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MODIFICATION OF SEPARATE RATIO TYPE EXPONENTIAL ESTIMATOR: A POST-STRATIFICATION APPROACH

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ABSTRACT

In this research, modification of separate ratio type exponential estimator introduced in an earlier study is proposed. Expressions for the bias and mean square error (MSE) of the proposed estimator up to first degree of approximation are derived. The optimum value of the constant which minimize the MSE of the suggested estimator is also obtained. In the same vein, efficiency comparisons between the proposed estimator and some related existing ones under the case of post-stratification is conducted. Empirical studies have been conducted to demonstrate the efficiencies of the suggested estimators over other considered estimators. The proposed MSE and Percentage Relative Efficiency (PRE) were used to evaluate the achievement of the modified estimator.

Keywords: Ratio-Type estimator, Bias, Mean Square Error, Post-stratification.

INTRODUCTION

In sampling scheme, the auxiliary information is used to improve the precision of estimators of the population of the main quantity under study. Ratio type and product type estimators have been extensively used when auxiliary information is obtainable. In estimating population means Tailor et al (2016); we use sample mean $\overline{X}$ which is unbiased and has a large amount of variation. We now, seek an estimator with minimal variance or may be biased but with a minimum MSE as compared to sample mean. Classical Ratio estimator is considered mostly in practices when the correlation between interested variable Y and supplementary variable X is positive as suggested by Lone and Tailor (2014). The product type is adopted when the relationship between the interesting variable and the additional variable is negative as suggested by Lone and Tailor (2015). The post-stratification problem was first discussed by Hansel et al. (1953) and Cochran (1940) and Robson (1957) visualized classical ratio and product estimators which were studied in the case of post-stratification by Ige and Tripathi (1989). Chouhan (2012) the proposed class of ratio type estimators using various known parameters of auxiliary variate in case of post-stratification. Lone et al. (2016) proposed a ratio-cum-product type estimator in the case of post-stratification. Singh (1967) used the information on the population mean of two auxiliary variates and the proposed ratio-cum-product type estimator for population means in simple random sampling. The research conducted by Singh (1967) and Chouhan (2012) motivated us to propose the class of ratio-cum-product type estimators in case of post-stratification. Lone and Taylor (2014) as well as Lone and Taylor (2015) proposed dual to separation ratio type exponential estimator and dual to separation product type exponential estimator respectively in case of post-stratification. Many Researchers including Holt and Smith (1979), Jagers et al. (1985),Jagers et al. (1985) Jagers (1986), Ige and Tripathi (1989),Agrawal and Panda (1993), Singh & Espejo (2003),Vishwakarma and Singh (2011), Tailor et al. (2011),Lone and Tailor (2015) contributed significantly to this area of research.

Materials and Methods

Consider a finite population $Z = (Z_1, Z_2, \dots, Z_N)$ of size N. Let the population be divided to L strata with hth stratum containing N_h unit, h=1,2,…,L, so that $\sum_{h=1}^{L} N_h = N$ and $W_h = \frac{N_h}{N}$ is the stratum weight. Suppose a sample of size n is draw from population Z using simple random sampling without replacement. After selecting the sample it is observed that which units belong to h^{th}stratum. Let $n_h$be the size of the sample falling in h^{th}stratum such that $\sum_{h=1}^{L} n_h$. Here it is assumed that n is so large that possibility of being zero is very small .Let x_{hi}be the observation of i^{th}observation or unit that falls h^{th}stratum for auxiliary variate x and y_{hi} be the i^{th}observation or unit that falls h^{th}stratum for study variate y then,

$$\overline{X}_h = \frac{1}{N}\sum_{i=1}^{N_h} x_{hi} \qquad ;h^{th}\text{stratum mean for auxiliary variate x}$$

$$\bar{Y}_h = \frac{1}{N}\sum_{i=1}^{N_h} y_{hi}$$; h^{th} stratum mean for study variate y

$$\bar{X} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} x_{hi} = \frac{1}{N}\sum_{h=1}^{L} N_h \bar{X}_h$$; Population mean of the auxiliary variate x

$$\bar{Y} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} y_{hi} = \frac{1}{N}\sum_{h=1}^{L} N_h Y_h = \sum_{h=1}^{L} W_h \bar{Y}_h$$; Population mean of the study variate y

In the case post-stratification, usual unbiased estimator of $\bar{Y}$ is describe as:

$\bar{Y}_{ps} = \sum_{h=1}^{L} W_h \bar{y}_h$ and $\bar{y}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} y_{hi}$ is sample mean of n_h sample unit that unit fall in the h^{th}stratum.

Using the Stephen (1945), the variance of $\bar{y}_{ps}$to the first degree of approximation is obtained as

$$\operatorname{var}\left(\bar{y}_{ps}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h S_{yh}^2 + \frac{1}{n^2}\sum_{h=1}^{L}\left(1 - W_h\right) S_{yh}^2 \quad (1)$$

Where $S_{yh}^2 = \frac{1}{N_h - 1}\sum_{h=1}^{N_h}\left(y_{hi} - \bar{Y}_h\right)^2$

Lone and Taylor (2014) defined some classes of modified dual exponential estimators of finite $\bar{Y}$ in case of post-stratification as:

$$\hat{\bar{y}}_{pps}^{S} = \sum_{h=1}^{L} W_h \bar{y}_h \left(\frac{\bar{X}_h}{\bar{x}_h^*}\right) \quad (2)$$

The MSE of the estimator provided in equation (2) is obtained as follows:

$$MSE\left(\hat{\bar{y}}_{pps}^{*}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\left[\sum_{h=1}^{L} W_h S_{yh}^2 + \sum_{h=1}^{L} W_h R_h^2 S_{xh}^2 - 2\sum_{h=1}^{L} W_h R_h S_{yxh}\right] \quad (3)$$

$$\hat{\bar{y}}_{pps}^{*} = \sum_{h=1}^{L} W_h \bar{y}_h \left(\frac{\bar{x}_h^*}{\bar{X}_h}\right) \quad (4)$$

Where $\bar{x}_h^* = \frac{N_h \bar{X}_h - n_h \bar{x}_h}{N_h - n_h}$

bias and MSE of (4) are obtained as follows respectively:

$$B\left(\hat{\bar{Y}}_{p}^{*ps}\right) = -\left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} a_h \bar{Y}_h \rho_{yxh} C_{xh} C_{yh} \quad (5)$$

and

$$MSE\left(\hat{\bar{Y}}_{p}^{*ps}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\left(\sum_{h=1}^{L} W_h S_{yh}^2 + \sum_{h=1}^{L} W_h R_h^2 a_h^2 S_{xh}^2 - 2\sum_{h=1}^{L} W_h a_h R_h S_{yxh}\right) \quad (6)$$

Where

$$S_{xh}^{2}=\frac{1}{N_{h}-1}\sum_{h=1}^{N_{h}}\left(x_{hi}-\bar{X}_{h}\right)^{2}\quad R_{h}=\frac{\bar{Y}_{h}}{\bar{X}_{h}},\ a_{h}=\frac{n_{h}}{N_{h}-n_{h}}$$

$$\hat{\bar{Y}}_{ps}^{*spe}=\sum_{h=1}^{L}W_{h}\bar{y}_{h}\left(\frac{\bar{X}_{h}-x_{h}^{*}}{\bar{X}_{h}+x_{h}^{*}}\right) \tag{7}$$

MSE of the estimator in equation (7) is obtained as:

$$MSE\left(\hat{\bar{Y}}_{ps}^{*pse}\right)=\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_{h}\left(S_{yh}^{2}+\frac{1}{4}R_{h}^{2}S_{sh}^{2}-R_{h}S_{yxh}\right) \tag{8}$$

$$\hat{\bar{Y}}_{ps}^{*}pe=\sum_{h=1}^{L}W_{h}\bar{y}_{h}\exp\left(\frac{\bar{x}_{h}^{*}-\bar{X}_{h}}{\bar{x}_{h}^{*}+\bar{X}_{h}}\right) \tag{9}$$

Where $\bar{x}_{h}^{*}=\dfrac{N_{h}\bar{X}_{h}-n_{h}\bar{x}_{h}}{N_{h}-n_{h}}$

bias and MSE of (9) is obtained as follows respectively:

$$B\left(\hat{\bar{Y}}_{ps}^{*\mathrm{Re}}\right)=\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}\frac{1}{4\bar{X}_{h}}\left(-\frac{1}{2}R_{h}a_{h}^{2}S_{xh}^{2}-2a_{h}S_{yxh}\right) \tag{10}$$

$$MSE\left(\hat{\bar{Y}}_{ps}^{*\mathrm{Re}}\right)=\left(\frac{1}{n}-\frac{1}{N}\right)\left(\sum_{h=1}^{L}W_{h}S_{yh}^{2}+\frac{1}{4}\sum_{h=1}^{L}W_{h}a_{h}^{2}R_{h}^{2}S_{xh}^{2}-\sum_{h=1}^{L}W_{h}a_{h}R_{h}S_{xyh}\right) \tag{11}$$

Modified separate ratio exponential type estimator

Following Srivenkatarama(1980) and Bandyonpadhyay(1980) transformation we suggested modification of separate ratio type exponential estimator for population mean $\bar{Y}$ in case of post stratification.

$$\hat{\bar{Y}}_{ps}^{**R}=\sum_{h=1}^{L}W_{h}\bar{y}_{h}\left(\frac{\bar{x}_{h}^{*}}{\bar{X}_{h}}\right)^{\alpha}\exp\left(\frac{\bar{x}_{h}^{*}-\bar{X}_{h}}{\bar{x}_{h}^{*}+\bar{X}_{h}}\right) \tag{12}$$

Where $\bar{x}_{h}^{*}$ is as defined earlier and α is an unknown constant

To obtain the bias and MSE of the proposed estimator $\hat{\bar{Y}}_{ps}^{**R}$, we have

$\bar{y}_{h}=\bar{Y}_{h}(1+e_{0h}),\ \bar{x}_{h}=\bar{X}_{h}(1+e_{1h}),\ \bar{Y}=\sum_{h=1}^{L}W_{h}\bar{y}_{h},\ \bar{X}=\sum_{h=1}^{L}W_{h}\bar{X}_{h}$ such that

$$E(e_{0h})=E(e_{1h})=0$$

$$E(e_{0h}^{2})=\left(\frac{1}{nW_{h}}-\frac{1}{N_{h}}\right)C_{yh}^{2}$$

$$E(e_{0h}^{2})=\left(\frac{1}{nW_{h}}-\frac{1}{N_{h}}\right)C_{yh}^{2}$$

$$E(e_{0h}e_{1h}) = \left(\frac{1}{nW_h} - \frac{1}{N_h}\right)\rho_{yxh} C_{yh} C_{xh}$$

Where

$C_{yh} = \dfrac{S_{yh}}{\overline{Y}_h}$: Coefficient of variation of the study variate,

$C_{xh} = \dfrac{S_{xh}}{\overline{X}_h}$: Coefficient of variation of the auxiliary variate and

$\rho_{xh} = \dfrac{S_{yxh}}{S_{yh}S_{xh}}$: Coefficient of correlation between study variate and auxiliary variate.

$$\hat{\overline{Y}}_{ps}^{**R} = \sum_{h=1}^{L} W_h \bar{y}_h \left[\frac{N_h \overline{X}_h - n_h \bar{x}_h}{\overline{X}_h (N_h - n_h)}\right]^{\alpha} \exp\left[\left(\frac{N_h \overline{X}_h - n_h \bar{x}_h}{N_h - n_h} - \bar{x}_h\right) \Big/ \left(\frac{N_h \overline{X}_h - n_h \bar{x}_h}{N_h - n_h} + \bar{x}_h\right)\right]$$

$$\hat{\overline{Y}}_{ps}^{**R} = \sum_{h=1}^{L} W_h \overline{Y}_h (1 + e_{0h})(1 - a_h e_{1h})^{\alpha} \exp\left[-\left(\frac{a_h e_{1h}}{2}\right)\left(1 - \frac{a_h e_{1h}}{2}\right)^{-1}\right] \tag{13}$$

$$(1 - a_h e_{1h})^{\alpha} = 1 - \alpha a_h e_{1h} + \frac{\alpha(\alpha-1) a_h^2 e_{1h}^2}{2} - \frac{\alpha(\alpha-1)(\alpha-2) a_h^3 e_{1h}^3}{6} + \cdots \tag{14}$$

$$\left(1 - \frac{a_h e_{1h}}{2}\right)^{-1} = 1 + \frac{a_h e_{1h}}{2} + \frac{a_h^2 e_{1h}^2}{4} + \cdots \tag{15}$$

Substitute (14) and (15) into (13) and retaining terms up to the second power of e_s'

$$\hat{\overline{Y}}_{ps}^{**R} = \sum_{h=1}^{L} W_h \overline{Y}_h (1 + e_{0h})\left[1 - \alpha a_h e_{1h} + \frac{\alpha(\alpha-1) a_h^2 e_{1h}^2}{2}\right] \exp\left[-\left(\frac{a_h e_{1h}}{2}\right)\left(1 + \frac{a_h e_{1h}}{2} + \frac{a_h^2 e_{1h}^2}{4}\right)\right]$$

$$\hat{\overline{Y}}_{ps}^{**R} = \sum_{h=1}^{L} W_h \overline{Y}_h (1 + e_{0h})\left[1 - \alpha a_h e_{1h} + \frac{\alpha(\alpha-1) a_h^2 e_{1h}^2}{2}\right]\left[\left(1 - \frac{a_h e_{1h}}{2} - \frac{a_h^2 e_{1h}^2}{8}\right)\right] \tag{16}$$

Expanding the right hand side of equation (16) and retaining terms up to the second power of e_s'we have

$$\hat{\overline{Y}}_{ps}^{**R} - \overline{Y}_h = \overline{Y}_h \left[\begin{array}{l} -\dfrac{a_h e_{1h}}{2} - \dfrac{a_h^2 e_{1h}^2}{8} - \alpha a_h e_{1h} + \dfrac{\alpha a_h^2 e_{1h}^2}{2} + \dfrac{\alpha(\alpha-1) a_h^2 e_{1h}^2}{2} + e_{0h} - \dfrac{a_h e_{0h} e_{1h}}{2} \\ \quad - \dfrac{a_h^2 e_{0h} e_{1h}^2}{8} - \alpha a_h e_{0h} e_{1h} + \alpha a_h e_{0h} e_{1h}^2 + \dfrac{\alpha(\alpha-1) a_h e_{0h} e_{1h}}{2} \end{array}\right] \text{(1}$$

To obtain bias we take expectation of both side of equation (17) then

$$E[\hat{\bar{Y}}_{ps}^{**R}-\bar{Y}_h]=\sum_{h=1}^{L}W_h\bar{Y}_h\begin{bmatrix}-\frac{a_he_{1h}}{2}-\frac{a_h^2E(e_{1h}^2)}{8}-\alpha a_hE(e_{1h})+\frac{\alpha a_h^2E(e_{1h}^2)}{2}+\\ \frac{\alpha(\alpha-1)a_h^2E(e_{1h}^2)}{2}+E(e_{0h})-\frac{a_hE(e_{0h}e_{1h})}{2}\\ -\frac{a_h^2E(e_{0h}e_{1h}^2)}{8}-\alpha a_hE(e_{0h}e_{1h})+\alpha a_hE(e_{0h}e_{1h}^2)+\\ \frac{\alpha(\alpha-1)a_hE(e_{0h}e_{1h})}{2}\end{bmatrix}$$

$$E[\hat{\bar{Y}}_{ps}^{**R}-\bar{Y}_h]=\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}\bar{Y}_h\begin{bmatrix}-\frac{a_h^2S_{xh}^2}{8}+\frac{\alpha a_h^2S_{xh}^2}{2\bar{X}_h^2}+\frac{\alpha(\alpha-1)a_h^2\bar{Y}_hS_{xh}^2}{\bar{X}_h}-a_hS_{yxh}\\ -2\alpha a_hS_{yxh}+\alpha(\alpha-1)a_hS_{yxh}\end{bmatrix}$$

Therefore the bias of the proposed estimator ($\hat{\bar{Y}}_{ps}^{**R}$) is:

$$B\left(\hat{\bar{Y}}_{ps}^{**R}\right)=\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}\frac{a_h}{2\bar{X}_h}\begin{bmatrix}\alpha\, a_hR_hS_{xh}^2+\alpha\, a_hR_hS_{xh}^2(\alpha-1)-\frac{a_hR_hS_{xh}^2}{4}-S_{yxh}^2\\ -2a_hR_h\;S_{yxh}+R_h\alpha(\alpha-1)S_{yxh}\end{bmatrix} \quad (18)$$

By squaring both side of (17), taking expectation and retaining terms up to the second power of e_s' , we get the mean square error of the proposed estimator ($\hat{\bar{Y}}_{ps}^{**R}$) up to the first degree of approximation by saying :

$$E[\hat{\bar{Y}}_{ps}^{**R}-\bar{Y}_h]^2=\bar{Y}_h^2E\left[\frac{-a_he_{1h}}{2}-\alpha_ha_he_{1h}+e_{0h}\right]^2$$

$$E[\hat{\bar{Y}}_{ps}^{**R}-\bar{Y}_h]^2=\bar{Y}_h^2\begin{bmatrix}\frac{a_h^2E(e_h^2)}{4}+\alpha^2{}_ha_h{}^2E(e^2{}_{1h})+E(e^2{}_{0h})+\alpha_ha_h^2E(e_{1h}^2)-a_ha_hE(e_{0h}e_{1h})\\ -2\,\alpha a_hE(e_{0h}e_{1h})\end{bmatrix}$$

The MSE of the proposed estimator ($\hat{\bar{Y}}_{ps}^{**R}$) is

$$MSE\left(\hat{\bar{Y}}_{ps}^{**R}\right)=\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_h\begin{bmatrix}S_{xh}^2+\alpha^2a_h^2R_h^2S_{xh}^2+\alpha\, a_h^2R_h^2S_{xh}^2+\frac{a_h^2R_h^2S_{xh}^2}{4}-a_hR_hS_{yxh}\\ -2\,\alpha\, a_hR_hS_{yxh}\end{bmatrix} \quad (19)$$

We obtain the optimum value (α) to minimize $MSE(\hat{\bar{Y}}_{ps}^{**R})$ by differentiating $MSE(\hat{\bar{Y}}_{ps}^{**R})$ with respect to α and equating the derivative to zero we have:

$$\alpha_{opt}=\frac{2\,s_{yxh}-a_hR_hS_{xh}^2}{2\,a_hR_hS_{xh}^2}$$

By substituting the value of α_{opt} into (19) we get the minimum value of $MSE(\hat{\bar{Y}}_{ps}^{**R})$ as

$$MSE_{\min}\left(\hat{\bar{Y}}_{ps}^{**R}\right)=\left(\frac{1}{n}-\frac{1}{N}\right)\left(\sum_{h-1}^{L}W_h S_{yh}^2-\sum_{h-1}^{L}W_h\frac{S_{yxh}^2}{S_{xh}^2}\right) \tag{20}$$

2.2 Efficiency comparison

Theoretical comparison of the proposed estimators has been made with the competing estimators of the population mean $\bar{Y}$. Therefore, comparison of (1), (6), (8) and (11) shows that the recommended estimators $\hat{\bar{Y}}_{ps}^{**R}$ under ratio type exponential estimators would be more efficient than

$\bar{y}_{ps}$ If : $\beta > -\sum_{h-1}^{L} W_h S_{yh}^2$

$\hat{\bar{Y}}_{ps}^{*}$ If: $\beta > \sum_{h-1}^{L} W_h a_h R_h \left(2S_{yxh} - S_{xh}^2\right)$

$\hat{\bar{Y}}_{ps}^{SRe}$ If: $\beta > \sum_{h-1}^{L} W_h R_h \left(S_{yxh} - \frac{1}{4} R_h S_{xh}^2\right)$

$\hat{\bar{Y}}_{ps}^{Re}$ If: $\beta > \sum_{h-1}^{L} W_h a_h R_h \left(S_{yxh} - \frac{1}{4} R_h S_{xh}^2\right)$

Where $\beta = \sum_{h-1}^{L} \frac{S_{yxh}^2}{S_{xh}^2}$

1. Empirical study

To study the merits of the proposed estimators we are going to consider two natural population data sets. The descriptions of the populations are given below:

Table 1: Population I

Constants	Stratum I	Stratum II
n_h	4	4
N_h	10	10
$\bar{X}_h$	149.7	91
$\bar{Y}_h$	142.8	102.6
S_{xh}^2	181.17	43.16
S_{yh}^2	37.08	158.76
ρ_{yxh}	0.22	0.28
S_{yxh}^2	18.44	23.3

Source: Johnston et al. (1972), p.171

Table 2: Population II

Constants	Stratum I	Stratum II
n_h	2	2
N_h	5	5
$\bar{X}_h$	37.60	46.40
$\bar{Y}_h$	45.60	58.40
S_{xh}^2	5.04	10.24
S_{yh}^2	21.0	15.84
ρ_{yxh}	0.39	0.27
S_{yxh}^2	4.04	3.44

Source: Johnston et al. (1972)

Population I

Table 3: MSE and PRE of proposed and some conventional related estimators over sample mean

Table 3

Estimators	MSE	PRE
$\bar{y}_{ps}$	2.2128	100
$\hat{\hat{Y}}_{ps}^{*}$	2.10365	105.19
$\hat{\hat{Y}}_{ps}^{SRe}$	2.01365	109.89
$\hat{\hat{Y}}_{ps}^{Re}$	2.0012	110.59
$\boldsymbol{\hat{\hat{Y}}_{ps}^{**R}}$	**1.9492**	**113.53**

Population II

Table 4: MSE and PRE of proposed and some conventional related estimators over sample mean

Table 4

Estimators	MSE	PRE
$\bar{y}_{ps}$	5.8752	100
$\hat{\hat{Y}}_{ps}^{*}$	7.0504	83.33

$\hat{\hat{Y}}_{ps}^{SRe}$	6.2074	94.65
$\hat{\hat{Y}}_{ps}^{Re}$	5.7304	102.53
$\hat{\hat{Y}}_{ps}^{**R}$	5.4415	107.97

CONCLUSION

In table 4.3 and 4.4 above the proposed and conventional estimators considered over the population mean are presented. The MSE and Percentage Relative Efficiency (PRE) are also presented in the table. For modified separation ratio type exponential estimator using population I and population II the PRE values 113.53and107.97 respectively for the proposed estimator as compared to the other conventional estimators shows, that the suggested estimator $\hat{\bar{Y}}_{ps}^{**R}$ perform better than the existing estimators with minimum MSE of 1.949158) and 5.441537 respectively for datasets I and II. Thus the proposed estimator is recommended for use in practice if the conditions obtained in section (2.2) are satisfied

References

Agrawal,M.,&Panda, K. (1993). An efficient estimator in post stratification.Metron,51(3-4),179–187.

Bandyopadhyay,S. (1980). Improved ratio and product estimators. Sankhya,42(1-2),45–49.

Chouhan, S. (2012). Improved estimation of parameters using auxiliary information in sample surveys. Unpublished Ph.D.Thesis, VikramUniversity, Ujjain, MP India.

Gamze, Ö. (2015). Modified exponential type estimator for population mean using auxiliary variables in stratified randomsampling.AlphanumericJournal,3(2),49–56.

Holt, D.,and Smith, T. F. (1979). Post stratification. Journal of the Royal Statistical Society: SeriesA(General),142(1),33–46.

Ige,A.,and Tripathi,T. (1989). Estimating of population mean using post stratification and auxiliary information. Abacus,18(2),265–276.

Jagers, P., Oden, A.,anf Trulsson, L. (1985). Post-stratification and ratio estimation: usages of auxiliary information in survey sampling and opinion polls. International Statistical Review/Revue InternationaledeStatistique,221-238.

Johnston,J. P., Halleent, R.M.,&Lezius,D.K. (1972).Effects of span wise rotation on the structure of two-dimensional fully developed turbulent channel flow. Journal of FluidMechanics,56(3),533–557.

Lone,H.A.,and Tailor, R. (2014). Dual to separate ratio type exponential estimator in post-stratification. Journal of Statistics Applications&Probability,3(3),425.

Lone,H.A.,&Tailor, R. (2015). Dual to separate product type exponential estimator in post-stratification. Journal of Statistics Applications&Probability,4(3),429.

Lone,H.A., Tailor, R., &Singh, H.P. (2016). Generalized ratio-cum-product type exponential estimator in stratified random sampling Communications in Statistics-TheoryandMethods,45(11),3302–3309.

Tailor, R., & Mishra, A. (2018). Ratio and ratio type separate estimator for population mean when coefficient of variation is known. JOURNAL OF RELIABILITY AND STATISTICAL STUDIES, 11(1),45–5

Singh,H.P.,&Espejo,M.R.(2003).Onlinearregressionandratio–productestimation of a finite population mean. Journal of the Royal Statistical Society: Series D(The Statistician),52(1),59–67.

Srivenkataramana,T. (1980). A dual to ratio estimator in sample surveys. Biometrika, 67(1),199–204.

Vishwakarma, G.K.,& Singh,H.P. (2011). Separate ratio-product estimator for estimating population mean using auxiliary information. Journal of Statistical Theory and ApplicationsISSN,1538,7887.

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