



APPLICATION OF HOMOTOPY PERTURBATION METHOD TO THE MATHEMATICAL MODELLING OF TEMPERATURE RISE DURING MICROWAVE HYPERTHERMIA

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ABSTRACT

We study a one dimensional non-linear model of multi-layered human skin exposed to microwave heating during cancer therapy. The model is analyzed using homotopy perturbation method and the fact that there are variations in specific heat, thermal conductivity and blood perfusion from one individual to another were considered. The purpose of this study was to investigate the effect of variable blood perfusion, microwave heating and thermal conductivity on the temperature field during microwave hyperthermia. By varying the parameters, we were able to determine maximum rise of temperature as an individual undergoes cancer therapy. The results were presented in graphs and it was discovered that the temperature of the tumor increases with increase in the microwave heating index while the blood perfusion remain constant.

Keywords: Biological tissue, perfusion, hyperthermia therapy, microwave, mathematical model

INTRODUCTION

Hyperthermia (or thermotherapy) is a cancer treatment that involves heating tumor cells within the body. Elevating the temperature of tumor cells results in cell membrane damage, which in turn leads to the destruction of the cancer cells. Hyperthermia treatment of cancer requires directing a carefully controlled dose of heat to the cancerous tumor and surrounding body tissue.

Hyperthermia treatment is an important therapeutic option in biomedical cancer medicine. It is promising method to treat various types of cancer by heating the tumor to about 41°C using electro-magnetic energy, thereby inducing preferential apoptosis of cancerous cells. It makes the tumor more susceptible to an accompanying radio or chemotherapy (Christen and Shenk, 2007).

The application of heat during hyperthermia may be done in different ways:

- External: High energy waves are aimed at a tumour near the body surface from a machine outside the body.
- Internal: A thin needle or probe is put right into the tumor.

However, there are cases in which the body temperature rises to the level of temperature that a patient would have if they have fever, this is called the fever-range whole body hyperthermia.

Studies suggest that this may cause certain immune cells to become more active for the next few hours and raise the levels of cell killing compounds in the blood (Alexander and Lawrence, 2008).

Electro-magnetic radiation is the source of microwaves, this implies that microwaves are waves of electrical and mechanical energy that moves together in space (Olanrewaju P.O., Ayeni, R.O. and Adebimpe O., 2007). Microwave energy is very effective in heating cancerous tumors, this is because tumors contain high level of water. When a microwave thermotherapy antenna is turned on, body tissue with high water content that are irradiated with significant amount of microwave energy are heated (Erinle, 2005).

According to Olayiwola et al. (2012), Mathematical models based on the underlying mechanisms of cancer can help the medical/scientific community understand and anticipate the effect of parameters on temperature rise during microwave hyperthermia. Model can be used to improve our understanding of essential relationships between the social and biological mechanism that influence the growth of tissues in the body. However, this study provided a numerical solution to the equation of temperature rise during microwaves hyperthermia and also considered the effect of variable blood perfusion, microwave heating and thermal conductivity on the temperature field during microwave hyperthermia.

Method of Solution

Homotopy Perturbation Method

We devote this section to the discussion undertaking the general method to derive the special solution of

$$\partial_{x^n}^n \partial_{y^m}^m \dots, \partial_i^i [u(x, y, \dots, t)] + L[u(x, y, \dots, t)] + N[u(x, y, \dots, t)] = f(x, y, \dots, t) \tag{1}$$

We will assume that $H(x, \dots, t)$ is the solution of the linear part of ; we can record an illustration to appropriate the value of the selected singular points for example at $X(x, y, \dots, t)$ and then the corrected solution can be written as follows

$$U(\alpha, \beta, \dots, i) = H(\alpha, \beta, \dots, i) + \int_0^\alpha \dots \int_0^t \lambda(x, y, \dots, t) \\ X(\partial_{x^n}^n \partial_{y^m}^m \dots, \partial_i^i [u(x, y, \dots, t)] + L[u(x, t, \dots, t)] + N[u(x, y, \dots, t)] - f(x, y, \dots, t)) dx, \dots, dt \tag{2}$$

We will point out that $u(x, y, \dots, t)$ is the Lagrange multiplier and the second expression on the right is called the modification. The method has been modified into an iteration method in the subsequent approximants.

$$U_{n+1}(\alpha, \beta, \dots, i) = H(\alpha, \beta, \dots, i) + \int_0^\alpha \dots \int_0^\alpha \lambda(x, y, \dots, t) \\ X(\partial_{x^n}^n \partial_{y^m}^m \dots, \partial_i^i [u(x, y, \dots, t)] + L[u(x, t, \dots, t)] + N[u(x, y, \dots, t)] - f(x, y, \dots, t)) dx, \dots, dt \tag{3}$$

Beside $H(\alpha, \beta, \dots, i)$ as preliminary guestimate with likely nonentities and $u(x, y, \dots, t)$ is pondered as a circumscribed adaptation meaning $\delta u(x, y, \dots, t) = 0$.

Indeed for random (α, β, \dots) , the above equation can be reformulated as follows,

$$U_{n+1}(x, y, \dots, t) = H(x, y, \dots, t) + \int_0^x \dots \int_0^i \lambda(x, y, \dots, t) \\ X(\partial_{x^n}^n \partial_{y^m}^m \dots, \partial_i^i [u_n(x, y, \dots, t)] + L[u(x, y, \dots, t)] + N[u(x, y, \dots, t)] - f(x, y, \dots, t)) dx, \dots, dt. \tag{4}$$

1. Mathematical Formulation

Following Adebile et al. (2004) and Erinle (2005), the governing equations for our problems are

$$\rho c_p \frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial x^2} + \rho_b w_b c_p (T_b - T) + Q, \dots, (x, t) \in D(0, T) \tag{5}$$

$$T(x, 0) = \frac{T_c x}{L} \quad x \in D \tag{6}$$

$$T(0, t) = T_a \quad T(1, t) = T_c \quad t \in (0, T) \tag{7}$$

Where Q is the electromagnetic energy, $\rho_b w_b$ is blood perfusion, c_b is the specific heat capacity of blood, c_p is the specific heat capacity of tissue, K is the thermal conductivity, ρ_b is the density of blood, p is the density of tissue, T_b is the temperature of aerial blood, T is the temperature.

Non-Dimensionalization

Using dimensionless variable

$$\eta = \frac{x}{L} \quad \theta = \frac{T - T_0}{T_b - T_0} \tag{8}$$

From equation (4)

$$T = \theta(T_b - T_0) + T_0$$

Then equation (1) becomes

$$\rho c_p \frac{\partial(\theta(T_b - T_0) + T)}{\partial t} = K \frac{\partial^2(\theta(T_b - T_0) + T)}{\partial x^2} + \rho_b w_b c_p (T_b - (\theta(T_b - T_0) + T)) + Q \quad (9)$$

$$\rho c_p (T_b - T_0) \frac{\partial \theta}{\partial t} = K (T_b - T_0) \frac{\partial^2 \theta}{\partial x^2} + \rho_b w_b c_p (T_b (1 - \theta) + T_0 (\theta - 1)) + Q \quad (10)$$

$$\rho c_p \frac{\partial \theta}{\partial t} = K \frac{\partial^2 \theta}{\partial x^2} + \frac{\rho_b w_b c_p ((1 - \theta)(T_b + T_0))}{(T_b - T_0)} + \frac{Q}{(T_b - T_0)} \quad (11)$$

$$\frac{\partial \theta}{\partial t} = \frac{K}{\rho c_p} \frac{\partial^2 \theta}{\partial x^2} + \frac{\rho_b w_b c_p (1 - \theta)}{\rho c_p} + \frac{Q}{\rho c_p (T_b - T_0)} \quad (12)$$

Steady state

$$\frac{K}{\rho c_p} \frac{\partial^2 \theta}{\partial x^2} + \frac{\rho_b w_b c_p (1 - \theta)}{\rho c_p} + \frac{Q}{\rho c_p (T_b - T_0)} = 0 \quad (13)$$

$$\frac{d}{dx} = \frac{d}{d\eta} \frac{d\eta}{dx} = \frac{1}{L} \frac{d}{d\eta} \quad (14)$$

$$\frac{Q}{\rho c_p (T_b - T_0)} = E_0 e^{-k\eta} (\beta\theta + T_0)^m \quad (15)$$

$$\frac{1}{\rho_r} \frac{\partial^2 \theta}{\partial \eta^2} - \alpha_0 (p + q\theta)^w (\theta - 1) + E_0 e^{-k\eta} (\beta\theta + T_0)^m = 0 \quad (16)$$

$$\theta(0) = \chi \quad \theta(1) = \lambda$$

Examples:

case 1

$$\frac{d^2 \theta}{d\eta^2} + \text{Pr } E_0 E^{-\eta} (\theta + 1) = 0 \quad (17)$$

$$\theta(0) = 0 \quad \theta(1) = 1$$

Let $\text{Pr } E_0 = a$ therefore (1)

$$\frac{d^2 \theta}{d\eta^2} = -a e^{-\eta} (\theta + 1) \quad (18)$$

Integrate twice

$$\theta(\eta) = H + C\eta - a \int \int e^{-\eta} (\theta + 1) \quad (19)$$

Where H and C are constants of Integration

Let

$$\theta(\eta) = \theta_0 + \varepsilon \theta_1 + \varepsilon^2 \theta_2 \quad (20)$$

Then (2) becomes

$$\theta_0 + \varepsilon \theta_1 + \varepsilon^2 \theta_2 = H + C\eta - a \int \int e^{-\eta} (\theta_0 + \varepsilon \theta_1 + \varepsilon^2 \theta_2 + 1) \quad (21)$$

By comparing both sides

$$\varepsilon^0 : \theta_0 = H + C\eta$$

Recall $\theta(0)=0 \quad \theta(1)=1$

That is when $\eta=0$

$$\theta_0(0)=H+0=0 \Rightarrow H=0$$

When $\eta=1$

$$\theta_0(1)=H+C(1)\Rightarrow 0+C=1\Rightarrow C=1$$

Thus

$$\varepsilon^0 : \theta(\eta)=0+\eta=\eta$$

$$\varepsilon^1 : \theta_1 = -a \int \int e^{-\eta} (\theta_0 + 1)$$

$$\theta_1 = -a \int \int e^{-\eta} (3 + \eta) + C_1\eta + C_2$$

Recall

$$\theta_1(0) = -3a + C_2 = 0 \Rightarrow C_2 = 3a$$

$$\theta_1(1) = -4ae^{-1} + C_1 + 3a = 0 \Rightarrow C_1 = 4ae^{-1} - 3a$$

Thus

$$\theta_1(\eta) = -ae^{-\eta} (3 + \eta) + (4ae^{-1} - 3a)\eta + 3a \tag{22}$$

$$\varepsilon^2 : \theta_2 = -a \int \int e^{-\eta} \theta_1 = -a \int \int e^{-\eta} (-ae^{-\eta} (3 + \eta) + (4ae^{-1} - 3a)\eta + 3a)$$

$$\theta_2 = \frac{1}{4} a^2 (4e^{-2\eta} + C^{-2\eta}\eta - 16e^{-\eta-1} \eta - 32e^{-\eta-1} + 12e^{-\eta}\eta + 12e^{-\eta}) + C_1\eta + C_2$$

At

$$\theta_2(0) = \frac{1}{4} a^2 (16 - 32e^{-1}) + C_2 = 0 \Rightarrow C_2 = -\frac{1}{4} a^2 (16 - 32e^{-1})$$

$$\theta_2(1) = C_1 + \frac{1}{4} a^2 (-43e^{-2} + 24e^{-1}) - \frac{1}{4} a^2 (16 - 32e^{-1})$$

$$= C_1 - \frac{1}{4} a^2 (-43e^{-2} + 24e^{-1}) + \frac{1}{4} a^2 (16 - 32e^{-1})$$

$$\theta_2 = \frac{1}{4} a^2 (4e^{-2\eta} + e^{-2\eta}\eta - 16e^{-\eta-1} \eta - 32e^{-\eta-1} + 12e^{-\eta}\eta + 12e^{-\eta}) +$$

$$\left(-\frac{1}{4} a^2 (-43e^{-2} + 24e^{-1}) + \frac{1}{4} a^2 (16 - 32e^{-1}) \right) \eta - \frac{1}{4} a^2 (16 - 32e^{-1}) \tag{23}$$

⋮
⋮

Therefore

$$\theta(\eta) = \theta_0 + \theta_1 + \theta_2 + \dots \tag{24}$$

Case 2

Take $m = 2, w = 0, \alpha_0 = 0, k = 0, \beta = T_0 = 1, \chi = 0, \lambda = 1$

So the equation becomes

$$\frac{d^2\theta}{d\eta^2} + \rho_r E_0 (\theta+1)^2 = 0 \quad (25)$$

$$\theta(0) = 0, \theta(1) = 1$$

$$\frac{d^2\theta}{d\eta^2} + a(\theta+1)^2 = 0$$

$$\theta(0) = 0, \theta(1) = 1 \quad (26)$$

$$\frac{d^2\theta}{d\eta^2} = -a(\theta+1)^2$$

Integrating twice

$$\theta(\eta) = H + c\eta \iint a(\theta+1)^2 d\eta \quad (27)$$

Where H and c are constants

$$\text{Let } \theta(\eta) = \theta_0 + \xi\theta_1 + \xi^2\theta_2, \dots \quad (28)$$

Then (2) becomes

$$\theta_0 + \xi\theta_1 + \xi^2\theta_2, \dots = H + c\eta - a \iint ((\theta_0 + \xi\theta_1 + \xi^2\theta_2 + 1)^2 d\eta$$

Introducing perturbation parameter, then we have

$$\theta_0 + \xi\theta_1 + \xi^2\theta_2, \dots = H + c\eta - a\xi \iint ((\theta_0 + \xi\theta_1 + \xi^2\theta_2 + 1)^2 d\eta \quad (29)$$

By comparing both sides

$$\xi^0 : \theta_0 = H + c\eta$$

$$\text{Recall } \theta(0) = 0, \theta(1) = 1$$

That is when $\eta = 0$

$$\theta(0) = H + 0 = 0 \Rightarrow H = 0$$

When $\eta = 1$

$$\theta_0(1) = H + c(1) = 0 + c = 1 \Rightarrow c = 1$$

Then

$$\xi^0 : \theta_0 = 0 + \eta = \eta$$

$$\xi^1 : \theta_1 = -a \iint (\theta_0 + 1)^2 d\eta$$

$$\theta_1 = -a \iint (\eta + 1)^2 d\eta$$

$$\theta_1 = -a \frac{1}{12} (\eta + 1)^4 + c_1\eta + c_2$$

Recall

$$\theta_1(0) = -\frac{a}{12} + c_2 = 0 \quad \Rightarrow c_2 = \frac{a}{12}$$

$$\theta_1(1) = -\frac{16a}{12} + c_1 + \frac{a}{12} = 0 \quad \Rightarrow c_1 = \frac{15a}{12}$$

Thus

$$\theta_1 = -a \frac{1}{12} (\eta + 1)^4 + \left(\frac{15a}{12} \right) \eta + \frac{a}{12}$$

$$\xi^2 : \theta_2 = \iint (\theta_1 + e^\eta \theta_1^3) d\eta d\eta$$

⋮
⋮

Therefore

$$\theta(\eta) = \theta_0 + \theta_1 + \theta_2 + \dots$$

$$\theta(\eta) = \eta - a \frac{1}{12} (\eta + 1)^4 + \left(\frac{15a}{12} \right) \eta + \frac{a}{12} + \dots \tag{30}$$

case 3

$$\frac{d^2\theta}{d\eta^2} - (\theta - 1) + ae^\eta (\theta + 1)^3 = 0 \tag{31}$$

$$\theta(0) = 0, \theta(1) = 1$$

$$\frac{d^2\theta}{d\eta^2} = (\theta - 1) - ae^\eta (\theta + 1)^3$$

Integrating twice

$$\theta(\eta) = H + c\eta + \iint (\theta - 1) - ae^\eta (\theta + 1)^3 \tag{32}$$

Where H and c are constants

Let $\theta(\eta) = \theta_0 + \xi\theta_1 + \xi^2\theta_2, \dots$

Then (2) becomes

$$\theta_0 + \xi\theta_1 + \xi^2\theta_2, \dots = H + c\eta + \iint (\theta_0 + \xi\theta_1 + \xi^2\theta_2 - 1) - e^\eta (\theta_0 + \xi\theta_1 + \xi^2\theta_2 + 1)^3$$

Introducing perturbation parameter, then we have

$$\theta_0 + \xi\theta_1 + \xi^2\theta_2, \dots = H + c\eta + \xi \iint (\theta_0 + \xi\theta_1 + \xi^2\theta_2 - 1) - ae^\eta (\theta_0 + \xi\theta_1 + \xi^2\theta_2 + 1)^3$$

By comparing both sides

$$\xi^0 : \theta_0 = H + c\eta$$

Recall $\theta(0) = 0, \theta(1) = 1$

That is when $\eta = 0$

$$\theta(0) = H + 0 = 0 \Rightarrow H = 0$$

When $\eta = 1$

$$\theta_0(1) = H + c(1) = 0 + c = 1 \Rightarrow c = 1$$

Then

$$\xi^0 : \theta_0 = 0 + \eta = \eta$$

$$\xi^1 : \theta_1 = \iint (\theta_0 - 1) - ae^\eta (\theta_0 + 1)^3$$

$$\theta_1 = \iint (\eta - 1) - ae^\eta (\eta + 1)^3$$

$$\theta_1 = \frac{\eta^3}{6} - \frac{\eta^2}{2} + a(-e^\eta \eta^3 + 3e^\eta \eta^2 - 9e^\eta \eta + 11e^\eta) + c_1 \eta + c_2$$

Recall

$$\theta_1(0) = 11a + c_2 = 0 \Rightarrow c_2 = -11a$$

$$\theta_1(1) = \frac{-34}{3} + 4ek_1 = 1 \Rightarrow c_1 = 1 + \frac{34}{3} - 4e$$

$$c_1 = \frac{37}{3} - 4e$$

Thus

$$\theta_1 = \frac{\eta^3}{6} - \frac{\eta^2}{2} - e^\eta \eta^3 + 3e^\eta \eta^2 - 9e^\eta \eta + 11e^\eta + \left(\frac{37}{3} - 4e\right)\eta - 11$$

$$\xi^2 : \theta_2 = \iint (\theta_1 + e^\eta \theta_1^3) d\eta d\eta$$

⋮
⋮

Therefore

$$\theta(\eta) = \theta_0 + \theta_1 + \theta_2 + \dots$$

$$\theta(\eta) = \eta + \frac{\eta^3}{6} - \frac{\eta^2}{2} - e^\eta \eta^3 + 3e^\eta \eta^2 - 9e^\eta \eta + 11e^\eta + \left(\frac{37}{3} - 4e\right)\eta - 11 + \dots \tag{33}$$

Numerical Simulation

This work showed the application of homotopy perturbation method to solve the non-linear model. The numerical simulation was done using MAPLE 13 software and the values of parameter used were adopted from Erinle (2005). The solutions obtained were shown graphically as follows.

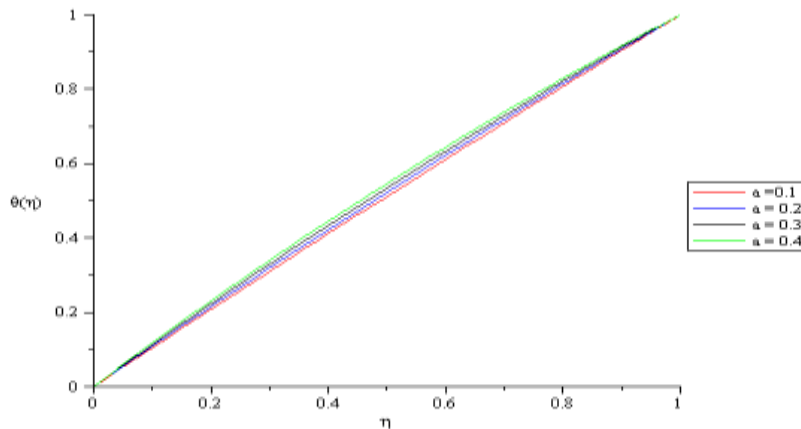


Figure1: the graph of θ against η for fixed values of $k = 1, \beta = T_0 = 1, x = 0, \lambda = 1$

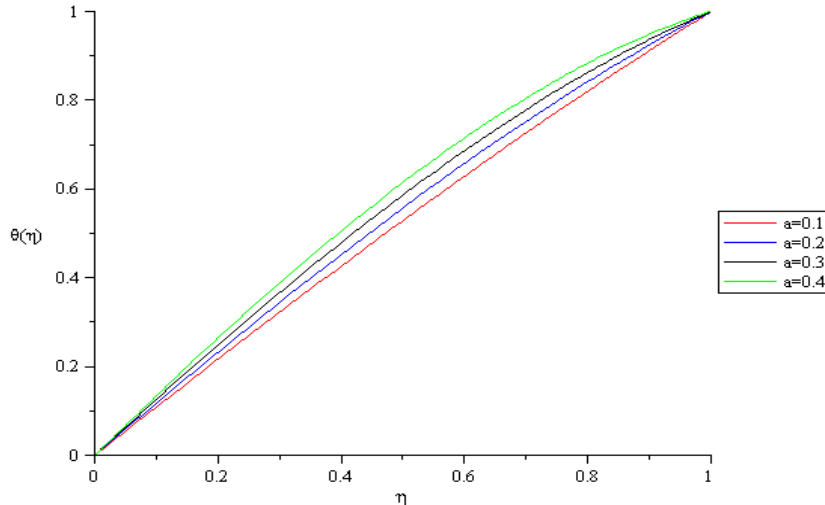


Figure2: the graph of θ against η for fixed values of $k = 0, \beta = T_0 = 1, x = 0, \lambda = 1, \alpha_0 = 0$

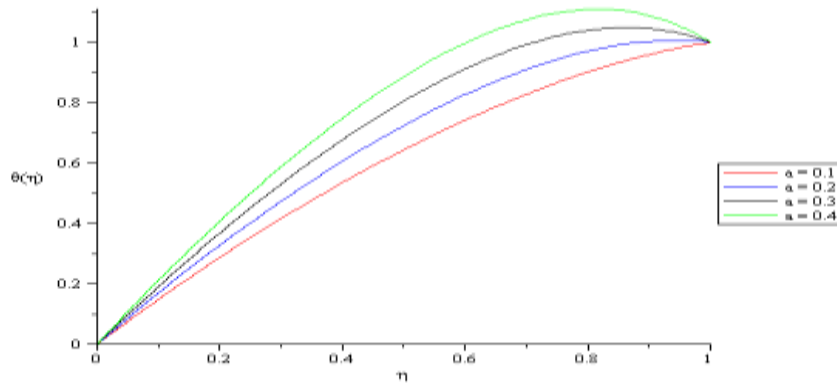


Figure3: the graph of θ against η for fixed values of $k = 1, \beta = T_0 = 1, x = 0, \lambda = 1, m = 3, w = 0, \alpha_0 = 1$

DISCUSSION OF FINDINGS

In this study, we examined the effect of parameters on temperature rise during microwave hyperthermia. Numerical simulations was done for the three cases using MAPLE 13 software. The results in figure 1 to 3 show that the temperature of the tumor rises as the heating surface increases with increase in microwave heating index while blood perfusion index remains constant.

CONCLUSION

We have examined the effect of parameters on temperature rise during microwave hyperthermia. We have successfully used Homotopy perturbation method to solve the equation of temperature rise during microwaves hyperthermia. The study showed that Homotopy Perturbation Method is an effective approximation method to solving the equations of temperature rise during microwaves hyperthermia. Numerical simulations were done by varying the parameters and we were able to determine maximum rise of temperature as an individual undergoes cancer therapy. According to the graphical illustration, we could see that the temperature rise depend on microwave heating and surface heating even when blood perfusion remain constant.

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Appendix:**Code used in generating graphs**

In the simulation code, x was used to replaced η , while u, v, w, z represent the graph of θ at a=1, a=2, a=3 and a=4 respectively.

Other examples follow the same procedure.

```

> y := x + (-a·e-x·(3 + x) + (4·a·e-1 - 3·a)·x + 3·a)
      +  $\frac{1}{4} a^2 (4 e^{-2x} + e^{-2x} x - 16 e^{-x-1} x - 32 e^{-x-1}
      + 12 e^{-x} x + 12 e^{-x}) - \frac{1}{4} a^2 (16 - 32 e^{-1}) - \left( \frac{1}{4} a^2 (
      -43 e^{-2} + 24 e^{-1}) - \frac{1}{4} a^2 (16 - 32 e^{-1}) \right) \cdot x$ 

> a := 0.4
                                     a :=0.4

> v := x - 0.2 e-x (3 + x) + (0.8 e-1 - 0.6) x + 0.4400000000
      + 0.0400000000e-2x + 0.0100000000e-2x x
      - 0.1600000000e-x-1 x - 0.3200000000e-x-1
      + 0.1200000000e-x x + 0.1200000000e-x + 0.3200000000e-1
      - (-0.4300000000e-2 + 0.5600000000e-1 - 0.1600000000) x

> u := x - 0.1 e-x (3 + x) + (0.4 e-1 - 0.3) x + 0.2600000000
      + 0.0100000000e-2x + 0.0025000000e-2x x
      - 0.0400000000e-x-1 x - 0.0800000000e-x-1
      + 0.0300000000e-x x + 0.0300000000e-x
      + 0.0800000000e-1 - (-0.1075000000e-2
      + 0.1400000000e-1 - 0.0400000000) x

> w := x - 0.3 e-x (3 + x) + (1.2 e-1 - 0.9) x + 0.5400000000
      + 0.0900000000e-2x + 0.0225000000e-2x x
      - 0.3600000000e-x-1 x - 0.7200000000e-x-1
      + 0.2700000000e-x x + 0.2700000000e-x + 0.7200000000e-1
      - (-0.9675000000e-2 + 1.2600000000e-1 - 0.3600000000) x

> z := x - 0.4 e-x (3 + x) + (1.6 e-1 - 1.2) x + 0.5600000000
      + 0.1600000000e-2x + 0.0400000000e-2x x
      - 0.6400000000e-x-1 x - 1.2800000000e-x-1
      + 0.4800000000e-x x + 0.4800000000e-x + 1.2800000000e-1
      - (-1.7200000000e-2 + 2.2400000000e-1 - 0.6400000000) x

> plot([u, v, w, z], x = 0 .. 1, linestyle = [dot, dash, dot, dash], color
      = [red, blue, black, green])

```