COMPUTATIONAL SOLUTION OF TEMPERATURE DISTRIBUTION IN A THIN ROD OVER A GIVEN INTERVAL
$$I = \{x|0 < x < 1\}$$

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ABSTRACT

In this paper, two analytical–numerical algorithms are formulated based on homotopy perturbation method and new iterative method to obtain numerical solution for temperature distribution $u(x,t)$ in a thin rod over a given finite interval. The effects of different parameters such as the coefficient β which accounts for the heat loss and the diffusivity constant ϕ are examined when initial temperature distribution $u(x,0)$ (trigonometry and algebraic functions) are considered. The error in both algorithms approaches to zero as the computational length P increases. The proposed algorithms have been demonstrated to be quite flexible, robust and accurate. Thus, the algorithms are established as good numerical tools to solve several problems in applied mathematics and other related field of sciences.

Keywords: Homotopy Perturbation Method, New Iterative Method, Temperature Distribution, Diffusivity Constant, Computational Algorithms

INTRODUCTION

The heat equations model the flow of heat through materials. They arise from interpretation of models of physical phenomena to reality in applied sciences and engineering such as in fluid mechanics, thermodynamics, vibration analysis, electrical engineering and applied physics. In a thin rod with non-uniform temperature, thermal energy is transferred from regions of higher temperature to regions of lower temperature. Heat diffusion equation is a parabolic partial differential equation (PDE) which describes the heat distribution in a given region and provides the basic tool for heat conduction analysis. The numerical solutions for this type of equation provide understanding knowledge of the temperature distribution which may then be used with Fourier's law to determine the heat flux as elucidated in the works of Guenther and Lee (1996), Cannon and Browder (1984) and

Bergman *et al.* (2011). Seeking of analytic-numerical methods has gained the interest of researchers for finding approximate solutions to PDEs. This interest is driven by demand of applications both in industries and sciences which leads to investigate analytical and numerical methods for solving initial and boundary value problems as expressed by Cheniguel (2014). In this paper, the temperature distribution $u(x,t)$ is sought in a thin rod over the finite interval $I = \{x|0 < x < 1\}$ whose lateral surface is not insulated. The rod is experiencing a heat loss proportional to the difference between the rod temperature and the surrounding temperature at zero degrees. Adopting the form of the homogeneous partial differential equation expressed by Articolo (2009), we have the second order heat equation as:

Figure 1: A thin rod of length L not insulated on its sides along x-coordinate at time t.

$$\frac{\partial u(x,t)}{\partial t} = \phi\left(\frac{\partial^2 u(x,t)}{\partial x^2}\right) - \beta u(x,t) \qquad k, \beta > 0, \quad 0 \le x \ge 1, \quad 0 \le t \ge T \tag{1}$$

with the initial temperature used in the works of Articolo (2009) and Moore (2005):

$$u(x,0) = \begin{cases} 4\sin(x) + 2\sin(2x) + 7\sin(3x) \\ \qquad\qquad 1 - \dfrac{x^2}{3} \end{cases} \tag{2}$$

The initial and boundary conditions are:

$$\begin{cases} u_t(x,0) = 0 \\ u(1,t) + u_t(1,t) = 0 \end{cases} \tag{3}$$

The left end of the rod is insulated and the right end is losing heat by convection into a zero temperature surrounding where coefficient β accounts for the heat loss and ϕ is called the thermal diffusivity of the medium under consideration. This constant is equal:

$$\phi = \frac{K}{\rho C_p} \tag{4}$$

where C_p the specific heat of the medium at constant pressure, ρ is the mass density and K is the thermal conductivity of the medium. By convention, it could be said that heat is a diffusion process whereby heat flows from high-temperature regions to low-temperature regions similar to how salt in a water solution diffuses from high-concentration regions to low-concentration regions. The magnitude of the thermal diffusivity ϕ is an indication of the ability of the medium to conduct heat from one region to another. In this paper, $u(x,t)$ denotes the temperature at the point x-metres along the rod at time t (in seconds) and the authors are interested in how the temperature along the rod varies with time which is subjected to initial temperature distribution $u(x,0)$ (in the forms of trigonometry, exponential and logarithm functions).

Recently, Wegrzyn-Skrzypczak and Skrzypczak (2017) presented the results of analytical and numerical solutions for the problem of heat transport in the rod of finite length employing Fourier series and Finite Element Method (FEM). They addressed the compatibility of both solutions and distribution of the temperature for selected time moments. Among many researchers who investigated the heat transfers in solid media is Makhtoumi (2017). He presented a numerical method for the heat equation over one dimensional rod region. Al-Huniti *et al.* (2002) studied dynamic response of a rod due to a moving heat source under the hyperbolic heat conduction model. Also, Haggkvisk (2009) considered a practical and theoretical approach to the problem of plate thermometer, which is a robust and simple instrument to be used to handle fierce environmental conditions, as a mean of calculating incident heat radiation. He (1999) presented a study for the numerical solution of three-dimensional second order partial differential equation that occurs in a thin plate vibration behaviour using homotopy perturbation algorithm.

The aim of this paper is to formulate and employ two computational algorithms based on homotopy perturbation method and new iterative method to obtain temperature distribution $u(x,t)$ in a thin rod over a given finite interval whose lateral surface is not insulated.

COMPUTATIONAL TECHNIQUES
Homotopy Perturbation Method (HPM)
He (1999) proposed a new perturbation technique coupled with the homotopy technique, which is called the homotopy perturbation method (HPM). Hemeda (2012) and Falade *et al.* (2020) respectively applied homotopy perturbation method for solving systems of nonlinear coupled equations and solve some systems of partial differential equations, viz: the systems of coupled Burgers' equations in one and two dimensions and the system of Laplace's equation. Furthermore, Xiao *et al.* (2015) proposed a local fractional homotopy perturbation method which was an extended form of the classical homotopy perturbation method. Singh and Chatterjee (2017) presented numerical solutions of one dimensional space and time fractional advection- dispersion equation by homotopy perturbation method.

In this paper, we present the homotopy perturbation method (HPM) as a numerical technique for solving functional equation of the form:

$$A(u) - f(\sigma) = 0, \qquad \sigma \epsilon \Omega \tag{6}$$

with boundary conditions:

$$B\left(u, \frac{\partial u}{\partial \sigma}\right) = 0, \qquad \sigma \epsilon \Gamma \tag{7}$$

where A is a general operator, B is a boundary operator, $f(\sigma)$ is a known analytic function, and Γ is the boundary of the domain Ω. A can further be divided into two parts L and N where L is the linear part of A and N if the nonlinear part of A. Eq. (6) can now be expressed as:

$$L(u) + N(u) - f(\sigma) = 0, \quad \sigma \epsilon \Omega \tag{8}$$

By constructing the homotopy $v(\sigma, p): \Omega \times [0,1] \to \mathbb{R}$, which satisfies:

$$H(v, p) = (1 - p)[L(v) - L(u_0)] + p[A(v) - f(\sigma)] = 0, \quad p\epsilon[0,1], \sigma\epsilon\Omega \tag{9}$$

u_0 is the initial approximation Eq. (6), the homotopy parameter is then used to expand;

$$v = v_0 + pv_1 + p^2 v_2 + \cdots = \sum_{j=0}^{\infty} p^j v_j \tag{10}$$

Take $p = 1$, then

$$u(\sigma) = \lim_{p=1} v = \sum_{j=0}^{\infty} p^j v_j \tag{11}$$

Formulation of six steps homotopy perturbation algorithm (HPA)

In order to formulate homotopy perturbation algorithm (HPA), we consider Eq. (1) coupled with initial and boundary conditions and Eq. (6) - Eq. (11) using MAPLE 18 commands as follows:

```
restart:
Step 1:
Digits ≔ 30;
ϕ ≔ [0.1554 , 0.0554 ]
β ≔ 0.3333;
P ≔ 20;
```
$$u(x,0) := \begin{cases} 4\sin(x) + 2\sin(2x) + 7\sin(3x) \\ 1 - \dfrac{x^2}{3} \\ 2\sin\left(\dfrac{\pi x}{2}\right) + \sin(2\pi x) + 4\sin(2\pi x) \\ 7\log_{10}(x) \end{cases};$$

```
u_t(x, 0) ≔ 0;
u(1, t) + u_t(1, t):=0;
u[0] ≔ u(x, 0) + t * (u(1, t) + u_t(1, t));

Step 2:
for i from 0 to 0 do
A[1] ≔ diff(u[0], x) − ϕ * diff(u[0], x, x) − β * u[0];
A[2] ≔ −Int(A[1], t);
u[i + 1] ≔ value(A[2]);
end do

Step 3:
for i from 1 to P do
B[1] ≔ −ϕ * diff(u[i], x, x) − β * u[i];
B[2] ≔ −Int(B[1], t);
u[i + 1] ≔ value(B[2]);
end do

Step 4:
u[HPA] ≔ evalf(sum(u[j], j = 0 ... P + 1));                    (12)
eval(u[HPA], [x = 0.0, t = 0.0]);
 eval(u[HPA], [x = 0.1, t = 0.1]);
eval(u[HPA], [x = 0.2, t = 0.2]);
eval(u[HPA], [x = 0.3, t = 0.3]);
eval(u[HPA], [x = 0.4, t = 0.4]);
eval(u[HPA], [x = 0.5, t = 0.5]);
eval(u[HPA], [x = 0.6, t = 0.6]);
eval(u[HPA], [x = 0.7, t = 0.7]);
eval(u[HPA], [x = 0.8, t = 0.8]);
```

$eval(u[HPA], [x = 0.8, t = 0.8]);$
$eval(u[HPA], [x = 0.9, t = 0.9]);$
$\ eval(u[HPA], [x = 1.0, t = 1.0]);$

Step 5:
$T[0] \coloneqq eval(u[HPA], [x = 0]);$
$T[2] \coloneqq eval(u[HPA], [x = 2]);$
$T[4] \coloneqq eval(u[HPA], [x = 4]);$
$T[6] \coloneqq eval(u[HPA], [x = 6]);$
$T[8] \coloneqq eval(u[HPA], [x = 8]);$
$T[10] \coloneqq eval(u[HPA], [x = 10]);$

Step 6:
$m[3Dplot] \coloneqq plot3d(u[HPA], t = -10\pi \dots 10\pi, x = -10\pi \dots 10\pi, grid = [100,100], color);$
$m[2Dplot] \coloneqq plot([T[0], T[2], T[4], T[6], T[8], T[10]]), t = 0 \dots 10, color =$
$[red, blue, green, yellow, black, purple], axes = BOXED, title = cases),$
$m[Polarplot] \coloneqq (u[HPA], t = -10\pi \dots 10\pi, x = -10\pi \dots 10\pi);$
$m[Densityplot] \coloneqq (u[HPA], t = -10\pi \dots 10\pi, x = -10\pi \dots 10\pi);$
Output: Tables 1,2,3 and Figures:1,,2,3....,16
where P is the computational length.

New Iterative Method
New iterative method (NIM) was proposed by Daftardar-Gejji and Jafari (2006). NIM is simple in its principles and easy to implement on computer using symbolic computation packages such as Maple. This method is better a numerical method as it is free from rounding off errors and does not require large computer space as evident in the works of Bhalekar and Gejji (2008) and Allawee (2018).

Consider new iterative method (NIM) as a numerical technique for solving functional equation of the form
$$u(\bar{x}) = f(\bar{x}) + N(u(\bar{x})) \tag{13}$$
where N a nonlinear operator from a Banach space $B \to B$ and $f(\bar{x})$ is a known function.
$\bar{x} = (x_1, \ x_2, \ x_3, \dots, x_n)$. We need to obtain the solution $u(\bar{x})$ of Eq. (13) having the series of the form:
$$u(\bar{x}) = \sum_{i=0}^{\infty} u_i(\bar{x}) \tag{14}$$
The nonlinear operator which is on the right-hand side of Eq. (13) can be decomposed as follow:
$$N\left(\sum_{n=0}^{\infty} u_i(\bar{x})\right) = N(u_0) + \sum_{i=1}^{\infty} \left\{ N\left(\sum_{j=0}^{i} u_j\right) - N\left(\sum_{j=0}^{i-1} u_j\right) \right\} \tag{15}$$
Substituting equations Eq. (13) and Eq. (14) into the equation Eq. (15) and becomes:
$$\sum_{i=0}^{\infty} u_i(\bar{x}) = f(\bar{x}) + N(u_0) + \sum_{i=1}^{\infty} \left\{ N\left(\sum_{j=0}^{i} u_j\right) - N\left(\sum_{j=0}^{i-1} u_j\right) \right\} \tag{16}$$
The recurrence relation is given by
$$\begin{cases} u_0 = f, \\ \quad u_1 = N(u_0) \\ \qquad \vdots \\ u_{m+1} = N(u_0 + u_1 + \cdots + u_m) - N(u_0 + u_1 + \cdots + u_{m-1}) \\ \qquad m = 1,2,3, \dots \end{cases} \tag{17}$$
Then
$$(u_1 + u_2 + \cdots + u_{m+1}) = N(u_0 + u_1 + \cdots + u_m), \qquad m = 1,2,3, \dots \tag{18}$$

and
$$\sum_{i=0}^{\infty} u_i = f + N\left(\sum_{i=0}^{\infty} u_i\right) \tag{19}$$
The q −term approximate solution of Eq. (13) is given by;
$$u = u_0 + u_1 + \cdots + u_{q-1} \tag{20}$$

Formulation of seven steps New Iterative Algorithm (NIA)
In this section, we consider Eq. (1) coupled with initial conditions (2) and Eq. (13) to Eq. (20) and formulate five steps algorithm using MAPLE 18 commands to solve Eq. (1) as follows:

```
restart:
```
Step 1:
$Digits := 30;$
$\phi := [0.1554 , 0.0554];$
$\beta := 0.3333;$
$P := 20;$

$$u(x,0):= \begin{cases} 4\sin(x) + 2\sin(2x) + 7\sin(3x) \\ 1 - \dfrac{x^2}{3} \\ 2\sin\left(\dfrac{\pi x}{2}\right) + \sin(2\pi x) + 4\sin(2\pi x) \\ 7\log_{10}(x) \end{cases};$$

$u_t(x,0) := 0;$
$u(1,t) + u_t(1,t) := 0;$
$u[0] := u(x,0) + t * \big(u(1,t) + u_t(1,t)\big);$

Step 2:
for n from 0 to P do
$u[n+1] := value(int(\phi * diff(u[n], x, x) + \beta * u[n], t = 0 .. t));$
end do

Step 3:
for n from 0 to P + 1 do
$u[n] := u[n];$
end do
Step 4:
$Sum\ \boldsymbol{U} := sum(u[j]), j = 0 \ldots P + 1;$
$Simp\boldsymbol{U} := simplify(sum\boldsymbol{U});$
$u[NIA] := evalf(simp\boldsymbol{U});$
end do

Step 5:
$eval(u[NIA], [x = 0.0, t = 0.0]);$ (21)
$eval(u[NIA], [x = 0.1, t = 0.1]);$
$eval(u[NIA], [x = 0.2, t = 0.2]);$
$eval(u[NIA], [x = 0.3, t = 0.3]);$
$eval(u[NIA], [x = 0.4, t = 0.4]);$
$eval(u[NIA], [x = 0.5, t = 0.5]);$
$eval(u[NIA], [x = 0.6, t = 0.6]);$
$eval(u[NIA], [x = 0.7, t = 0.7]);$
$eval(u[NIA], [x = 0.8, t = 0.8]);$
$eval(u[NIA], [x = 0.8, t = 0.8]);$
$eval(u[NIA], [x = 0.9, t = 0.9]);$
$eval(u[NIA], [x = 1.0, t = 1.0]);$
Step 6:
$T[0] := eval(u[NIA], [x = 0]);$
$T[2] := eval(u[NIA], [x = 2]);$
$T[4] := eval(u[NIA], [x = 4]);$
$T[6] := eval(u[NIA], [x = 6]);$
$T[8] := eval(u[NIA], [x = 8]);$
$T[10] := eval(u[NIA], [x = 10]);$

Step 7:
$m[3Dplot] := plot3d(u[NIA], t = -10\pi \ldots 10\pi, x = -10\pi \ldots 10\pi, grid = [100,100], color);$
$m[2Dplot] := plot\big([T[0], T[2], T[4], T[6], T[8], T[10]]\big), t = 0 \ldots 10, color =$
$[red, blue, green, yellow, black, purple], axes = BOXED, title = cases),$
$m[Polarplot] := (u[NIA], t = -10\pi \ldots 10\pi, x = -10\pi \ldots 10\pi);$
$m[Densityplot] := (u[NIA], t = -10\pi \ldots 10\pi, x = -10\pi \ldots 10\pi);$
Output: Tables 1,2,3 and Figures:1,,2,3....,16

where P is the computational length.

Absolute error (E_t)

In order to determine the error involved in the proposed algorithms, we consider the absolute error as follows:

$$E_t = |u(x,t)_{exact} - u(x,t)_{numerical}|. \tag{22}$$

DISCUSSION OF RESULTS

In order to demonstrate the two proposed algorithms, we consider ϕ thermal diffusivity of the medium under consideration as $\phi = 0.1554w/mk$ *and* $\phi = 0.0554w/mk$ and heat loss coefficient $\beta = 0.3333$. Applying equations (12) and (21), we obtain the numerical solutions in Table 1 and Table 2.

Table 1: The temperature distribution $u(x,t)$ in a thin rod over the finite interval $I = \{x|0 < x < 1\}$.

$u(x,t)$	Solutions	Case 1 $\phi = 0.1554, \beta = 0.3333$ $u(x,0) = 4\sin(x) + 2\sin(2x) + 7\sin(3x)$ $u_t(x,0) = 0, u(1,t) + u_t(1,t) = 0$	Case 2 $\phi = 0.0554, \beta = 0.3333$ $u(x,0) = 4\sin(x) + 2\sin(2x) + 7\sin(3x)$ $u_t(x,0) = 0, u(1,t) + u_t(1,t) = 0$
(0.1, 0.1)	Exact	2.6521495757683692957631574165882245 28	2.8470310610423122839312338943999425 70
	HPA	2.6521495757683692957631574165882245 29	2.8470310610423122839312338943999425 69
	NIA	2.6521495757683692957631574165882245 28	2.8470310610423122839312338943999425 68
(0.2, 0.2)	Exact	4.7527034017185775369731269841014812 14	5.4601641759423524317848345637003049 58
	HPA	4.7527034017185775369731269841014812 12	5.4601641759423524317848345637003049 55
	NIA	4.7527034017185775369731269841014812 11	5.4601641759423524317848345637003049 57
(0.3, 0.3)	Exact	6.2659174475573756492635159066203969 57	7.6699378794545702579881498724132611 05
	HPA	6.2659174475573756492635159066203362 38	7.6699378794545702579881498724132611 05
	NIA	6.2659174475573756492635159066203362 37	7.6699378794545702579881498724132611 02
(0.4, 0.4)	Exact	7.2115995775098075243473737357642549 9025	9.3468028848632106577934514086405571 62
	HPA	7.2115995775098075243473737357638518 1928	9.3468028848632106577934514086405571 62
	NIA	7.2115995775098075243473737357638518 1927	9.3468028848632106577934514086405571 61
(0.5, 0.5)	Exact	7.6521552765184040966424789380615598 69	10.410183696467747173157871796631054 48
	HPA	7.6521552765184040966424789322403620 89	10.410183696467747173157871796631054 49
	NIA	7.6521552765184040966424789322403620 84	10.410183696467747173157871796631054 48
(0.6, 0.6)	Exact	7.6784300245412195399669305979391214 75	10.832997289631767630095848054454393 58
	HPA	7.6784300245412195399669302856085437 74	10.832997289631767630095848054454393 58
	NIA	7.6784300245412195399669302856085437 82	10.832997289631767630095848054454393 58
(0.7, 0.7)	Exact	7.3958526975964538328634346456049992 67	10.641408294311936313072481080792559 98
	HPA	7.3958526975964538328634264588534944 21	10.641408294311936313072481080792559 97
	NIA	7.3958526975964538328634264588534944 14	10.641408294311936313072481080792559 98
(0.8, 0.8)	Exact	6.9120629350461463969382682956341977 1	9.9100232431716364346401291681761450 72
	HPA	6.9120629350461463969381482727529215 11	9.9100232431716364346401291681761450 60
	NIA	6.9120629350461463969381482727529215 12	9.9100232431716364346401291681761450 62
(0.9, 0.9)	Exact	6.3268395559802494119265891626973337 282	8.7531169080980379891754499888918878 09
	HPA	6.3268395559802494119264879820906180 314	8.7531169080980379891754499888918876 49
	NIA	6.3268395559802494119264879820906180 314	8.7531169080980379891754499888918876 49

Table 2: The temperature distribution $u(x,t)$ in a thin rod over the finite interval $I = \{x|0 < x < 1\}$.

$u(x,t)$	Solutions	Case 3 $\phi = 0.1554 , \beta = 0.3333$ $u(x,0) = 1 - \dfrac{x^2}{3}$ $u_t(x,0) = 0, u(1,t) + u_t(1,t) = 0$	Case 4 $\phi = 0.0554 , \beta = 0.3333$ $u(x,0) = 1 - \dfrac{x^2}{3}$ $u_t(x,0) = 0, u(1,t) + u_t(1,t) = 0$
(0.1,0.1)	Exact	1.019734243972716468494282330684589162	1.026596056743723131053823183345899768
	HPA	1.019734243972716468494282330684589162	1.026596056743723131053823183345899768
	NIA	1.019734243972716468494282330684589162	1.026596056743723131053823183345899768
(0.2,0.2)	Exact	1.032531282501371545306117024025157942	1.046720903756481453457365224141398932
	HPA	1.032531282501371545306117024025157943	1.046720903756481453457365224141398932
	NIA	1.032531282501371545306117024025157943	1.046720903756481453457365224141398932
(0.3,0.3)	Exact	1.037656701780686252993428477575609077	1.059664525010099770187399767045015571
	HPA	1.037656701780686252993428477575609078	1.059664525010099770187399767045015572
	NIA	1.037656701780686252993428477575609078	1.059664525010099770187399767045015572
(0.4,0.4)	Exact	1.034326089885997016541275218117628917	1.064668070766782892609132404320329611
	HPA	1.034326089885997016541275218117628917	1.064668070766782892609132404320329612
	NIA	1.034326089885997016541275218117628918	1.064668070766782892609132404320329613
(0.5,0.5)	Exact	1.021702213894932897667976583727712352	1.060921087918769668521863156759094001
	HPA	1.021702213894932897667976583727712352	1.060921087918769668521863156759094001
	NIA	1.021702213894932897667976583727712352	1.060921087918769668521863156759094001
(0.6,0.6)	Exact	0.998892053692859697178763107923441174	1.047558609385673953174190057993126009
	HPA	0.998892053692859697178763107923441177	1.047558609385673953174190057993126010
	NIA	0.998892053692859697178763107923441177	1.047558609385673953174190057993126014
(0.7,0.7)	Exact	0.964943685671467128640632964723325906	1.023658095883069631314040326999792559
	HPA	0.964943685671467128640632964723325974	1.023658095883069631314040326999792576
	NIA	0.964943685671467128640632964723325973	1.023658095883069631314040326999792573
(0.8,0.8)	Exact	0.918843009223403815439357111250741531	0.988236222307391869231411070659671884
	HPA	0.918843009223403815439357111250742810	0.988236222307391869231411070659671918 8
	NIA	0.918843009223403815439357111250742809	0.988236222307391869231411070659671918 9
(0.9,0.9)	Exact	0.859510308616053748177622094402613561	0.940245506799336893560988696551675838
	HPA	0.859510308616053748177622094402630813	0.940245506799336893560988696551680575
	NIA	0.859510308616053748177622094402630813	0.940245506799336893560988696551680575

Table 3: <u>Absolute error</u>

$u(x,t)$	Algorithm	**Case 1**	**Case 2**	**Case 3**	**Case 4**
(0.1,0.1)	HPA_{E_t}	$1.1000 - E36$	$1.1000 - E36$	$2.1000 - E37$	0.000000000
	NIA_{E_t}	$1.1000 - E36$	$1.1000 - E36$	0.000000000	0.000000000
(0.2, 0.2)	HPA_{E_t}	$2.1000 - E36$	$3.1000 - E36$	0.000000000	0.000000000
	NIA_{E_t}	$3.1000 - E36$	$1.1000 - E36$	$1.1000 - E36$	0.000000000
(0.3, 0.3)	HPA_{E_t}	$6.0719 - E32$	$3.1000 - E36$	$1.1000 - E37$	0.000000000
	NIA_{E_t}	$6.0720 - E32$	$3.1000 - E36$	$1.1000 - E36$	0.000000000
(0.4, 0.4)	HPA_{E_t}	$2.0317 - E29$	0.00000000	0.000000000	$1.1000 - E36$
	NIA_{E_t}	$4.0317 - E29$	$1.1000 - E36$	$1.0000 - E36$	$1.1000 - E36$
(0.5, 0.5)	HPA_{E_t}	$5.8211 - E27$	$1.1000 - E35$	0.000000000	0.000000000
	NIA_{E_t}	$5.8212 - E27$	0.00000000	0.000000000	0.000000000
(0.6, 0.6)	HPA_{E_t}	$3.1233 - E23$	0.00000000	$2.6000 - E36$	$1.1000 - E36$
	NIA_{E_t}	$3.1233 - E25$	0.00000000	$3.1000 - E36$	$1.1000 - E36$
(0.7, 0.7)	HPA_{E_t}	$8.1868 - E24$	$1.1000 - E35$	$6.7500 - E35$	$1.1000 - E35$
	NIA_{E_t}	$8.1868 - E24$	0.00000000	$6.7500 - E35$	$1.1000 - E35$
(0.8, 0.8)	HPA_{E_t}	$1.2036 - E22$	$1.2100 - E35$	$1.2788 - E33$	$3.4310 - E34$
	NIA_{E_t}	$1.2036 - E22$	$1.1000 - E35$	$1.2783 - E33$	$3.4311 - E34$
(0.9, 0.9)	HPA_{E_t}	$1.0118 - E21$	$1.6000 - E34$	$1.7252 - E32$	$4.7362 - E33$
	NIA_{E_t}	$1.0118 - E21$	$1.6000 - E34$	$1.7252 - E32$	$4.7363 - E33$

Plots' Representation

In this section, we present behavioural pattern of temperature distribution $u(x, t)$ in a thin rod in the 3D, 2D, polar plot, and density plots phase.

3D plots temperature distribution $u(x, t)$ solutions in a thin rod

Figure 1. 3D plot U(x,t) when φ=0.1554w/mk β=0.3333   Case 1

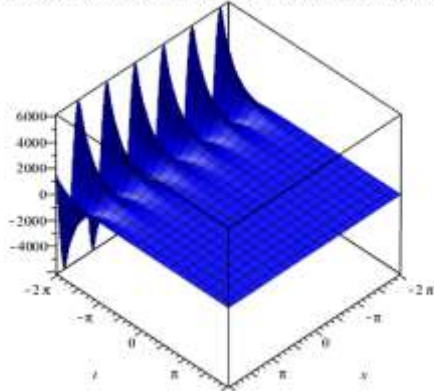Figure 2. 3D plot U(x,t) when φ=0.0554w/mk β=0.3333   Case 2
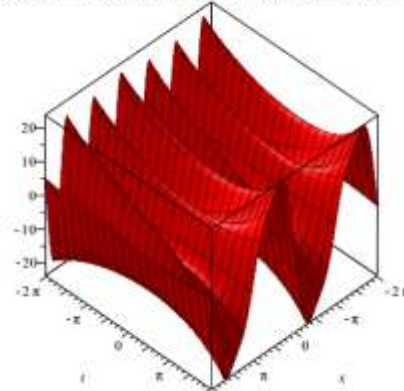
Figure 3. 3D plot U(x,t) when φ=0.1554w/mk β=0.3333   Case 3
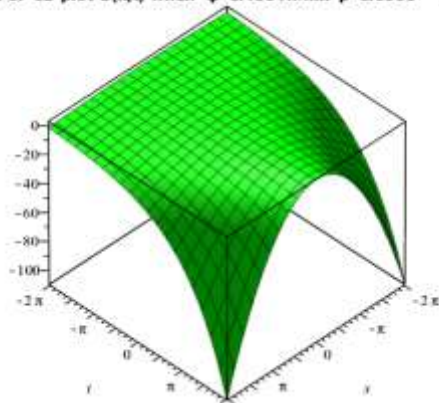
Figure 4. 3D plot U(x,t) when φ=0.0554w/mk β=0.3333   Case 4
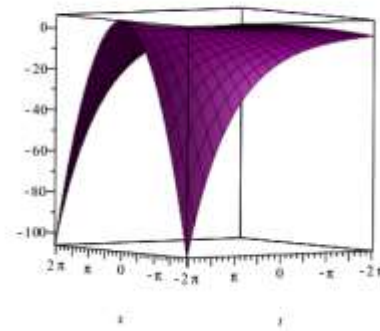
2D plots temperature distribution $0 \leq t \leq 10$

Figure 5. Temperature distributions at several times for case 1

Figure 6. Temperature distributions at several times for case 2

Figure 7. Temperature distributions at several times for case 3

Figure 8. Temperature distributions at several times for case 4
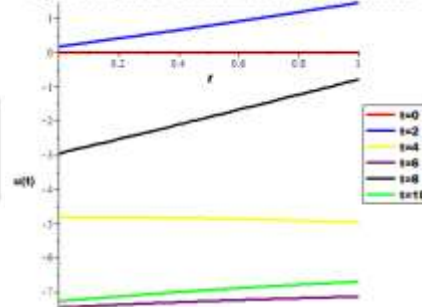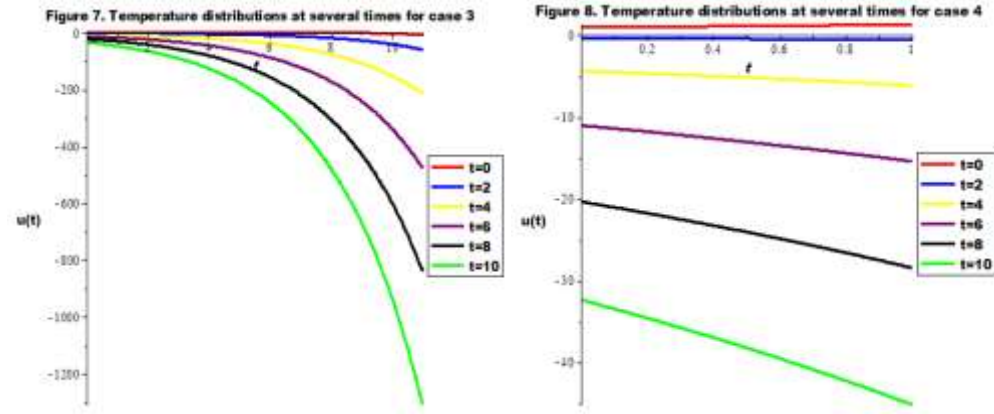
Polar plot temperature distribution $u(x, t)$ in a thin rod
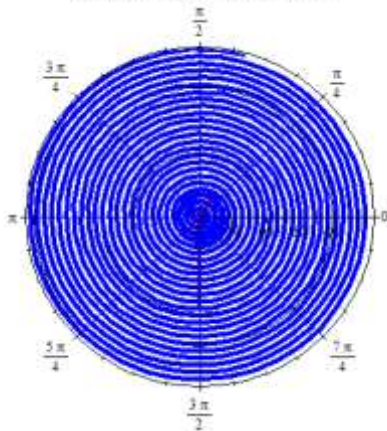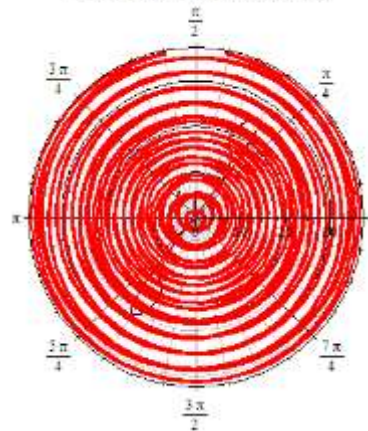

Figure 9. Polarplot for case 1

Figure 10. Polarplot for case 2
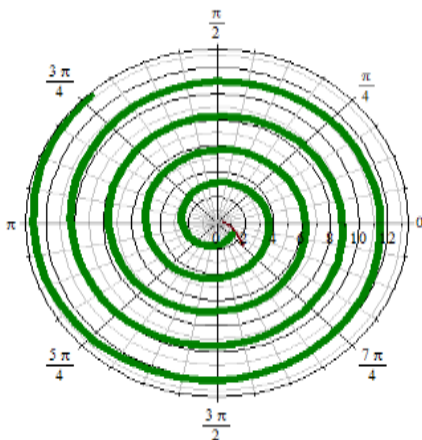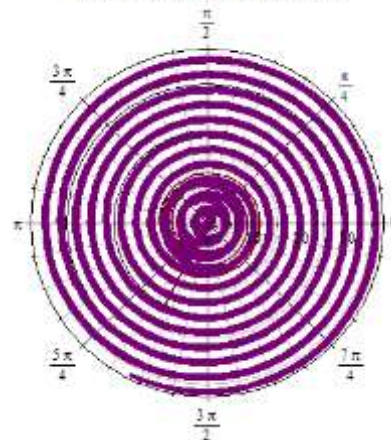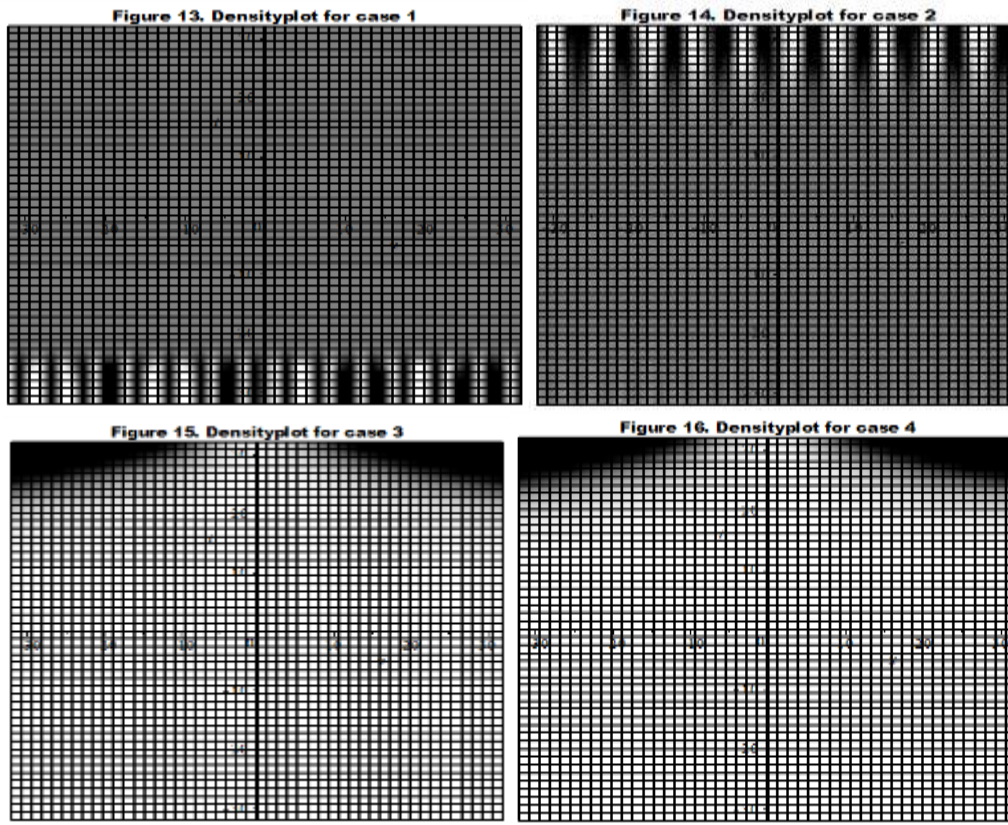

Figure 11. Polarplot for case 3

Figure 12. Polarplot for case 4

Density plot temperature distribution u(x, t) in a thin rod

Figure 13. Densityplot for case 1

Figure 14. Densityplot for case 2

Figure 15. Densityplot for case 3

Figure 16. Densityplot for case 4

CONCLUSION

The parabolic heat equation (1) was used to determine temperature distribution $u(x,t)$ in a thin rod whose lateral surface is not insulated and rod is experiencing a heat loss proportional to the difference between the rod temperature and the surrounding temperature at zero degrees. Four numerical experiments are considered to examine the relationship between coefficient β which accounts for the heat loss in equation (1) and ϕ thermal diffusivity of the thin rod over the finite interval $I = \{x|0 < x < 1\}$. The proposed algorithms have been demonstrated to be quite flexible, robust, efficiency and accurate in which the results obtained confirm the versatility, simplicity and efficiency of proposed techniques. Thus, the algorithms are suggested as good numerical tools to solve several problems in applied mathematics and other related field of sciences.

The aim of this paper is achieved from computational solutions of temperature distribution $u(x,t)$ in a thin rod which are presented in Table 1 and Table 2 for four cases of trigonometric and algebraic functions that are considered for initial temperature conditions. The following observations are made as follows:

i.   Table 1 shows the numerical solutions obtained at higher the value of $\phi = 0.1554\ \mathrm{k}/sm^2$, thermal diffusivity gives less temperature distribution $u(x,t)$ in a thin rod under the same initial and boundary conditions. Conversely, as the value $\phi = 0.0554\ \mathrm{k}/sm^2$ decreases, thermal

diffusivity gives higher temperature distribution $u(x,t)$ in a thin rod;

ii.   In the same vain, Table 3 shows the numerical solutions obtained at higher the value of $\phi = 0.1554\ \mathrm{k}/sm^2$. In this case, thermal diffusivity gives less temperature distribution $u(x,t)$ in a thin rod under the same initial and boundary conditions. In contrast, as the value of $\phi = 0.0554\ \mathrm{k}/sm^2$ becomes less, thermal diffusivity gives higher temperature distribution $u(x,t)$ in a thin rod;

iii.   The absolute error Table 3 indicates the efficiency of the two proposed algorithms;

iv.   The plot representations depict the numerical solutions of temperature distribution $u(x,t)$ in a thin rod in 3D plot solutions of Figure 1, Figure 2, Figure 3 and Figure 4;

v.   2D plot solutions in Figure 5, Figure 6, Figure 7 and Figure 8 of temperatures ranges $0^0 \leq t \leq 10^0$ are presented;

vi.   Temperature distribution $u(x,t)$ in a thin rod is presented in polar-plots (Figure 9, Figure 10, Figure 11 and Figure 12) and the density plot is displayed in (Figure 13, Figure 14, Figure 15 and Figure 16).

Hence, the reduction of ϕ thermal diffusivity enhances more temperature distribution $u(x,t)$ in a thin rod.

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