FUDMA Journal of Sciences (FJS)
ISSN online: 2616-1370
ISSN print: 2645 - 2944
Vol. 5 No. 2, June, 2021, pp 567-573
DOI: https://doi.org/10.33003/fjs-2021-0502-569

STOCHASTIC TRANSMISSION DYNAMICS OF COVID-19 WITHIN A DENSITY DEPENDENT POPULATION

Kwaghkor L. M.[a*], Onah E. S.[b], Bassi G. I.[c] and Danjuma, T.[d]

[a]School of Basic and Remedial Studies (SBRS), Federal University of Lafia, Nigeria
[b]National Mathematical Centre, Abuja, Nigeria
[c]Department of Mathematics, Federal University of Lafia, Nigeria
[d]Department of Mathematical Sciences, Federal University Gusau, Nigeria
[*]Corresponding Author's email: matlubem@gmail.com, +2348061263095

ABSTRACT

Coronavirus diseases (COVID-19) is a respiratory disease. Most infected people are known to develop mild to moderate symptoms and recover without requiring special treatment except for those who have underlying medical conditions and the elderly have a higher risk of developing severe disease. This research is aimed at studying the probable spread of the COVID-19 virus within a completely susceptible density-dependent population using a modified exponential distribution function. The modified exponential distribution function was extended to include the Basic Reproduction Number $\mathcal{R}_0$ which was computed using the Nigerian COVID-19 index cases from 27th February to 18th April, 2020 to be $\mathcal{R}_0 = 3.5$. Various interesting results were obtained for the $\mathcal{R}_0 = 3.5$ including the time period for the spread for different population sizes. The duration of the spread of the virus is from 4 to 7 hours with an average of 5.5 hours. This indicates that, for one infectious person with $\mathcal{R}_0 = 3.5$ to enter a completely susceptible population of size $50 \leq n \leq 2000$, the virus can spread through the entire population in about 5.5 hours if no control measures are in place.

Keywords: COVID-19, Transmission, Dynamics, Density-dependent-Population, Modified-Exponential-Distribution, Basic-Reproduction-Number.

INTRODUCTION

Corona Virus Disease (COVID -19) which causes various symptoms such as pneumonia, fever, breathing difficulty, and lung infection etc, belongs to a family of viruses that are common in animals worldwide; even though, very few cases have been known to affect humans (Wuhan Municipal Health and Health Commission (WMHC), 2020; Adhikari *et al.,* 2020). COVID-19 has now been declared as a public health emergency of international concern and that the virus affects different people in different ways. COVID-19 is a respiratory disease and most infected people develop mild to moderate symptoms and recover without requiring special treatment. People who have underlying medical conditions and the elderly have a higher risk of developing severe disease and death. Common symptoms include (WHO, 2020a): fever, tiredness, dry cough, shortness of breath, aches and pains, sore throat, and very few people will report diarrhoea, nausea or a runny nose. The COVID-19 is caused by the virus SARS-CoV-2. The source(s) and transmission routine(s) of SARS-CoV-2 remain elusive. Human-to-human transmission of SARS-CoV-2 occurs mainly between family members, including relatives and friends who intimately have contact with patients (Guo *et al.,* 2020) and crowded areas.

Among many nations battling with the spread of COVID-19 is Nigeria which has recorded a number of confirmed cases which is increasing slowly for now (NCDC, 2020a). The President of the Federal Republic of Nigeria has declared COVID-19 a dangerous infectious disease (NCDC, 2020b). According to NCDC (2020c), Mass gatherings are highly visible events with the potential for serious public health consequences if they are not planned and managed carefully. Mass gatherings can amplify the spread of coronavirus disease (COVID-19). This is true for COVID-19 because once infected, one automatically becomes infectious as one common mode of transmission is through droplets. According to current evidence, COVID-19 virus is primarily transmitted between people through respiratory droplets and contact routes. Droplet transmission occurs when a person is in close contact (within 1 meter) with someone who has respiratory symptoms (e.g., coughing or sneezing) and is therefore at risk of having his/her mucosae (mouth and nose) or conjunctiva (eyes) exposed to potentially infective respiratory droplets. Transmission may also occur through fomites in the immediate environment around the infected person. Therefore, transmission of the COVID-19 virus can occur by direct contact with infected people and indirect contact with surfaces in the immediate environment or with objects used on the infected person (WHO, 2020b).

The aim of this research is to study the probable spread of the COVID-19 virus within a density dependent population (where every person in the population is susceptible as a result of lack of control measure) using a modified exponential distribution. This modified exponential distribution was proposed by Kwaghkor *et al.* (2019) to study long term unemployment rate and was also used by Kwaghkor (2020) to study the effect of deforestation in Nigeria. In this work, the modified exponential distribution function will extended to include the Basic Reproduction Number $\mathcal{R}_0$ which as a way of application is computed using the current available data on the Nigerian COVID-19 index cases for the period of 27th February to 18th April, 2020 provided by The Nigeria Centre for Disease Control (NCDC).

After the introduction, the remaining part of the paper is organized as follows: In Section 2, the modified exponential distribution function was stated and the cumulative distribution function derived. In Section 3, the basic reproduction number was presented. In Section 4, the results were presented followed by their discussions. Finally, the conclusion and recommendations are presented in Section 5.

MATERIALS AND METHODS

This modified exponential distribution function was first proposed by Kwaghkor *et al.* (2019) to study long term unemployment rate and was presented as

$$f(t;\lambda) = \begin{cases} (1-\exp(-J/n))\exp(-(1-\exp(-J/n))t), & t>0 \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

where $\lambda = 1-\exp(-J/n)$, $J =$ Number of persons entering unemployment state at a given time interval, t and $n =$ Number of persons from the source of entering unemployment state at a given time interval, t.

The Cumulative Distribution Function (CDF) of the modified exponential distribution function written as the probability of lifetime being less than some value, t, is

$$F(t;\lambda) = P(T \le t) = 1 - exp[-(1-\exp(-J/n))t] \quad (2)$$

Equation (2) was used to give a probable unemployment rate if J and n are known.

In this paper, n is redefined as the total number of persons in the population, while J is redefined as the number of infected persons at a particular term of the sequence (of infected) per time interval (in this case one hour) under the following additional parameters and assumptions.

Parameters

$\boldsymbol{x}$: is the number of persons an infected person can produce within the infectious period (x in this case is the Basic Reproduction Number, $\mathcal{R}_0$),

$\boldsymbol{p}$: is the number of infected persons within an hour,

$\boldsymbol{i}$: is the duration of the spread (i.e. the number of hours that the spread lasts).

$\boldsymbol{n}$: is the total number of persons in the population.

Assumptions

i. The population in this paper is not the number of persons in a locality but a closed population of Mass gatherings such as parties, religious activities, rallies, protests etc,
ii. Mass gatherings are expected to last between zero to eight hours: This is $0 \le i \le 8$,
iii. There are no control measures put in place yet (which means every person in the population is susceptible),
iv. Screening for new cases is conducted at the interval of every one hour.

Based on the parameters and the assumptions above, J is redefined as the number of infected persons at a particular term of the sequence (of infected) per time interval (in this case one hour) as follows

$T_1 = 1 + x = p$

In the next one hour period,

$T_2 = T_1 x = px$, (assuming that each infected person is capable of infecting x persons within one hour). So that, $T_3 = T_2 x = px^2$, $T_4 = T_3 x = px^3, \cdots, T_i = T_{i-1} x = px^{i-1}$

Hence,

$$J = T_i = px^{i-1} = (1+x)x^{i-1} \quad (3)$$

where $1+x$ is the number of primary infections within an hour. x, i are as defined above.

Now, substituting equation (3) into equation (2) gives equation (4) below

$$F(t;\lambda) = P(T \le t) = 1 - exp\left[-\left(1-\exp\left(-((1+x)x^{i-1})/n\right)\right)t\right] \quad (4)$$

By the assumption that screening for infected persons is carried out in every one hour ($t = 1$ hour), then equation (4) will be written as

$$F(t;\lambda) = P_{SI}(T \le t) = 1 - exp\left[-\left(1-\exp\left(-((1+x)x^{i-1})/n\right)\right)\right] \quad (5)$$

Equation (5) will be used to study the probable spread of the COVID-19 virus (from susceptible population S to an infectious population I) within a density-dependent population.

Application: The Basic Reproduction Number ($\mathcal{R}_0$) for Nigeria as it relates to contact rate and infectious period

The basic reproduction number, $\mathcal{R}_0$, is defined as the expected number of secondary cases produced by a single infection in a completely susceptible population (Jones, 2007; Van Den Driessche and Watmough, 2008).

Relating to contact rate and infectious period, the basic reproduction number ($\mathcal{R}_0$) is directly proportional to the infectious period (τ). That is

$$\mathcal{R}_0 \propto \tau$$

$$\mathcal{R}_0 = \beta\tau \quad (6)$$

where, β is the infection-producing contacts per unit time or the rate of spread of the infection within a time period.

Table 1: Daily COVID-19 Cases in Nigeria from 27th Feb. – 18th April, 2020 (NCDC, 2020d; Worldometers, 2020)

Date	New Cases in Nigeria	Cumulative
27-Feb	1	1
28	0	1
29	0	1
1-Mar	0	1
2	0	1

3	0	1
4	0	1
5	0	1
6	0	1
7	0	1
8	0	1
9	1	2
10	0	2
11	0	2
12	0	2
13	0	2
14	0	2
15	0	2
16	0	2
17	1	3
18	5	8
19	4	12
20	0	12
21	10	22
22	8	30
23	10	40
24	4	44
25	7	51
26	14	65
27	5	70
28	27	97
29	14	111
30	20	131
31	4	135
1-Apr	39	174
2	10	184
3	26	210
4	4	214
5	18	232
6	6	238
7	16	254
8	22	276
9	12	288
10	17	305
11	13	318
12	5	323
13	20	343
14	30	373

15	34	407
16	35	442
17	51	493
18	49	542

Using the data of Table 1 and equation (6), the average $\mathcal{R}_0$ within 24 hours ($\tau = 24$ hours) is 10.4231, and $\beta = 0.4343$. Substituting the value of $\beta = 0.4343$ back into equation (6) gives equation (7) below

$$\mathcal{R}_0 = 0.4343\tau \qquad (7)$$

Therefore, the basic reproduction number ($\mathcal{R}_0$) for a time period of 8 hours (8 hours is the maximum period for the Mass gathering) is

$$\mathcal{R}_0 = 0.4343 \times 8 = 3.4744 \cong 3.5 = x$$

$\mathcal{R}_0 = 3.5 = x$ is substituted into equation (5) to give

$$F(t;\lambda) = P_{SI}(T \le t) = 1 - exp\left[-\left(1 - \exp\left(-\left((1+3.5)3.5^{i-1}\right)/n\right)\right)\right] \qquad (8)$$

A MATLAB programme is written to help in the computation of the probable transmission dynamics path using equation (8). The table generated from the computation is presented in Table 2 and the graph is presented in Figure 1 below.

RESULTS AND DISCUSSION

Results

The result of the model equation (5) for the Basic Reproduction Numbers $\mathcal{R}_0 = 3.5$ and various population densities, $n = 50, 100, 500, 1000$ and 2000 are presented in Figures 1.

Table 2: Transmission probabilities of COVID-19 within a density dependent population

i	$P_{SI}: n = 50$	I	Cum.(I)	$P_{SI}: n = 100$	I	Cum. (I)	$P_{SI}: n = 500$	I	Cum. (I)	$P_{SI}: n = 1000$	I	Cum. (I)	$P_{SI}: n = 2000$	I	Cum. (I)
0	0.0251	1	1	0.0127	1	1	0.0026	1	1	0.0013	1	1	0.0006	1	1
1	0.0825	4	5	0.043	4	5	0.0089	4	5	0.0045	5	6	0.0022	4	5
2	0.2368	12	17	0.1356	14	19	0.0305	15	20	0.0155	16	22	0.0078	16	21
3	0.4872	24	41	0.3454	35	54	0.0991	50	70	0.0522	52	74	0.0268	54	75
4	0.6243	31	72	0.5746	57	111	0.274	137	207	0.1609	161	235	0.0879	176	251
5	0.6321	32	104	0.6317	63	174	0.5233	262	469	0.388	388	623	0.2491	498	749
6	0.6321	32	136	0.6321	63	237	0.6288	314	783	0.5958	596	1219	0.5001	1000	1749
7	0.6321	32	168	0.6321	63	300	0.6321	316	1099	0.632	632	1851	0.6262	1252	3001
8	0.6321	32	200	0.6321	63	363	0.6321	316	1415	0.6321	632	2483	0.6321	1254	4265
9	0.6321	32	232	0.6321	63	426	0.6321	316	1731	0.6321	632	3115	0.6321	1254	5519

I: number of infected person, Cum. (I): Cumulative infected persons, $P_{SI}: n$ means the probability of infected persons within a population of n persons.

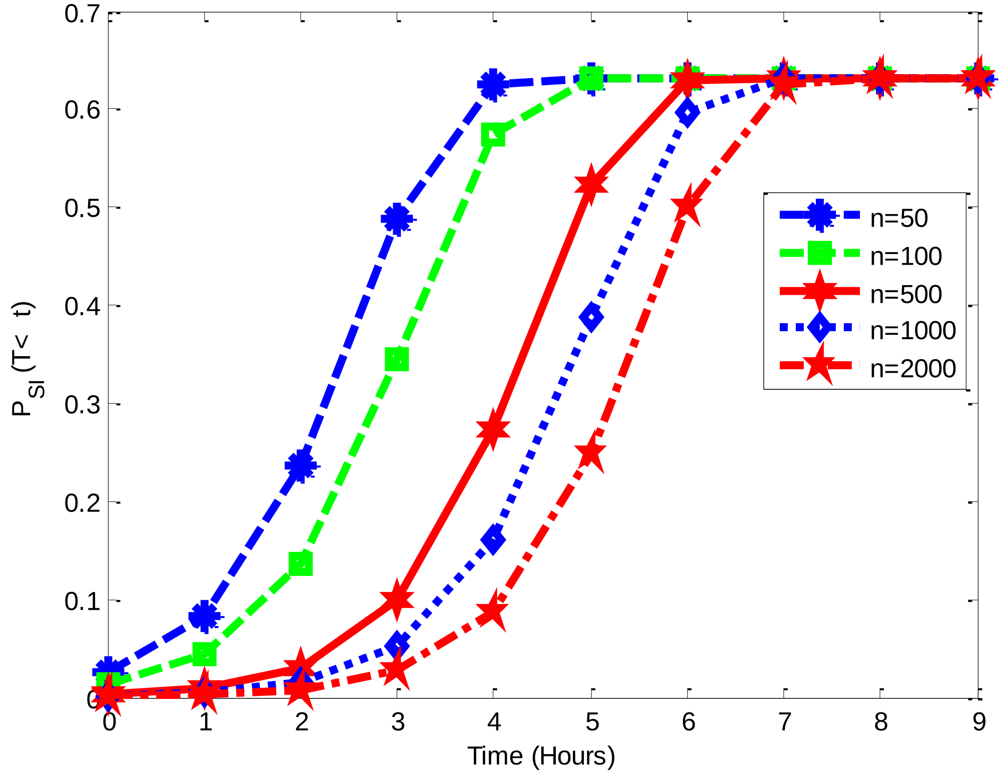

Figure 1: COVID-19 dynamics within a density dependent population for various population sizes

DISCUSSION

Figure 1 and Table 2 shows the dynamics of COVID-19 within a density dependent population of varying sizes ($n = 50, 100, 500, 1000, \text{and } 2000$) where everyone in the population is susceptible. The Basic Reproduction Numbers is calculated (using data of Table 1) to be, $\mathcal{R}_0 = 3.5$.

Particularly, Figure 1 and Table 2 shows that if one infected person enters a density dependent population of size 50 with $\mathcal{R}_0 = 3.5$, the probability of having 4 additional cases within one hour is 0.0825 with the cumulative of 5 infected persons. In two hours, the probability of having 12 additional cases is 0.2368 with a total of 17 infected persons. In three hours, the probability of having 24 additional cases is 0.4872 with a total of 41 infected persons. In four hours, the probability of having 31 additional cases is 0.6243 with a supposed total of 72 infected persons. It can also be noticed from both Figure 1 and Table 2 that after the fourth hour, the graph, the probability of transmission and number of infected persons are constants. This simply means that within four (4) hours, the virus is likely to spread through the entire population of size 50 with $\mathcal{R}_0 = 3.5$ if no control measures are in place.

Similarly, for a density dependent population of size 100 with $\mathcal{R}_0 = 3.5$, the probability of having 4 additional cases within one hour is 0.0430 with the cumulative of 5 infected persons. In two hours, the probability of having 14 additional cases is 0.1356 with a total of 19 infected persons. In four hours, the probability of having 57 additional cases is 0.5746 with a supposed total of 111 infected persons. Also, it can be noticed from the fifth hour that the graph, the probability of transmission and number of infected persons are constants. This simply means that within five (5) hours, the virus is likely to spread through the entire population of size 100 with $\mathcal{R}_0 = 3.5$ if no control measures are in place.

Also, for a density dependent population of size 500 with $\mathcal{R}_0 = 3.5$, the probability of having 4 additional cases within one hour is 0.0089 with the cumulative of 5 infected persons. In three hours, the probability of having 50 additional cases is 0.0991 with a total of 70 infected persons. In six hours, the probability of having 314 additional cases is 0.6288 with a supposed total of 783 infected persons. Also, it can be noticed that after the sixth hour, the graph, the probability of transmission and number of infected persons are constants. This simply means that within six (6) hours, the virus is likely to spread through the entire population of size 500 with $\mathcal{R}_0 = 3.5$ if no control measures are in place.

For a density dependent population of size 1000 with $\mathcal{R}_0 = 3.5$, the probability of having 5 additional cases within one hour is 0.0045 with the cumulative of 6 infected persons. In four hours, the probability of having 161 additional cases is 0.1609 with a total of 235 infected persons. In six hours, the probability of having 596 additional cases is 0.5958 with a supposed total of 1219 infected persons. Also, it can be noticed that after the sixth hour, the graph, the probability of transmission and number of infected persons are constants. This simply means that within six (6) hours, the virus is likely to spread through the entire population of size 1000 with $\mathcal{R}_0 = 3.5$ if no control measures are in place.

For a density dependent population of size 2000 with $\mathcal{R}_0 = 3.5$, the probability of having 4 additional cases within one hour is 0.0022 with the cumulative of 5 infected persons. In four hours, the probability of having 176 additional cases is 0.0879 with a total of 251 infected persons. In seven hours, the probability of having 1252 additional cases is 0.6262 with a supposed total of 3001 infected persons. Also, it can be noticed that after the seventh hour, the graph, the probability of transmission and number of infected persons are constants. This simply means that within seven (7) hours, the virus is likely to spread through the entire population of size 2000 with $\mathcal{R}_0 = 3.5$ if no control measures are in place.

It can also be observed from Figures 1 and Table 2 that for $\mathcal{R}_0 = 3.5$, the duration for the spread of the virus is from 4 to 7 hours with average of 5.5 hours. This is simply indicating that, for one infectious person with $\mathcal{R}_0 = 3.5$ to enter a completely susceptible population of size $50 \leq n \leq 2000$, the virus can spread through the entire population in about 5.5 hours; hence the name pandemic.

Also, it can be noticed from Figure 1 and Table2 that the curves have their stable values at $P_{SI} = 0.6321$. This is simply indicating that the probability of the COVID-19 virus spreading within a density dependent population of size $50 \leq n \leq 2000$ for $\mathcal{R}_0 = 3.5$ is 0.6321 (that is $P_{SI}(4 \leq t \leq 7: \bar{x} = 5.5) = 0.6321$) if no control measures are in place.

CONCLUSION

This research is aimed at studying the probable spread of the COVID-19 virus within a density-dependent population (where every person in the population is susceptible) using a modified exponential distribution function. The modified exponential distribution function was extended to include the Basic Reproduction Number $\mathcal{R}_0$ which is computed using the Nigerian COVID-19 index cases for the period of 27th February to 18th April, 2020. The following results were obtained for $\mathcal{R}_0 = 3.5$: for $n = 50$, the virus is likely to spread through the entire population within four (4) hours; for $n = 100$, the virus is likely to spread through the entire population within five (5) hours; for $n = 500$ the virus is likely to spread through the entire population within six (6) hours; for $n = 1000$, the virus is likely to spread through the entire population within six (6) hours; for $n = 2000$, the virus is likely to spread through the entire population within seven (7) hours; also, for $\mathcal{R}_0 = 3.5$, the duration for the spread of the virus is from 4 to 7 hours with an average of 5.5 hours. This is simply indicating that, for one infectious person to enter a completely susceptible population of size $50 \leq n \leq 2000$, the virus can spread through the entire population in about 5.5 hours; hence the name pandemic.

Also, the results indicates that the $P_{SI}(4 \leq t \leq 7: \bar{t} = 5.5) = 0.6321$ if one infectious person with $\mathcal{R}_0 = 3.5$ enters a density dependent population of size $50 \leq n \leq 2000$ where no control measures are in place.

It is recommended that further research can be carried out on the transmission dynamics of the COVID-19 virus by in-cooperating into the extended modified exponential distribution control measures (like: stay at home, no gathering of population sizes of 50 persons and above for up to four hours, washing of hands regularly with soap and running water, maintain one meter distance etc) to check the spread within a density dependent population.

REFERENCE

Adhikari, S. P., Meng, S., Wu, Y., Mao, Y., Ye, R., Wang, Q., Sun, C., Sylvia, S., Rozelle, S., Raat, H. and Zhou, H. (2020). Epidemiology, causes, clinical manifestation and diagnosis, prevention and control of coronavirus disease (COVID-19) during the early outbreak period: a scoping review. *Infectious Diseases of Poverty*, 9(29): 1-12.

Guo, Y.R., Cao, Q.D., Hong, Z.S., Tan, Y.Y. Chen, S.D., Jin, H.J., Sen Tan, K. Wang, D.Y. and Yan, Y. (2020). The origin, transmission and clinical therapies on coronavirus disease 2019 (COVID-19) outbreak - an update on the status. *Mil. Med. Res.*, 7 (2020):11.

Jones, J. H. (2007). "Notes on R_0" (PDF). https://web.stanford.edu/~jhj1/teachingdocs/Jones-on-R0.pdf.

Kwaghkor, L. M., Onah, E. S., Aboiyar, T. and Ikughur, J. A. (2019). A Modified Exponential Distribution for Predicting Long Term Unemployment Rate. *International Journal of Mathematical Analysis and Optimization: Theory and Applications*, 2019(2):599-609.

Kwaghkor, L. M. (2020). Stochastic Modelling of deforestation effect in Nigeria: A Comparative Study. *FUW Trends in Science & Technology Journal*, 5(3): 919-923.

NCDC (2020a). Case Summary in Nigeria as at April 3rd 2020. https://covid19.ncdc.gov.ng/

NCDC (2020b). The Quarantine Act (CAP Q2 LFN 2004)- COVID-19 Regulations, 2020, Federal Republic of Nigeria. https://covid19.ncdc.gov.ng/

NCDC (2020c). VERSION 1- COVID-19 GUIDANCE FOR SAFE MASS GATHERINGS IN NIGERIA 15th March 2020. https://covid19.ncdc.gov.ng/ on 3/4/2020. Pp.1-2.

NCDC, (2020d). An update of COVID-19 outbreak in Nigeria. https://ncdc.gov.ng/diseases/sitreps /?cat=14&name=An%20update%20of%20COVID-19%20outbreak%20in%20Nigeria.

Van Den Driessche, P.; Watmough, James (2008). "Further Notes on the Basic Reproduction Number". Mathematical Epidemiology. Lecture Notes in Mathematics. 1945: 159–178.

WHO, (2020a). Coronavirus disease (COVID-19) outbreak. *www.who.int › health-topics › coronavirus.*

WHO (2020a). Statement on the second meeting of the International Health Regulations (2005) Emergency Committee regarding the outbreak of novel coronavirus (2019-nCoV). https://www.who.int/news-room/detail/30-01-2020-statement-on-the-second-meeting-of-the-int-ernational-healthregulations-(2005)-emergency-committee-regarding-the-outbreak-of-novelcoro-navirus-(2019-ncov).

WHO, (2020b). Modes of transmission of virus causing COVID-19: implications for IPC precaution recommendations. https://www.who.int/news-room/commentaries/detail/ modes-of-transmission-of-virus-causing-covid-19-implications-for-ipc-precaution-recommendations.

WMHC (2020). Wuhan Municipal Health and Health Commission's Briefing on the Current Pneumonia Epidemic Situation in Our City. 2020. http://wjw.wuhan.gov.cn/front/web/ show Detail/2019123108989.

Worldometers (2020). Coronavirus: Daily New Cases in Nigeria. https://www.worldometers.info/coronavirus/country/nigeria/