

Point Queue Analysis of Customers Services in South - Western Nigeria

*¹Shobanke Emmanuel Kayode and ²Adekola Olanrewaju Olumide

¹Federal University of Technology, Ilaro, Ogun State, Nigeria.

²Bells University of Technology, Ota, Ogun State, Nigeria.

*Corresponding authors' email: emmanuel.shobanke@federalpolyilaro.edu.org

ABSTRACT

Long wait times and congestion are still problems in Nigerian banking rooms, despite improvements in digital banking services. With a focus on M/M/1 and M/M/c queuing models, this study used point queue analysis to assess the effectiveness of customer service delivery in South-Western Nigeria. Over the course of five weeks, organized observation was used to gather primary data on customer attendance, service, departure, and waiting times. 177 valid observations were examined using statistical and queuing methods following data cleaning. The mean service time was 45.271 minutes, indicating a service rate of 1.325 clients per hour per server, while the mean inter-arrival time was 10.243 minutes, indicating an arrival rate of 5.858 customers per hour. The mean system time was 60.17 minutes, but the empirical mean waiting time was 14.90 minutes. The current four-server system has a utilization rate of 1.105 under the M/M/c model, indicating an unstable system since demand outpaced service capacity. The system was stabilized and utilization was lowered to 0.884 when the number of servers was increased to five, but the anticipated waiting time was still rather high. With six servers, the average waiting time decreased to approximately 11 minutes and system time to about 57 minutes. The results further showed mean waiting times of 81.47, 8.66 and 1.92 minutes for four, five and six servers, respectively. The study concludes that the existing four-server configuration is inadequate, while increasing staffing and reducing service-time variability can substantially improve service efficiency and customer satisfaction.

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INTRODUCTION

In order to manage customer flow, shorten wait times, and enhance service efficacy, queues are a crucial field of operation research. Long client queues in Nigeria's banking industry, especially at peak times, can lower customer satisfaction and show wasteful use of service resources. For the best possible service delivery, accurate analysis of arrival patterns, service rates, and system capacity is crucial. In order to guaranty optimal service delivery, the goal of this study is to use point queuing analysis to analyze the service pattern and its efficiency in a banking system. The specific goals are to: (i) use the Poisson probability distribution to model customer arrivals and service times in the banking industry; (ii) estimate important performance metrics like average waiting time, average queue length, and system utilization rate; (iii) investigate the single-server and multi-server queue models in the evaluation of service capacity on customers waiting time; and (iv) forecast how long a customer will normally wait in a point queue.

An effective queuing system in the banking sector can aid in efficient service delivery, cost-efficient operations and customer satisfaction. A queue is the situation that arises when the demand for a service is greater than the capacity to meet it in the immediate future. When the demand occurs in an uncertain way, there are models for predicting the behaviour of a system, Erlang (2021). These can be also applied in the health sector, where patients are brought in, wait, get a treatment and either depart or want more treatment. The decisions taken with regard to the limitation on beds, staffing, and service points, etc., can be supported with queuing analysis. In general, units that are not served in real-

time are called "arriving units", and may suffer from a taste of discomfort and dissatisfaction if waiting times are long. When demand fluctuates, therefore, queueing theory is applied to evaluate waiting times, to optimize staff schedules, and to increase system resilience. The companies are, therefore, interested in allocating resources efficiently because long wait times have been linked to customer dissatisfaction (Mogilner et al (2022)).

Queues are commonplace in everyday operations, such as lining up at a grocery store to pay, ordering food at a restaurant, and waiting for medical appointments, among other scenarios (Afolalu and Babaremu, 2021). If the wait times are extended sometimes it can lead to higher consumption, but most often it helps to reduce staff costs and customer wait times. Some banking applications of queue scheduling include Automated Teller Machines (ATMs), service counters and deposit facilities. Management can install additional ATM machines or deposit machines or install separate counters for specific services to make one long line of customers into shorter, more manageable lines of customers. In the health sector, the same can be true with multiple service points, which can enhance the use of resources and can help to manage trade-offs between service quality and waiting time.

The issues of waiting time estimation and impact of change of staff members are also significant in airport check-in and security inspections. To support the development of queue scheduling systems, technologies like image processing, artificial neural networks and machine learning can be applied. Marin and Rossi (2020) mentioned that automated queue management technology is broadly adopted such as in

banking to track customer arrivals and direct them to the right service area. But, it is not the case with many existing real world systems that it has been taken care of customer waiting time. The queue length and duration can thus be estimated and the waiting experience enhanced by implementing better queue management using queuing models Olawale O. O. et al (2026).

The queues are studied in organizations that have a line of people who are stuck waiting, and service efficiency and customer satisfaction are balanced. There are various scheduling techniques such as longest job first, shortest job first and most profitable work first. Queues can be structured or unstructured; a structured queue is an ordered line of customers or a queue in a specific location, such as a bank or post office or grocery store. The authors Kyritsis and Deriaz (2020) described the problem of queue organization as a problem due to the unpredictable human behavior. The concepts of queue management are Shortest Processed First (SPF), First Come First Serve (FCFS), Single Queue (SQ), Multiple Queue (MQ), Diffuse Queue (DQ) and Head of Queue (HQ). SPF assigns jobs to be processed first based on processing time, FCFS serves customers based on their arrival. While SQ offers a single line to minimise the crowding and for customers to feel confident that they are being treated fairly, MQ offers multiple lines and can be helpful when catering for large groups.

Queuing theory is a mathematical tool used to study waiting lines in service industries like the financial sector. It focuses mainly on probabilistic modelling of customer waiting times, service mechanisms, and arrival processes and includes performance measures such as the average number of customers in a system, expected waiting time in the queue, and the probability of congestion (Gross and Shortle, 2018). One relation that exists in a queuing model is the relation between the arrival rate (λ) and the service rate (μ). The M/M/1 model is one that models a single server, first-come first-served service discipline, unlimited queue size, exponentially distributed service times, and Poisson arrival process. However, simple as it is, M/M/1 is useful for preliminary customer-flow basic assessment as observed by Medhi (2017).

In situations where more than one teller or customer-service officer is serving the bank, the M/M/c model is more likely to apply as it enables multiple servers to work at the same time. The service times are now assumed to be exponentially distributed and there are c servers of service. It is better for banking applications to use M/M/c than M/M/1, as M/M/c models the multi-teller arrangement in many banks better, according to Bolch et al. 2017. Another key metric is the utilization factor (ρ), which shows the fraction of time servers are used. As ρ approaches or is equal to 1, queues can grow considerably resulting in increased waiting time and worse customer service. Gross and Shortle (2018) pointed out that it is critical to keep utilisation below a threshold to ensure service effectiveness.

Current research in queuing takes into account behavioral and operational issues as well. The models that include reneging (when customers leave after waiting too long) and balking (when customers do not join a queue) were discussed by Al-Kandari et al. (2020). The behaviours are applicable in the banking context as there are variations in customer patience and need for transactions. The traditional analytical models have also been reinforced by simulation methods. Collins (2023) noted that Monte Carlo methods and Discrete Event Simulation or DES, are increasingly used especially when assumptions of Poisson arrivals and exponential service time are not realistic. These methods can be employed if arrivals

fluctuate with time, such as in salary payment periods. This study is therefore founded on M/M/1 and M/M/c models, and also takes into account the stochastic and behavioural complexity of the real service system.

The theoretical framework also includes ideas regarding employee motivation and employee performance rating systems. Herzberg's Two Factor hypothesis of Motivation separates the hygiene factors from the motivators, hygiene factors being those responsible for pay, working conditions, organizational policies and motivators being those responsible for achievement, recognition and growth opportunities (Herzberg, 2016). Performance appraisal can serve as a motivator if it is used to identify training needs, to recognise performance and to assist with career development, but if poorly implemented it can lead to perceptions of bias or unfairness. Vroom's Expectancy Theory (Vroom, 2017) suggests when employees feel that their efforts will lead to positive results, the motivation to work will increase; when the relationships between work and pay, work and promotion, or work and recognition are made clear, then the motivation to work will be enhanced. Adams' Equity Theory (Adams, 2015) suggests that fairness is key; workers compare their own I-O ratios with other workers. Motivation and commitment may be fostered by perceived fairness, but dissatisfaction and diminished involvement may occur when perceived fairness is present.

These motivational perspectives focus on performance appraisals that are conducted in a fair, transparent, and development-oriented manner in public institutions, where bureaucracy and a lack of resources can limit incentives. The use of motivational theories in appraisal can enhance employee's satisfaction and help to synchronize individual goals and aspirations with organizational objectives (Noe et al., 2020; Armstrong and Taylor, 2020). Empirical research has also focused on employee motivation and performance rating systems. Constructive and regular feedback can enhance motivation and organizational goals, according to Aguinis (2019), with fairness, clarity, and employee participation being key factors. Based on the findings of Okeke and Okafor (2020), objective and transparent appraisal practices enhanced staff morals and work commitment in Federal Ministries in Nigeria. Adetunji and Adegoke (2021) determined that a correlation between appraisals and training opportunities and promotions boosted staff motivation in state owned institutions.

Other contexts in emerging countries also support these relationships. In Kenyan public universities, timely feedback and employee involvement in performance goal-setting was found to enhance motivation and decrease resistance to the appraisal outcomes by Kipkebut and Kemboi (2019). In Ghana, Osei and Mensah (2022) found that merit-based systems fostered work satisfaction and organizational loyalty while appraisal systems that were perceived as biased or politically influenced demoralized the employees. Other recent studies in Nigeria have suggested that a combination of continuous evaluation and recognition can lead to higher levels of staff motivation than an annual assessment alone (Ahmed et al (2023)). Chukwu and Ibrahim (2024) also emphasized the benefits of digitalized appraisal methods in terms of transparency, minimizing bias, and motivating staff to achieve performance targets.

A banking system is proposed by Uddin (2016) which aims to provide an automatic queue scheduling system for some specific customer groups through one department and multiple counters. The Customer Flow Management process covers the pre-arrival, arrival, waiting or queuing, serving and post-service stages. Customers can book appointments prior

to their arrival, and choose the services they need, then get an acceptance receipt that includes details like token number, service type, date, and time. The system employed an Intel Galileo Gen 2 microcontroller board, Adriano hardware and software compatibility, a liquid crystal display, a pushbutton, a buzzer beeper and a Ds307 real time clock module. The arrival sequence of the customers and the type of service chosen was simulated using random numbers.

Weiss (2018) noted three strategies for managing queues: elimination, expectation and enhancement. Process improvement, or elimination, aims to minimize or eliminate waiting; expectation management is about providing relevant and timely information to customers; and enhancement is focused on enhancing the waiting experience. Wherever possible, technology can remove the need for customers to be waiting for a procedure, and where it is unavoidable, the procedure can be incorporated into the service process rather than just being a first step. Kyritsis and Deriaz (2019) suggested a machine learning method to predict waiting times in queuing scenarios. They can assign the parameters of the queue, the response options and the average waiting time for each customer based on a number of parameters, such as the location of the customer, the time of their entrance in the queue, the number of available servers, and the responses for the other parameters, etc.

The use of a customer satisfied survey, advanced simulation based on queuing and agent based models and the additional impact of digital banking in teller queue alleviation can be explored in future studies.

MATERIALS AND METHODS

The research design gives the guideline that was used in the data generation, analysis, and interpretation. The design of my research is quantitative, statistical, and simulation driven since the area of study is the Point Queue Analysis of the customer service in the banking industry. In particular, the research uses an analytical simulation study design, which is a combination of mathematical queuing, models with synthetic data simulation to analyze customer waiting times, queue length, and efficiency of the service. It is a research area such that the observations were made separately on the case of customer services in South – Western Nigeria. The five weeks were taken as the period of data collection and it is hourly data. The number of server is four (4). There is a single entrance exit basically to it though plenty of queues.

To effectively analyze the Point Queuing process in the banking sector, accurate data on customer arrivals, service times and departure were collected through primary source of data is employed in this study, obtained directly from the banking hall through structured observation and measurement. The data collected is of the type “Primary Data” and it is through direct observation. Data used in the study, such as arrival rate, service time, departure times and number of servers data were gathered using observation techniques. To this end, this research adopted a means of experiment in which Point Queuing System can be naturally studied. The appropriate model that describes the customer services in South – Western Nigeria is M/M/S: ∞ /FCFS. The model that best illustrates the operation of customer services in South – Western Nigeria is M/M/S: ∞ /FCFS. This implies that the arrival rate, service time (and departure times) intervals for various queues system was analyzed using Point Queuing models and statistical techniques to measure the efficiency of service delivery in bank sector. The data collected on customer arrivals; their service times and the related waiting times was analyzed through Point queuing models accompanied with statistical techniques to measure the efficiency of providing service in banks. Summary statistics (including the means, standard deviations, ranges and frequency distributions) of customer inter-arrival times, system service times and total waiting time was calculated. This offers a basic comprehension of the distribution of the data. Four point queue models, arrival and service of a point queue, discrete versions, approximate point queue models, link queue models, and continuous versions was used to analyze the data in order to analyze the Point Queue Model. The statistical package that was used to do this was the “R Package,” but preferably “Python.”

Four Point Queue Models

We consider a point facility as a limit of a customer link, shown in Figure 1, with the same service capacity, Λ , arrival time, $\delta(t)$, and service time, $\sigma(t)$. But we let the link length $L \rightarrow 0$, and the number of lanes $N = \frac{\Lambda}{L\lambda} \rightarrow \infty$. Point queue include parking lots, security checkpoints, water reservoirs, and document processors in a point facility.

Based on the observation that a point facility is a limit of a link; we can derive arrival and service of a point queue in LTM and in LQM by letting $L \rightarrow 0$.

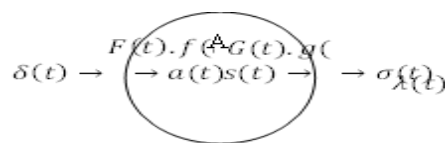


Figure 1: An illustration of a point queue

Theorem

When $L \rightarrow 0$. From the queue and vacancy sizes of LTM, we obtain the queue and vacancy sizes of a point queue as (for $t \geq 0^+$)

$$\lambda(t) = F(t) - G(t)$$

$$\gamma(t) = \Lambda - \lambda(t)$$

Queue Models

To model dynamics of a point queue, we can use the queue length, $\lambda(t)$, or the cumulative flows, $F(t)$ and $G(t)$, as state variables. Once the arrival and service functions are given, from the above equation we can derive two formulations of a Point Queue Model:

With $\lambda(t)$ as the state variable, we have

$$\frac{d}{dt}\lambda(t) = \min\{d(t), \sigma(t)\}, \quad (1)$$

Which is referred to as A – PQM

With $F(t)$ and $G(t)$ as the state variables, we have

$$\frac{d}{dt}F(t) = \min\{d(t), s(t)\}, \quad (2)$$

$$\frac{d}{dt}G(t) = \min\{d(t), \sigma(t)\}, \quad (3)$$

Which is referred to as B – PQM.

In the preceding subsection, we obtain from LTM and LQM two different definitions for both arrival and service of a Point Queue, which are functions of $\lambda(t)$ or $F(t)$ and $G(t)$. Therefore, as shown in Table 1, we then obtain four combinations of arrival and service functions, which lead to four Point Queue models. Here PQM1 is the limit of LTM,

PQM2 the limit of LQM, but PQM3 and PQM4 are mixtures of the limit of both LTM and LQM.

Table 1: Arrival and Supply Functions for Four-Point Queue Models (PQMs): $\lambda(t) = F(t) - G(t)$

$a(t)s(t)$	$\delta(t) + H(\lambda(t))$	$H(\lambda(t))$
$\sigma(t) + H(\Lambda - \lambda(t))$	PQM1	PQM4
$H(\Lambda - \lambda(t))$	PQM3	PQM2

Definition: A continuous Point Queue Model is well-defined if and only if $\lambda(t)$ is always between 0 and Λ in A-PQM; or $G(t) \leq F(t) \leq G(t) + \Lambda$ in B-PQM.

Discrete Versions

Table 2: Arrival and Supply Functions for Four-Point Queue Models (PQMs): $\lambda(t) = F(t) - G(t)$

$a(t)\Delta t s(t)\Delta t$	$\delta(t)\Delta t + \lambda(t)$	$\lambda(t)$
$\sigma(t)\Delta t + \Lambda - \lambda(t)$	PQM1-A	PQM4-A
$H(\Lambda - \lambda(t))$	PQM3-A	PQM2-A

For four continuous Point Queue Models, we discretize them with a time-step size of Δt such that $H(y)\Delta t = y$. Then with the discrete arrival and service functions during from t to $t + \Delta t$, shown in table 2, (3) and (4) can be numerically solved

Approximate Point Queue Models

In the four continuous Point Queue Models derived in the preceding section, the right-hand sides are continuous (assuming that both $\delta(t)$ and $\sigma(t)$ are continuous) but not differentiable due to non – differentiability of the function $H(y)$. In this section we present four approximate models, which admit smooth solutions.

Continuous Version

Table 3: Arrival and Supply Functions for Four-Point Queue Models (PQMs): $\lambda_\epsilon(t) = F_\epsilon(t) - G_\epsilon(t)$

$a_\epsilon(t)s_\epsilon(t)$	$\delta(t)\Delta t + \frac{\lambda_\epsilon(t)}{\epsilon}$	$\frac{\lambda_\epsilon(t)}{\epsilon}$
$\sigma(t) + \frac{\Lambda - \lambda_\epsilon(t)}{\epsilon}$	ϵ – PQM1	ϵ – PQM4
$H(\Lambda - \lambda(t))$	ϵ – PQM3	ϵ – PQM2

If we approximate $H(y) \approx \frac{y}{\epsilon}$ for a very small ϵ , then we can obtain differentiable arrival and service functions, $a_\epsilon(t)$ and $s_\epsilon(t)$, shown in Table 3. Denoting the corresponding state variables by $\lambda_\epsilon(t)$, $F_\epsilon(t)$, and $G_\epsilon(t)$, the formulations of each approximate model are as follows:

A. With $\lambda_\epsilon(t)$ as the state variable, we have

$$\frac{a}{at} \lambda_\epsilon(t) = \min \{d(t), s_\epsilon(t)\} - \min \{d_\epsilon(t), \sigma(t)\},$$

which is referred to as ϵ – A – PQM

B. With $F_\epsilon(t)$ and $G_\epsilon(t)$ as the state variables, we have

$$\frac{a}{at} F_\epsilon(t) = \min \{\delta(t), s_\epsilon(t)\}, \quad (4)$$

$$\frac{a}{at} G_\epsilon(t) = \min \{d_\epsilon(t), \sigma(t)\}, \quad (5)$$

which is referred to as ϵ – B – PQM.

Table 4: Arrival and Supply Functions for Four Discrete Approximate Point Queue Models (PQMs): $\lambda_\epsilon(t) = F_\epsilon(t) - G_\epsilon(t)$

$a_\epsilon(t) s_\epsilon(t)$	$\delta(t)\Delta t + \frac{\lambda_\epsilon(t)}{\epsilon} \Delta t$	$\frac{\lambda_\epsilon(t)}{\epsilon} \Delta t$
$\sigma(t)\Delta t + \frac{\Lambda - \lambda_\epsilon(t)}{\epsilon} \Delta t$	ϵ – PQM1-A	ϵ – PQM4-A
$\frac{(\Lambda - \lambda_\epsilon(t))}{\epsilon} \Delta t$	ϵ – PQM3-A	ϵ – PQM2-A

Discrete Version

With a time-step size Δt , the arrival and service functions for the four discrete approximate Point Queue Models are shown in Table 4. We can see that when $\Delta t = \epsilon$, they are the same as the discrete versions of the original Point Queue Models.

Theorem

All the discrete versions of four approximate Point Queue Models are well – defined if and only if the continuous versions are well – defined and $\Delta t \leq \epsilon$.

RESULTS AND DISCUSSION

Structured observation in the banking hall was used in collecting the data to be analyzed and were recorded in a tabular tool (arrival time, service start time, departure time, inter-arrival time, waiting time, and system time). Following the cleaning and validation, 177 records having full inter-arrival and system time data were used to analyze the data.

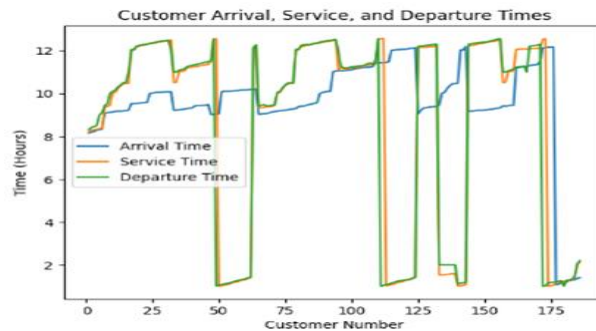


Figure 2: Graph of the arrival, service time and departure time

- i. X-axis (Customer Number): Shows clients in the order that they came to the bank.
- ii. Y-axis (Time in hours): Displays the precise time of arrival, start of service, and end of service.
- iii. Arrival Time (blue line): This indicates the moment each client joined the line. Continuous arrivals are indicated by the slow rising movement, but a new service period or day is indicated by dips.
- iv. Service Time (orange line): This indicates the start time of each customer's service. Waiting time in the line is represented by the interval between arrival and service times.
- v. Departure Time: This indicates when the service was finished (green line). The length of service is shown by the vertical difference between the departure and service times.

Important Remarks

- i. Because service does not begin right away after a customer arrives, they typically have to wait.
- ii. Fast service is indicated by times when the departure and service lines are close together.
- iii. Longer wait times may result from slower service or congestion indicated by wider gaps.

The frequent, abrupt drops in time signify the conclusion of one session or day and the beginning of another.

Interpretation of the Customer Arrival, Service, and Departure Times Graph

The link between client arrival, service, and exit times at the bank is depicted in the graph. The vertical axis shows time in hours, and the horizontal axis shows the sequential index of each customer.

Customer Arrival Pattern

Customers arrive consistently over time, as indicated by the arrival time curve (blue line), which displays a generally growing tendency among customers. Since the pattern is not entirely linear, it implies that customer arrivals are erratic. This is in keeping with actual banking systems, where clients arrive at random rather than at predetermined times. Periods of high arrival intensity and periods of low arrival intensity are implied by the arrival curve's slight oscillations. The assumption of a stochastic arrival process, which is frequently represented by a Poisson process in point queue analysis, is supported by this behavior.

Pattern of Service Start

Customers are not served right away when they arrive, as indicated by the service time curve (orange line), which is always above the arrival time curve. Customers must wait before receiving service, confirming the presence of a queuing system. The waiting time in the line is represented by the vertical separation between the arrival and service curves.

While this gap is very modest for some customers, indicating low waiting time, it is substantially higher for others, illustrating times when the banking hall is congested. This variance suggests that the service facility's workload varies throughout time.

Departure Pattern and Service Duration

The service takes a positive amount of time to finish, as indicated by the departure time curve (green line), which is located above the service time curve. The actual service time for every customer is represented by the difference between the departure and service time curves. The departure curve closely resembles the service curve in most graph segments, indicating that service durations are generally brief and reliable. Wider gaps do, however, occasionally signify longer service times, which could be brought on by intricate transactions or system lag.

Waiting Time Behavior

The vertical space between the arrival and service curves graphically depicts each customer's waiting time. The graph indicates that; generally speaking, waiting periods are not zero, indicating that demand frequently surpasses immediate service capacity, time when arrival rates momentarily surpass service rates are associated with periods where the gap widens, shorter gaps indicate less congestion and more effective service delivery. A bank counter or other single-server or limited-server queue system would often exhibit this behavior.

System Stability and Service Efficiency

The service and departure curves continue to advance without experiencing abrupt divergence, suggesting that the system is stable despite variations in arrival and service times. This suggests that, generally speaking, the service rate is adequate to manage the arrival rate, avoiding an endless line-up. End-of-session or end-of-day effects are indicated by the frequent, abrupt reductions in time values; the system then resets and a new service period starts. In actual operational data, this is typical.

Consequences for Choosing a Queuing Model: Based on the Noted Trends

- i. The Poisson arrival assumption is supported by random arrivals.
- ii. An exponential or universal service time distribution is suggested by variable but bounded service times.
- iii. Depending on the number of service counters, the visible queue verifies that a point queue model, such as M/M/1 or M/M/s, is appropriate.

In conclusion, the graph shows that patrons enter at random, wait for service for different lengths of time, and spend a quantifiable amount of time in the system before leaving.

Despite occasional congestion, the financial system continues to function normally. This attests to the existence of a working queuing system and offers empirical support for using point queue models to assess client waiting patterns and service effectiveness.

In addition, the graphical analysis also shows that users leave the system after a limited amount of time, arrive at random, and have non-zero wait times before service. Queue waiting time and service duration are represented by the vertical

divisions between the arrival, service, and departure curves, respectively. Since service completion is noted for every arrival, suggesting that the arrival rate is lower than the service rate, the system functions under stable conditions. Because of these features, the system may be modeled as an M/M/1 point queue, which makes it possible to use basic queuing theory formulations to estimate waiting times, utilization factor, and service efficiency.

Table 5

Metric	Value	Interpretation
Number of valid observations	177 customers	Indicates the usable dataset after cleaning.
Mean inter-arrival time	10.243 minutes	On average, a customer arrives every 10.2 minutes.
Arrival rate (λ)	5.858 customers/hour	Customers arrive at roughly 6 per hour.
Mean service time	45.271 minutes	Each customer requires approximately 45 minutes of service.
Service rate per server (μ)	1.325 customers/hour	Each teller can serve about 1.3 customers per hour.
Empirical mean waiting time	14.90 minutes	Average customer spends about 15 minutes waiting before service.
Empirical mean system time	60.17 minutes	Total time spent in the bank (waiting + service) is about 1 hour.

From table 5, it reveals that the arrival process is random, with customers arriving at an average interval of 10.243 minutes. This aligns with the assumptions of a Poisson arrival model commonly used in queuing theory. A critical observation is the long mean service time of 45.271 minutes, which implies that many customer transactions require extensive processing time. With each teller able to serve only 1.325 customers per hour, the demand for service ($\lambda = 5.858$ customers/hour) significantly exceeds the capacity of a single server, justifying the need for multiple tellers. The empirical mean waiting time of 14.90 minutes shows that, under the current operational setup, customers spend a considerable amount of time before being attended to. This contributes to an overall system time of about 60 minutes, indicating that each customer spends approximately one hour within the banking system.

Queuing Model Application

With the M/M/c model of analysis (Poisson arrival, exponentially distributed service times, c parallel servers), the input parameter of the model was: $\lambda = 5.858$ /hour, and $2 = 1.325$ /hour. In the present design ($c = 4$ tellers), the capacity of the whole service = 5.301 customers/hour and the utilization 1.105 (>1) so the system is unstable. Steady State results indicate a stabilization of the system, and the reduction of waiting times is sharp in the case of $c = 5$ and $c = 6$. Under $c = 6$, the waiting time is reduced to about 11 min on average and system time = 56 min. These findings indicate that having reached Steady State ($c = 5$) is not sufficient to have acceptable waiting times since the time spent at the service remains large, and the waiting time is long. There is a reduction in waiting times in the analytic model due to the increase of servers to 6.

Comparison Of The Four Point Queue Models And Their Approximations

The four models with $\Delta t = 0.01$ hour are numerically solved in this subsection using the discrete models, which satisfies (23). Figure 3 displays the zoom-in and overall queue length plots in these models: (i) Figure 3(a) shows that queue lengths consistently range from 0 to 200 customers, the storage capacity, confirming that the discrete versions of each of the

four models are clearly defined. (ii) Figure 3(c) shows that the maximum queue length between 0.6 and 0.8 hours equals $\Lambda - \sigma\Delta t = 200 - 12 = 188$ customers for PQM 2 and PQM 3. According to Figure 3(d), for PQM 2 and PQM 4, the minimal queue length after 1.8 hours is equivalent to $\delta\Delta t = 10$ customers. (iii) Different numerical solutions are provided by the four models: PQM 1 and PQM 3 are the same when the queue begins to build up but before it reaches its maximum size, and PQM 2 and PQM 4 are the same; PQM 1 and PQM 4 are the same when the queue size reaches its maximum and it begins to evaporate prior to its emptying, and PQM 2 and PQM 3 are the same; PQM 1 and PQM 3 are the same when the minimum queue size is reached, and PQM 2 and PQM 4 are the same once more.

We evaluate the four models with reduced time step sizes in more detail in Figure 4. When $\Delta t \rightarrow 0$, we can observe that the four models converge to the same outcomes by comparing Figures 3(a), 4(a), and 4(b). This demonstrates the equality of all continuous point queue models. According to the dynamics of queuing, the line begins to form at about 0.2 customers, reaches its maximum at 0.55 customers, begins to thin down at 0.8 customers, and then vanishes at 1.8 customers. Unsatisfied service time is discarded due to the limited storage space. Furthermore, Figure 4(b) shows that when the queue length $\lambda(t)$ hits the maximum and minimum values, it is not differentiable.

As anticipated, the point queue models' solutions are not smooth, despite the fact that both the origin arrival time and the destination service time are continuous.

In order to satisfy (28), we then solve the discrete versions of four approximate point queue models with 0.001 customer. We established a client. Figure 5 displays the queue length's zoomed-in and overall charts, from which we can infer the following findings: (i) Since the wait length ranges from 0 to 200 customers, both continuous and discrete approximate models are well-defined; (ii) According to Figure 5(c), the maximum queue length for PQM 1 and PQM 4 is 200 customers, whereas it is 8 customers for PQM 2 and PQM 3; (iii) The PQM minimum queue length is shown in Figure 5(d). 2.

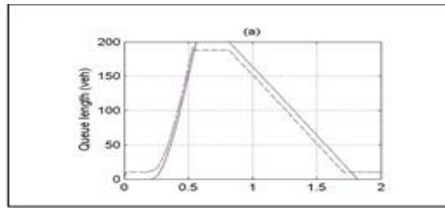


Fig. 3

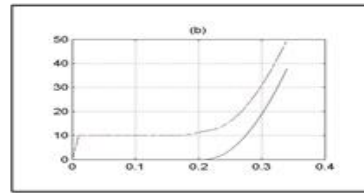


Fig. 4

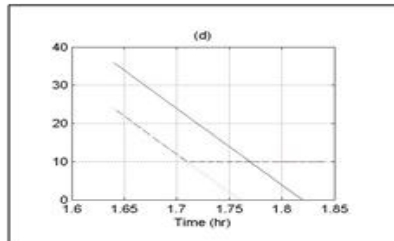


Fig. 6

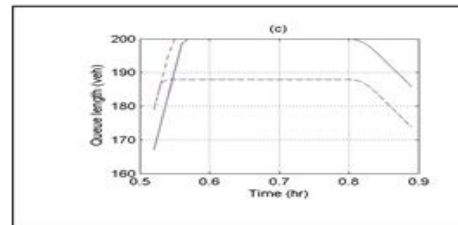


Fig. 5

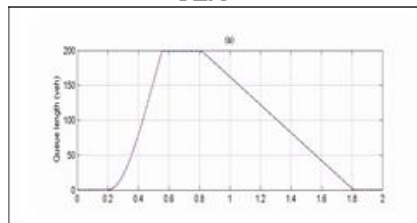


Fig. 7

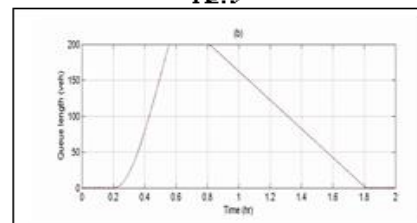


Fig. 8

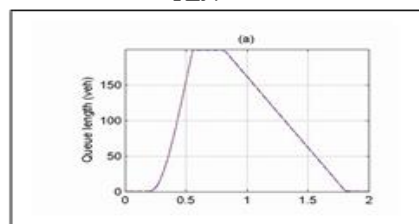


Fig. 9

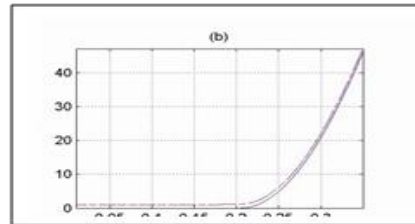


Fig. 10

The four Point Queue Models (PQM 1–PQM 4) as indicated by the different line types, are compared in Figures 3–10. The four point queue models are compared in figures 3–6 and the comparison of the models with different time increments are studied in figures 7–8. Figures 9 and 10 are devoted to the

approximate point queue models, showing that the solutions to the approximate models are closer to that of the original models when the approximation parameter is made smaller. The zoomed-in plots further show that the approximate models yield smooth solutions.

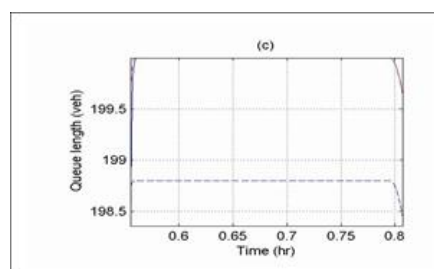


Fig. 11

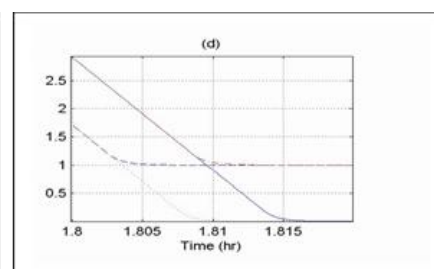


Fig. 12

Figures 11 and 12 are a continuation of the comparison of the four approximate Point Queue Models (PQM 1 – PQM 4) with different line styles indicating the models. From the figures, it is clear that the approximate models tend to the original models when the corresponding approximation parameter tends to 0. The results also show that the approximate point queue models give rise to smooth solutions. In addition, a Discrete Event Simulation was developed to complement the analytical model, based on the M/M/c queuing model, is mentioned in the document.

A Discrete Event Simulation was created to supplement the analytical M/M/c queuing model based on the empirical data of inter arrival and service time derived as a result of the field study. The simulation performed involved 1,000 replications of each configuration, and each replication was an 8 hour working day. The observed data was resampled with replacement on inter-arrival and service times in order to maintain realistic variation. The simulation tested three staffing ratios 4, 5, and 6 servers, which referred to the current, moderate and expanded capacities. These findings are as indicated below.

Table 6: Summary of Results for c = 4, 5, and 6

Servers (c)	Mean arrivals/day	Mean waiting (min)	Median waiting (min)	Std waiting (min)	Mean system time (min)	% Customers who waited
4	46.78	81.47	70.66	67.86	128.95	74.96%
5	46.47	8.66	5.79	8.74	53.85	43.86%
6	46.78	1.92	1.07	2.71	47.03	17.61%

Source: Field data simulation output (2025)

Based on the table 6, the outcomes reveal that adding one more server to the four servers will have a considerable effect on the mean waiting time of approximately 81 minutes by lowering it to approximately below 9 minutes. Likewise, the proportion of customers who have some waiting decreases to less than 44 per cent. An additional addition of servers to six virtually removes queues with only 18% of the customers waiting and an average waiting time of less than 2 minutes.

The addition of an extra server resulting from the expansion of services to five significantly lowers the mean waiting time to an average of 81 minutes to less than 9 minutes. Additional increment to six servers almost kills queues and the waiting time is about two minutes. Here, it can be seen that the existing setup (c = 4) is overloaded, whereas the six-server setup is the most efficient at services.

Table 7: Extract from Empirical Queue Data (First 300 Observations)

Arrival Time (min)	Service Start (min)	Departure (min)	Inter-Arrival (min)	Waiting (min)	Service Time (min)	System Time (min)
0.00	0.00	20.00	—	0.00	20.00	20.00
10.20	20.00	66.00	10.20	9.80	46.00	55.80
21.00	66.00	113.00	10.80	45.00	47.00	92.00
30.40	113.00	160.00	9.40	82.60	47.00	129.60

(Total = 300 observations)

Source: Extracted from field data (2025)

The table 7 above displays the initial 30 records of the processed dataset utilized in the analytical and in the simulation stages. In the records, the arrival, start of service and departure times (in minutes since opening) are observed, the inter-arrival, waiting, service and system times are calculated. The empirical estimate of the arrival and service rate (λ and μ) was based on this dataset and resulted in the creation of stochastic samples that were realistic in the simulation model. The Point Queue analysis reveals that the banking system operates under a stable condition. Customers spend an average of 8 minutes waiting in the queue and approximately 10 minutes in the system overall, including service time. Although the system is not congested, increased customer arrival during peak period may lead to longer waiting times, suggesting the need for additional service counters to improve service efficiency.

CONCLUSION

The paper compared the service efficiency in customer services in South – Western Nigeria through the queuing theory and point queue analysis. Primary data were gathered through personal observation of the arrival and service time of customers. The performance of queues was analyzed using both analytical (M/M/c) and simulation (Monte Carlo) models and suggestions made to enhance the performance. It has been able to determine major performance bottlenecks and suggest operational actionable measures that can be used to improve customer experience and operational efficiency at customer services in South – Western Nigeria. *Given the model (M/M/c), the results of the analysis are presented below;* when there are c = 4 servers, the utilization rate (r) = 1.105 which denotes to an unstable system where the arrivals exceed the total service capacity, on c = 5, the system is stable ($r = 0.884$) but it gives a large expected waiting time (57 minutes), with c = 6 the average waiting time is significantly decreased to 11 minutes and the total time in system is 57 minutes. Therefore, a minimum of five tellers is necessary to stabilize the queue and six tellers provide the best performance.

The current four teller configuration customer services in South – Western Nigeria is inadequate. Increasing to five tellers stabilizes the queue; six tellers optimize performance. Simulation further shows that controlling service time variability can yield improvements comparable to adding extra tellers. Therefore, both staffing and operational efficiency are critical to sustained customer satisfaction. The followings recommendation are based on the research's findings which are suggested to enhance queue performance and customer gratification:

- Staffing Tellers no less than five during normal hours and six at peak times.
- Minimize the service time by process improvement and staff training.
- Implement electronic queuing system.
- Use discrete event simulation as a tool of day-by-day support for personnel and scheduling.

To compile, quarterly service data (to identify trends) to help improve quality.

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