}\{H_t + [X(t)r(t) + \pi_s(t)\upsilon\eta(t) + \pi_s(t)\sigma_s\lambda_r k_1 r(t) + \pi_l(t)b_1\lambda_r k_1 r(t)]H_x \\ &+ \frac{1}{2}(\pi_s^2(t)\eta(t) + \left[\pi_s(t)\sigma_s\sigma_r\sqrt{r(t)} + \pi_l(t)b_1\sigma_r\sqrt{r(t)}\right]^2)H_{xx} - [\pi_s(t)\sigma_s\sigma_r^2 r(t) \\ &+ \pi_l(t)b_1\sigma_r^2 r(t)]H_{xr} + \big[\rho\pi_s(t)\sigma_\eta\eta(t)\big]H_{x\eta} + [a - br(t)]H_r + \frac{1}{2}\sigma_r^2 r(t)H_{rr} \\ &+ \alpha[\delta - \eta(t)]H_\eta + \frac{1}{2}\sigma_\eta^2\eta(t)H_{\eta\eta}\} = 0 \end{aligned} \quad (11)$$

$$H(T,r,\eta,x)=U(x) \quad (12)$$

where $H_t, H_\eta, H_x, H_r, H_{xx}, H_{rr}, H_{\eta\eta}, H_{x\eta}$ and H_{xr} denote partial derivatives of first and second orders with respect to t, r, η and x respectively.

Differentiating (11) with respect to $\pi_s(t)$ and $\pi_l(t)$, we obtain

$$(\upsilon\eta+\sigma_s\lambda_r k_1 r)H_x+(\pi_s(t)\eta+(\pi_s(t)\sigma_s^2\sigma_r^2 r+\pi_l(t)b_1\sigma_s\sigma_r^2 r)H_{xx}$$
$$-\sigma_s\sigma_r^2 rH_{xr}+\rho\sigma_\eta\eta H_{x\eta}=0 \quad (13)$$

and

$$b_1\lambda_r k_1 rH_x+(\pi_s(t)b_1\sigma_s\sigma_r^2 r+\pi_l(t)b_1^2\sigma_r^2 r)H_{xx}-b_1\sigma_r^2 rH_{xr}=0 \quad (14)$$

Solving (13) and (14) for $\pi_s(t)$ and $\pi_l(t)$ give the first order maximizing conditions for the optimal strategy $\left(\pi_s^*(t),\pi_s^*(t)\right)$.

From equation (14), we have

$$\pi_l(t)=\frac{H_{xr}}{b_1H_{xx}}-\frac{\lambda_r k_1 H_x}{b_1\sigma_r^2 H_{xx}}-\frac{\pi_s(t)\sigma_s}{b_1} \quad (15)$$

Substituting for $\pi_l(t)$ in equation (13) and simplifying, we obtain

$$\pi_s^*(t)=-\upsilon\frac{H_x}{H_{xx}}-\rho\sigma_\eta\frac{H_{x\eta}}{H_{xx}} \quad (16)$$

Substituting **(**16**)** in **(**15**)** gives

$$\pi_l^*(t)=\frac{H_{xr}}{b_1H_{xx}}-\frac{\lambda_r k_1 H_x}{b_1\sigma_r^2 H_{xx}}-\frac{\sigma_s}{b_1}\left(-\upsilon\frac{H_x}{H_{xx}}-\rho\sigma_\eta\frac{H_{x\eta}}{H_{xx}}\right)$$
$$=\frac{H_{xr}}{b_1H_{xx}}+\frac{(\upsilon\sigma_s\sigma_r^2-\lambda_r k_1)H_x}{b_1\sigma_r^2H_{xx}}+\frac{\rho\sigma_\eta\sigma_s H_{x\eta}}{b_1H_{xx}} \quad (17)$$

Substituting (16) and (17) in (11) gives the partial differential equation (PDE) for the value function.

$$H_t+xrH_x-\left(\frac{\upsilon^2\eta}{2}+\frac{\lambda_r^2k_1^2r}{2\sigma_r^2}\right)\frac{H_x^2}{H_{xx}}-\rho^2\sigma_\eta^2\eta\frac{H_{x\eta}^2}{2H_{xx}}-\sigma_r^2r\frac{H_{xr}^2}{2H_{xx}}-\rho\sigma_\eta\eta\upsilon\frac{H_xH_{x\eta}}{H_{xx}}$$
$$+\lambda_r k_1 r\frac{H_xH_{xr}}{H_{xx}}+(a-br)H_r+\frac{1}{2}\sigma_r^2rH_{rr}+\alpha(\delta-\eta)H_\eta+\frac{1}{2}\sigma_\eta^2\eta H_{\eta\eta}=0 \quad (18)$$

The problem now is solving (18) for the value function and replace it in (16) and (17).

3 The Solution of the Portfolio Optimization Problem

In the case of CRRA utility function, we conjecture a solution to the equation (18) in the following form:

$$H(t,r,\eta,x)=\frac{x^\beta}{\beta}f(t,r,\eta),\quad \beta<1,\beta\neq 0 \quad (19)$$

With the boundary condition:

$$f(T,r,\eta)=1 \quad (20)$$

From (19), we have

$$\left.\begin{aligned}&H_t=\frac{x^\beta}{\beta}f_t, H_x=x^{\beta-1}f,\quad H_r=\frac{x^\beta}{\beta}f_r,\ \ H_\eta=\frac{x^\beta}{\beta}f_\eta, H_{xx}=(\beta-1)x^{\beta-2}f\\ &\qquad H_{xr}=x^{\beta-1}f_r, H_{x\eta}=x^{\beta-1}f_\eta,\quad H_{rr}=\frac{x^\beta}{\beta}f_{rr},\ \ H_\eta=\frac{x^\beta}{\beta}f_{\eta\eta}\end{aligned}\right\} \quad (21)$$

Where $H_t, H_x, H_r, H_\eta, H_{xx}, H_{xr}, H_{x\eta}, H_{rr}$ and $H_{\eta\eta}$ are first order and second order partial derivatives of H with respect to t, r and η, $f_t, f_r, f_\eta,$ f_{rr} and $f_{\eta\eta}$ represent the first order and second order partial derivatives of f with respect to t, r and η.

Introducing these derivatives in (21) into (18) and dividing through by $\frac{x^\beta}{\beta}$ yields

$$f_t+r\beta f-\left(\frac{\upsilon^2\eta}{2}+\frac{\lambda_r^2k_1^2r}{2\sigma_r^2}\right)\frac{\beta f}{(\beta-1)}-\rho^2\sigma_\eta^2\eta\frac{\beta f_\eta^2}{2(\beta-1)f}-\sigma_r^2r\frac{\beta f_r^2}{2(\beta-1)f}-\rho\sigma_\eta\eta\upsilon\frac{\beta f_\eta}{\beta-1}$$
$$+\lambda_r k_1 r\frac{\beta f_r}{\beta-1}+(a-br)f_r+\frac{1}{2}\sigma_r^2rf_{rr}+\alpha(\delta-\eta)f_\eta+\frac{1}{2}\sigma_\eta^2\eta f_{\eta\eta}=0$$
$$f_t+\left[r\beta-\left(\frac{\beta\upsilon^2\eta}{2(\beta-1)}+\frac{\beta\lambda_r^2k_1^2r}{2\sigma_r^2(\beta-1)}\right)\right]f-\frac{\beta\rho^2\sigma_\eta^2\eta f_\eta^2}{2(\beta-1)f}-\frac{\beta\sigma_r^2rf_r^2}{2(\beta-1)f}$$
$$+\left[\alpha(\delta-\eta)-\frac{\beta\rho\sigma_\eta\eta\upsilon}{\beta-1}\right]f_\eta+\left[\frac{\beta\lambda_r k_1 r}{\beta-1}+(a-br)\right]f_r+\frac{1}{2}\sigma_r^2rf_{rr}+\frac{1}{2}\sigma_\eta^2\eta f_{\eta\eta}=0 \quad (22)$$

We conjecture $f(t,r,\eta)$ as the following:

$$\left.\begin{aligned}&f(t,r,\eta)=e^{D_1(t)+D_2(t)r+D_3(t)\eta}\\ &D_1(t)=D_2(t)=D_3(t)=0\end{aligned}\right\} \quad (23)$$

From (23)

$$\left.\begin{aligned} f_t &= (D_1'(t) + D_2'(t)r + D_3'(t)\eta)f \\ f_r &= D_2(t)f,\ \ f_\eta = D_3(t)f \\ f_{rr} &= D_2^2(t)f,\ \ f_{\eta\eta} = D_3^2(t)f \end{aligned}\right\} \quad (24)$$

Hence substituting for $f_t, f_r, f_\eta,\ f_{rr}$ and $f_{\eta\eta}$ in (22) gives:

$$[D_1'(t) + aD_2(t) + \alpha\delta D_3(t)]f + rf\left[D_2'(t) + \left(\beta - \frac{\beta\lambda_r^2 k_1^2}{2\sigma_r^2(\beta-1)}\right) + \left(\frac{1}{2}\sigma_r^2 - \frac{\beta\sigma_r^2}{2(\beta-1)}\right)D_2^2(t)\right.$$
$$\left. + \left(\frac{\beta\lambda_r k_1}{\beta-1} - b\right)D_2(t)\right] + \eta f\left[D_3'(t) - \left(\frac{\beta\upsilon^2}{2(\beta-1)}\right) + \left(\frac{1}{2}\sigma_\eta^2 - \frac{\beta\rho^2\sigma_\eta^2}{2(\beta-1)}\right)D_3^2(t)\right.$$
$$\left. - \left(\alpha + \frac{\beta\rho\sigma_\eta\upsilon}{\beta-1}\right)D_3(t)\right] = 0 \quad (25)$$

Eliminating the dependency on r and η, we decompose (25) into:

$$D_1'(t) + aD_2(t) + \alpha\delta D_3(t) = 0 \quad (26)$$

$$D_2'(t) + \left(\frac{1}{2}\sigma_r^2 - \frac{\beta\sigma_r^2}{2(\beta-1)}\right)D_2^2(t) + \left(\frac{\beta\lambda_r k_1}{\beta-1} - b\right)D_2(t) + \left(\beta - \frac{\beta\lambda_r^2 k_1^2}{2\sigma_r^2(\beta-1)}\right) = 0 \quad (27)$$

$$D_3'(t) + \left(\frac{1}{2}\sigma_\eta^2 - \frac{\beta\rho^2\sigma_\eta^2}{2(\beta-1)}\right)D_3^2(t) - \left(\alpha + \frac{\beta\rho\sigma_\eta\upsilon}{\beta-1}\right)D_3(t) - \left(\frac{\beta\upsilon^2}{2(\beta-1)}\right) = 0 \quad (28)$$

Observe that (27) and (28) are the general Ricotta equations.

Now, we turn to solving the above three equations. From (26), we have

$$D_1'(t) = -aD_2(t) - \alpha\delta D_3(t)$$

$$D_1(t) = -\left(a\int_t^T D_2(t)\,dt + \alpha\delta\int_t^T D_3(t)\,dt\right) \quad (29)$$

From (27), we have that

$$\frac{dD_2(t)}{dt} = \left(\frac{\beta\sigma_r^2}{2(\beta-1)} - \frac{1}{2}\sigma_r^2\right)D_2^2(t) + \left(b - \frac{\beta\lambda_r k_1}{\beta-1}\right)D_2(t) + \left(\frac{\beta\lambda_r^2 k_1^2}{2\sigma_r^2(\beta-1)} - \beta\right) \quad (30)$$

Observe that the RHS of (30) is a quadratic function. Therefore,

$$M_{1,2} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

where

$$A = \left(\frac{\beta\sigma_r^2}{2(\beta-1)} - \frac{1}{2}\sigma_r^2\right), B = \left(b - \frac{\beta\lambda_r k_1}{\beta-1}\right), C = \left(\frac{\beta\lambda_r^2 k_1^2}{2\sigma_r^2(\beta-1)} - \beta\right)$$

The discriminant $= B^2 - 4AC = b^2 + \dfrac{\beta(2b\lambda_r k_1 - 2\sigma_r^2 - \lambda_r^2 k_1^2)}{1-\beta}$ since $\beta < 1$

Let $\Delta_0 = b^2 + \dfrac{\beta(2b\lambda_r k_1 - 2\sigma_r^2 - \lambda_r^2 k_1^2)}{1-\beta}$, then

$$M_{1,2} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{\left(b + \frac{\beta\lambda_r k_1}{1-\beta}\right) \pm \sqrt{\Delta_0}}{\left(\frac{\sigma_r^2}{1-\beta}\right)}, \qquad \beta < 1$$

Equation (27) has different solutions depending on whether $\Delta_0 > 0, \Delta_0 = 0$ and $\Delta_0 < 0$. Now, let $\Delta_0 > 0$ then the quadratic function has two different roots denoted by M_1 and M_1 respectively such that

$$\frac{dD_2(t)}{dt} = A[(D_2(t) - M_1)(D_2(t) - M_2)] \quad (31)$$

Therefore, equation (31) becomes

$$\frac{1}{M_1 - M_2}\left(\frac{1}{D_2(t) - M_1} - \frac{1}{D_2(t) - M_2}\right)dD_2(t) = Adt \quad (33)$$

The integral of (33) with respect to t, from t to T is:

$$\frac{1}{M_1 - M_2}\int_t^T\left(\frac{dD_2(s)}{D_2(s) - M_1} - \frac{dD_2(s)}{D_2(s) - M_2}\right) = A\int_t^T ds$$

$$D_2(t) = \frac{M_1 M_2 - M_1 M_2 e^{A(M_1 - M_2)(T-t)}}{M_1 - M_2 e^{A(M_1 - M_2)(T-t)}}$$

Note that

$$A=\left(\frac{\beta\sigma_r^2}{2(\beta-1)}-\frac{1}{2}\sigma_r^2\right)=-\left(\frac{\beta\sigma_r^2}{2(1-\beta)}+\frac{1}{2}\sigma_r^2\right)\text{ for }\beta<1$$

Therefore,

$$D_2(t)=\frac{M_1M_2-M_1M_2e^{-\left(\frac{1}{2}\sigma_r^2+\frac{\beta\sigma_r^2}{2(1-\beta)}\right)(M_1-M_2)(T-t)}}{M_1-M_2e^{-\left(\frac{1}{2}\sigma_r^2+\frac{\beta\sigma_r^2}{2(1-\beta)}\right)(M_1-M_2)(T-t)}} \tag{34}$$

Next we solve for $D_3(t)$ in (28)

$$D_3'(t)=\left(\frac{\beta\rho^2\sigma_\eta^2}{2(\beta-1)}-\frac{1}{2}\sigma_\eta^2\right)D_3^2(t)+\left(\alpha+\frac{\beta\rho\sigma_\eta\upsilon}{\beta-1}\right)D_3(t)\ +\left(\frac{\beta\upsilon^2}{2(\beta-1)}\right) \tag{35}$$

Now,

$$M_{3,4}=\frac{-B_1\pm\sqrt{B_1^2-4A_1C_1}}{2A_1}$$

From (35), we have

$$A_1=\left(\frac{\beta\rho^2\sigma_\eta^2}{2(\beta-1)}-\frac{1}{2}\sigma_\eta^2\right),B_1=\left(\alpha+\frac{\beta\rho\sigma_\eta\upsilon}{\beta-1}\right),C_1=\left(\frac{\beta\upsilon^2}{2(\beta-1)}\right)$$

the discriminant $=B_1^2-4A_1C_1=\alpha^2-\frac{2\beta\rho\sigma_\eta\upsilon\alpha}{1-\beta}-\frac{\beta\upsilon^2\sigma_\eta^2}{1-\beta},\qquad \beta<1$

Again, let $\Delta_1=\alpha^2-\frac{2\beta\rho\sigma_\eta\upsilon\alpha}{1-\beta}-\frac{\beta\upsilon^2\sigma_\eta^2}{1-\beta}$

Then,

$$M_{3,4}=\frac{\left(\alpha-\frac{\beta\rho\sigma_\eta\upsilon}{1-\beta}\right)\pm\sqrt{\Delta_1}}{\left(\sigma_\eta^2+\frac{\beta\rho^2\sigma_\eta^2}{1-\beta}\right)},\qquad \beta<1$$

Equation (28) has different solution depending on whether $\Delta_1>0,\Delta_1=0$ and $\Delta_1<0$. Let $\Delta_1>0$, then the quadratic function has two distinct roots denoted by M_3 and M_4 respectively such that

$$\frac{dD_3(t)}{dt}=A_1[(D_3(t)-M_3)(D_3(t)-M_4)] \tag{36}$$

From (36), we have

$$\frac{1}{M_3-M_4}\left(\frac{1}{D_3(t)-M_3}-\frac{1}{D_3(t)-M_4}\right)dD_3(t)=A_1dt \tag{37}$$

The integral of (37) from t to T with respect to t is:

$$\frac{1}{M_3-M_4}\int_t^T\left(\frac{1}{D_3(s)-M_3}-\frac{1}{D_3(s)-M_4}\right)dD_3(s)=A_1\int_t^T ds$$

$$D_3(t)=\frac{M_3M_4-M_3M_4e^{A_1(M_3-M_4)(T-t)}}{M_3-M_4e^{A_1(M_3-M_4)(T-t)}}$$

Observe that

$$A_1=\left(\frac{\beta\rho^2\sigma_\eta^2}{2(\beta-1)}-\frac{1}{2}\sigma_\eta^2\right)=-\left(\frac{\beta\rho^2\sigma_\eta^2}{2(1-\beta)}+\frac{1}{2}\sigma_\eta^2\right),\ \ \beta<1$$

Therefore,

$$D_3(t)=\frac{M_3M_4-M_3M_4e^{-\left(\frac{1}{2}\sigma_\eta^2+\frac{\beta\rho^2\sigma_\eta^2}{2(1-\beta)}\right)(M_3-M_4)(T-t)}}{M_3-M_4e^{-\left(\frac{1}{2}\sigma_\eta^2+\frac{\beta\rho^2\sigma_\eta^2}{2(1-\beta)}\right)(M_3-M_4)(T-t)}} \tag{38}$$

Theorem 1

From equations (16), (17), (21) and (24), the optimal proportion of capital invested in security, loan and treasury under stochastic interest rates and stochastic volatility framework, and for the case of CRRA utility function is given by:

$$\pi_{sp}^*(t)=\frac{\nu}{1-\beta}+\frac{\rho\sigma_\eta D_3(t)}{1-\beta}$$

$$\pi_{lp}^*(t)=\frac{(\lambda_rk_1-\upsilon\sigma_s\sigma_r^2)}{b_1\sigma_r^2(1-\beta)}-\frac{D_2(t)}{b_1(1-\beta)}-\frac{\rho\sigma_s\sigma_\eta D_3(t)}{b_1(1-\beta)}$$

$$\pi_{0p}^*(t)=1-\pi_{sp}^*(t)-\pi_{lp}^*(t)$$

$$= 1 + \frac{\upsilon\sigma_s\sigma_r^2 - \upsilon b_1\sigma_r^2 - \lambda_r k_1}{b_1\sigma_r^2(1-\beta)} + \frac{1}{b_1(1-\beta)}D_2(t) + \frac{\rho\sigma_\eta(\sigma_s - b_1)}{b_1(1-\beta)}D_3(t)$$

The Dynamics of Total Risk – Weighted Assets (TRWA) and Basel II CAR

The dynamics of the total risk – weighted assets at time t, can be described by the stochastic differential equation:

$$dY_{rw}(t) = 0 \times \left(X(t) - \pi_s(t) - \pi_l(t)\right)\frac{dS_0(t)}{S_0(t)} + 0.2 \times \pi_s(t)\frac{dS(t)}{S(t)} + 0.5 \times \pi_l(t)\frac{dL(t)}{L(t)}$$

where, $0, 0.2$ and 0.5 are the risk weights associated with the treasury, security and loan under Basel III Accord respectively. Therefore,

$$dY_{rw}(t) = 0.2 \times \pi_s(t)\frac{dS(t)}{S(t)} + 0.5 \times \pi_l(t)\frac{dL(t)}{L(t)}$$

$$\begin{aligned} dY_{rw}(t) &= 0.2\pi_s(t)\left[\left(r(t) + \upsilon\eta(t) + \sigma_s\lambda_r k_1 r(t)\right)dt + \sigma_s\sigma_r\sqrt{r(t)}dw_r(t) + \sqrt{\eta(t)}dw_s(t)\right] \\ &\quad +0.5\pi_l(t)\left[\left(r(t) + b_1\lambda_r k_1 r(t)\right)dt + b_1\sigma_r\sqrt{r(t)}dw_r(t)\right] \\ &= \{0.2\pi_s(t)\left(r(t) + \upsilon\eta(t) + \sigma_s\lambda_r k_1 r(t)\right) + 0.5\pi_l(t)\left(r(t) + b_1\lambda_r k_1 r(t)\right)\}dt \\ &\quad +0.2\pi_s(t)\sqrt{\eta(t)}dw_s(t) + \left[0.2\pi_s(t)\sigma_s\sigma_r\sqrt{r(t)} + 0.5\pi_l(t)b_1\sigma_r\sqrt{r(t)}\right]dw_r(t) \end{aligned} \quad (39)$$

The capital adequacy ratio dynamics can be described as:

$$\text{CAR} = \frac{K(t)}{Y_{rw}(t)} \quad (40)$$

where, $K(t)$ is the total capital and $Y_{rw}(t)$ is the total risk – weighted assets of the financial institution respectively. Let $CAR = Z(t)$, then from (40)

$$Z(t) = \frac{K(t)}{Y_{rw}(t)} \quad (41)$$

Proposition 1 (SDE for Basel II CAR)

Let the dynamics of the total capital of the financial institution be $dK(t) = k(t)dt$ and the total risk – weighted assets $Y_{rw}(t)$ be described by (39). The dynamics of the Basel III capital adequacy ratio $Z(t)$ satisfies the following stochastic differential equation:

$$\begin{aligned} dZ(t) &= f\left(Y_{rw}(t)\right)dK(t) + K(t)df\left(Y_{rw}(t)\right) \\ &= \frac{k(t)dt}{Y_{rw}(t)} + \left\{\frac{K(t)}{Y_{rw}^3(t)}\left(\left[0.2\pi_s(t)\sqrt{\eta(t)}\right]^2 + \right.\right. \\ &\left[0.2\pi_s(t)\sigma_s\sigma_r\sqrt{r(t)} + 0.5\pi_l(t)b_1\sigma_r\sqrt{r(t)}\right]^2\Big) - [0.2\pi_s(t)\left(r(t) + \upsilon\eta(t) + \sigma_s\lambda_r k_1 r(t)\right) \\ &+0.5\pi_l(t)\left(r(t) + b_1\lambda_r k_1 r(t)\right)]\frac{K(t)}{Y_{rw}^2(t)}\Big\}dt - \left[\frac{K(t)}{Y_{rw}^2(t)}0.2\pi_s(t)\sqrt{\eta(t)}\right]dw_s(t) \\ &-\left[\frac{K(t)}{Y_{rw}^2(t)}\left(0.2\pi_s(t)\sigma_s\sigma_r\sqrt{r(t)} + 0.5\pi_l(t)b_1\sigma_r\sqrt{r(t)}\right)\right]dw_r(t) \end{aligned} \quad (42)$$

Proof:

Let $f\left(Y_{rw}(t)\right) = \frac{1}{Y_{rw}(t)}$, $dK(t) = k(t)dt$, then

$$Z(t) = K(t)f\left(Y_{rw}(t)\right)$$

$$dZ(t) = d\left[K(t)f\left(Y_{rw}(t)\right)\right] \quad (43)$$

Applying Ito product rule to the RHS (right hand side) of (43) yields

$$dZ(t) = f\left(Y_{rw}(t)\right)dK(t) + K(t)df\left(Y_{rw}(t)\right) \quad (44)$$

From Ito Lemma,

$$\begin{aligned} df\left(Y_{rw}(t)\right) &= f'(t)dt + f'\left(Y_{rw}(t)\right)dY_{rw}(t) + \frac{1}{2}f''(Y_{rw}(t))[dY_{rw}(t)]^2 \\ &= 0.dt + \left(-\frac{1}{Y_{rw}^2(t)}\right) \times dY_{rw}(t) + \frac{1}{2}\left(2 \times \frac{1}{Y_{rw}^3(t)}\right)[dY_{rw}(t)]^2 \\ &= -\frac{dY_{rw}(t)}{Y_{rw}^2(t)} + \frac{[dY_{rw}(t)]^2}{Y_{rw}^3(t)} \end{aligned} \quad (45)$$

From (39), we have

$$[dY_{rw}(t)]^2 = \left[0.2\pi_s(t)\sqrt{\eta(t)}\right]^2 dt + \left[0.2\pi_s(t)\sigma_s\sigma_r\sqrt{r(t)} + 0.5\pi_l(t)b_1\sigma_r\sqrt{r(t)}\right]^2 dt$$

Hence,

$$df(Y_{rw}(t)) = -\frac{dY_{rw}(t)}{Y_{rw}^2(t)} + \frac{[dY_{rw}(t)]^2}{Y_{rw}^3(t)}$$

$$= \Big\{\tfrac{1}{Y_{rw}^3(t)}\Big(\big[0.2\pi_s(t)\sqrt{\eta(t)}\big]^2 + \big[0.2\pi_s(t)\sigma_s\sigma_r\sqrt{r(t)} + 0.5\pi_l(t)b_1\sigma_r\sqrt{r(t)}\big]^2\Big)$$

$$-[0.2\pi_s(t)\big(r(t)+\upsilon\eta(t)+\sigma_s\lambda_r k_1 r(t)\big) + 0.5\pi_l(t)\big(r(t)+b_1\lambda_r k_1 r(t)\big)]\frac{1}{Y_{rw}^2(t)}\Big\}dt$$

$$-\frac{1}{Y_{rw}^2(t)}\{0.2\pi_s(t)\sqrt{\eta(t)}dw_s(t) + \big[0.2\pi_s(t)\sigma_s\sigma_r\sqrt{r(t)} + 0.5\pi_l(t)b_1\sigma_r\sqrt{r(t)}\big]dw_r(t)\}$$

Now, returning back to (44) we have

$$dZ(t) = f(Y_{rw}(t))dK(t) + K(t)df(Y_{rw}(t))$$

$$= \frac{k(t)dt}{Y_{rw}(t)} + \Big\{\tfrac{K(t)}{Y_{rw}^3(t)}\Big(\big[0.2\pi_s(t)\sqrt{\eta(t)}\big]^2 + \big[0.2\pi_s(t)\sigma_s\sigma_r\sqrt{r(t)} + 0.5\pi_l(t)b_1\sigma_r\sqrt{r(t)}\big]^2\Big)$$

$$-[0.2\pi_s(t)\big(r(t)+\upsilon\eta(t)+\sigma_s\lambda_r k_1 r(t)\big) + 0.5\pi_l(t)\big(r(t)+b_1\lambda_r k_1 r(t)\big)]\frac{K(t)}{Y_{rw}^2(t)}\Big\}dt$$

$$-\Big[\frac{K(t)}{Y_{rw}^2(t)}0.2\pi_s(t)\sqrt{\eta(t)}\Big]dw_s(t)$$

$$-\Big[\frac{K(t)}{Y_{rw}^2(t)}\Big(0.2\pi_s(t)\sigma_s\sigma_r\sqrt{r(t)} + 0.5\pi_l(t)b_1\sigma_r\sqrt{r(t)}\Big)\Big]dw_r(t) \qquad (46)$$

Proposition 2 (SDE for the Capital Required to Maintain Total Capital Ratio at 8% and 15%)

Given that the capital adequacy ratio is:

$\text{CAR} = Z(t) = \dfrac{K(t)}{Y_{rw}(t)}$, then the dynamics of the capital required to maintain the total capital ratio at 8% and 15% are:

$$dK_1(t) = \{0.016\pi_s(t)\big(r(t)+\upsilon\eta(t)+\sigma_s\lambda_r k_1 r(t)\big) + 0.04\pi_l(t)\big(r(t)+b_1\lambda_r k_1 r(t)\big)\}dt$$
$$+0.016\pi_s(t)\sqrt{\eta(t)}dw_s(t) + \big[0.016\pi_s(t)\sigma_s\sigma_r\sqrt{r(t)} + 0.04\pi_l(t)b_1\sigma_r\sqrt{r(t)}\big]dw_r(t) \qquad (47)$$

and

$$dK_2(t) = \{0.03\pi_s(t)\big(r(t)+\upsilon\eta(t)+\sigma_s\lambda_r k_1 r(t)\big) + 0.075\pi_l(t)\big(r(t)+b_1\lambda_r k_1 r(t)\big)\}dt$$
$$+0.03\pi_s(t)\sqrt{\eta(t)}dw_s(t) + \big[0.03\pi_s(t)\sigma_s\sigma_r\sqrt{r(t)} + 0.075\pi_l(t)b_1\sigma_r\sqrt{r(t)}\big]dw_r(t) \qquad (48)$$

respectively.

Proof:

For 8% total capital ratio

$\dfrac{K_1(t)}{Y_{rw}(t)} = 0.08$, this implies that $K_1(t) = 0.08Y_{rw}(t)$

Therefore, the dynamics of the capital required to maintain total capital ratio at 8% is:

$$dK_1(t) = 0.08dY_{rw}(t)$$

$$= 0.08(0.2\pi_s(t)\big[\big(r(t)+\upsilon\eta(t)+\sigma_s\lambda_r k_1 r(t)\big)dt + \sigma_s\sigma_r\sqrt{r(t)}dw_r(t)$$
$$+\sqrt{\eta(t)}dw_s(t)] + 0.5\pi_l(t)[\big(r(t)+b_1\lambda_r k_1 r(t)\big)dt + b_1\sigma_r\sqrt{r(t)}dw_r(t)])$$

$$= 0.08\{[0.2\pi_s(t)\big(r(t)+\upsilon\eta(t)+\sigma_s\lambda_r k_1 r(t)\big) + 0.5\pi_l(t)\big(r(t)+b_1\lambda_r k_1 r(t)\big)]dt$$
$$+0.2\pi_s(t)\sqrt{\eta(t)}dw_s(t) + \big[0.2\pi_s(t)\sigma_s\sigma_r\sqrt{r(t)} + 0.5\pi_l(t)b_1\sigma_r\sqrt{r(t)}\big]dw_r(t)\}$$

$$= \{0.016\pi_s(t)\big(r(t)+\upsilon\eta(t)+\sigma_s\lambda_r k_1 r(t)\big) + 0.04\pi_l(t)\big(r(t)+b_1\lambda_r k_1 r(t)\big)\}dt$$
$$+0.016\pi_s(t)\sqrt{\eta(t)}dw_s(t) + \big[0.016\pi_s(t)\sigma_s\sigma_r\sqrt{r(t)} + 0.04\pi_l(t)b_1\sigma_r\sqrt{r(t)}\big]dw_r(t) \qquad (49)$$

Similarly, the dynamics of the capital required to maintain total capital ratio at 15% is:

$$dK_2(t) = 0.15dY_{rw}(t)$$

$$dK_2(t) = \{0.03\pi_s(t)\big(r(t)+\upsilon\eta(t)+\sigma_s\lambda_r k_1 r(t)\big) + 0.075\pi_l(t)\big(r(t)+b_1\lambda_r k_1 r(t)\big)\}dt$$
$$+0.03\pi_s(t)\sqrt{\eta(t)}dw_s(t) + \big[0.03\pi_s(t)\sigma_s\sigma_r\sqrt{r(t)} + 0.075\pi_l(t)b_1\sigma_r\sqrt{r(t)}\big]dw_r(t) \qquad (50)$$

Numerical Examples

Here, we present the numerical simulation for the evolution of the optimal investment strategy, Total risk weighted – assets, Basel II CAR and, capital required to maintain CAR at 8% and 15%. We assume that the investment period $T = 10$ years, $k = 0$. The remaining parameters: $a = 0.0187, b = 0.2339, r_0 = 0.05, \eta_0 = 1, \beta = -2, \lambda_r = 1, k_1 = 0.0073, \sigma_r = 0.0854, \alpha = 2,$ $\delta =$

$0.3, \rho = 0.5, \sigma_\eta = 1, \upsilon = 1.5, b_1 = 0.7, \sigma_s = 0.02, K(0) = 1, Y_{rw}(0) = 1.4, Z_1(0) = 0.08, Z_2(0) = 0.15$ are gotten from (Deelstra *et al*, 2003, Hui *et al*, 2013, Grant and Peter, 2014 and Ugo, 2014) . Graphs plotted from the numerical examples are given below:

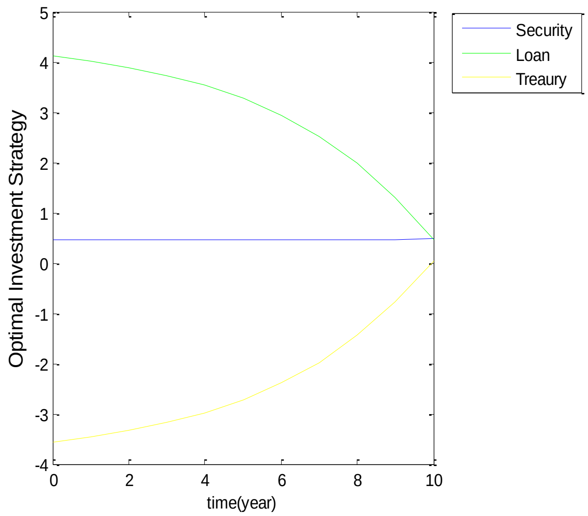

Fig.1 The effect of time on the optimal investment strategy

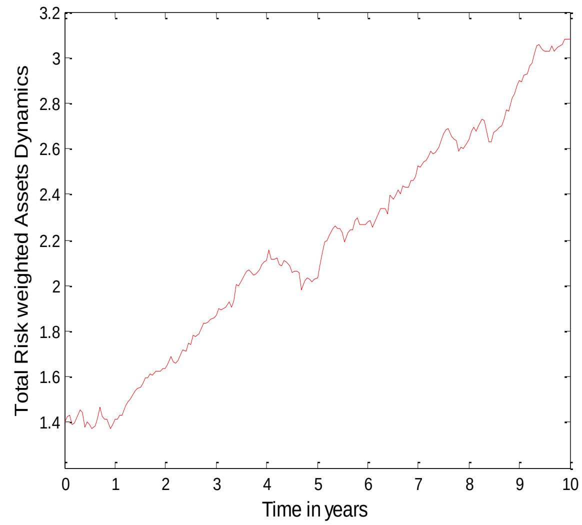

Fig.2 The total risk – weighted assets, $Y_{rw}(t)$

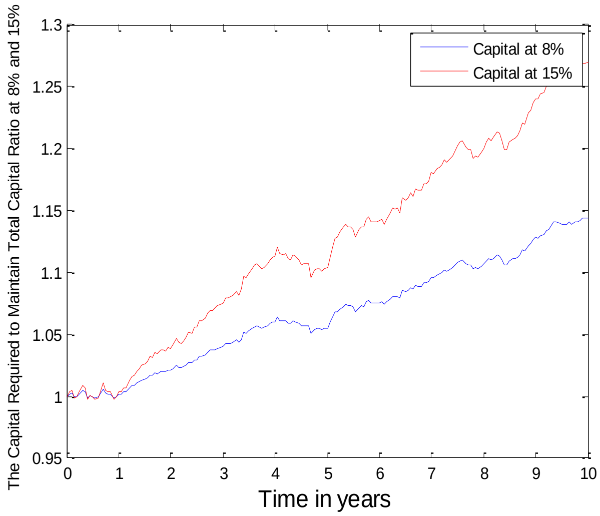

Fig.3 The capital, $K(t)$, required to maintain the capital adequacy ratio at 8% and 15%

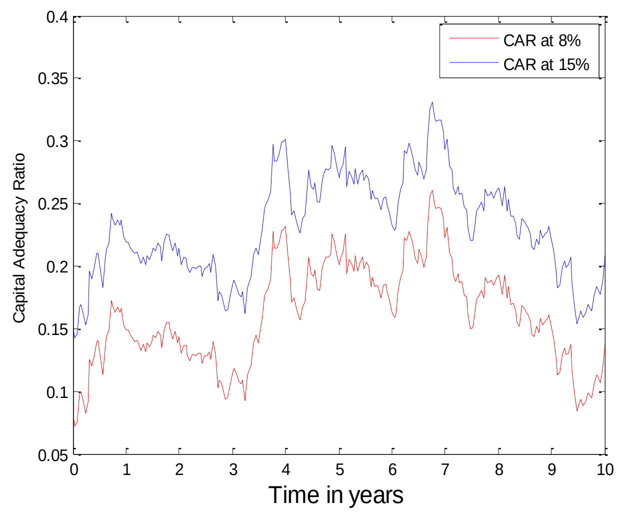

Fig. 4 The behavior of the capital adequacy ratio at 8% and 15%

Figure 1 illustrates the trends of how the optimal proportion of the wealth invested in the three assets change with time. From Figure 1, there is a positive relationship between optimal investment in the treasury and time. That is, as time increases so also the optimal investment in the treasury. However, the optimal proportion invested in the security almost remains unchanged and the optimal proportion invested loan decreases. It also shows that the optimal proportion invested in the treasury is negative at the beginning of the investment horizon which indicates that the investor takes a short position within this period but toward the end of the investment period, the investor invests more in the treasury to reach the optimal investment strategy. Hence, the optimal investment strategy is to diversify the financial institution portfolio away from the risky assets and towards the riskless asset.

Figure 2 illustrates how the evolution of the risk weighted – asset is affected by the stochastic variables characterizing the economy. By Basel II standard and Nigeria CBN, the financial institution is considered to be strongly capitalized and guaranteed the ability to absorb unexpected losses as shown in Figure 4. Therefore, as shown in Figure 4, the higher the CAR the more resilient the financial institution but this also has its down side as shown in figure 3. From Figure 3, we observed that more capital is needed to maintain the capital adequacy ratio at 15% than 8%. Therefore, the higher the percentage of the capital adequacy ratio, the more capital needed to maintain the prescribed capital adequacy ratio by the financial institution. This would tie up capital needed for investment by the investor. Therefore, prescribed capital adequacy ratio should be kept in a range such that the financial institution is well capitalized and guarantee that the financial institution can absorb reasonable unexpected losses, and also relieve fund for investor for investment which is important to the shareholders and the economy.

CONCLUSION

In this paper, we considered portfolio optimization problem of a financial institution where the interest rate is driven by stochastic affine interest rate model and the volatility of the security is described by the Heston stochastic volatility model. Therefore, the investor has to deal with the risk of both interest rate and volatility. Here, the investor objective is to maximize the terminal wealth. Under the portfolio optimization problem, the financial market consists of three assets namely; security, loan and treasury. We derived the optimal investment strategy for the case of CRRA utility function, obtained the explicit solution of the resulted Hamilton – Jacobi – Bellman equation for the optimal asset allocation problem. We also derived an explicit stochastic differential equation (SDE) for the capital adequacy ratio (CAR) which is the ratio of the financial institution total capital to the total risk – weighted assets under Basel II Accord, SDE for TRWA, and SDE for the capital needed to maintain the capital adequacy ratio at 8% and 15% and solved the SDEs numerically using Euler – Maruyama method. Analyze the behavior of the optimal portfolio and CAR via some numerical examples with interpretation of its economic meanings in the real market.

REFERENCES

Basel Committee on Banking Supervision (2004). International Convergence of Capital Measurements and Capital Standard: A revised Framework. Bank for International Settlements. www.bis.org/bcbs

Debajyoti, G. R., Bindya, K. and Swati, K. (2013). Basel I to Basel II to Basel III: A risk management journey in Indians Banks. *AIMA Journal of Management and Research*. **7**(2/4): 474 – 497.

Dangl, J. P. and Lehar, B. (2004). Value at risk vs. building block regulation in banking. *Journal of Financial Intermediation*. **13**: 132 – 155.

Decamps, J. P., Rochet, J. C. and Roger, B. (2004). The three pillars of Basel II: Optimizing the mix. *Journal of Financial Intermediation*. **13**: 96 – 131.

Deelstra, G., Grasselli, M. and Koehl, P. F. (2003). Optimal investment strategies in the presence of a minimum guarantee. *Insurance: Mathematics and Economics*. **33**: 189 – 207.

Diamond, D.W. and Rajan, R.G. (2000). A theory of bank capital. *The Journal of Finance*. **55**(6): 2431 – 2465.

Fouche, C. H., Mukuddem – Petersen, J. and Petersen, M. A. (2006). Continuous – time stochastic modeling of capital adequacy ratio for banks. *Applied Stochastic Model in Business and Industry*. **22**(1): 41 – 71.

Grant, E. M. and Peter, J. W. (2014). An optimal portfolio and capital management strategy for Basel III Compliant Commercial Banks. *Journal of Applied Mathematics*. Vol. 2014, Article ID 723873, 11 pages.

Hui, Z., Ximm, R. and Yonggan, Z. (2013). Optimal excess – of – loss reinsurance and investment problem for an insurer with jump – diffusion risk process under Heston model. *Insurance: Mathematics and Economics*. **53**: 504 – 514.

Investopedia (2019). Basel Accord. Available at http://www.investopedia.com/items/b/basel_accord.asp (Accessed 27 November 2019).

Mukuddem – Petersen, J. and Petersen, M. A. (2008). Optimizing asset and capital adequacy management in banking. *Journal of Optimization Theory and Applications*. **137**(1): 205 – 230.

Munk, C., Sorensen, C. and Vinther, T. N. (2004). Dynamic asset allocation under mean – reverting returns, stochastic interest rates and inflation uncertainty. Are popular recommendations consisted with rational behavior? *International Review of Economics and Finance*. **13**: 141 – 166.

Merton, R. C. (1969). Lifetime portfolio selection under uncertainty: The continuous case. *Review of Economics and Statistics*. **51**: 247 – 257.

Merton, R. C. (1971). Optimal consumption and portfolio rules in a continuous time model. *Journal of Economics Theory*. **3**: 373 – 413.

Peter, J. W., Garth, J. V. S. and Grant, E. M. (2011). An optimal investment strategy in bank management. *Mathematical Methods in the Applied Sciences.* **34**: 1606 - 1617.

Von Thadden, E. L. (2004). Bank capital adequacy regulation under the new Basel accord. *Journal of Financial Intermediation*. **13**(2): 90 – 95.

Ugo Obi – Chukwu (2014). Meaning of Capital Adequacy Ratio as Defined by Central Bank of Nigeria. www.nairametrics.com

Wachter, J. A. (2002). Portfolio and consumption decisions under mean – reverting returns: An exact solution for complete markets. *Journal of Financial and Quantitative Analysis*. **37**(1): 63 – 91.