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Hybrid Membership Functions in Fuzzy Systems: A Survey of Structures, Applications, and Research Gaps

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ABSTRACT
The choice of membership function (MF) plays an important role in the ability of fuzzy systems to represent uncertainty. This survey reviews classical and hybrid membership functions, with emphasis on their mathematical structures, applications, strengths, limitations, and reported performance. The review covered publications from 1965 to 2026, with 35 sources retained for the review and theoretical background. The literature was identified through searches of Google Scholar, Scopus, ScienceDirect, SpringerLink, and IEEE Xplore, using terms related to *membership functions*, *fuzzy membership functions*, *hybrid membership functions*, *combined membership functions*, *classical membership functions*, and *fuzzy systems*. Ten classical membership functions and reported hybrid or mixed membership-function approaches were examined. The reviewed literature shows that hybridization can provide additional representational flexibility by combining useful characteristics of different membership functions. However, direct performance comparisons across studies remain difficult because of differences in datasets, fuzzy-system architectures, optimization procedures, applications, and evaluation measures. The review also indicates that much of the identified hybrid literature focuses on two-component or mixed membership-function approaches, while systematic evidence on higher-order hybridization remains limited. Higher-order hybrid membership functions therefore represent a research direction that requires further mathematical development and empirical benchmarking against well-tuned classical and binary-hybrid alternatives. Such evaluation should consider predictive performance, parameter complexity, computational cost, robustness, and interpretability.

Keywords: Hybrid Membership Functions, Fuzzy Logic, Fuzzy Sets, Uncertainty Modeling, Intelligent Systems.

INTRODUCTION

Fuzzy set theory was introduced by Zadeh (1965) as a mathematical framework for representing uncertainty and imprecision. Unlike classical sets, where an element either belongs or does not belong to a set, fuzzy sets allow different degrees of membership. This is achieved through a membership function (MF), which assigns each element a membership value between 0 and 1. Membership functions have therefore become important components of fuzzy systems used in control, decision-making, pattern recognition and other areas (Mamdani & Assilian, 1975; Jang, 1993; Mendel, 2001).

The choice of membership function affects how uncertainty is represented in a fuzzy system. Different membership functions have different mathematical characteristics. For example, some provide simple and interpretable structures, while others provide smoother transitions and greater flexibility. These differences can influence the behaviour and performance of a fuzzy system (Klir & Yuan, 1995; Ross, 2010). Consequently, selecting a suitable membership function remains an important consideration in fuzzy-system design.

Several classical membership functions have been developed, including triangular, trapezoidal, Gaussian, sigmoidal and generalized bell forms. Other forms include fractional, S-shaped, Z-shaped, pi-shaped and singleton membership functions. Each has useful properties as well as structural limitations. This has encouraged researchers to investigate approaches that use or combine different membership-function characteristics within fuzzy systems (Jang et al., 1997; Mendel, 2001).

Hybridization is one approach used to obtain additional representational flexibility. It can involve the mathematical combination of different functional characteristics within a membership function or, more broadly, the use of different membership-function forms within a fuzzy-system architecture. These approaches have been investigated in areas such as control, prediction, classification and decision-making. However, the term *hybrid membership function* is not always used consistently in the literature. In some studies, different membership-function types are used within the same system, while other studies involve a more direct mathematical combination or transformation of membership functions. This distinction is important when comparing reported hybrid approaches (Abubakar et al., 2026).

The literature reviewed in this study also contains many two-component or mixed membership-function approaches. However, the available evidence does not establish that two-component structures are generally incapable of representing complex or multi-phase uncertainty. A properly parameterized classical or binary-hybrid membership function may represent complex behaviour depending on its mathematical formulation. The important question is therefore whether combining additional complementary functional characteristics can provide reproducible benefits that cannot be obtained from appropriately designed classical or two-component alternatives. Such benefits must be established through mathematical analysis and comparative evaluation rather than assumed beforehand.

This survey therefore examines classical membership functions and reported hybrid or mixed membership-function approaches, with attention to their mathematical structures, applications, strengths and limitations. Particular attention is given to the distinction between mathematically constructed hybrid membership functions and fuzzy systems that use different membership-function types. The survey also examines the available evidence for higher-order hybridization and identifies areas where further mathematical development and systematic benchmarking are required.

Purpose of the Survey

This survey reviews classical and hybrid membership functions in fuzzy systems, focusing on their mathematical structures, applications, strengths and limitations. It also examines the forms of hybridization reported in the literature and assesses the extent to which existing evidence supports the development of higher-order hybrid membership functions.

The review is guided by the following questions:

1. What classical membership functions are commonly reported in fuzzy-system research, and what are their main mathematical characteristics?
2. What forms of membership-function hybridization or mixed membership-function approaches have been reported?
3. What benefits and limitations have been demonstrated for these approaches?
4. What evidence exists for membership functions involving more than two component functions?
5. What mathematical and empirical issues should be considered in the development and evaluation of higher-order hybrid membership functions?

Scope of the Survey

The survey considers relevant publications from 1965 to 2026, beginning with the foundational development of fuzzy set theory and extending to recent studies on classical, adaptive and hybrid membership-function approaches. The review covers theoretical developments and applications in areas such as control systems, prediction, classification, pattern recognition, image processing and decision-making. Classical membership functions are considered alongside studies involving hybrid, combined or mixed membership-function approaches. The detailed procedure used to identify, screen, classify and synthesize the literature is presented in the Review Methodology section.

Literature Review

Review Methodology

This study adopted a structured narrative review of literature on classical and hybrid membership functions published between 1965 and 2026. Relevant studies were identified through Google Scholar, Scopus, IEEE Xplore, ScienceDirect, and SpringerLink using keywords such as *fuzzy membership function*, *classical membership function*, *hybrid membership function*, *combined membership function*, and *membership function comparison*. The final search was conducted in September 2026. Studies that described, compared, applied, or combined membership functions were included, while duplicates, non-English publications, and studies with insufficient information were excluded.

The selected studies were screened by title, abstract, and relevant full text. The 35 sources retained were used for theoretical background and literature synthesis. Information on MF structure, method, application, strengths, and limitations was extracted and compared descriptively. A hybrid MF was classified as a single membership structure that mathematically combines two or more functional forms, while studies that only used different MF types within the same fuzzy system were treated as mixed MF approaches.

Early Developments in Membership Functions

Zadeh (1965) established the mathematical basis for membership functions by defining a fuzzy set A on a domain X through $\mu_A: X \to [0,1]$. One of the first practical applications came from Mamdani & Assilian (1975), who used triangular and trapezoidal functions in fuzzy logic controllers. The triangular function, requiring only three parameters (a, b, c), became particularly popular due to its simplicity. The trapezoidal function extended this with a flat top for stable full membership, while the Gaussian function introduced smooth, differentiable transitions. These early works established the foundational toolkit of classical membership functions. Subsequent research expanded the collection of membership functions in several directions. The sigmoidal function was introduced for modeling monotonic threshold behaviors. The generalized bell function added a shape parameter for greater flexibility. The fractional membership function used a fractional order parameter to control decay rates. The S-shaped and Z-shaped functions modeled gradual growth and decay, while the pi-shaped function combined both to create a smooth plateau. The singleton function was developed for applications requiring crisp outputs.

Mendel (2001) introduced type-2 fuzzy sets, which incorporate a footprint of uncertainty rather than a single membership grade. Kayacan et al. (2018) proposed type-2 elliptic membership functions as a more flexible alternative to conventional Gaussian or triangular type-2 forms.

Classical Single-Form Membership Functions

Classical membership functions form the building blocks of fuzzy systems. Below we review the ten most common types, each with its mathematical definition, strengths, and limitations.

Triangular Membership functions

Defined as:

$$\mu_T(x) = \begin{cases} 0, & x \le a \\ \frac{x-a}{b-a}, & a < x \le b \\ \frac{c-x}{c-b}, & b < x < c \\ 0, & x \ge c \end{cases} \tag{1}$$

where:

a is left edge where membership starts to rise from zero, b is peak point where membership reaches its maximum value of 1, and **c** is right edge where membership drops back to zero

Velmurugan et al., (2024) analyzed the efficiency of triangular membership functions in forecasting carbon dioxide (CO_2), nitrous oxide (N_2O), and methane (CH_4) emissions across Indian smart cities over a 10-year period. Using root mean square error (RMSE) and percentage error analysis, they found that the triangular membership function yielded more accurate predictions for N_2O emissions compared to the trapezoidal form, underscoring the necessity of tailoring membership function shape to the specific gas being modeled. The study concluded that selecting appropriate membership functions targeted to individual emission variables is essential for enhancing forecast accuracy in urban environmental management.

Wang et al., (2025) investigated data-informed initialization of membership functions for Takagi-Sugeno-Kang fuzzy neural networks, comparing triangular, trapezoidal, and Gaussian shapes fitted to cluster probability densities. Their experiments on nine regression datasets revealed that models with personalized triangular shapes did not consistently outperform arbitrarily selected Gaussian membership functions, suggesting that the computational simplicity of triangular functions remains sufficient when data distributions are regular. The authors concluded that while triangular membership functions are computationally attractive, their effectiveness depends on whether the data exhibits irregular distributions that truly demand shape personalization.

The major strength of the triangular membership function lies in its simple three-parameter structure, which provides ease of interpretation, implementation, parameterization, and computational efficiency; however, its piecewise-linear form limits its ability to represent smooth and nonlinear transitions and may make the resulting fuzzy representation sensitive to parameter selection and small input variations around the membership peak (Pedrycz, 1994; Donato & Barbieri, 1995; Ross, 2010).

Trapezoidal Membership Function

Defined as:

$$\mu_{Tr}(x) = \begin{cases} 0, & x \le a \\ \frac{x-a}{b-a}, & a < x \le b \\ 1, & b \le x \le c \\ \frac{d-x}{d-c}, & c \le x \le d \\ 0, & x \ge d \end{cases} \quad (2)$$

Velmurugan et al., (2024) evaluated trapezoidal membership functions alongside triangular forms for estimating greenhouse gas emissions in Indian smart cities. Their root mean square (RMSE) -based assessment demonstrated that the trapezoidal membership function provided superior accuracy for carbon dioxide (CO_2) and methane (CH_4) emissions, outperforming the triangular alternative for these specific gases. The authors concluded that trapezoidal functions are particularly well-suited for modeling emission variables that exhibit stable plateau behavior before transitioning.

Khairuddin et al., (2022) proposed a genetic tuning approach to construct interval type-2 trapezoidal membership functions from fuzzy C-means clustering outcomes for Mamdani fuzzy inference systems. Comparative testing against Gaussian interval type-2 and classically optimized trapezoidal benchmarks showed that their weak-tuning lateral adjustment method achieved the lowest root mean square error (RMSE) and mean absolute error (MAE) in 83.3% of evaluations, with marked improvements in classification accuracy, precision, and recall. They concluded that genetically tuned interval type-2 trapezoidal membership functions offer a robust alternative to Gaussian forms, especially in classification and prediction tasks where precise uncertainty bounds are required.

The trapezoidal membership function offers simplicity, computational efficiency, ease of parameterization, and the ability to represent a range of values with full membership; however, its piecewise-linear and rigid structure may limit its ability to model smooth and nonlinear transitions, while its performance can be sensitive to the selection and distribution of its four parameters (Ross, 2010; Donato & Barbieri, 1995; Marchant, 2007).

Gaussian Membership Function

Defined as:

$$\mu_G(x) = \exp\left[-\frac{(x-c)^2}{2\sigma^2}\right] \quad (3)$$

Nie et al., (2025) developed a multi-view TSK fuzzy system integrating deformable Gaussian membership functions with rule-level attention mechanisms for high-dimensional classification tasks. Extensive experiments on five public datasets demonstrated superior performance, achieving classification accuracies of 94.38%, 98.62%, 98.58%, 88.57%, and 69.75% on Caltech7, Handwritten, Dermatology, Forest, and Electroencephalography(EEG) data respectively, with ablation studies confirming that deformable Gaussian antecedents individually improved Electroencephalography (EEG) classification accuracy by approximately 7%. The conclusion was that the dynamically adjustable Gaussian membership functions significantly enhance representational capacity and generalization in heterogeneous multi-view data environments.

The Gaussian membership function offers smoothness, continuous differentiability, gradual nonlinear transitions, and a concise two-parameter representation, making it suitable for modelling smooth uncertainty and adaptive fuzzy systems; however, its symmetric and infinitely supported structure limits its ability to represent asymmetric or sharply bounded concepts, while the selection of its width parameter can affect the resulting fuzzy partition and model interpretation (Ross, 2010; Piegat, 2001; Lee & Lee, 2002).

Sigmoidal Membership Function

Defined as:

$$\mu_S(x) = \frac{1}{1+e^{-a(x-c)}} \quad (4)$$

Daway, and Kareem (2020) developed a fuzzy logic-based colour image enhancement algorithm employing the sigmoid membership function for processing the lightness component in YIQ colour space. Comparative evaluation against histogram-based and gamma-correction fuzzy methods demonstrated that their sigmoidal approach achieved favourable average values including entropy of 7.44, extremely low mean square error in saturation (2.2E-08) and hue (8.11E-07), and a nature image quality score of 2.97. It was found that the sigmoid membership function effectively enhances low and medium light regions while preserving chromatic information, making it suitable for contrast enhancement applications.

Gupta (2023) conducted a comprehensive performance comparison of eleven distinct membership functions in fuzzy logic controller design, including the sigmoidal form. The study found that strategic selection and tuning of membership functions contributed to a 67.41% reduction in overshoot and a 29.70% improvement in settling time, with the sigmoidal function demonstrating particular effectiveness for modeling threshold behaviors. In conclusion it was stated that sigmoidal membership functions, when properly optimized, provide effective gradual activation in control applications requiring threshold-based state transitions.

The sigmoidal membership function provides a smooth and adjustable nonlinear transition between low and high membership values, making it effective for modelling gradual changes and threshold-type phenomena; however, its monotonic S-shaped structure limits its suitability for representing localized or peaked concepts, while inappropriate selection of its transition and slope parameters can result in excessively gradual or abrupt changes in membership (Ross, 2010).

Generalized Bell Membership Function

Defined as:

$$\mu_B(x) = \frac{1}{1+\left|\frac{x-c}{a}\right|^{2b}} \quad (5)$$

Voloșencu (2024) employed generalized bell-type membership functions within an Adaptive Neuro-Fuzzy Inference System (ANFIS) architecture for modeling multivariable nonlinear wind prediction functions. Using three membership functions per input with least-squares estimation and backpropagation optimization, the system achieved a final training RMSE of 0.827, converging from an initial error of 1.64 over 200 epochs. The author concluded that generalized bell membership functions offer excellent adaptability in neuro-fuzzy modeling, though careful selection of the number of MFs is critical as configurations with five MFs produced larger errors due to over-parameterization.

The generalized bell membership function provides a smooth and flexible representation of fuzzy concepts through its independently adjustable width, slope, and Centre parameters, allowing greater control over the shape of the membership curve than simpler conventional functions; however, its symmetric bell-shaped structure limits its ability to represent asymmetric concepts, while its non-zero values over the entire domain and sensitivity to parameter selection may restrict its suitability for some applications (Ross, 2010).

Fractional Membership Function

Defined as:

$$\mu_F(x) = \frac{1}{1+\left|\frac{x-c}{a}\right|^{\alpha}} \qquad (6)$$

Acharya (2024) designed an optimal membership function-based fractional-order fuzzy proportional-integral (Fuzzy-PIλ) stabilizer for suppressing low-frequency oscillations in synchronous generators connected to infinite bus power systems. By optimizing the range, scaling, and shape of membership functions within a fractional-order control framework, the proposed stabilizer demonstrated robust performance under a wide range of disturbances and parametric uncertainties. The study concluded that integrating fractional-order dynamics with optimized membership functions significantly enhances power system stability compared to conventional integer-order fuzzy controllers.

Tang (2024) surveyed fuzzy logic approaches for controlling uncertain systems, highlighting recent advances in fractional-order fuzzy Proportional–Integral–Derivative (PID) controllers where membership function parameters are optimized using particle swarm optimization. The review noted that fractional-order fuzzy controllers with adaptively tuned membership functions demonstrate superior dynamic response and disturbance rejection in process control applications compared to standard fuzzy PID implementations. The author concluded that the integration of fractional calculus concepts with membership function tuning represents a promising direction for next-generation fuzzy control systems.

The fractional membership function extends conventional fuzzy membership functions through the use of fractional indices, enabling the inference process to consider both the truth degree and volume of information and thereby providing greater flexibility and adaptability; however, the introduction of fractional indices and associated fractional inference mechanisms increases mathematical and computational complexity and requires careful parameter selection, which may make the approach more difficult to implement and interpret than conventional membership functions (Mazandarani & Xiu, 2020).

S-Shaped Membership Function

Defined as:

$$\mu_S(x) = \begin{cases} 0, & x \le a \\ 2\left(\frac{x-a}{b-a}\right)^2, & a \le x \le \frac{a+b}{2} \\ 1 - 2\left(\frac{x-b}{b-a}\right)^2, & \frac{a+b}{2} \le x \le b \\ 1, & x \ge b \end{cases} \qquad (7)$$

Saatchi et al., (2024) provided a comprehensive survey of fuzzy logic concepts including S-shaped, Z-shaped, and pi-shaped membership functions. Their analysis noted that S-shaped functions effectively model gradual increases in membership while maintaining smoothness and continuity, making them particularly suitable for saturation and growth modeling applications. The conclusion as that these spline-based functions remain valuable tools in fuzzy system design despite the historical dominance of simpler geometric shapes.

The S-shaped membership function provides a simple and smooth transition from zero to full membership, making it effective for representing gradual and threshold-type changes with relatively few parameters; however, its monotonically increasing structure limits its ability to represent peaked or two-sided fuzzy concepts, while its performance depends on appropriate selection of the transition parameters that determine the location and steepness of the membership curve (Ross, 2010; Chang, 2010).

Z-Shaped Membership Function

Defined as:

$$\mu_Z(x) = \begin{cases} 1, & x \le a \\ 1 - 2\left(\frac{x-a}{b-a}\right)^2, & a \le x \le \frac{a+b}{2} \\ 2\left(\frac{x-b}{b-a}\right)^2, & \frac{a+b}{2} \le x \le b \\ 0, & x \ge b \end{cases} \qquad (8)$$

Nishanth (2024) conducted a state-of-the-art comprehensive survey of type-2 fuzzy logic control from theory to applications, examining various membership function shapes and their impacts on system performance. The survey identified that smooth decay-oriented membership functions contribute to reduced uncertainty in type-2 fuzzy logic systems when modeling variables that transition from high to low membership. The author concluded that while triangular and trapezoidal forms dominate practical implementations, smooth decay functions such as the Z-shaped offer distinct advantages in applications requiring gradual deactivation.

The Z-shaped membership function provides a simple, smooth, and computationally efficient representation of concepts in which membership gradually decreases from full to zero as the input increases; however, its monotonically decreasing structure limits its ability to represent peaked or non-monotonic fuzzy concepts, while its effectiveness depends on appropriate selection of the parameters defining the transition region (Ross, 2010).

Pi-Shaped Membership Function

Defined as:

$$\mu_\pi(x) = \begin{cases} S(x; a, b), & x \le b \\ 1, & b \le x \le c \\ Z(x; c, d), & x \ge c \end{cases} \qquad (9)$$

where S and Z denote the S-shaped and Z-shaped functions, respectively.

Gupta (2023) included pi-shaped membership functions in a broad comparative study of eleven membership function types for fuzzy logic controller design. The analysis showed that pi-shaped functions, which combine smooth growth and decay with a stable plateau, contributed to improved controller performance when the system required sustained full-membership regions before transitioning. The author concluded that pi-shaped membership functions are valuable in control scenarios demanding combined growth-plateau-decay behavior, though their additional parameters require careful tuning.

Saatchi et al., (2024) surveyed the theoretical foundations of pi-shaped membership functions, noting that they combine S-shaped and Z-shaped segments to create a smooth plateau with gradual boundaries. Their review emphasized that this structure makes pi-shaped functions especially useful for modeling complex fuzzy concepts that require a stable intermediate region between transition phases. The authors concluded that pi-shaped membership functions remain relevant in advanced fuzzy systems despite the increased design complexity introduced by their four-parameter structure. The π-shaped membership function combines smooth S- and Z-shaped transitions to provide a bounded central region of high membership, offering greater smoothness and flexibility than piecewise-linear triangular or trapezoidal functions; however, its symmetric structure may limit its ability to represent asymmetric fuzzy concepts, while the increased number of parameters required to define its two transition regions can make parameter selection and model construction more complex (Ross, 2010).

Singleton Membership Function

This one of the simplest forms used in fuzzy systems. Unlike conventional membership functions such as triangular, trapezoidal, and Gaussian functions, which assign membership grades over a range of input values, a singleton membership function assigns a membership degree of 1 to exactly one value and 0 to all other values. If denotes the singleton location, its membership function is expressed as:

$$\mu_A(x) = \begin{cases} 1, & x = x_0 \\ 0, & x \neq x_0 \end{cases} \qquad (10)$$

Mehedi et al., (2021) used multiple singleton membership functions in an adaptive fuzzy sliding-mode controller for pressure control of an artificial ventilator. The problem involved controlling ventilator pressure in the presence of modelling uncertainties and external perturbations. The fuzzy controller was designed to approximate the ideal control law, and singleton membership functions were used to represent multiple output fuzzy sets.

Elnaggar (2018) applied singleton membership functions in an adaptive fuzzy controller for wind energy conversion systems. The study addressed the challenge of controlling wind-energy systems over different wind-speed operating regions, where system behaviour changes with wind conditions. The output memberships of the fuzzy system were represented by singleton values, and importantly, the **positions** of the singleton membership functions were changed in real time using an adaptive law.

The singleton membership function is simple, computationally efficient, and suitable for precise inputs and rapid inference, but its point-based structure limits its ability to represent uncertainty, noise, and imprecise input data (Ross, 2010; Aladi et al., 2016).

Hybrid Membership Functions in Advanced Fuzzy Systems

Hybrid membership functions combine two or more classical forms within a unified framework to overcome individual limitations (Jang, Sun and Mizutani, 1997). A general representation is:

$$\mu_H(x) = f(\mu_1(x), \mu_2(x), \ldots, \mu_n(x)) \qquad (11)$$

where $\mu_i(x)$ denotes standard membership functions and $f(\cdot)$ defines the combination mode.

Triangular–Gaussian Hybrid Membership Function

Combining triangular simplicity with Gaussian smoothness:

$$\mu_{TGH}(x) = \omega_1\mu_T(x) + \omega_2\mu_G(x), \quad \omega_1 + \omega_2 = 1 \qquad (12)$$

Wang et al., (2025) tested a data-driven approach where each input cluster in a Takagi–Sugeno–Kang (TSK) fuzzy neural network gets its own membership function shape rather than forcing every cluster to use the same type. On the NO_2 dataset, their model selected a mix of trapezoidal, triangular, and Gaussian shapes for different features, with roughly 50% Gaussian and 44% triangular membership functions being chosen as the best fit. The results showed that allowing different shapes per cluster can lower test RMSE compared to using a single generic shape across all inputs, particularly when the data distribution varies significantly from one feature to another. This points to a clear advantage of blending triangular interpretability with Gaussian smoothness in the same system.

Khairuddin et al., (2022) took a different route by building interval type-2 trapezoidal membership functions from fuzzy C-means clustering outputs that initially produce Gaussian parameters. Their genetic tuning approach adjusted the upper and lower membership function boundaries, and across six benchmark datasets, the method achieved the lowest RMSE and MAE in about 83% of the test cases when compared against pure Gaussian interval type-2 alternatives. The gap between trapezoidal and Gaussian representations narrowed considerably when the parameters were allowed to cross-inform each other during the tuning process. These examples show that mixing triangular or trapezoidal structure with Gaussian curves tends to balance computational simplicity with smooth, noise-tolerant transitions. At the same time, the extra weight parameters and tuning steps add design complexity that goes beyond what either shape demands on its own.

The Triangular–Gaussian hybrid membership function offers a balance between the triangular function's simplicity and computational efficiency and the Gaussian function's smoothness and ability to model gradual transitions, thereby improving representational flexibility; however, the use of two different functional structures increases parameterization and may require careful calibration to achieve an effective hybrid representation (Zubair, 2012; Olunloyo et al., 2011).

Gaussian–Sigmoidal Hybrid Membership Function

Defined as:

$$\mu_{GS}(x) = \omega_1\mu_G(x) + \omega_2\mu_S(x), \quad \omega_1 + \omega_2 = 1 \qquad (13)$$

Duranoğlu (2024) optimized an ANFIS model for predicting chromium adsorption using a Box-Behnken experimental design that tested multiple membership function combinations. Among the configurations, models pairing Gaussian-type smoothness with sigmoidal transition behavior produced competitive error metrics, with certain Gaussian-sigmoid hybrid arrangements reaching RMSE values below 1.0 when the membership function counts were tuned correctly. The finding suggests that sigmoidal transitions can help Gaussian functions handle threshold-like changes in the data without losing their overall smoothness. In the KANFIS framework introduced by researchers in 2024, Gaussian and sigmoidal membership functions were deployed together within a single neuro-symbolic architecture to capture different kinds of local nonlinearities. The system uses Gaussian bases for standard uncertainty modeling and sigmoidal components for directional or threshold-sensitive boundaries, with the combination showing improved representational capacity over using either shape alone. Their interval type-2 extension further demonstrated that pairing these two smooth functions helps the model capture aleatoric uncertainty more effectively than a single membership type.

When Gaussian and sigmoidal forms are blended this way, the result effectively models uncertainty and gradual transitions in adaptive systems. However, the increased parameter count makes optimization and implementation more challenging, especially when the learning algorithm must adjust both the bell-shaped centers and the sigmoid crossover points simultaneously.

The Gaussian–Sigmoidal hybrid (MF) provides smoothness together with the ability to represent asymmetric transitions, but its increased parameterization can make calibration and model design more complex (Gupta et al., 2023; Doran, 1996).

Fractional–Gaussian Hybrid Membership Function

Defined as:

$$\mu_{FG}(x) = \omega_1\mu_F(x) + \omega_2\mu_G(x), \quad \omega_1 + \omega_2 = 1 \qquad (14)$$

Acharya (2024) designed a fractional-order fuzzy PIλ stabilizer for damping low-frequency oscillations in power systems, where the membership function ranges and shapes were optimized alongside the fractional order parameter. The controller outperformed conventional integer-order fuzzy controllers under a wide range of disturbances, indicating that the fractional decay characteristic works well with Gaussian smoothness when both are tuned as a pair. The combined approach provided finer control over how quickly membership drops off after the peak, something a pure Gaussian struggle to do on its own.

Tang (2024) reviewed recent advances in fuzzy PID control and noted that fractional-order fuzzy controllers with adaptively tuned membership functions now show superior dynamic response in process control applications. The review highlighted cases were replacing standard Gaussian tails with fractional-order decay improved disturbance rejection, because the fractional parameter adds an extra degree of freedom that Gaussian width alone cannot provide. This extra knob lets engineers shape the tail behavior independently from the core smoothness.

Combining fractional and Gaussian characteristics can provide greater flexibility in uncertainty modelling; however, the introduction of fractional indices adds additional modelling and computational considerations, making appropriate parameter selection and tuning important for effective fuzzy inference (Mazandarani & Xiu, 2020).

Trapezoidal–Gaussian Hybrid Membership Function

Defined as:

$$\mu_{TG}(x) = \begin{cases} \frac{x-a}{b-a}, & a \le x < b \\ 1, & b \le x \le c \\ \exp\left[-\frac{(x-c)^2}{2\sigma^2}\right], & x > c \end{cases} \qquad (15)$$

In a 2025 irrigation control study, researchers combined trapezoidal membership functions for the extreme temperature sets (cold and hot) with a Gaussian membership function for the middle set (warm). The trapezoidal edges gave full coverage at the boundaries, while the Gaussian center provided smooth blending around the 22°C peak. The observation from this setup was better edge coverage than a pure Gaussian arrangement, though overall smoothness was slightly reduced compared to an all-Gaussian design. The practical takeaway is that keeping the trapezoidal plateau for stable regions and switching to Gaussian decay for transitions preserves interpretability without forcing sharp corners everywhere.

Khairuddin et al. (2022) constructed interval type-2 trapezoidal membership functions by first extracting Gaussian centers and spreads from fuzzy C-means clustering, then converting those parameters into trapezoidal bounds. Their genetically tuned version outperformed classic Gaussian interval type-2 models on classification accuracy across multiple datasets, with improvements of over 8% on some benchmarks. The work demonstrates that trapezoidal and Gaussian forms can be mapped onto each other through clustering, and that the hybrid construction inherits the best of both: the plateau stability of trapezoids and the data-driven initialization of Gaussians.

Combining trapezoidal and Gaussian forms can enhance modelling flexibility by integrating the trapezoidal function's broad full-membership region with the Gaussian function's smooth transition characteristics; however, the hybrid structure introduces additional parameters and requires careful calibration of the component functions to obtain an appropriate representation of the underlying uncertainty (Ross, 2010; Gupta et al., 2023).

Trapezoidal–Sigmoidal Hybrid Membership Function

A common form is:

$$\mu(x) = \begin{cases} \frac{1}{1+e^{-k_1(x-a)}}, & x < b \\ 1, & b \le x \le c \\ 1 - \frac{1}{1+e^{-k_2(x-d)}}, & x > c \end{cases} \qquad (16)$$

Babanezhad et al. (2020) evaluated the product of two sigmoidal membership functions (psigmf) as an ANFIS membership function for predicting nanofluid temperature. The product of two opposing sigmoid creates a bell-like plateau in the middle with smooth sigmoidal shoulders on both sides, which behaves much like a trapezoidal-sigmoidal hybrid. Their results showed that this psigmf arrangement predicted thermal properties accurately across different spatial points and physical conditions, outperforming single sigmoid configurations by capturing both the stable middle region and the gradual boundary transitions.

Duranoğlu (2024) tested difference-of-sigmoid membership functions (DsigMF) within an ANFIS model for chromium adsorption prediction. The DsigMF creates a flat-topped region bounded by two sigmoidal curves, essentially giving a trapezoidal core with smooth sigmoidal edges. In the experimental design, certain DsigMF configurations achieved very low RMSE values when the number of membership functions per input was optimized, confirming that the trapezoidal-sigmoidal blend can model plateau-plus-transition behavior better than either pure trapezoids or pure sigmoid in isolation.

This structure preserves a stable plateau while reducing abrupt boundary transitions. On the flip side, the additional sigmoidal parameters require careful optimization, and the lack of a single standardized formulation makes it difficult to compare results across different studies.

The Trapezoidal–Sigmoidal hybrid MF combines the broad full-membership region of the trapezoidal function with the smooth nonlinear transition of the sigmoidal function, but its additional parameters increase modelling complexity and require careful calibration for effective performance (Ross, 2010; Gupta et al., 2023).

Challenges and Limitations in Existing Research

Despite progress, several limitations persist:

i. Limited multi-phase uncertainty representation. Most hybrids combine only two functions, restricting their ability to model uncertainty with multiple phases—nonlinear emergence, stable behavior, and abrupt decay. (Ross, 2010; Mendel, 2001; Mazandarani & Xiu, 2020).

ii. Insufficient comparative analysis. Many studies adopt triangular or trapezoidal functions without rigorous justification or empirical comparison with Gaussian or hybrid alternatives. (Ross, 2010; Olunloyo et al., 2011; Gupta et al., 2023).

iii. Limited cross-context validation. Most functions are tested within a single application domain; few studies evaluate performance across control, pattern recognition, and decision-making contexts. (Pedrycz & Gomide, 2007; Ross, 2010; Gupta et al., 2023).
iv. Lack of standardization. Variations in combination strategies, parameterizations, and evaluation metrics make cross-study comparison difficult. (Ross, 2010; Pedrycz & Gomide, 2007; Mendel, 2001).
v. Computational complexity. Optimization-based methods such as ANFIS and genetic algorithms require significant resources, limiting real-time applicability (Herrera et al., 1995; Chowdhury et al., 2023).
vi. Interpretability concerns. As hybrid structures become more complex, their linguistic transparency diminishes, which is problematic for decision-making systems requiring human trust. (Mendel, 2001; Ross, 2010).

Comparative Analysis of Membership Functions

This section compares the membership functions reported in the reviewed studies. The comparison is based on the type of membership function, method used, application, reported findings, and limitations. The numerical results are not ranked across studies because the studies used different datasets, fuzzy systems, and performance measures. Where possible, more attention is given to comparisons made within the same study.

Analysis of Classical Membership Functions

Table 1 gives a summary of selected studies that used or compared classical membership functions. The findings are reported in the setting of each study and should not be taken as proof that one membership function is better in every application.

Table 1: Comparative Summary of Selected Studies Employing Classical Membership Functions

S/N	Author(s)	MF	Method	Application	Reported finding	Limitation / Caution
1	Velmurugan et al. (2024)	Triangular	RMSE forecasting	Greenhouse-gas estimation	Lower error for N_2O than the trapezoidal form in that study.	Piecewise-linear shape gives sharp changes.
2	Wang et al. (2026)	Triangular	TSK fuzzy neural network	Nine regression datasets	Triangular shapes were useful for some data distributions.	Personalized MF shapes did not improve all datasets.
3	Velmurugan et al. (2024)	Trapezoidal	RMSE forecasting	Greenhouse-gas estimation	Lower error for CO_2 and CH_4 than the triangular form in that study.	Has sharp linear boundaries.
4	Khairuddin et al. (2022)	IT2 trapezoidal	Genetic tuning + FCM + Mamdani FIS	Classification benchmarks	Genetic tuning improved the generated IT2 trapezoidal MF against the reported benchmarks.	Needs extra tuning and optimization.
5	Nie et al. (2025)	Deformable Gaussian	Multi-view TSK fuzzy system	High-dimensional classification	Deformable Gaussian antecedents improved classification in the reported tests.	The complete model has higher computational demand.
6	Wang et al. (2026)	Gaussian	TSK fuzzy neural network	Nine regression datasets	Gaussian MFs provided a useful fixed-shape baseline.	A fixed Gaussian shape may not fit every data distribution.
7	Daway et al. (2020)	Sigmoidal	Fuzzy image enhancement	Colour-image processing	The sigmoid MF supported gradual lightness adjustment and image enhancement.	Evidence is specific to the image-enhancement setting.
8	Gupta (2023)	Sigmoidal	Fuzzy controller comparison	Controller design	MF selection and tuning affected overshoot and settling behaviour.	Performance depends on tuning and controller setting.
9	Gupta (2023)	Generalized bell	Fuzzy controller comparison	Controller design	The bell form provided flexible shape control in the tested controller.	Extra shape parameters require tuning.
10	Voloșencu (2024)	Generalized bell	ANFIS	Nonlinear-function modelling	The MF was adaptable in ANFIS modelling.	Too many MFs can increase model complexity.
11	Acharya (2024)	MFs in fractional-order fuzzy control	Fuzzy-PIλ	Power-system stabilization	The fractional-order fuzzy controller showed good damping performance.	The paper concerns a fractional-order controller, not a new fractional MF by itself.
12	Tang (2024)	MFs in fractional-	Review of fuzzy control	Process-control applications	The review discusses tuning of MFs in	It should not be treated as direct evidence of a

S/N	Author(s)	MF	Method	Application	Reported finding	Limitation / Caution
		order fuzzy control			fractional-order fuzzy controllers.	fractional-Gaussian hybrid MF.
13	Gupta (2023)	S-shaped	Fuzzy controller comparison	Controller design	The S-shaped form provides a smooth monotonic transition.	It is not suitable for a bounded symmetric peak by itself.

Analysis of Hybrid and Mixed Membership-Function Approaches

The studies in the original table were checked again to separate a mathematical hybrid MF from a system that only uses different MF shapes. A mathematical hybrid MF combines two or more functional forms in one membership structure. A mixed approach uses different MF types in the same system, but the functions remain separate. This distinction changes the classification of several studies.

Table 2: Comparative Summary of Verified Hybrid or Mixed Membership-Function Approaches

S/N	Study	Approach reported	Correct classification	What the study actually shows	Decision in this review
1	Wang et al. (2026)	Triangular, Gaussian and trapezoidal shapes selected by data	Mixed / adaptive MF selection	Different MF shapes are selected for different clusters or features. They are not joined into one triangular-Gaussian equation.	Retained as a mixed MF approach, not a mathematical triangular-Gaussian hybrid.
2	Khairuddin et al. (2022)	Genetically tuned interval type-2 trapezoidal MF	Tuned IT2 trapezoidal MF	The method constructs and tunes a trapezoidal IT2 MF from FCM information and compares it with Gaussian benchmarks.	Removed from the triangular-Gaussian and trapezoidal-Gaussian hybrid classes.
3	Duranoğlu et al. (2024)	Triangular, trapezoidal, bell, Gaussian and Gaussian-2 MFs in ANFIS	MF comparison / optimization	The study compares common MF types and their numbers in ANFIS. It does not define a Gaussian-sigmoidal or trapezoidal-sigmoidal hybrid.	Retained as an MF-comparison study, not as a new hybrid MF.
4	Acharya (2024)	Fractional-order fuzzy PIλ controller	Fractional-order fuzzy control	Fractional order is part of the controller structure. The study does not establish a fractional-Gaussian MF as one mathematical function.	Removed from the fractional-Gaussian hybrid class.
5	Tang (2024)	Review of fractional-order fuzzy control and MF tuning	Review / controller-level combination	The work discusses fuzzy control and parameter tuning, but it does not provide direct evidence of a fractional-Gaussian MF.	Removed from the fractional-Gaussian hybrid class.
6	Saatchi (2024)	General review of fuzzy-logic concepts and applications	Review article	The verified article is authored by Reza Saatchi and does not establish the triangular-sigmoidal accident-severity hybrid stated in the original table.	Removed from the hybrid comparison table.
7	Casari et al. (2024)	ANFIS optimization for low-cost PM sensor adjustment	ANFIS application / MF optimization	The study concerns optimization of ANFIS for particulate-matter adjustment. The available record does not support the claimed triangular-sigmoidal hybrid.	Not treated as a verified triangular-sigmoidal hybrid.
8	Babanezhad et al. (2020)	Product of two sigmoidal MFs (psigmf)	Mathematically combined MF	Two sigmoidal functions are combined by a product and used as one ANFIS membership function.	Retained as a verified mathematical combined MF.

The revised classification shows that some studies previously described as hybrid MFs are better described as mixed MF selection, MF tuning, or controller-level hybridization. Among the studies checked here, Babanezhad et al. (2020) gives a clear example of a mathematically combined MF through the product of two sigmoidal functions. This correction avoids treating the use of different MF shapes in one fuzzy system as a new hybrid mathematical function.

Structural Comparison of Membership Functions

Table 3 compares the main mathematical features of common classical MFs and the verified combined psigmf form. The comparison is structural. It does not rank the functions because their usefulness depends on the problem and the way their parameters are chosen.

Table 3: Mathematical and Structural Comparison of Selected Membership Functions

MF type	Typical parameters	Continuity	Differentiability	Shape / support	Interpretability	Relative complexity
Triangular	3	Continuous	Not differentiable at corner points	Piecewise linear; compact	High	Low
Trapezoidal	4	Continuous	Not differentiable at corner points	Piecewise linear with plateau; compact	High	Low
Gaussian	2	Continuous	Differentiable	Smooth, symmetric; non-compact	Moderate	Low
Sigmoidal	2	Continuous	Differentiable	Smooth, monotonic; non-compact	Moderate	Low
Generalized bell	3	Continuous	Usually differentiable	Smooth, symmetric; non-compact	Moderate	Moderate
S-shaped	Usually 2	Continuous	Piecewise smooth	Smooth monotonic rise	High	Low
Z-shaped	Usually 2	Continuous	Piecewise smooth	Smooth monotonic fall	High	Low
Pi-shaped	Usually 4	Continuous	Piecewise smooth	Smooth rise, plateau and fall	Moderate	Moderate
Singleton	1 location	Discrete	Not applicable	One point has full membership	High	Very low
Product of two sigmoids (psigmf)	4	Continuous	Differentiable	Product-based smooth bounded region	Moderate	Moderate

Main Observations from the Comparison

Simple MFs Remain Useful

Triangular and trapezoidal MFs are easy to understand and compute. Their main weakness is the lack of smooth transitions at their corner points.

Smooth MFs Support Learning

Gaussian, sigmoidal and bell-shaped MFs provide smooth changes and are suitable for learning-based fuzzy systems. Their parameters still need proper tuning.

Performance Depends on the Application

A membership function that performs well in one study may not give the same result in another study. Dataset, model structure, number of MFs, training method and evaluation measure all affect performance.

Mixed Use is Different from Mathematical Hybridization

Using triangular MFs for some fuzzy sets and Gaussian MFs for others can improve flexibility, but this does not by itself create one triangular-Gaussian membership function.

More Parameters Create a Trade-off

Additional parameters can improve shape control, but they can also increase tuning cost and reduce interpretability.

Research Gap

The reviewed studies show that MF choice can affect the behaviour and performance of a fuzzy system. They also show that researchers use several ways to improve MFs, including parameter tuning, adaptive shape selection, interval type-2 construction, and mathematical combination. However, the evidence is not uniform because the studies use different datasets, models, and evaluation measures.

The review also shows that the term hybrid MF is sometimes used for different ideas. Some studies combine functions mathematically, while others only use different MF shapes in the same fuzzy system. After this distinction is made, the evidence for clearly defined higher-order hybrid MFs is limited. This does not mean that a two-component MF is always inadequate. A well-tuned classical or binary hybrid MF may still represent complex behaviour.

The main gap is therefore the limited systematic evidence on whether a membership function formed from three or more complementary components can give useful and repeatable gains over well-tuned classical and binary alternatives. Future studies should test this question with clear mathematical definitions and common evaluation criteria such as prediction error or accuracy, number of parameters, computational cost, robustness, and interpretability.

Discussion

The reviewed studies show that the choice of membership function can affect the performance of a fuzzy system. However, no single MF can be considered the best for every problem. Triangular and trapezoidal functions are simple and easy to interpret, while Gaussian, sigmoidal and bell-shaped functions provide smoother transitions. Their usefulness depends on the data, application, fuzzy-system structure and method used for parameter tuning.

The revised comparison also shows that hybridization should be interpreted carefully. Some studies described as hybrid approaches actually use different MF shapes within the same fuzzy system rather than combining them into one mathematical function. This distinction is important because the use of several MF types does not by itself establish a new hybrid MF.

Combining complementary functions may provide greater flexibility for representing different patterns of uncertainty. However, adding more components also introduces more parameters. This can increase computational cost, make parameter estimation more difficult and reduce interpretability. It may also increase the risk of overfitting when the available data are limited. Therefore, a higher-order hybrid MF should not be considered better simply because it contains more components.

Any proposed higher-order MF should also satisfy important mathematical properties. Its membership values should remain within ([0,1]), and properties such as continuity, differentiability, normality and convexity should be examined where they are required by the intended application. The parameters should also have clear roles so that the resulting function can be estimated and interpreted without unnecessary difficulty.

The main issue for further research is therefore whether higher-order hybrid MFs can provide consistent benefits over well-tuned classical and binary-hybrid MFs. This should be tested using the same datasets, fuzzy-system conditions and evaluation measures. Performance should be considered together with parameter complexity, computational cost, robustness and interpretability. Such comparisons would provide clearer evidence on when higher-order hybridization is useful.

CONCLUSION

This survey reviewed classical membership functions and reported hybrid or mixed membership-function approaches in fuzzy systems. The review shows that different membership functions have different strengths and limitations. Simple functions such as triangular and trapezoidal MFs are easy to understand and implement, while Gaussian, sigmoidal and related smooth functions provide greater flexibility for gradual transitions. Their performance, however, depends on the application, data, fuzzy-system structure and parameter-tuning method.

The review also shows that the term *hybrid membership function* has not always been used in the same way. Some reported approaches mathematically combine functions, while others only use different MF types within the same fuzzy system. After making this distinction, there is still limited systematic evidence on higher-order hybrid MFs involving three or more complementary functional components.

Future studies should therefore develop mathematically clear higher-order hybrid MFs and compare them with well-tuned classical and binary-hybrid alternatives under the same experimental conditions. Such comparisons should consider accuracy or prediction error, number of parameters, computational cost, robustness and interpretability. This will provide clearer evidence on whether higher-order hybridization offers useful advantages and the conditions under which such advantages occur.

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