

Mathematical Modeling of Business Growth Dynamics in the Banking Sector: The Role of Customer Acquisition, Conversion, and Retention

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ABSTRACT

In this paper, we provide a mathematical framework of business development in the banking industry, using three variables, namely, customer acquisition (L), conversion (C) and retention (M). The easiest formula to use is a basic core formula that indicates that growth is a product of these variables where a low score on any will drastically lower the overall performance. The equation is extended into a differential equation to be able to describe the time-dependent development of the customer base with references to the growth and loss (churn) of the customer base. Numerical solution shows that acquisition and conversion can be added to the growth linearly, whereas retention can be added to the sustainability of growth much more significantly. The results highlight that enhancing customer retention by virtue of enhancing service delivery is the most crucial determinant of business growth that is long term and which is therefore most important in profitability and the stability of the organization.

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INTRODUCTION

The issue of business growth has taken centre stage as organizations in a highly competitive and fast-changing business environment, especially in the banking sector, grapple with it. With the high rate of introducing financial technologies, particularly under the umbrella term of Financial Technology (FinTech), banking practices have changed drastically through the incorporation of digital platforms, mobile banking, and automated financial services. This, along with growing customer expectations and the increasing ease of switching between service providers, has intensified competition among banks, making sustainable growth more challenging than ever before (Zhengmeng et al., 2024; Gomber et al., 2017). As a result, organizations are shifting from conventional product-based models towards customer-focused approaches that emphasize customer acquisition, conversion, and retention as building blocks of long-term growth and profitability (Kotler & Keller, 2021; Lemon & Verhoef, 2016).

Retaining customers has generally been recognized as a key factor in the success of an organization. The cost of retaining existing customers is often lower than that of acquiring new customers, as customer retention can reduce marketing and acquisition costs while increasing customer lifetime value and loyalty (Reichheld & Sasser, 1990; Kotler & Keller, 2021). In the banking sector, where services can be relatively similar and easily replicated, gaining a competitive advantage requires greater attention to service quality, trust-building, and customer relationship management (Zeithaml et al., 2018). Empirical research has also shown that customer satisfaction, perceived value, trust, and customer experience are important factors influencing customer retention and loyalty (Zhengmeng et al., 2024; Homburg et al., 2017). Moreover, improvements in customer retention can have important implications for business performance,

highlighting the strategic importance of reducing customer churn.

Beyond retaining existing customers, business growth also relies on acquiring new customers and converting them into active clients. Customer acquisition refers to the process of identifying, attracting, and engaging potential customers, whereas conversion focuses on transforming those prospects into actual customers. By applying segmentation, targeting, and positioning strategies effectively, organizations can improve their marketing efficiency and conversion performance (Kotler & Keller, 2021). Recent advances in data analytics, artificial intelligence, and machine learning have further enhanced acquisition and conversion activities by enabling organizations to analyse customer behaviour, identify potential high-value customers, and develop more personalized marketing approaches (Ibitoye et al., 2025; Davenport et al., 2020). In addition, the Customer Lifetime Value (CLV) framework provides a basis for evaluating customers according to their long-term value rather than focusing solely on short-term acquisition outcomes (Kumar & Reinartz, 2012).

Mathematical and predictive modelling is increasingly being used to examine business growth patterns and customer behaviour. Approaches such as churn prediction models, customer lifetime value frameworks, and differential equation models can provide valuable insights into the factors influencing customer retention and business expansion. For instance, Silveira et al. (2021) developed a predictive churn model for a banking organization, demonstrating the potential of predictive methods for identifying customers at risk of leaving. Similarly, de Brito et al. (2025) examined the use of open banking data to improve customer lifetime value estimation. Furthermore, artificial intelligence provides opportunities for improving marketing analysis, customer understanding, personalization, and strategic decision-making (Davenport et al., 2020; Huang & Rust, 2021).

However, many existing predictive and data-intensive approaches may require substantial amounts of high-quality customer data, which can limit their applicability in environments where reliable data are difficult to obtain. Moreover, the use of differential equations in business modelling has attracted growing interest because of their capacity to represent dynamic relationships among key variables as they evolve over time. In contrast to static models, differential equation frameworks enable the continuous examination of growth processes, allowing researchers to assess stability conditions, equilibrium points, and the long-term behaviour of systems (Brauer & Castillo-Chavez, 2012). While these methods have been widely applied in epidemiology and population dynamics, their mathematical structure can also be adapted to business and economic contexts to examine customer movement, market entry, customer conversion, and sustained business growth. Although substantial research exists on customer management and business growth, a notable gap remains in the development of simplified and analytically manageable models that integrate customer acquisition, conversion, and retention within a single unified framework. Many existing studies examine these components separately, thereby limiting a comprehensive understanding of how they interact dynamically. This research addresses this gap by introducing a mathematical model that consolidates these three core components into an integrated structure. By combining a deterministic multiplicative framework with a differential equation approach, the model provides insights into the dynamics of customer growth and enables the investigation of how locating, conversion, and retention rates influence the long-term behaviour of the customer population, particularly within the banking sector.

MATERIALS AND METHODS

Simple model (Core formula)

Let L (Locating), represents how many people or individuals that can be reached.

C (Converting), represents how many of the reached individuals can be converted as customers, that is those that say yes to the banking officials proposal.

M (Maintaining), represents how many of the converted customers you can keep to be doing business with the organization.

If any of these variables is low, the growth drops drastically. Mathematically, Business Growth(G) = $L \times C \times M$.

Mathematical Analysis of the Core Formula Model

Suppose a banker reaches 1,000 individuals and successfully convinces 10% of them to open an account with their organization. If the organization then retains 80% of those newly acquired customers through quality service, this scenario can be expressed mathematically as follows:

$L = 1000$ (People reached) VERSION

$C = 10\%$ Of $1000 = \frac{10}{100} \times 1000 = 100$ customers.

$M = 80\%$ Of the converted customers $\frac{80}{100} \times 100 = 80$ customers.

This now implies that, 80 customers were retained or maintained by the bank through service.

Banks have common products but differ in service delivery. As your retention rate increases, business grows over the years.

Now let us consider a more realistic model for Business growth.

Nonlinear Differential Equation Model

We equally assume the following:

L (Locating) to be the rate of acquiring leads or rate of reach = $L(t)$.

C (Converting) to be fraction of those reached that is converted as customers per unit time = $L(t) \cdot C$.

M (Maintenance) to be the retention rate of the converted customers.

Where $(1 - M)$ = the churn rate (that is those that did not accept business proposal and then refuse to show up).

So that, the customers lost per unit time will be $(1 - M) \cdot N(t)$.

Putting the aforementioned assumption together, we have a non-linear ordinary differential equation that represents a mathematical model for business growth as given below:

$$\frac{dN}{dt} = L(t) \cdot C - (1 - M)N(t) \quad (1)$$

Where

$N(t)$ Is the number of customers at time(t).

$L(t)$ Is the rate of locating customers at time(t).

C Is the conversion rate.

M Is the maintenance or retention rate.

RESULTS AND DISCUSSION

We rewrite model (2) as:

$$\frac{dN}{dt} + (1 - M)N = L(t) \cdot C \quad (2)$$

This is the first order linear differential equation.

Let $\beta = (1 - M)$, so that we have:

$$\frac{dN}{dt} + \beta N = L(t) \cdot C \quad (3)$$

The integrating factor (IF) from equation (4) becomes:

$$I.F = e^{\int \beta dt} \quad (4)$$

$I.F = e^{\beta t}$ But $\beta = (1 - M)$,

$$I.F = e^{(1-M)t} \quad (5)$$

Now, we multiply through, the left hand side becomes:

$$\frac{d}{dt} [N(t)e^{(1-M)t}] = C \cdot L(t)e^{(1-M)t} \quad (6)$$

Integrating both sides of (7) we have:

$\int \frac{d}{dt} [N(t)e^{(1-M)t}] dt = \int C \cdot L(t)e^{(1-M)t} dt + K$. Where K is an integral constant.

$N(t)e^{(1-M)t} = \int C \cdot L(t)e^{(1-M)t} dt + K$.

The general solution now becomes:

$$N(t) = e^{-(1-M)t} \left[\int C \cdot L(t)e^{(1-M)t} dt + K \right].$$

Model Analysis

If $L(t) = L$ (Constant), then, $\int CL(t)e^{(1-M)t} dt =$

$$\frac{C \cdot L}{1-M} e^{-(1-M)t}.$$

$$\text{So that, } N(t) = \frac{C \cdot L}{1-M} + K e^{-(1-M)t}. \quad (7)$$

This result mathematically implies,

i. Long term growth (Steady state) as

$$t \rightarrow \infty, N(t) = \frac{C \cdot L}{1-M}. \quad (8)$$

Which represents the maximum sustainable customer's base.

ii. Conversion rate (C) implies effectiveness, that is, it increases effectiveness and productively.

M (Retention rate) is a very powerful variable that keeps business growing but yet negligible by many.

Mathematically, if $M=0.8$, then,

$1 - M = 0.2$ Therefore, equation (9) becomes:

$$N^* = \frac{C \cdot L}{0.2} \\ \Rightarrow N^* = 5C \cdot L \quad (9)$$

This result again, mathematically implies that,

- i. We have improved maintenance (M) slightly, it implies huge growth.
- ii. If we improve conversion rate (C), we have a linear improvement and finally,
- iii. If we improve location, that is, the reach rate (L), we also have a linear improvement.

But M remains an exponential leverage and should be targeted towards profit maximization and organization sustainability.

To cap it, service remains the key to business growth.

Numerical Simulations

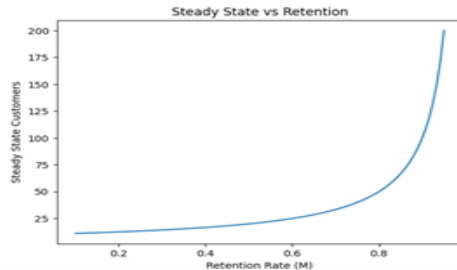


Figure 1: Simulation of the effect retention rate (M) against the steady state of customers over time

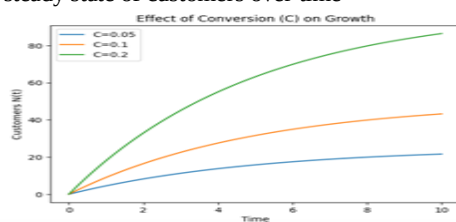


Figure 3: Simulation of effect of conversion (C) on Growth of customers over time

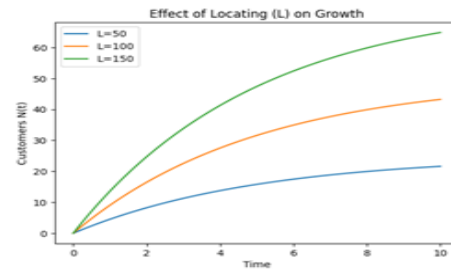


Figure 2: Simulation of effect of Locating on Growth of customers over time

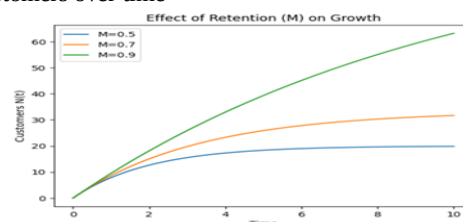


Figure 4: Simulation of effect of Retention (M) on Growth of customers over time

Discussion and Comparison of Simulation Results

Figure 1: Effect of the Retention Rate on the Equilibrium Customer Population

Figure 1 illustrates the effect of the retention rate on the equilibrium customer population in the business growth model. The simulation shows that an increase in the retention rate produces a substantial nonlinear increase in the equilibrium number of customers. This result indicates that improvements in the ability of a firm to retain existing customers can considerably strengthen the long-term customer base. The nonlinear response further suggests that the effect of retention becomes increasingly important as the retention rate improves, because customers who remain with the firm continue to contribute to the customer population over successive periods.

This finding is consistent with the customer relationship literature. Reichheld and Sasser (1990) emphasized that reducing customer defections can produce substantial improvements in business performance and profitability, demonstrating the economic importance of retaining existing customers. Similarly, Verhoef (2003) found that customer commitment and loyalty-oriented strategies positively influence customer retention and customer-share development. Therefore, the result obtained in Figure 1 agrees with previous research by demonstrating that customer retention is an important mechanism through which a firm can sustain and increase its long-term customer population.

However, the present model provides an additional mathematical perspective by showing that the relationship between retention and equilibrium customer population is nonlinear. Thus, the model suggests that retention strategies may generate increasingly larger effects on the long-run customer population rather than producing a simple proportional response. From a managerial perspective, this supports investment in customer satisfaction, loyalty programmes, service quality, and relationship management as

mechanisms for reducing customer loss and strengthening sustainable business growth.

Figure 2: Effect of the Locating Rate on the Customer Population

Figure 2 presents the effect of the locating rate on the customer population. The simulation demonstrates that increasing the locating rate leads to an increase in the customer population. The locating rate can be interpreted as the rate at which potential customers are identified or reached by the firm through marketing, advertising, customer targeting, and other forms of market outreach. Consequently, an increase in this parameter expands the pool of potential customers available to enter the customer acquisition process. The result is consistent with the general customer-management literature, which recognizes customer acquisition and relationship development as important components of firm growth. Gupta and Lehmann (2003), for example, conceptualized customers as important intangible assets whose value contributes directly to the value of the firm. The present result therefore agrees with the argument that increasing the number of customers entering the firm's customer base can improve the long-term value and growth potential of the business.

An important feature of Figure 2 is that the increase in customers does not continue indefinitely at the same rate. The eventual tendency toward a plateau can be interpreted in terms of market saturation. As more potential customers are reached, the remaining pool of prospective customers becomes smaller, thereby reducing the marginal benefit of additional locating efforts. This implies that simply increasing advertising expenditure may not produce unlimited customer growth. Instead, firms may need to combine customer-location strategies with effective conversion and retention mechanisms. Thus, the model suggests that customer acquisition should be accompanied by

strategies that transform identified prospects into customers and subsequently retain them.

Figure 3: Effect of the Conversion Rate on the Customer Population

Figure 3 depicts the effect of the conversion rate on the customer population. The numerical simulation shows that an increase in the conversion rate results in a corresponding increase in the customer population over time. The conversion rate represents the effectiveness with which potential or located customers are transformed into actual customers. Hence, a higher conversion rate means that a greater proportion of the potential customer population becomes part of the firm's active customer base.

This finding complements the result obtained for the locating rate in Figure 2. While locating increases the number of potential customers available to the firm, conversion determines how effectively those potential customers are transformed into actual customers. Consequently, a firm may reach a large number of potential customers without achieving substantial growth if its conversion mechanism is weak. The present model therefore demonstrates mathematically that customer acquisition involves both reaching potential customers and successfully converting them.

The result is also consistent with Gupta and Lehmann (2003), who emphasized the importance of customer relationships and customer lifetime value in determining firm value. In addition, Verhoef (2003) showed that relationship-management activities can influence customer-share development, although the magnitude of the effect depends on the specific strategy employed. The agreement between these studies and the present simulation suggests that improving customer engagement, service quality, communication, and the effectiveness of the sales process can increase the proportion of potential customers who become active customers.

The result also highlights an important implication of the model: locating customers alone is insufficient for sustainable business expansion. Firms must develop effective mechanisms for converting prospects into customers. Therefore, marketing strategies should not focus exclusively on increasing the number of leads but should also improve the quality of customer interaction and the efficiency of the conversion process.

Figure 4: Effect of the Retention Rate on the Customer Population over Time

Figure 4 further examines the influence of the retention rate on the customer population over time. The simulation indicates that a higher retention rate produces a larger customer population and allows the business to maintain a stronger customer base throughout the simulation period. This result demonstrates that customer retention has a cumulative effect on business growth because customers who are retained remain within the system and continue contributing to the customer population.

The result agrees strongly with Reichheld and Sasser (1990), who showed that customer defections have important financial consequences and that reducing customer loss can substantially improve business performance. It is also consistent with Verhoef (2003), whose empirical analysis found that affective commitment and economically oriented loyalty programmes positively influence customer retention and customer-share development. Thus, the present simulation supports previous empirical findings by demonstrating, from a mathematical modelling perspective,

that increasing customer retention can substantially improve the size and stability of the customer population.

Comparing Figures 1 and 4 provides an important additional insight. Figure 1 demonstrates the effect of retention on the equilibrium customer population, whereas Figure 4 illustrates its effect on the dynamic evolution of the customer population over time. Although both figures show a positive relationship between retention and customer population, they examine different aspects of the model. Figure 1 emphasizes the long-run equilibrium consequence of retention, while Figure 4 demonstrates how the benefit of retention accumulates during the transition toward the long-run state.

Overall, the four figures demonstrate that sustainable business growth depends on the interaction between customer locating, conversion, and retention. The locating rate expands the pool of potential customers, the conversion rate determines how effectively these prospects become customers, and the retention rate determines how long customers remain within the business system. The results therefore suggest that concentrating on only one component may not be sufficient to maximize long-term growth. Instead, an integrated strategy that improves customer acquisition, conversion, and retention is likely to produce a more sustainable customer base. This interpretation is consistent with the broader literature that treats customers as valuable assets and emphasizes the importance of managing customer relationships throughout the customer life cycle.

CONCLUSION

In this study, we developed a mathematical framework to analyze business growth within the banking sector by integrating three core components: customer acquisition (locating), conversion, and retention (maintenance). The baseline model showed that business growth is a multiplicative function of these variables, meaning that underperformance in any single component can significantly hinder overall outcomes. By extending the model to a differential equation framework, the study offered deeper insights into how the customer base evolves over time. The analytical solution revealed that while acquisition and conversion rates contribute linearly to growth, retention has a much stronger influence on long-term sustainability. Specifically, the steady-state analysis demonstrated that even minor improvements in customer retention can produce substantial increases in the total customer base. These results emphasize that retention largely driven by service quality is the most critical factor for sustained growth and profitability in banking. Although expanding outreach and improving conversion tactics are important, they must be supported by robust customer relationship management to reduce churn. In summary, this research highlights the need for a balanced growth strategy in which banks not only seek to attract new customers but also prioritize keeping existing ones. The proposed model offers a valuable quantitative tool for decision-making and can be adapted to other service-oriented industries aiming for sustainable business growth.

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