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THEORETICAL FOUNDATIONS AND APPLICATION OF THE WESTCOTT FORMALISM IN k_0-STANDARDIZED INSTRUMENTAL NEUTRON ACTIVATION ANALYSIS AT THE LOW-ENRICHED URANIUM NIGERIA RESEARCH REACTOR-1 (LEU-NIRR-1)

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ABSTRACT

Instrumental neutron activation analysis (INAA) based on the k_0-standardization method is a powerful multi-element technique whose accuracy depends on a sound theoretical treatment of neutron reaction rates and precise characterization of the irradiation facility. This study examines the theoretical foundations of the Westcott formalism and its integration with the k_0-standardization method, with specific application to the Low-Enriched Uranium Nigeria Research Reactor-1 (LEU-NIRR-1). The work reviews the nuclear physics of neutron capture, contrasting the Høgdahl convention (valid for $1/v$ nuclides) with the Westcott formalism required for non-$1/v$ nuclides. Modified Westcott expressions incorporating the epithermal shape factor $\alpha$and neutron temperature $T_n$are linked to the practical k_0 concentration equation. Reactor-physics aspects of LEU-NIRR-1 are examined, focusing on the 2018 HEU-to-LEU conversion. Experimental and computational determinations of the spectrum parameters f and α are reviewed for both cores. Results show only modest spectral hardening, with inner-channel values of $f \approx 19.7$ and $\alpha \approx -0.054$, and outer-channel values of $f \approx 48.6$ and $\alpha \approx +0.029$. These parameters, together with detector-efficiency calibration and true-coincidence summing corrections, enable reliable application of both Høgdahl and Westcott forms of the $k_0$method. Advantages of the combined approach are weighed against limitations from nuclear-data uncertainties. Current research trends, including post-conversion validation and refinement of $g(T_n)$ factors, are surveyed. The study demonstrates that LEU-NIRR-1 retains a stable, well-thermalised spectrum fully compatible with high-quality k_0-INAA, providing a robust foundation for multi-element analysis and a reference for other converted Miniature Neutron Source Reactors.

Keywords: Westcott formalism; k_0-standardization; Instrumental neutron activation analysis; Neutron spectrum parameters; Theoretical foundations; HEU-to-LEU conversion, Non-$1/v$ nuclides

INTRODUCTION

Instrumental Neutron Activation Analysis (INAA) is one of the most accurate, precise, and reliable nuclear analytical techniques for qualitative and quantitative elemental determination in environmental, geological, biological, archaeological, forensic, industrial, and nuclear materials. Since the pioneering work of Hevesy and Levi (1936) on neutron-induced radioactivity, neutron activation analysis has evolved into one of the International Atomic Energy Agency (IAEA)-recommended reference methods for multi-element analysis because of its excellent analytical sensitivity, high accuracy, outstanding precision, and non-destructive nature (De Corte, 1987; Molnár, 2004; Blaauw et al., 2023). Unlike most spectroscopic techniques, INAA depends on nuclear rather than chemical properties, thereby minimizing matrix effects and eliminating the need for extensive chemical digestion or calibration standards (De Corte, 1987).

The analytical principle of INAA is based on neutron-induced nuclear reactions, predominantly the radiative neutron capture reaction (n,γ), in which stable nuclei absorb neutrons to form radioactive isotopes. These radionuclides decay according to their characteristic half-lives while emitting gamma rays with unique energies that serve as elemental fingerprints. Measurement of these gamma rays using high-resolution High-Purity Germanium (HPGe) detectors enables simultaneous qualitative and quantitative determination of numerous elements over concentration ranges extending from major constituents to sub-ng g^{-1} levels (Molnár, 2004; Knoll, 2010).

Research reactors remain the most important neutron sources for INAA because they provide intense, stable, and reproducible neutron fluxes ranging from approximately 10^{11} to 10^{14} n cm^{-2} s^{-1}. Among the various reactor designs, the Miniature Neutron Source Reactor (MNSR) has become particularly attractive owing to its compact design, inherent safety characteristics, operational simplicity, and excellent neutron flux stability. These features have facilitated the establishment of neutron activation laboratories in many developing countries for environmental monitoring, geological exploration, agriculture, medicine, archaeology, and industrial quality assurance (IAEA, 2020; Jonah et al., 2005).

Nigeria entered the era of research reactor utilization following the commissioning of the Nigeria Research Reactor-1 (NIRR-1) at the Centre for Energy Research and Training (CERT), Ahmadu Bello University, Zaria, in 2004. NIRR-1 is a 30-kW Chinese-designed Miniature Neutron Source Reactor (MNSR) that has become the country's primary neutron irradiation facility for neutron activation analysis, neutron radiography, isotope production, reactor physics investigations, neutron beam research, and postgraduate training. Since its commissioning, NIRR-1 has supported numerous studies involving environmental pollution, medicinal plant characterization, food safety, geology, archaeology, industrial materials, and nuclear analytical science (Jonah et al., 2005; Alhassan, 2024; Jonah et al., 2019).

A major milestone in the operational history of NIRR-1 was its successful conversion from a Highly Enriched Uranium (HEU) core to a Low-Enriched Uranium (LEU) core under the Global Threat Reduction Initiative (GTRI) coordinated by the International Atomic Energy Agency (IAEA). The LEU conversion significantly strengthened global nuclear security while preserving the reactor's neutron analytical capabilities.

Nevertheless, the conversion required renewed characterization of reactor neutron spectrum parameters because the neutron energy distribution directly influences neutron activation calculations and analytical accuracy. Consequently, precise reactor spectrum characterization has become indispensable for maintaining analytical quality and metrological traceability in LEU-NIRR-1 applications (IAEA, 2020; Jonah et al., 2005).

Historically, three principal standardization methods have been employed in neutron activation analysis: the absolute method, the relative comparator method, and the k₀-standardization method. Although the absolute method is theoretically rigorous, its practical implementation requires accurate knowledge of neutron flux, detector efficiency, nuclear constants, and irradiation geometry, making it susceptible to cumulative uncertainties. The relative comparator method substantially reduces these uncertainties through simultaneous irradiation of standards and unknown samples but requires preparation and maintenance of multiple elemental standards (De Corte, 1987).

To overcome these limitations, Simonits, De Corte, and Hoste (1975) introduced the k₀-standardization method, which utilizes a single comparator, typically ^{197}Au, together with accurately evaluated nuclear constants and experimentally determined reactor spectrum parameters. This innovation eliminated the need for multiple elemental standards while maintaining excellent analytical accuracy, reproducibility, and international traceability. Since its introduction, the k₀ method has become the most widely adopted standardization procedure for reactor-based INAA and forms the theoretical basis of software packages such as k₀-IAEA, KayZero, k₀-INRIM, k₀-IPEN, and k₀-DALAT (Simonits et al., 1975; De Corte, 1987; Blaauw et al., 2023).

Like it has been stated, the successful implementation of the k₀-standardization method depends fundamentally on accurate characterization of the reactor neutron spectrum. Research reactor neutron fields comprise thermal, epithermal, and fast neutron components whose relative contributions vary according to reactor design, moderator properties, irradiation position, and operating conditions. Consequently, two reactor-specific neutron spectrum parameters: the thermal-to-epithermal neutron flux ratio (f) and the epithermal neutron spectrum shape factor (α) must be accurately determined for each irradiation facility. These parameters constitute essential inputs to modern k₀ calculations and significantly influence analytical accuracy (De Corte & Simonits, 2003; Akaho & Nyarko, 2002; Jonah et al., 2005).

The theoretical basis of k₀-INAA is rooted in the Høgdahl convention, which assumes that neutron capture cross sections obey the ideal 1/v absorption law in the thermal neutron region. However, several analytically important nuclides, including Eu, Lu, Sm, Dy, Yb, and Hf all deviate significantly from ideal 1/v behaviour because of resonance absorption (De Corte et al., 1994; van Sluijs et al., 2015). To account for these deviations, Westcott (1960) developed a generalized neutron capture formalism incorporating neutron temperature, resonance absorption, and departures from Maxwellian neutron distributions through the introduction of the Westcott g-factor and associated correction parameters. This formalism has since become one of the cornerstones of modern neutron activation analysis, reactor dosimetry, and neutron spectrum characterization (Westcott, 1960; Holden, 1991; De Corte & Simonits, 2003).

The incorporation of the Westcott formalism into the k₀-standardization methodology represents one of the most significant theoretical advances in neutron activation analysis. Modern k₀ software integrates Westcott correction factors with reactor neutron spectrum parameters, detector efficiency calibration, and coincidence summing corrections, updated nuclear databases, and rigorous uncertainty propagation models. The 2021 IAEA software intercomparison demonstrated that differences among international k₀ laboratories are now dominated primarily by detector efficiency characterization and neutron spectrum parameter determination rather than by the k₀ formalism itself, underscoring the maturity and robustness of the method (Blaauw et al., 2023; Di Luzio et al., 2022).

For the scientists and analysts working at the NIRR-1 laboratory, a thorough understanding of the theoretical relationship between the Westcott formalism and the k_0-standardization method is indispensable. Accurate application of the Westcott formalism underpins their ability to characterize the reactor neutron spectrum, determine reliable f and α parameters, validate methods using certified reference materials, estimate uncertainties properly, and maintain robust quality assurance in routine k_0-INAA. Furthermore, continued refinement of Westcott-based protocols directly enhances the capacity of the NIRR-1 laboratory team to deliver high-quality analytical services in critical national areas such as environmental monitoring, food safety, geological exploration, medicinal plant analysis, nuclear forensics, and advanced materials characterization. Consequently, mastering these theoretical foundations remains essential for the LEU-NIRR-1 scientific community to sustain internationally traceable measurements and to strengthen Nigeria's nuclear analytical capabilities.

Fundamental Nuclear Physics of Neutron Activation

Neutron activation analysis rests on the controlled induction of radioactivity through neutron–nucleus interactions, primarily radiative capture(n,γ). When a target nucleus absorbs a neutron, the resulting compound nucleus is formed in an excited state and de-excites by emitting one or more prompt gamma rays, leaving a radioactive product nuclide. The subsequent radioactive decay of this product, typically accompanied by characteristic delayed gamma rays, constitutes the measurable signal used for elemental quantification (De Corte, 1987; Simonits et al., 1975).

Neutron–Nucleus Interaction and Reaction Rate

The probability of a neutron interacting with a nucleus is quantified by the microscopic cross-section$\sigma(E)$, which is strongly energy-dependent. For the majority of analytically useful nuclides the thermal capture cross-section follows the $1/v$ law:

$$\sigma(v) = \sigma_0 \frac{v_0}{v} \quad (1)$$

Where σ_0 is the cross-section at the reference velocity $v_0 = 2200\ \mathrm{m\ s^{-1}}$(corresponding to $E_0 = 0.0253$ eV) and v is the neutron velocity. Deviations from this behaviour in the thermal region are accounted for by the Westcott $g(T_n)$ factor (Westcott, 1960; Westcott et al., 1958).

The reaction rate R per target nucleus is the integral of the product of the neutron flux density $\phi(E)$ and the energy-dependent cross-section:

$$R = \int_0^\infty \sigma\,(E)\ \phi(E)\ dE \quad (2)$$

In practical reactor spectra this integral is conventionally partitioned into thermal (sub-cadmium) and epithermal (epi-cadmium) contributions.

Reactor Neutron Spectra

The neutron spectrum inside a thermal research reactor such as NIRR-1 can be idealised as the superposition of three components (Beckurts & Wirtz, 1964; De Corte, 1987):

i. A Maxwellian thermal distribution characterised by a neutron temperature T_n:

$$\phi_{\mathrm{th}}(E) \propto E \exp\left(-\frac{E}{kT_n}\right) \quad (3)$$

ii. An epithermal (slowing-down) spectrum that approximates$1/E^{1+\alpha}$, where the parameter αquantifies departure from the ideal $1/E$ shape caused by neutron leakage and absorption.

iii. A fast (fission) spectrum that is of secondary importance for most (n,γ) reactions used in INAA.

The thermal-to-epithermal flux ratio $f = \phi_{\mathrm{th}}/\phi_{\mathrm{e}}$ and the epithermal shape factor α are therefore the principal spectrum parameters that must be determined experimentally for accurate activation calculations.

Activation Equation

Consider a thin target containing N nuclei of a given isotope irradiated for a time t_{irr} in a constant neutron flux. The number of radioactive product nuclei present at the end of irradiation is

$$N^* = \frac{N\,R}{\lambda}\left(1 - e^{-\lambda t_{\mathrm{irr}}}\right) \quad (4)$$

Where $\lambda = \ln 2\,/T_{1/2}$ is the decay constant of the product. After a decay (cooling) interval t_{d} and during a counting intervalt_{c}, the net number of counts recorded in a full-energy gamma-ray peak is

$$N_p = \frac{N\,R\,\varepsilon_p\,I_\gamma}{\lambda}\left(1 - e^{-\lambda t_{\mathrm{irr}}}\right)e^{-\lambda t_{\mathrm{d}}}\left(1 - e^{-\lambda t_{\mathrm{c}}}\right) \quad (5)$$

Here ε_p is the full-energy peak detection efficiency and I_γ is the absolute gamma-ray emission probability. Solving for the number of target nuclei N(and hence for the elemental mass) yields the fundamental equation of neutron activation analysis.

Resonance Integrals and Non-$1/v$ Behaviour

In the epithermal region the capture cross-section is dominated by discrete resonances. The resonance integral is defined as

$$I_0 = \int_{E_{\mathrm{Cd}}}^{\infty} \frac{\sigma(E)}{E}\,dE \quad (6)$$

Where E_{Cd} is the effective cadmium cut-off energy (approximately 0.55 eV for a 1 mm Cd cover). When the epithermal spectrum is non-ideal, the effective resonance integral becomes$I_0(\alpha)$. For nuclides that deviate from the $1/v$ law, the Westcott formalism replaces the simple thermal cross-section σ_0 by the effective cross-section $g(T_n)\sigma_0$ and introduces a modified spectral index that couples the thermal and epithermal contributions (De Corte et al., 1994; Westcott, 1960).

Nuclear Data Requirements

Accurate absolute or k_0-standardised analysis demands high-quality nuclear data for every target–product pair: isotopic abundance, thermal capture cross-section σ_0(or $k_0$factor), resonance integral I_0(or $Q_0 = I_0/\sigma_0$), half-life of the product nuclide, absolute gamma-ray intensities, and Westcott $g(T_n)$ factors for non-$1/v$ nuclides.

These data are compiled in evaluated libraries (e.g., ENDF/B, JEFF) and in dedicated $k_0$databases (De Corte & Simonits, 2003). Uncertainties in any of these quantities propagate directly into the final elemental concentrations and therefore constitute a fundamental limit on the accuracy of the method.

Theory of the Westcott Formalism

The Westcott formalism provides a rigorous framework for calculating neutron reaction rates in thermal reactor spectra when the capture cross-section of the target nuclide deviates from pure $1/v$ behaviour in the thermal energy region. Originally developed by Westcott and collaborators in the late 1950s for reactor physics applications (Westcott et al., 1958; Westcott, 1960), the formalism was later adapted and refined for neutron activation analysis, particularly within the k_0-standardization method (De Corte et al., 1993; De Corte et al., 1994). It remains the standard theoretical treatment for non-$1/v$ nuclides in research-reactor INAA.

Basic Concepts and Definitions

In an ideal moderator the thermal neutron density follows a Maxwell–Boltzmann distribution characterised by a neutron temperatureT_n. Westcott expressed the reaction rate per target nucleus in the compact form

$$R = n\,v_0\,\hat{\sigma} \quad (7)$$

Where n is the neutron density, $v_0 = 2200\ \mathrm{m\,s^{-1}}$ is the conventional reference velocity, and $\hat{\sigma}$ is an *effective* cross-section. The effective cross-section is written

$$\hat{\sigma} = \sigma_0\left[g(T_n) + r\sqrt{\frac{T_n}{T_0}}\,s_0\right] \quad (8)$$

Here, σ_0 is the 2200 m s^{-1} capture cross-section, $g(T_n)$ is the Westcott g-factor that accounts for departure from the $1/v$ law in the thermal region, r is a measure of the relative strength of the epithermal flux (related to the spectral index), s_0 is a reduced resonance integral parameter, $T_0 = 293.6$ K is the reference temperature corresponding to v_0.

It has been reported that for a pure $1/v$ absorber, $g(T_n) = 1$ for all temperatures and the expression reduces to a simple multiple of σ_0.

The Westcott g-Factor

The g-factor is defined as the ratio of the reaction rate averaged over a pure Maxwellian spectrum at temperature T_n to the reaction rate that would be obtained if the cross-section followed a strict $1/v$ law with the sameσ_0:

$$g(T_n) = \frac{\int_0^\infty \sigma(v)\,v\,n(v)\,dv}{\sigma_0\,v_0\int_0^\infty n(v)\,dv} \quad (9)$$

Where $n(v)$ is the Maxwellian density distribution equivalently, in energy representation,

$$g(T_n) = \frac{1}{\sigma_0}\sqrt{\frac{4T_n}{\pi T_0}}\int_0^\infty \sigma(E)\,\frac{E}{(kT_n)^2}\exp\left(-\frac{E}{kT_n}\right)dE \quad (10)$$

Values of $g(T_n)$ are tabulated or calculated from evaluated cross-section libraries for all analytically important non-$1/v$ nuclides (e.g., ^{151}Eu, ^{176}Lu, ^{191}Ir, ^{197}Au at elevated temperatures). When $g(T_n)$ differs appreciably from unity, neglect of this factor introduces systematic bias in the derived elemental concentrations.

Epithermal Contribution and the Spectral Index

The parameter $r$characterises the relative magnitude of the epithermal flux. In the original Westcott formulation, the epithermal spectrum is assumed to follow a pure $1/E$ shape above a suitable joining energy. The reduced resonance integral s_0 is defined by

$$s_0 = \frac{2}{\sqrt{\pi}}\frac{I_0'}{\sigma_0} \quad (11)$$

Where I_0' is the resonance integral after subtraction of the $1/v$ tail.

In real reactor spectra the epithermal flux more closely follows$1/E^{1+\alpha}$. De Corte and co-workers therefore generalised the Westcott expressions by replacing $s_0$with an α-dependent quantity $s_0(\alpha)$ and adjusting the definition of

the spectral index (De Corte et al., 1994). The modified reaction-rate formula becomes

$$R = n\, v_0\, \sigma_0 \left[g(T_n) + r(\alpha) \sqrt{\frac{T_n}{T_0}}\, s_0(\alpha) \right] \qquad (12)$$

This modified Westcott formalism is the version employed in modern k_0-NAA when non-$1/v$ nuclides are to be determined.

Relationship to the Høgdahl Convention

For pure $1/v$ nuclides ($g = 1$) the Westcott expression can be shown to be mathematically equivalent to the Høgdahl convention under appropriate definitions of the thermal and epithermal fluxes (De Corte et al., 1993). The Høgdahl thermal flux ϕ_{th} and epithermal flux per unit lethargy ϕ_{e} are related to the Westcott parameters by

$$\phi_{\text{th}} = n\, v_0 \left(1 + \frac{r}{\sqrt{\pi}} \frac{2}{\sqrt{T_n/T_0}} \cdots \right),\ f = \frac{\phi_{\text{th}}}{\phi_{\text{e}}} \propto \frac{1}{r} \qquad (13)$$

Therefore, the Høgdahl convention is recovered as a limiting case of the Westcott formalism when $g(T_n) = 1$. The advantage of retaining the Westcott framework is that the same set of spectrum parameters can be used consistently for both $1/v$ and non-$1/v$ nuclides.

Determination of the Westcott Spectrum Parameters

Application of the formalism requires experimental knowledge of three quantities at each irradiation site:

i. the epithermal shape factor α,
ii. the spectral index $r(\alpha)$ (or equivalently the flux ratio f),
iii. The neutron temperature T_n.

It has been reported that the first two parameters are commonly obtained by the cadmium-ratio multi-monitor method using a set of both $1/v$ and non-$1/v$ monitors (De Corte et al., 1981). The neutron temperature is determined either by the use of temperature-sensitive monitors (e.g., ^{176}Lu/ ^{197}Au) or by direct measurement of the moderator temperature combined with calculated thermalisation corrections. Once these parameters are known, the effective reaction rate for any nuclide can be evaluated from its nuclear data (σ_0, $g(T_n)$, $s_0(\alpha)$) and inserted into the concentration equation of the $k_0$method.

Practical Implications for k_0-NAA

Within the k_0-standardization framework, the Westcott reaction-rate expression is substituted into the general concentration formula, yielding

$$\rho_x = \rho_c \frac{N_{p,x}/t_{m,x}}{N_{p,c}/t_{m,c}} \frac{1}{k_{0,c}(x)} \frac{\varepsilon_{p,c}}{\varepsilon_{p,x}} \frac{f + Q_{0,c}(\alpha)}{f + Q_{0,x}(\alpha)} \frac{G_{\text{th},c}}{G_{\text{th},x}} \frac{G_{\text{e},c}}{G_{\text{e},x}} \qquad (14)$$

With appropriate Westcott replacements for the factors $f + Q_0(\alpha)$ when non-$1/v$ nuclides are involved. The necessity of measuring an additional parameter (T_n) and of possessing reliable $g(T_n)$ data constitutes the principal practical overhead of the Westcott approach relative to the simpler Høgdahl convention. Nevertheless, for a reactor such as LEU-NIRR-1, whose spectrum is stable and well characterised, this overhead is modest and is more than offset by the ability to analyse the full suite of analytically important elements.

The k_0-Standardization Method

The k_0-standardization method is a single-comparator approach to neutron activation analysis that combines the experimental simplicity of the relative method with the multi-element capability of the absolute method. Introduced by Simonits, De Corte and Hoste in 1975, it has become the preferred standardization technique for instrumental neutron activation analysis (INAA) at many research reactors, including Miniature Neutron Source Reactors such as NIRR-1 (Simonits et al., 1975; De Corte, 1987; Jonah et al., 2005; Joseph et al., 2017). The method relies on a set of composite nuclear constants, the k_0 factors which are independent of the irradiation facility and the detection system once a small number of reactor spectrum parameters and detector efficiency ratios have been determined.

Fundamental Principle

In the classical relative method, a standard containing a known mass of the analyte element must be co-irradiated with every sample. In the absolute method all nuclear data and absolute flux values must be known with high accuracy. The k_0 method occupies an intermediate position: a single comparator element (almost always gold) of known mass is co-irradiated with the sample, and the concentration of any other element is calculated from the ratio of the measured peak areas, the detector efficiency ratio, and a pre-determined k_0 factor that encapsulates the relevant nuclear data of both the analyte and the comparator.

The k_0 factor of an analyte nuclide $x$relative to the gold comparator is defined as

$$k_{0,\text{Au}}(x) = \frac{M_{\text{Au}}\, \theta_x\, \sigma_{0,x}\, I_{\gamma,x}}{M_x\, \theta_{\text{Au}}\, \sigma_{0,\text{Au}}\, I_{\gamma,\text{Au}}} \qquad (15)$$

Where M is the molar mass, θ the isotopic abundance, σ_0 the 2200 m s^{-1} capture cross-section, and $I_\gamma$the absolute gamma-ray emission probability. Because the k_0 factors are ratios, many systematic uncertainties that affect absolute nuclear data cancel, yielding constants that can be determined experimentally to high precision and are universally applicable (De Corte & Simonits, 2003).

Concentration Equation in the Høgdahl Convention

For nuclides that obey the $1/v$ law in the thermal region the reaction rate is expressed in the Høgdahl convention. The concentration ρ_x (mass fraction) of the analyte is then given by

$$\rho_x = \rho_c \frac{(N_p/t_m)_x}{(N_p/t_m)_c} \frac{1}{k_{0,c}(x)} \frac{\varepsilon_{p,c}}{\varepsilon_{p,x}} \frac{f + Q_{0,c}(\alpha)}{f + Q_{0,x}(\alpha)} \frac{G_{\text{th},c}\, G_{\text{e},c}}{G_{\text{th},x}\, G_{\text{e},x}} \qquad (16)$$

where, subscripts x and c denote analyte and comparator, N_p is the net peak area, t_m is the live counting time, ε_p is the full-energy peak detection efficiency, $f = \phi_{\text{th}}/\phi_{\text{e}}$ is the thermal-to-epithermal flux ratio, $Q_0(\alpha) = I_0(\alpha)/\sigma_0$ is the resonance integral to thermal cross-section ratio corrected for the epithermal shape factor α, while G_{th} and G_{e} are the thermal and epithermal neutron self-shielding factors.

The only reactor-dependent quantities that must be measured at each irradiation site are therefore f and α. Once these parameters and the relative detection efficiencies are known, any number of elements can be determined from a single co-irradiated gold monitor.

Extension to the Westcott Formalism

When non-$1/v$ nuclides are analysed, the Høgdahl factors $f + Q_0(\alpha)$ are replaced by the corresponding Westcott expressions involving the $g(T_n)$ factor and the modified spectral index as earlier emphasized. The concentration equation retains the same formal structure, but the reaction-rate ratio is now evaluated with the full Westcott formula:

$$\frac{R_x}{R_c} = \frac{\sigma_{0,x}\left[g_x(T_n) + r(\alpha)\sqrt{T_n/T_0}\, s_{0,x}(\alpha)\right]}{\sigma_{0,c}\left[g_c(T_n) + r(\alpha)\sqrt{T_n/T_0}\, s_{0,c}(\alpha)\right]} \qquad (17)$$

This extension allows the k_0 method to cover essentially the entire set of analytically useful elements while preserving the single-comparator advantage (De Corte et al., 1994).

Experimental Determination of k_0 Factors and Spectrum Parameters

Recommended k_0 factors are obtained from carefully designed activation experiments in well-characterised reactor positions and are compiled in internationally accepted libraries (De Corte & Simonits, 2003). The spectrum parameters f and α are most accurately determined by the cadmium-ratio multi-monitor method, in which a set of monitors spanning a wide range of Q_0 values are irradiated with and without cadmium covers (De Corte et al., 1981). For reactors with highly stable flux, such as NIRR-1, these parameters need to be re-measured only after significant core modifications (e.g., the HEU-to-LEU conversion).

Advantages and Practical Considerations

The principal advantages of the k_0 method are:

i. elimination of multi-element standards and the associated matrix-matching problems,
ii. high sample throughput,
iii. straightforward implementation of quality-assurance protocols based on certified reference materials,
iv. Direct traceability to a single, well-characterised comparator.

Practical requirements include accurate relative efficiency calibration of the gamma-ray spectrometer (usually performed with standard sources or by the efficiency-transfer method), correction for true-coincidence summing, and evaluation of neutron self-shielding when samples are not infinitely dilute. For the LEU-NIRR-1 these procedures have been standardised for both inner and outer irradiation channels, enabling routine multi-element analysis with typical expanded uncertainties of 3–5 % for most elements (Jonah et al., 2005; Anas et al., 2023).

Reactor Physics of LEU-NIRR-1

The Nigeria Research Reactor-1 (NIRR-1) is a tank-in-pool type Miniature Neutron Source Reactor (MNSR) designed and supplied by the China Institute of Atomic Energy (Plate 1). It is operated by the Centre for Energy Research and Training (CERT) at Ahmadu Bello University, Zaria, and is dedicated primarily to neutron activation analysis and limited radioisotope production. Understanding its reactor physics, particularly after the conversion from highly enriched uranium (HEU) to low-enriched uranium (LEU), is essential for the accurate re-implementation of the k_0-standardization method and the Westcott formalism.

Core Design and Conversion History

NIRR-1 reached first criticality in 2004 with a core fuelled by UAl_4 alloy enriched to approximately 90.2 % ^{235}U (HEU). Under the Reduced Enrichment for Research and Test Reactors (RERTR) programme and an IAEA Coordinated Research Project on MNSR conversion, the core was replaced in 2018 with LEU fuel consisting of $U0_2$ pellets enriched to 12.5–13.0 % ^{235}U (International Atomic Energy Agency, 2018; Jonah et al., 2014).

The LEU core retains the same overall geometry as the original HEU core: a cylindrical array of fuel pins surrounded by a beryllium reflector annulus, with a central control rod and additional beryllium shim plates for reactivity management. The nominal thermal power remains 30 kW, corresponding to a thermal neutron flux of approximately 1×10^{12} n cm^{-2} s^{-1} in the inner irradiation channels. Because MNSRs operate with lifetime cores and very low burn-up (<1 %), fuel management is essentially non-existent; the principal change introduced by the conversion is a modest alteration of the neutron spectrum caused by the increased ^{238}U content and the different fuel-matrix density (Anas et al., 2023; Jonah et al., 2009a).

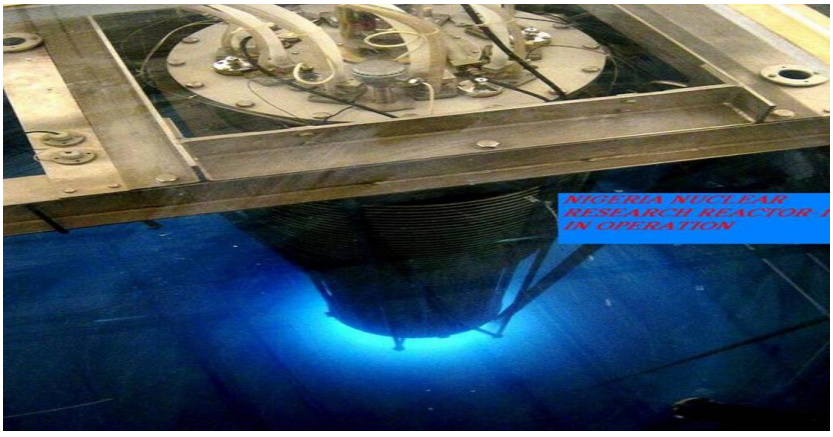

Plate 1: Nigeria Research Reactor-1 (NIRR-1) in operation

Neutron Flux Distribution and Irradiation Channels

NIRR-1 is substantially equipped with ten irradiation channels: five inner channels located inside the beryllium annulus (high thermal flux, well-thermalised spectrum) and five outer channels located outside the annulus (lower flux, harder spectrum). For routine k_0-INAA the most frequently used positions are the inner channels (B1–B3) and selected outer channels (A2, B4).

Typical thermal neutron flux values are:

i. Inner channels: $\phi_{th} \approx (5–10) \times 10^{11}$ n cm^{-2} s^{-1}at 15–30 kW,
ii. Outer channels: $\phi_{th} \approx (1–2) \times 10^{11}$ n cm^{-2} s^{-1}.

The flux is highly stable because of the large negative temperature coefficient of reactivity and the absence of significant xenon poisoning at the low power level. This stability is a key advantage for the k_0 method, which assumes constant spectrum parameters during irradiation (Jonah et al., 2005).

Neutron Spectrum Parameters

The two principal spectrum parameters required by the Høgdahl and Westcott formalisms are the thermal-to-epithermal flux ratio f and the epithermal shape factor α were measured both before and after conversion using the cadmium-ratio multi-monitor method.

HEU core (pre-2018)

i. Inner channels: $f \approx 18.4$–19.8, $\alpha \approx -0.052$ to -0.046
ii. Outer channels: $f \approx 49.5$, $\alpha \approx +0.024$ (Jonah et al., 2005; Sadiq et al., 2010).

LEU core (post-2018)

i. Inner channel B3: $f = 19.67 \pm 0.30$, $\alpha = -0.054 \pm 0.003$
ii. Outer channel B4: $f = 48.6 \pm 0.4$, $\alpha = +0.029 \pm 0.004$ (Anas et al., 2023).

Monte-Carlo calculations (MCNP) performed prior to conversion predicted a slight spectral hardening in the LEU core, consistent with the experimental observation that α becomes marginally more negative in the inner channels and the flux ratio f changes by only a few percent (Jonah et al., 2009b). These small shifts lie within typical experimental uncertainties and do not compromise the suitability of the reactor for k_0-INAA.

Criticality and Reactivity Control

The LEU core was designed to achieve a cold clean excess reactivity comparable to that of the HEU core (approximately 3.5–4.0 mk). Reactivity control is accomplished by a single central cadmium control rod and by the addition or removal of beryllium reflector shim plates. The large negative moderator-temperature coefficient ensures inherent safety; a loss-of-coolant accident or reactivity insertion transient is terminated by the temperature rise of the light-water moderator without the need for active safety systems. Safety analyses confirmed that the conversion introduced no new accident categories and did not increase the consequences of previously analysed design-basis accidents (Jonah et al., 2014).

Implications for Neutron Activation Analysis

From the standpoint of reactor physics, the LEU-NIRR-1 retains the essential characteristics that make MNSRs ideal for k_0-standardised INAA, particularly for the:

i. high thermal-to-epithermal flux ratios in the inner channels,
ii. excellent flux stability,
iii. well-thermalised spectra that minimise corrections for non-$1/v$ nuclides,
iv. Negligible burn-up-induced spectral drift over the core lifetime.

The modest spectral changes accompanying conversion require only a one-time re-determination of f, α and, if the full Westcott formalism is employed, the neutron temperature T_n. Once these parameters are established, the same k_0 library and efficiency calibration procedures used with the HEU core remain valid, ensuring continuity of analytical quality (Anas et al., 2023).

Characterization of the Reactor Neutron Spectrum and Parameters

Accurate implementation of both the Høgdahl convention and the Westcott formalism in k_0-standardised INAA requires precise knowledge of the neutron spectrum parameters at each irradiation site. Like we have mentioned, for the LEU-NIRR-1, these parameters are the thermal-to-epithermal flux ratio f, the epithermal shape factor α, and, when non-$1/v$ nuclides are analysed, the neutron temperature T_n. Characterization of the spectrum therefore constitutes a critical experimental and computational step that links the nuclear-physics foundations to the practical utilization of the reactor.

Spectrum Parameters and Their Physical Meaning

The thermal-to-epithermal flux ratio is defined as

$$f = \frac{\phi_{\text{th}}}{\phi_{\text{e}}} \qquad (18)$$

Where $\phi_{\text{th}}$the conventional thermal is flux and ϕ_{e} is the epithermal flux per unit lethargy. High values of f (typically 15–50 in MNSRs) indicate a well-thermalised spectrum and reduce the relative contribution of resonance activation.

The epithermal shape factor αquantifies the departure of the actual epithermal spectrum from the ideal $1/E$ form:

$$\phi_{\text{e}}(E) \propto \frac{1}{E^{1+\alpha}} \qquad (19)$$

A negative α corresponds to a spectrum that is softer than $1/E$ (more thermalised), while a positive αindicates a harder spectrum. Both f and α enter the effective resonance integral $I_0(\alpha)$ and the $Q_0(\alpha)$ factor that appear in the k_0 concentration equation.

When the full Westcott formalism is employed, the neutron temperature T_n must also be known in order to evaluate the $g(T_n)$ factors of non-$1/v$ nuclides effectively.

Experimental Determination: The Cadmium-Ratio Multi-Monitor Method

The most widely recommended experimental technique for reactors with stable flux is the cadmium-ratio multi-monitor method (De Corte et al., 1981; De Corte, 1987). A set of flux monitors spanning a wide range of Q_0 values (commonly Au, Zr, Mo, Co, Zn, Fe, etc.) is irradiated both with and without a cadmium cover (usually 1 mm thick). The cadmium ratio of each monitor i is

$$R_{\text{Cd},i} = \frac{A_{\text{bare},i}}{A_{\text{Cd},i}} \qquad (20)$$

Where Adenotes the specific activity corrected for decay and counting geometry. From the set of cadmium ratios, the parameters f and α are extracted by iterative solution of the defining equations

$$\log \frac{\bar{E}_{r,i}^{-\alpha}}{(R_{\text{Cd},i}-1)Q_{0,i}(\alpha)} = -\alpha \log \bar{E}_{r,i} + \text{constant} \qquad (21)$$

Or equivalent linearised forms. The slope of the resulting plot yieldsα, after which f is obtained from any monitor of well-known Q_0.

For the determination of neutron temperature, monitors with strongly temperature-dependent $g(T_n)$ factors (especially ^{176}Lu) are co-irradiated with a $1/v$ monitor such as ^{197}Au; the activity ratio then providesT_n.

Computational Approaches

Complementary to experiment, detailed Monte-Carlo simulations (MCNP or equivalent) of the reactor core are used to compute the multi-group neutron flux spectrum in the irradiation channels. Reaction rates for the monitor set are then calculated by folding the spectrum with evaluated cross-section libraries (ENDF/B or JEFF). Cadmium ratios and the parameters f and α are derived from the calculated reaction rates and compared with experimental values for validation (Jonah et al., 2009). Such calculations were particularly valuable prior to the HEU-to-LEU conversion, allowing prediction of the expected spectral shift.

Spectrum Parameters of LEU-NIRR-1

Extensive measurements had been performed on both the HEU and LEU cores of NIRR-1, and have established the following representative values (Jonah et al., 2005; Sadiq et al., 2010; Anas et al., 2023) are presented in Table 1:

Table 1: NIRR-1 Neutron Flux Parameters

Channel type	Core	f	α
Inner (e.g. B3)	HEU	18.4 – 19.8	–0.052 to –0.046
Inner (B3)	LEU	19.67 ± 0.30	–0.054 ± 0.003
Outer (e.g. B4)	HEU	≈ 49.5	+0.024 ± 0.002
Outer (B4)	LEU	48.6 ± 0.4	+0.029 ± 0.004

The conversion produced only a slight spectral hardening, visible as a small change in $\alpha$and a nearly unchanged$f$. These results confirm that the LEU core remains highly suitable for k_0-INAA. Neutron-temperature measurements, where performed, typically yield values close to the moderator temperature (approximately 300–320 K under operating conditions), consistent with the strong thermalisation provided by the light-water moderator and beryllium reflector.

Uncertainty and Quality Assurance

The dominant contributions to the uncertainty of f and α arise from counting statistics, monitor nuclear data, cadmium cut-off energy, and self-shielding corrections. With careful experimental design, expanded uncertainties of 2–5 % for f and 0.005–0.010 for α are routinely achieved. Periodic re-measurement after any significant core modification (or at multi-year intervals) forms an essential part of the quality-assurance programme for k_0-based analysis at NIRR-1.

Detector Physics

In instrumental neutron activation analysis, the induced radioactivity is quantified by high-resolution gamma-ray spectrometry. The accuracy of the k_0-standardization method therefore depends critically on a thorough understanding of the physical processes that govern the interaction of gamma rays with the detector, the formation of the full-energy peak, and the conversion of peak areas into absolute or relative disintegration rates.

High-Purity Germanium Detectors

Modern INAA laboratories almost exclusively use high-purity germanium (HPGe) semiconductor detectors (Knoll, 2010; Gilmore, 2008). When a gamma ray interacts in the active germanium volume it produces electron–hole pairs; the number of pairs is proportional to the energy deposited. An applied electric field sweeps the charge carriers to the electrodes, generating a current pulse whose amplitude is proportional to the deposited energy. After amplification and pulse processing, the events are sorted into a pulse-height spectrum in which the full-energy peaks correspond to complete absorption of the gamma-ray energy (Knoll, 2010; Gilmore, 2008).

The principal performance characteristics of an HPGe detector are:

i. energy resolution (typically 1.8–2.2 keV FWHM at 1332 keV for a coaxial detector),
ii. full-energy peak efficiency,
iii. peak-to-Compton ratio,
iv. Timing properties (important for coincidence-summing corrections).

At the LEU-NIRR-1 the post-conversion spectrometry system centres on a GEM30-7 p-type coaxial HPGe detector coupled to an ORTEC DSPEC-50A digital spectrometer, providing the resolution and stability required for multi-element $k_0$analysis (Anas et al., 2023).

Full-Energy Peak Efficiency

The full-energy peak efficiency $\varepsilon_p(E)$is defined as the probability that a gamma ray of energy Eemitted from a source in a given geometry will deposit its entire energy in the detector and be recorded in the corresponding full-energy peak. It is a function of gamma-ray energy, source–detector distance, source geometry and composition, and detector dimensions (Joseph et al., 2013, 2015).

In the k_0 concentration equation, the absolute efficiencies cancel when the analyte and comparator are counted in identical geometries; only the *ratio* of efficiencies appears:

$$\frac{\varepsilon_{p,c}}{\varepsilon_{p,x}} \quad (22)$$

Nevertheless, an accurate relative efficiency curve spanning the energy range of interest (typically 50–2000 keV) is indispensable. Efficiency calibration is performed with standardised multi-nuclide sources or by a combination of experimental points and Monte-Carlo efficiency-transfer calculations. For the non-point geometries commonly used in NAA (disc-shaped samples, vials), efficiency transfer from a point-source calibration is routinely applied.

True-Coincidence Summing

When a radionuclide emits two or more cascading gamma rays within the resolving time of the detector, true-coincidence summing occurs. A coincidence event removes counts from the individual full-energy peaks and may create sum peaks. The magnitude of the effect increases with detection efficiency and is therefore more pronounced at close source-to-detector distances, the geometries that is often preferred for low-activity NAA samples.

The coincidence-summing correction factor COI for a given peak is defined such that the true peak area is recovered by

$$N_p^{\text{true}} = N_p^{\text{observed}} \times COI \quad (23)$$

For the k_0 method the correction must be applied consistently to both the analyte and the comparator peaks. Modern digital spectrometers and dedicated software packages calculate COI factors from the known decay scheme, the total and peak efficiencies, and the source geometry.

Additional Measurement Corrections

Several other physical effects must be taken into account:

i. Pulse pile-up and dead-time losses – corrected by live-time circuitry or by the use of a pulse-generator peak (Knoll, 2010; Gilmore, 2008).
ii. Gamma-ray attenuation within the sample (self-absorption) – evaluated from the sample mass, composition and the mass-attenuation coefficients (Knoll, 2010; Gilmore, 2008).
iii. Geometry and positioning reproducibility – minimised by rigid sample holders and, when necessary, by rotational averaging (Gilmore, 2008).
iv. Background subtraction – performed by spectrum analysis software that accounts for both continuum and discrete background peaks (Knoll, 2010; Gilmore, 2008).

Practical Implementation at LEU-NIRR-1

Following the HEU-to-LEU conversion, a new gamma-ray spectrometry station was installed. Efficiency curves were established for the principal counting geometries (typically 5 cm and 10 cm source-to-detector distances) over the energy range 59–1840 keV using certified standard sources (Anas et al., 2023). The digital signal-processing chain provides excellent energy stability and throughput, allowing reliable analysis of both short-lived and long-lived activation products. Quality-control protocols include regular checks of energy calibration, resolution, and efficiency with a reference source (usually ^{152}Eu or ^{133}Ba) (Anas et al., 2023).

Because the k_0 method relies on efficiency *ratios* rather than absolute efficiencies, the dominant remaining uncertainties associated with the detection system are those arising from true-coincidence summing, sample self-attenuation, and small differences in counting geometry between samples and the gold comparator. With careful experimental design these contributions can be kept below 1–2 %, consistent with the overall expanded uncertainty target of 3–5 % for most elemental concentrations.

Application of the Westcott Formalism in LEU-NIRR-1

The modified Westcott formalism becomes necessary at the LEU-NIRR-1 whenever non-$1/v$ nuclides are included in the analytical protocol. Although the majority of elements determined by routine k_0-INAA obey the $1/v$ law sufficiently well for the simpler Høgdahl convention to be adequate, a number of analytically important radionuclides, particularly certain rare-earth elements, iridium and a few others that exhibit significant departures from $1/v$ behaviour in the thermal region. For these nuclides the Westcott $g(T_n)$ factor and the associated spectral index must be incorporated into the concentration calculation. We have described briefly how the formalism is applied under the specific neutron-spectrum and detection conditions of the LEU-NIRR-1.

Identification of Non-$1/v$ Nuclides Relevant to NIRR-1

The most frequently encountered non-$1/v$ target nuclides in NIRR-1 sample matrices (geological, environmental, biological and industrial materials) include:

i. ^{151}Eu $(n,\gamma)^{152}$Eu,
ii. ^{153}Eu $(n,\gamma)^{154}$Eu,
iii. ^{176}Lu $(n,\gamma)^{177}$Lu,
iv. ^{191}Ir $(n,\gamma)^{192}$Ir,
v. And, to a lesser extent, selected isotopes of Yb, Hf and Ta.

It should be noted that for these nuclides, the Westcott $g(T_n)$ factors at the neutron temperatures prevailing in the LEU-NIRR-1 irradiation channels (approximately 300–320 K) differ from unity by several percent to more than 10 % in the most pronounced cases. Neglect of the g-factor would therefore introduce systematic bias comparable to or larger than the target expanded uncertainty of the method.

Implementation of the Modified Westcott Concentration Equation

It is important to note that within the k_0 framework, the concentration of a non-$1/v$ analyte x relative to the gold comparator is evaluated by replacing the Høgdahl reaction-rate ratio with its Westcott counterpart:

$$\rho_x = \rho_c \frac{(N_p/t_m)_x}{(N_p/t_m)_c} \frac{1}{k_{o,c}(x)} \frac{\varepsilon_{p,c}}{\varepsilon_{p,x}} \frac{[g_x(T_n)+r(\alpha)\sqrt{T_n/T_0}\,s_{0,x}(\alpha)]}{[g_c(T_n)+r(\alpha)\sqrt{T_n/T_0}\,s_{0,c}(\alpha)]} \frac{G_{th,c}\,G_{e,c}}{G_{th,x}\,G_{e,x}} \quad (24)$$

We must emphasize that Gold itself is only mildly non-$1/v$; its $g(T_n)$ factor is close to unity and is routinely included for consistency. The spectrum parameters f (or the equivalent spectral index $r(\alpha)$) and $\alpha$are taken from the values established for the relevant irradiation channel of the LEU core. The neutron temperature T_n is either measured with a Lu–Au monitor pair or approximated by the measured moderator temperature with a small calculated thermalisation correction.

Experimental Protocol at LEU-NIRR-1

Application of the Westcott formalism at NIRR-1 follows a standardised sequence:

i. Irradiation – Samples and a gold comparator (typically a dilute Al–Au alloy wire or foil) are co-irradiated in a characterised inner or outer channel. For non-$1/v$ work an additional Lu monitor is often included to confirm T_n (Alhassan et al., 2024).
ii. Spectrum-parameter assignment – The channel-specific values of f, $\alpha$and $T_n$determined after the LEU conversion (Anas et al., 2023) are inserted into the calculation software.
iii. Gamma-ray spectrometry – Counting is performed on the GEM30-7 HPGe system at calibrated geometries. True-coincidence summing corrections are applied to all relevant peaks, including those of the non-$1/v$ radionuclides.
iv. Data reduction – Peak areas are converted to concentrations using a $k_0$software package that implements the full modified Westcott expression and the recommended nuclear data (including evaluated $g(T_n)$ and $s_0(\alpha)$ factors).

Validation and Performance

Validation is performed normally by repeated analysis of certified reference materials that contain measurable concentrations of Eu, Lu, Ir and other non-$1/v$ elements. Results obtained with the Westcott-corrected k_0 protocol typically agree with certified values within the combined expanded uncertainties (normally 4–7 % for these elements). Comparison with pure Høgdahl calculations on the same spectra demonstrates that the Westcott correction removes previously observed systematic biases of several percent for the strongest non-$1/v$ cases.

Because the neutron spectrum of the LEU-NIRR-1 is stable and the parameters$f$, α and T_n change only slowly (if at all) with time, the Westcott corrections remain valid for extended periods. Re-characterisation is required only after major core interventions or significant changes in operating temperature.

Practical Considerations and Limitations

The additional experimental burden imposed by the Westcott formalism at NIRR-1 is modest: one extra monitor (Lu) when T_n must be confirmed, and the use of software that correctly implements the modified reaction-rate expression. The principal remaining limitations are:

i. uncertainty in the evaluated $g(T_n)$ factors themselves,
ii. possible small gradients of neutron temperature across the irradiation channels,
iii. The need for accurate true-coincidence summing corrections for the often-complex decay schemes of non-$1/v$ activation products.

These contributions are incorporated into the overall uncertainty budget and do not preclude the reliable determination of non-$1/v$ elements at the concentration levels normally encountered in NIRR-1 applications.

Advantages and Limitations

The combination of the k_0-standardization method with the modified Westcott formalism at the LEU-NIRR-1 offers a powerful and flexible approach to multi-element instrumental neutron activation analysis. At the same time, the method is subject to a number of inherent and practical limitations that must be understood and managed if high-quality results are to be obtained. We have summarised the principal advantages and limitations in the specific context of the LEU-NIRR-1 facility.

Advantages

Single-comparator simplicity and multi-element capability: Only one well-characterised comparator (normally gold) needs to be co-irradiated with each batch of samples. Once the reactor spectrum parameters and the relative detection-efficiency curve are known, the concentrations of dozens of elements can be calculated from a single gamma-ray spectrum. This eliminates the preparation, irradiation and counting of multi-element standards and removes the associated matrix-matching difficulties (Simonits et al., 1975; De Corte, 1987).

Extension to non-$1/v$ nuclides: Incorporation of the modified Westcott formalism allows accurate determination of elements whose thermal capture cross-sections deviate from the $1/v$ law (e.g., Eu, Lu, Ir). The same experimental infrastructure and software platform can therefore be used for both $1/v$ and non-$1/v$ analytes, extending the analytical scope of the reactor without requiring separate relative calibrations (De Corte et al., 1994).

Suitability of the LEU-NIRR-1 spectrum: The neutron spectrum of the LEU core remains highly thermalised and stable. Measured values of f and α changed only marginally after the HEU-to-LEU conversion and lie well within the range for which the k_0 method was originally optimised (Anas et al., 2023; Jonah et al., 2005). Flux stability over a typical irradiation period is excellent, minimising one of the classical sources of uncertainty in activation analysis.

Metrological traceability and quality assurance: Results are directly traceable to the $k_0$nuclear-data library and to a single comparator whose mass and composition are known with high accuracy. Regular analysis of certified reference materials provides a straightforward and internationally recognised quality-control pathway.

Operational efficiency: High sample throughput, modest turnaround times and relatively low operational cost make the method attractive for both research and service analysis. The lifetime-core design of the MNSR further reduces the frequency of spectrum re-characterisation.

Limitations

Dependence on accurate spectrum parameters: The method requires reliable values of f, α and, for Westcott applications, T_n. Although these parameters are stable at NIRR-1, any unrecognised change (for example after an unplanned core intervention or a significant alteration in operating temperature) propagates directly into the calculated concentrations. Periodic experimental verification remains mandatory.

Nuclear-data uncertainties: The accuracy ultimately achievable is limited by the uncertainties in the recommended k_0 factors, $Q_0(\alpha)$ values and, especially, the Westcott $g(T_n)$ factors of non-$1/v$ nuclides. For the strongest non-$1/v$ cases these nuclear-data uncertainties can dominate the overall uncertainty budget.

Corrections for detection and sample effects: True-coincidence summing, gamma-ray self-attenuation and neutron self-shielding must be evaluated carefully. These corrections become more important for close counting geometries, large or dense samples, and radionuclides with complex decay schemes. Inadequate treatment introduces systematic bias that is not removed by the k_0 formalism itself.

Software and expertise requirements correct implementation of the full Westcott expression, coincidence-summing corrections and efficiency transfer demands specialised software and trained personnel. Laboratories that lack these resources may be restricted to the simpler Høgdahl approximation and therefore to a reduced set of elements.

Sample-type restrictions: The method assumes that the sample is homogeneous on the scale of the neutron mean free path and that self-shielding factors can be calculated or are negligible. Highly absorbing matrices (e.g., samples rich in Cd, B or rare-earth elements) or very large samples may require additional experimental corrections or may be better analysed by the classical relative method.

Limited applicability to very short-lived or pure beta emitters: The technique is optimised for gamma-emitting activation products with half-lives ranging from minutes to years. Extremely short-lived species or pure beta emitters fall outside the normal scope of instrumental k_0-NAA.

Balance of Advantages and Limitations at LEU-NIRR-1

For the great majority of sample types and elements routinely analysed at the LEU-NIRR-1, the advantages substantially outweigh the limitations. The reactor's stable, well-characterised spectrum, the availability of a modern HPGe spectrometry system, and the existence of established spectrum parameters allow the combined k_0–Westcott protocol to deliver accurate multi-element results with expanded uncertainties typically in the range 3–7 %. The limitations are manageable through careful experimental design, regular quality-control measurements and the judicious selection of irradiation and counting conditions. Consequently, the method remains the technique of choice for high-throughput, high-quality instrumental neutron activation analysis at the facility.

Current Research Trends

Research on the Westcott formalism and the k_0-standardization method continues to evolve, driven by the dual requirements of higher metrological accuracy and broader applicability. In the specific context of low-power research reactors such as the LEU-NIRR-1, current efforts focus on consolidating the gains made after HEU-to-LEU conversion, refining nuclear data, improving computational support, and extending the method to new analytical domains. We have outlined some of the principal research trends visible in the mid-2020s.

Post-Conversion Spectrum Re-characterisation and Validation

A major theme has been the systematic experimental and computational re-characterisation of MNSR cores after conversion to LEU fuel. Studies at NIRR-1 and sister facilities (e.g., GHARR-1) have quantified the modest spectral hardening that accompanies the increase in ^{238}U content and have confirmed that the resulting changes in f and α remain compatible with high-quality k_0-INAA (Anas et al., 2023; Jonah et al., 2009). Ongoing work emphasises long-term monitoring of spectrum stability, the influence of beryllium reflector poisoning, and the development of rapid re-characterisation protocols that can be applied after any future core intervention.

Refinement of Westcott $g(T_n)$ Factors and Nuclear Data

The accuracy of non-$1/v$ determinations are still limited by uncertainties in evaluated $g(T_n)$ factors and related resonance parameters. Current research therefore concentrates on:

i. re-evaluation of thermal-neutron capture cross-section shapes using new time-of-flight and pile-oscillation data,
ii. generation of temperature-dependent g-factors from modern evaluated libraries (ENDF/B-VIII, JEFF-3.3 and successors),
iii. Inter-comparison exercises organised under IAEA auspices to recommend consensus values for the most important non-$1/v$ nuclides (Eu, Lu, Ir, Yb, etc.).

These efforts directly benefit laboratories that apply the full Westcott formalism at reactors such as LEU-NIRR-1.

Advanced Computational Modelling and Efficiency Transfer

Monte-Carlo radiation-transport codes (MCNP, PHITS, OpenMC) are increasingly used both to predict spectrum parameters prior to experimental campaigns and to perform efficiency-transfer calculations for non-standard sample geometries. Hybrid approaches that combine a detailed reactor-core model with a validated detector model allow a priori estimation of self-shielding and coincidence-summing corrections, reducing the experimental burden. Research is also directed toward uncertainty propagation through these computational chains, bringing k_0 results closer to full metrological traceability.

Software Development and Automation

User-friendly software packages that implement the complete modified Westcott $k_0$formalism, automatic coincidence-summing corrections, and modern spectrum-analysis algorithms continue to be refined. Parallel developments include automated sample changers, digital signal-processing optimisations for high-count-rate measurements, and laboratory information-management systems that embed k_0 calculations within a quality-assured workflow. These tools lower the expertise barrier and improve reproducibility across different MNSR laboratories.

Expansion of Application Domains

While classical geological and environmental analysis remain core applications, recent work has extended k_0-Westcott protocols to:

i. forensic and authentication studies,
ii. characterisation of advanced materials and nuclear forensics samples,
iii. determination of rare-earth elements in secondary resources (e.g., coal ash, electronic waste),
iv. Nutritional and biomedical studies requiring high accuracy for selected non-$1/v$ elements.

The stability and well-characterised spectrum of the LEU-NIRR-1 make it particularly suitable for these emerging applications.

Alternative and Simplified Formalisms

A parallel research line explores whether the full Westcott treatment can be replaced, for many practical purposes, by simpler empirical or semi-empirical corrections. Proposals include effective g-factor approximations tied to a single monitor ratio or the use of "equivalent thermal cross-sections" derived from library folding. While these approaches have not displaced the classical Westcott formalism, they offer pragmatic options for laboratories that lack complete temperature characterisation.

Outlook for LEU-NIRR-1 and Similar Facilities

For the LEU-NIRR-1 the most immediate research priorities are (i) sustained experimental verification of spectrum parameters, (ii) incorporation of the latest recommended $g(T_n)$and $k_0$data into routine software, and (iii) participation in international inter-comparison programmes that benchmark Westcott-based results against independent methods. Broader trends, such as digital automation, advanced Monte-Carlo support and expanded application areas, will for sure further enhance the scientific and societal return of the facility in the coming decade.

CONCLUSION

This study has provided a comprehensive examination of the theoretical foundations of the Westcott formalism and its integration with the k_0-standardization method for instrumental neutron activation analysis, with particular reference to the Low-Enriched Uranium Nigeria Research Reactor-1 (LEU-NIRR-1). The work reviewed the fundamental nuclear physics of neutron capture and established the clear distinction between the Høgdahl convention, which is adequate for $1/v$ nuclides, and the more general Westcott formalism required for accurate treatment of non-$1/v$ nuclides. The modified Westcott expressions, incorporating the epithermal shape factor $\alpha$and the neutron temperatureT_n, were shown to integrate seamlessly into the $k_0$concentration equation, thereby extending the single-comparator method to the full range of analytically useful elements. Detailed consideration of the reactor physics of NIRR-1 demonstrated that the 2018 conversion from highly enriched uranium to low-enriched uranium fuel produced only modest spectral hardening. Experimental and computational determinations of the key spectrum parameters confirmed that values of the thermal-to-epithermal flux ratio f and the epithermal shape factor α in both inner and outer irradiation channels remain well within the range suitable for high-quality k_0-INAA. Complementary aspects of detector physics such as the full-energy peak efficiency, true-coincidence summing corrections, and related measurement factors, were shown to be manageable with modern HPGe spectrometry systems, ensuring that the nuclear and reactor parameters can be translated into reliable elemental concentrations. The combined k_0–Westcott approach offers clear advantages: operational simplicity, multi-element capability, metrological traceability, and applicability to non-$1/v$ nuclides, all of which are fully supported by the stable neutron spectrum of the LEU-NIRR-1. While limitations associated with nuclear-data uncertainties, the need for accurate spectrum characterization, and certain sample-related corrections remain, these are well understood and can be controlled through established experimental and quality-assurance protocols. Current research trends, like post-conversion validation, refinement of $g(T_n)$ factors, advanced Monte-Carlo modelling, and software development has continued to strengthen the method and expand its application domains. Collectively, the evidence presented in this study confirms that the LEU-NIRR-1 retains a neutron spectrum and operational characteristics fully compatible with accurate, high-throughput instrumental neutron activation analysis using both the Høgdahl and Westcott implementations of the k_0-standardization method. The theoretical and practical framework elaborated here provides a robust foundation for ongoing analytical work at the facility and serves as a useful reference for other Miniature Neutron

Source Reactors that have undergone or are undergoing HEU-to-LEU conversion.

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