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AN ANALYSES ON PATIENTS' QUEUING SYSTEM AT MUHAMMAD ABDULLAHI WASE SPECIALIST HOSPITAL, KANO

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ABSTRACT

A major cause for concern in hospitals is congestion, which brings about untoward hardship to patients due to long queues and delay in service delivery. This paper seeks to minimize the waiting time of patients by comparing the performance indicators of a single server and multi-server model at the Paediatrics Department of Muhammad Abdullahi Wase Specialist Hospital Kano (MAWSHK). In order to achieve this, primary data was obtained through direct observation which in turn is subjected to the test of goodness of fit to ascertain the distribution that best describes the data. The performance indicators comprising utilization factor, average number of patients in the queue, average number of patients in the system, average waiting time in queue and average waiting time in system for a single server and multi-server model were computed and analyzed respectively. Our findings indicate that the G/G/4 model performs better compared to the G/G/1 model as it minimizes the waiting time of patients.

Keywords: Single-server, Multi-server, Utilization factor, Service delivery, Patients

INTRODUCTION

Waiting line or queue is pervasive in our day-to-day activities. It is practically inevitable in our quest for survival. Take for instance, customers waiting for services at a financial institution, patients waiting for the services of a doctor or nurse in a health facility, customers waiting to be attended to at a Premium Motor Spirit (PMS) refilling station, Students placed in the queue for their dissertation presentation, waiting at air terminals, traffic light intersections and so forth. The waiting line occurs as a result of a certain entity requesting service upon landing at a facility but is unable to receive services promptly. This problem arises due to the shortfall in the service rate against an increasing arrival rate of customers. Excessive waiting time for a service causes dissatisfaction to customers and traffic in the facilities. Subsequently, queue or waiting line phenomenon is a pertinent issue for any entity in the business of rendering services. Queuing theory can be traced to Agner Krarup Erlang, a Danish mathematician, statistician and engineer that developed the field of queuing theory (Kembe et al., 2012)

Queuing theory is the method which solve waiting line phenomena mathematically, and it is one of the most seasoned and most broadly utilized quantitative investigation method (Botani and Hassan, (2017). The first form of study carried out on the waiting line or (queue) phenomenon was by Agner Krarup Erlang (Erlang, 1909). While working for the Copenhagen telephone exchange, Erlang was able to get his research on queuing theory published. In queuing modelling, the unit requesting for service is often viewed as a customer whether it is human or otherwise. While the unit rendering service is alluded to as the server (Bhat, 2015). A basic queuing system comprise of customers arriving for service, hanging tight for service if it is not prompt, and if endered, leaves the system. David George Kendall, a mathematician and statistician known for his work in probability and statistical shape analysis proposed some notations as $a/b/c$ to describing queuing system where the first notation signify the Inter-arrival time distribution, the second notation denotes service time distribution and the third notation is the number of servers. His work has been extended by (Lee, 1966) and (Taha, 1968) to incorporate $d/e/f$ *with* d being the capacity of the queue, e is the size of the population requiring service, and f is the queuing discipline. The Inter-arrival time and service time are often denoted by letter M and G. Here, M is Markovian (Poisson or exponential) and G is utilized for the general distribution of independent and identically distributed random variables. (Gross, 2008; Kendall, 1953).

In an attempt to proffer answers for the confined queuing line with long waiting times in the CT scan unit in Erbil Teaching Hospital, (Botani & Hassan, 2017) demonstrated that the distribution of arrival rate and departure (service) rate follows the Poisson distribution, utilizing the Kolmogorov-Smirnov one-sample and Anderson Darling test. The study considered the ($M/M/1$) ($GD/15/1$) queuing schedule and found that the waiting time in the queue is over one day, due to the high number of patients and the presence of a single CT scanner. In view of this, they proposed the ($M/M/2$) ($GD/15/1$), so as to shorten the waiting time in the system to

0.06 days (1.44 hours). The study posited that there is the need for the managers of Erbil Teaching Hospital to purchase one extra CT scanner to ameliorate the untoward hardship experienced by patients because of the excessive waiting time.

Goswami (2014) focused his research on the analysis of balking and reneging in finite-buffer discrete-time single server queue with single and multiple working vacations. The interarrival times and service times of customers was assumed to be independent and geometrically distributed. He derived explicit expressions for stationary state probabilities using recursive method and various performance measures were presented. Numerical results were used to display the impact of the system parameters on the performance measures. One of the findings of the research is that managers of service systems can assist customers in making right decision by providing them with precise estimates of expected time in the system. Also, a better trade-off of service rate and balking rate may improve decision making and reduce reneging. One drawback from this work is that a generalisation of the model for multiple servers with balking, reneging and working vacations was not made.

(Aslan, 2015) fixated his studies on two big hospitals in Istanbul, Turkey during 2013-2014 years by estimating waiting time, service time and systems during various occasions in a half year. The study discovered that the functioning of doctors are not at the expected level when total utilization is considered. It was observed that improvements could be made to the present health care system if patients can be classified based on appointments such as scheduled patients, checking patients, priority patients and new patient of day, serving different patients at district time of the day, combining systems of all the hospital in the same city for scheduling of surgery and other operations, decreasing waiting queues and workload of doctors.

The study conducted by Ikwunne and Onyesolu, (2016) sought to determine an optimum server level at minimum total cost by incorporating waiting and service cost in homogeneous servers so as to decrease patients congestions in twenty-three (23) teaching hospitals in Nigeria. They implemented the multi-server queuing model utilizing quantitative methods, Production and Operation Management (POM, QM) and Queuing theory calculator software to obtain the outcomes. The study established that the average queue length, waiting duration of patients as well as over-utilization of specialist doctors at the teaching hospitals could be decreased to an optimum server level and at minimum expense as against their present server level with a high cost which includes waiting and service costs. They posited that the outcome of the investigation calls for refocusing, in order to improve the overall patients, care in the cultural context, whilst meeting the patients' need in our environment.

In this research, we hope to solve the problem of congestion experienced at the Paediatric Department of Muhammad Abdullahi Wase Specialist Hospital, kano. This is why we considered the queueing system set up at the Paediatric Department of MAWSHK. We focused our investigation on the number of patients requiring a service at the paediatric department. It was targeted at the waiting and service times of patients to see how much time they spend in the paediatric department before they get consideration. This was done so as to structure a framework equipped for enhancing the situation. This investigation is critical as it won't just be an expansion to the plethora of writing on queuing theory but also help the officials of the Paediatric Department of Muhammad Abdullahi Wase Specialist Hospital Kano, in settling on educated choices about the activities regarding the centre. The findings of this research will immensely benefit the patients that frequent the hospital since an improvement in service delivery will considerably spare them some valuable time.

METHODOLOGY

Method of Data Collection

This study essentially sourced for data primarily through direct observation. A pen, notebook, and a wristwatch were utilized as pre-requisite to getting relevant information(s) that includes: service time, arrival time of patients, number of patients, and waiting time. We carried out these observations during the working long stretches of (8:00 am - 12:00 pm) for about four weeks barring weekends. This enabled us to evaluate the conduct and attributes of the queuing system set up at the Muhammad Abdullahi Wase Specialist Hospital, Kano.

Goodness-of-Fit Test

Here, we employ the Kolmogorov-Smirnov one sample test which is famous for having no restrictions about the sample size to ascertain the distribution of the arrival and service patterns. In essence, it looks at the observed cumulative distribution function for a variable $F_0(x)$ with a specified theoretical distribution function $S_N(x)$. The test utilizes the test statistic

$$D = Max|F_0(x) - S_N(x)| \quad (1)$$

Where D is the maximum value for $|F_0(x) - S_N(x)|$ called the greatest deviation, $F_0(x)$ is the cumulative distribution function of the observed variable and $S_N(x)$ is the cumulative distribution function of the specified distribution.

Arrival process

The arrival process depicts the manner in which the patients arrive and join the system and how these arrivals are distributed after some time. For the most part, patients will arrive in a random manner, because of which prediction ends up impossible, in light of the fact that the patient is an independent entity and the service facility has next to zero command over the patient. Since arrival is stochastic, the probability distribution portraying the time between consecutive arrivals, or the inter-arrival time distribution is important; it is often assumed to be independent and possesses common distribution that includes: M: Markovian or memoryless, D: Deterministic distribution and G: General

Distribution.

Service pattern

The service system is another component in the general queuing structure that talks about the structure and the service conveyance speed. The parameter μ is utilized to describe the mean service rate. The service time distribution gives us the distribution of the service time ofpatients. As found in the arrival process, it can follow a few defined models, some of which are: Markovian, Deterministic and General Distribution.

These models assumes that both inter-arrival times and service times follows a general distribution. The first G indicates that inter-arrival times follow some general distribution (not necessarily exponential), while the second G indicates that service times also follows a general distribution (not necessarily exponential). Exact result for the performance measures are non-existent but fortunately, the Allen-Cunneen approximation often gives a good approximation for the performance measures. The following approximation of the performance measures for the G/G/1 model derived by (Marchal, 1976) is given as:

G/G/1 and G/G/S model

Average number of patient in the queue

$$L_q \approx \frac{\rho^2(1+\varphi_\alpha)(\varphi_s+\rho^2\varphi_s)}{2(1-\rho)(1-\rho^2\varphi_s)} \quad (2)$$

Where

$$\varphi_s = \frac{\sigma_s^2}{(\frac{1}{\mu})^2} \quad (3)$$

$$\varphi_\alpha = \frac{\sigma_\alpha^2}{(\frac{1}{\lambda})^2} \quad (4)$$

φ_α = Coefficient of variation of inter-arrival time.
φ_s = Coefficient of variation of service time.
σ_s^2 = Variance of service time.
σ_α^2 = Variance of the inter-arrival time.

Average waiting time a patient is in the queue:

$$W_q = \frac{\rho^2(1+\varphi_\alpha)(\varphi_s+\rho^2\varphi_s)}{\lambda(1-\rho)(1-\rho^2\varphi_s)} \quad (5)$$

Average time a patient spends in the system:

$$W_s = \frac{\rho^2(1+\varphi_\alpha)(\varphi_s+\rho^2\varphi_s)}{\lambda(1-\rho)(1-\rho^2\varphi_s)} + \frac{1}{\mu} \quad (6)$$

Average number of patient in the system:

$$L_s = \lambda\left[\frac{\rho^2(1+\varphi_\alpha)(\varphi_s+\rho^2\varphi_s)}{\lambda(1-\rho)(1-\rho^2\varphi_s)} + \frac{1}{\mu}\right] \quad (7)$$

Kingman (1961) gave the approximate performance metrics for G/G/s queue as;

$$W_q^{G/G/s} \approx W_q^{M/M/s}\frac{C_\alpha^2+C_s^2}{2} \quad (8)$$

L_s, L_q and W_s can be obtained from littles equation as shown in (Little, 1961)

3. Results and Discussion

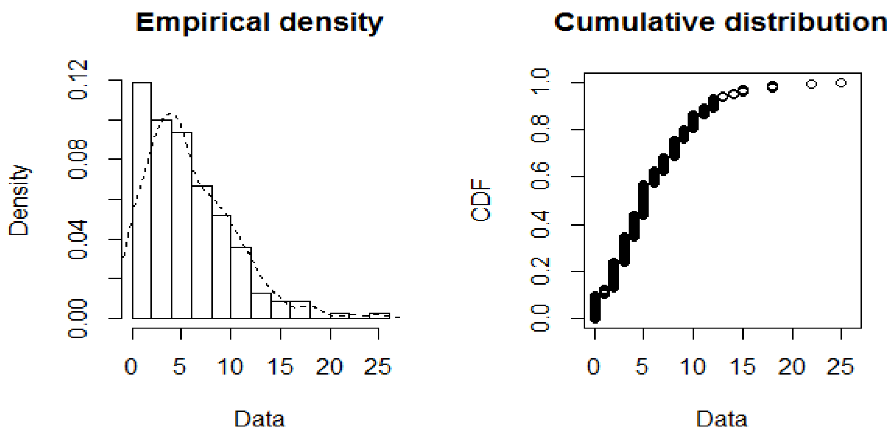

Figure 1: Histogram and CDF plot of an empirical distribution for Inter-arrival times of patients

We can infer from Figure 1 that the distribution of Inter-arrival time of patients is likely to come from a right tailed distribution since the density tilts to the right. However, we do not possess enough information to determine the type of right tail distribution that best describes our observed values. As a result of this we employed the services of the one sample Kolmogorov smirnov test to carry out a test for some underlining assumptions. The hypothesis is stated and the result presented below:

H_0: Inter-arrival comes from the exponential distribution

Vs

H_1: Inter-arrival does not come from the exponential distribution

Table 1: One-sample Kolmogorov-Smirnov test

data: Inter-arrival time (min)
D = 0.84808, p-value < 2.2e-6
alternative hypothesis: two-sided

Table 1 indicates strong evidence against the null hypothesis, since p-value is smaller than 0.05 significance level. What it entails in essence is that the Inter-arrival time does not follow the exponential distribution. Haven established from the Kolmogorov test conducted that our observed values doesn't satisfy the underlying assumption, we proceed to identifying suitable candidates by using the Cullen and Frey graph (Cullen and Frey, 1999)

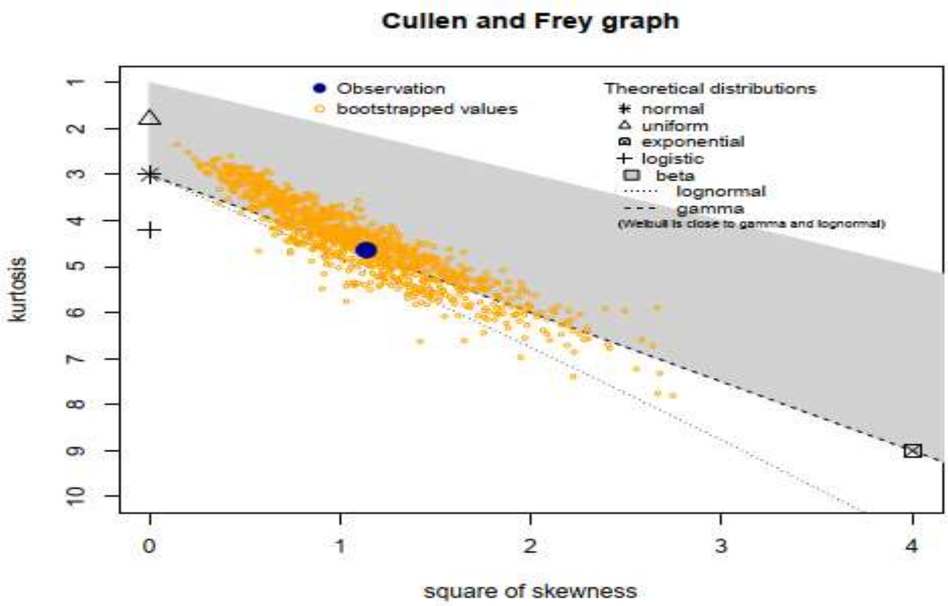

Figure 2: A plot of skewness-kurtosis for Inter-arrival time showing likely distribution candidates

Figure 2 displays values for some distribution in order to help the choice of distribution to fit our observation. For distributions like normal, logistic, exponential and uniform, are represented by one possible value for skewness and kurtosis. In essence, such distributions are characterized with a single point in the plot. Other distributions such as the gamma, lognormal and beta's area of possible values are represented by lines or larger areas respectively. From the plot above, positive values of skewness and kurtosis not far from 3 in comparison to the normal distribution were obtained.

A close inspection of the plot shows that the fit of 4 common distribution could be considered, which are lognormal, Weibull, gamma and normal because of how close they are to our observed values.

Table 2: Fitting a number of distribution for the inter-arrival times of patients

GOF statistics	Lognormal MGE	Gamma MGE	Weibull MGE	Normal MGE	Lognormal MLE	Gamma MLE	Weibull MLE	Normal MLE
KS	0.06382	0.23221	0.24108	0.10029	0.38509	0.30012	0.26071	0.14667
C-VM	9.31804	3.93478	4.43832	0.50324	0.15716	5.08191	3.75286	0.64982
AD	46.34271	25.2001	29.2469	5.65258	47.77664	28.40291	27.4032	3.93694
GOF Criteria								
AIC	1596.4	**1287**	1340.9	1419.3	1537.3	**1245.8**	1295.7	1393.9
BIC	1603.4	**1293.9**	1347.9	1426.3	1544.3	**1252.8**	1302.7	1400.9

The value in bold signifies the best model to fit for our observation

GOF: Goodness of fit

C-VM: Crammer-Von Misses

AD: Anderson Darling

Table 2 shows the fit of the 4 considered distributions with their corresponding statistics. Looking at the Goodness of fit criteria, it can be clearly seen that the gamma distribution has the smallest AIC and BIC. It is on this basis we have fitted the gamma distribution to our observed data as the best distribution that better describes our observed values.

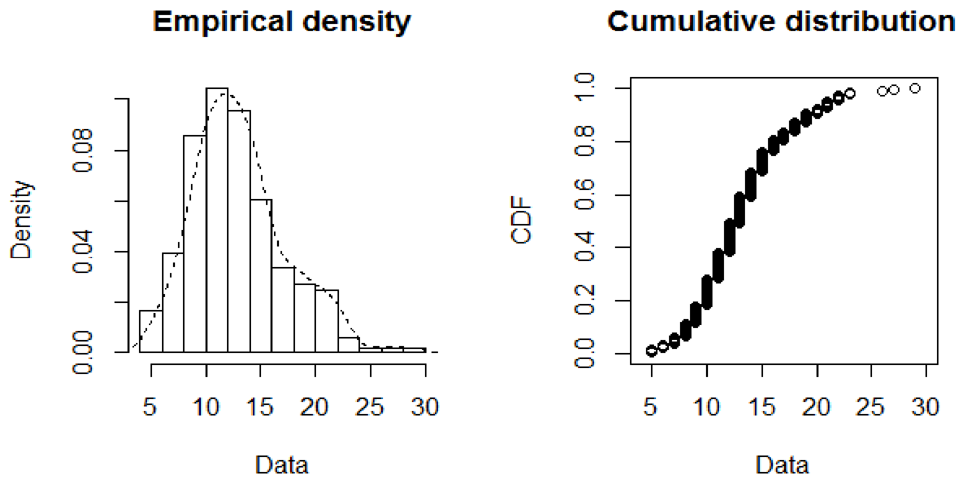

Figure 3: Histogram and CDF plot of an empirical distribution for Service time of patients

We can infer from Figure 3 that the distribution of Inter-arrival time of patients is likely to come from a right tailed distribution since the densities tilts to the right. However, we do not possess enough information to determine the type of right tail distribution that best describes our observed values. we employed the services of the one sample Kolmogorov smirnov test to carry out a test for some underlining assumptions. The hypothesis is stated and the result presented below:

H_0: Service time comes from the exponential distribution

Vs

H_1: Service time does not come from the exponential distribution

Table 3: One-sample Kolmogorov-Smirnov test

data: Service time (min)
D =1, p-value < 2.2e-14
alternative hypothesis: two-sided

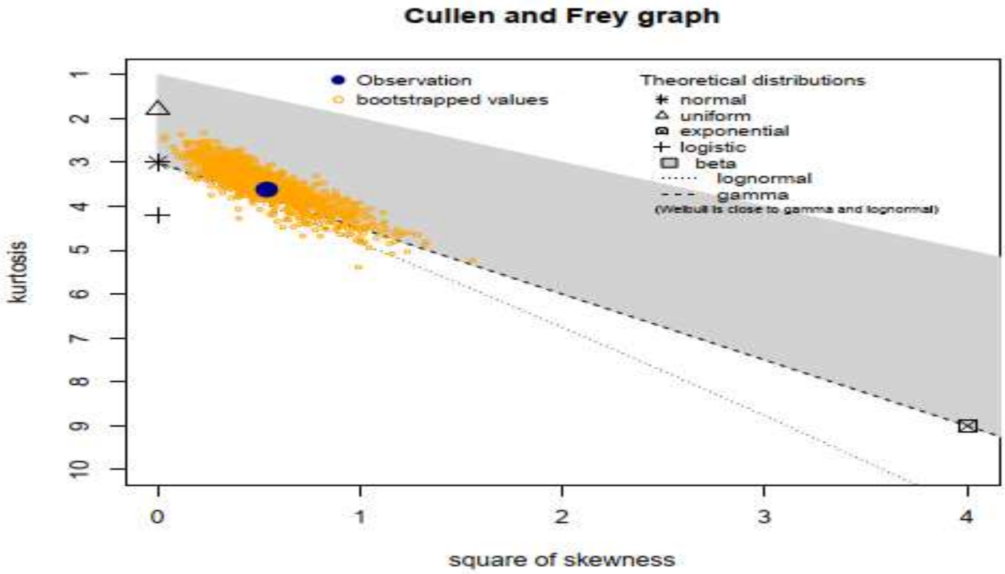

Figure 4: A skewness-kurtosis plot for Inter-arrival time showing likely distribution candidates

Table 4: Fitting a number of distribution for the service times of patients

GOF statistics	Lognormal MGE	Gamma MGE	Weibull MGE	Exp MGE	Lognormal MLE	Gamma MLE	Weibull MLE	Exp MLE
KS	0.06382	0.05832	0.101011	0.29865	0.06922	0.06744	0.10902	0.39232
C-VM	0.15436	0.15972	0.36362	8.36159	0.15716	0.174156	0.50459	10.7151
AD	0.93572	1.03755	6.01129	44.5099	0.94378	0.96881	2.91194	52.2969
GOF Criteria								
AIC	1364.6	**1363.6**	1438.2	1752.4	1364.6	**1362.9**	1385.1	1720.1
BIC	1371.6	**1370.5**	1445.2	1755.9	1371.5	**1369.8**	1392.1	1723.5

EXP: Exponential

Table 3 indicates strong evidence against the null hypothesis, since p-value is smaller than 0.05 significance level. What it entails in essence is that the Service time does not follow the exponential distribution and as such, Cullen and Frey graph was used in Figure 4 to help identify the choice of distribution for our observation. For distributions like normal, logistic, exponential and uniform, are represented by one possible value for skewness and kurtosis. In essence, such distributions are characterized with a single point in the plot. other distributions such as the gamma, lognormal and beta's area of possible values are represented by lines or larger areas respectively. From the plot above, positive values of skewness and kurtosis not far from 3 in comparison to the normal distribution were obtained. An inspection of the closeness of our observations and underlying assumption to the lognormal, Weibull, gamma and exponential distribution were the basis for considering how well they fit our Observed values. Table 4 displays the goodness of fit statistics for a number of distributions. From the Goodness of fit criteria in bold, it can be clearly seen that the gamma distribution has the smallest AIC and BIC. It is on this basis we have fitted the gamma distribution to our observed data as the best distribution that better describes our observed values.

Table 5: Evaluating Performance measures for single and multiple server queuing systems at the Paediatric Department of MAWSH.

PERFORMANCE METRICS	G/G/1	G/G/2	G/G/3	G/G/4
Utilization factor (%)	0.9745	0.8745	0.8001	0.6933
Mean inter-arrival rate	15.03	15.03	15.03	15.03
Mean service time	13.84	13.84	13.84	13.84
Avg. number of patients in queue (L_q)	5.3362	3.9653	2.7437	1.3335
Avg. number of patients in system (L_s)	6.213	4.883	3.664	1.2541
Avg. waiting time of patient in queue (W_q)	20.08	14.5095	11.178	5.0128
Avg.waiting time of patient in system (W_s)	33.92	28.3	25.01	18.85

CONCLUSION

In this study we have been able to identify that the inter-arrival and service time at the Paediatrics department of Muhammad Abdullahi Wase Specialist Hospital, Kano follow the general distribution with the First-come, First-serve (FCFS) queuing discipline. Looking at the performance metrics obtained in Table 5, we observed that the average number of patients in queue and system, average waiting time in queue and system improved when the number of servers were increased indicating that the multi-server G/G/s model performs better in comparison with the single-server G/G/1 model. In order to create a balance for the cost of providing an improved service by the management of the hospital, we recommend that the multi-server G/G/4 model should be implemented since it would minimize patients waiting time, thereby improving service delivery and also considerably save cost for the management.

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