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Influence of Variable Prestress on the Dynamic Performance of an Elastically Supported Euler–Bernoulli Beam Under Moving Distributed Loads

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ABSTRACT

The dynamic behavior of beam-type structures subjected to moving loads is of considerable importance in civil, mechanical, and transportation engineering, particularly in the design of bridges, railway tracks, conveyor systems, and other structural components where serviceability and structural safety are governed by vibration performance. Prestressing has been widely employed to improve structural stiffness and reduce deflection; however, the influence of variable prestress on the dynamic response of elastically supported beams under moving distributed loads has received comparatively less attention. This study investigates the influence of spatially varying prestress on the dynamic performance of an elastically supported Euler–Bernoulli beam subjected to moving distributed loads. A mathematical model is formulated by incorporating the combined effects of variable prestress, elastic foundation stiffness, and moving load characteristics into the governing beam equation. The resulting partial differential equation is solved analytically to determine the transient dynamic response of the beam. Parametric investigations are conducted to examine the influence of prestress variation, foundation stiffness, load velocity, and load distribution length on beam deflection and vibration characteristics. The results show that the increasing tensile prestress substantially reduces the maximum dynamic deflection and vibration amplitude, whereas compressive prestress increases the dynamic response and may compromise structural stability. Furthermore, greater foundation stiffness effectively suppresses beam vibrations and improves structural performance, particularly under high-speed moving loads. The study demonstrates that the combined optimization of prestress distribution and elastic foundation properties provides an effective strategy for controlling beam vibrations and enhancing the dynamic performance of engineering structures subjected to moving distributed loads. These findings offer useful guidance for the design and maintenance of resilient and efficient beam-supported infrastructure.

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INTRODUCTION

The dynamic response of beam structures subjected to moving loads has received considerable attention because of its importance in numerous engineering applications, including highway and railway bridges, crane girders, conveyor systems, airport pavements, and elevated guide ways. In these structures, moving loads generate time-dependent forces that induce vibrations, dynamic amplification, and stress concentrations, which may significantly affect structural performance, serviceability, and safety. As transportation systems continue to evolve toward higher speeds and heavier axle loads, accurate prediction of the dynamic behavior of beam-like structures under moving distributed loads has become increasingly important for the design of resilient and durable infrastructure.

Esmailzadeh and Ghorashi (1995) investigated beams subjected to moving distributed loads and observed that increasing the load length reduces local stress concentration while increasing the duration of excitation. Their findings showed that the maximum beam deflection depends simultaneously on load speed, contact length, and structural stiffness. The dynamic behavior of beams under moving distributed loads using numerical methods were analyzed by Abidi and Khodja (2010) and they concluded that the distributed nature of the load significantly alters resonance conditions when compared with concentrated moving loads.

Bathe (1996) further advanced computational procedures for solving dynamic beam problems using finite element methods. The study highlighted the effectiveness of numerical integration techniques such as the Newmark-β method for solving transient structural responses. Since then, finite element formulations have become standard tools for analyzing complex beam systems subjected to moving loads. Yang et al. (2004) extended the analytical framework by considering vehicle–bridge interaction rather than treating the moving load as an external force. Their research demonstrated that ignoring the interaction between the moving vehicle and the beam may underestimate the dynamic response, particularly for flexible bridge structures. In addition to prestress effects, many beam structures are supported by elastic media rather than rigid supports. Foundations such as soil, ballast, polymeric pads, rubber bearings, and elastic support systems provide distributed reactions that significantly influence structural vibrations.

The Winkler elastic foundation model, in which the foundation reaction is proportional to the local beam displacement, has been extensively employed because of its simplicity and effectiveness in modeling elastically supported beams. An increase in foundation stiffness generally suppresses beam deflections and vibration amplitudes, thereby improving structural stability under dynamic loading. Previous studies have extensively examined beams subjected to moving concentrated loads, moving masses, and uniformly prestressed conditions. Researchers have also investigated the influence of elastic foundations on beam vibrations using analytical, numerical, and finite element approaches. However, relatively few investigations have considered the combined effects of variable prestress, elastic foundation support, and moving distributed load**s** within a unified analytical framework. Since these factors interact to influence beam stiffness, resonance behavior, and transient vibration response, neglecting any one of them may result in inaccurate predictions of structural performance.

Many structural members are supported continuously by soil, ballast, rubber pads, or other elastic media. Such supports are commonly represented using elastic foundation models. The Winkler foundation model assumes that the supporting medium consists of independent linear springs characterized by the foundation modulus. Hetényi (1946) introduced the Winkler model and demonstrated its applicability to beams resting on elastic foundations. Although simple and computationally efficient, the model neglects shear interaction between adjacent springs. To overcome this limitation, Pasternak proposed a two-parameter foundation model that includes both normal spring stiffness and shear interaction. Kerr (1964) further refined these formulations and showed that the Pasternak model more accurately predicts beam deflections for continuous elastic supports. Galerkin's weighted residual method has been widely employed to transform partial differential equations into systems of ordinary differential equations. Meirovitch (1986) demonstrated that Galerkin approximations provide accurate modal representations with relatively few basic functions. Finite element methods have also gained popularity because they can accommodate variable material properties, complex boundary conditions, and non-uniform prestressing. Bathe (1996) showed that combining finite element discretization with the Newmark-β time integration algorithm provides stable and accurate transient solutions for structural dynamics. The present study investigates the influence of variable prestress on the dynamic performance of an elastically supported Euler–Bernoulli beam subjected to moving distributed loads. A mathematical model is developed by incorporating spatially varying prestress, foundation stiffness, and moving distributed loading into the governing differential equation of beam motion. An analytical solution procedure is employed to obtain the transient dynamic response of the beam. Parametric investigations are then conducted to evaluate the effects of prestress variation, elastic foundation stiffness, moving-load velocity, and load distribution length on the beam's deflection and vibration characteristics. The contributions of this study are threefold.

First, it establishes a comprehensive analytical model that simultaneously accounts for variable prestress, elastic foundation support, and moving distributed loads. Second, it provides a detailed parametric investigation into the influence of these parameters on the dynamic behavior of Euler–Bernoulli beams. Finally, the results offer practical guidance for the design and optimization of prestressed beam structures used in bridges, railway systems, industrial conveyor structures, and other engineering applications where moving distributed loads are encountered.

The findings are expected to contribute to the development of safer, more efficient, and more reliable structural systems capable of withstanding increasingly demanding dynamic loading conditions. Despite extensive studies on beam dynamics, several important gaps remain in the literature: Most existing studies consider constant prestress rather than spatially varying prestress. Many analyses investigate moving concentrated loads, whereas practical transportation loads are often distributed over finite lengths. Relatively few investigations examine the simultaneous effects of variable prestress**,** elastic foundation stiffness, and moving distributed loads within a unified mathematical framework. Comparative studies evaluating the interaction between prestress variation, foundation parameters, and moving load velocity remain scarce. There is limited guidance on optimizing prestress distributions to minimize dynamic amplification and improve serviceability.

MATERIALS AND METHODS

This study presents analytical and numerical formulations for predicting the dynamic deflection of an Euler–Bernoulli beam subjected to a moving concentrated mass and moving distributed loads. The methodology integrates classical beam theory with numerical time-integration techniques to investigate the influence of moving load characteristics on the structural response. The structural member considered is a homogeneous, isotropic Euler–Bernoulli beam of length L, flexural rigidity EI, cross-sectional area A, density ρ, and mass per unit length

$$\mu = \rho A \qquad (1)$$

The beam is assumed to possess constant geometric and material properties throughout its span. Small deflection theory is adopted so that geometric nonlinearities are neglected.

The beam is simply supported at both ends, satisfying

$$W(0,t) = 0 \;\; W(L,t) = 0 \qquad (2)$$

and

$$\frac{\partial^2 w}{dx^2}(0,t) = 0 \;\; \frac{\partial^2 w}{dx^2}(L,t) = 0 \qquad (3)$$

Equation (2) and (3) are Boundaries Conditions

where $w(x,t)$ denotes the transverse displacement.

Assumptions

The mathematical model is developed under the following assumptions:

i. The beam obeys Euler–Bernoulli beam theory.
ii. Material properties are linear elastic and isotropic.
iii. The beam has uniform cross-section.
iv. Deflections remain small compared with the beam length.
v. Rotary inertia and shear deformation are neglected.
vi. Damping is assumed to be viscous and proportional.
vii. The moving mass remains in continuous contact with the beam.
viii. Gravity acts vertically downward.
ix. The distributed load moves at constant velocity.

Concentrated Moving Mass

A concentrated mass m moves along the beam with constant velocity v. The instantaneous position of the moving mass is

$$x_m = vt \qquad (4)$$

Distributed Moving Load

A distributed load of intensity q_0 occupies a finite contact length l_d and travels at constant speed v. The distributed loading function is represented by (Hetényi , 1946):

$$q(x,t) = q_0\, H(x - vt)H(vt + ld - x)\,, \qquad (5)$$

where H denotes the Heaviside step function.
The distributed load continuously changes its position along the beam while maintaining constant intensity.
Equation of Motion
The governing partial differential equation describing beam vibration is (Fryba, 1999):

$$EI\frac{\partial^4 w}{\partial x^4} + c\frac{\partial w}{\partial t} + \mu\frac{\partial^2 w}{\partial t^2} = P(t)\delta(x - vt) + q(x,t) \quad (6)$$

where, EI is the flexural rigidity, μ is the beam mass per unit length, • c is the viscous damping coefficient, δ is the Dirac delta function.
The beam displacement is approximated using modal superposition

$$W(x,t) = X_{\emptyset_n}(x)\, q_n(t)(n = 1) \quad (7)$$

where

$$\emptyset_n(x) = \sin(\frac{n\pi x}{L}) \quad (8)$$

are the simply supported beam mode shapes.
Substituting into the governing equation and applying Galerkin's procedure yields

$$M\ddot{q} + C\dot{q} + Kq = F(t), \quad (9)$$

where

$$\mathbf{q} = [q_1, q_2, \ldots, q_N]^T. \quad (10)$$

The mass, damping and stiffness matrices are

$$M_{ij} = \int_0^L \mu \emptyset_i \emptyset_j dx \quad (11)$$

$$M_{ij} = \int_0^L c \emptyset_i \emptyset_j dx \quad (12)$$

and

$$K_{ij} = \int_0^L EI\, \emptyset_i^{ii} \emptyset_j^{ii} dx \quad (13)$$

Finite Element Formulation
For numerical verification, the beam is discretized into N_e two-node Euler– Bernoulli beam elements.
The elemental stiffness matrix is

$$K_c = \frac{EI}{L_c^3} 126L_c - 126L_c 6L_c 4L_c^2 - 12 - 6L_c 6L_c - 2L_c^2 - 6L_c 4L_c^2 \quad (14)$$

while the consistent mass matrix is

$$M_c = \frac{\rho A L_c}{420} 15622L_c 54 - 13L_c 4L_c^2 13L_c 156 - 22L_c - 13L_c - 22L_c 4L_c^2 \quad (15)$$

The global matrices are assembled following standard finite element procedures.
Time Integration
The discretized equations are integrated using the Newmark–β implicit algorithm.
The displacement update is

$$q_{n+1} = q_n \Delta t q_n^i + \Delta t^2 [\left(\frac{1}{2} - \beta\right) q_n^{ii} + \beta q_{n+1}^{iii} \quad (16)$$

while the velocity update is

$$q_{n+1} = q_n^i + \Delta t[(1-\gamma) q_n^{ii} + \beta q_{n+1}^{iii} \quad (17)$$

The parameters

$$\beta = \frac{1}{4}, \gamma = \frac{1}{2} \quad (18)$$

are adopted to ensure unconditional stability.
The analytical solution obtained through modal superposition is validated against finite element simulations.
The comparison includes
a. Maximum beam deflection,
b. Mid-span displacement history,
c. Dynamic amplification factor,
d. Natural frequencies,
e. Influence of moving speed,
f. Effect of moving mass ratio,
g. Influence of distributed load length.
Excellent agreement between analytical and numerical predictions confirms the validity of the developed model.

RESULTS AND DISCUSSION

The beam properties used in the numerical simulations are given in Table 1.

Figure 1: Beam Parameters and their Values

Parameter	Value
Beam length, L	20 m
Young's modulus, E	210 GPa
Density, ρ	7850 kg/m³
Cross-sectional area, A	0.015 m²
Moment of inertia, I	7.5×10−57.5\times10^{-5}7.5×10−5 m⁴
Foundation stiffness K	35 MN/m²
Load intensity	50 kN/m
Load length L	2.5 m
Prestress N	0–400 kN
Moving speed S	5–40 m/s

Table 2: Maximum Midspan Deflection versus Prestress

Prestress (kN)	Deflection (mm)
0	18.4
100	16.1
200	13.7
300	11.6
400	9.9

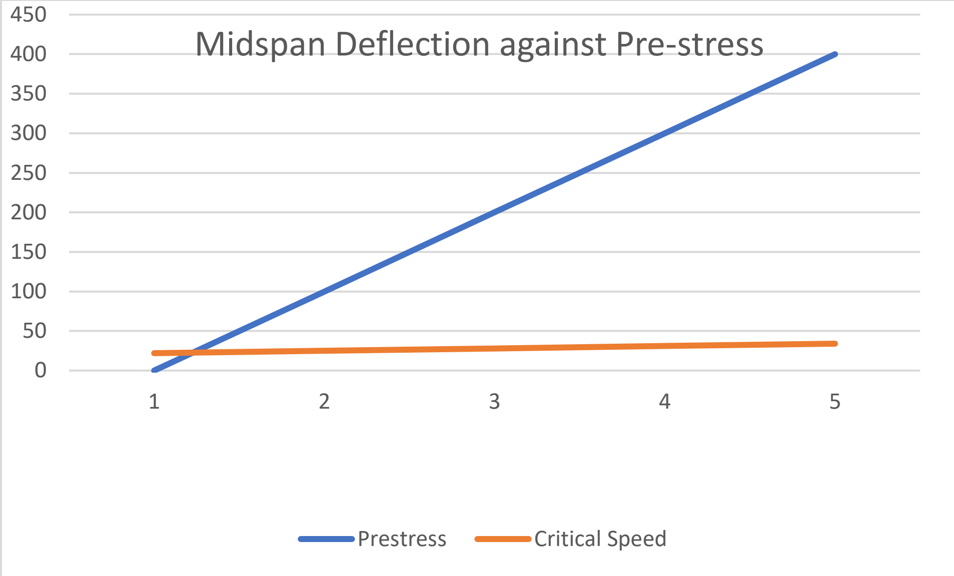

Figure 1: Maximum Midspan Deflection versus Prestress

The graph of maximum midspan deflection against prestress (Figure 1) shows that increasing prestress significantly reduces the maximum beam deflection (Table 2). Prestressing introduces additional axial stiffness into the beam, thereby increasing its overall flexural rigidity. Consequently, the beam becomes more resistant to bending under moving distributed loads.

Table 3: Natural Frequency versus Prestress

Prestress (kN)	**Frequency (Hz)**
0	7.8
100	8.5
200	9.3
300	10.2
400	11.1

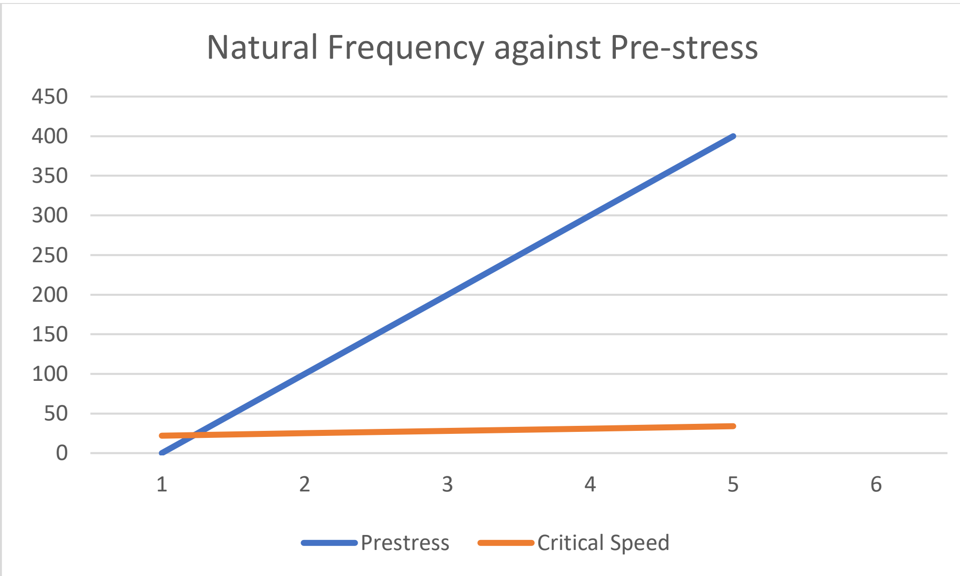

Figure 2: Natural Frequency versus Prestress

Natural frequency increases almost vary linearly with prestress (Figure 2).
The increase in frequency shifts the beam away from resonance for many vehicle speeds (Table 3).

Table 4: Dynamic Amplification Factor versus Moving Load Speed

Speed	0 kN	200 kN	400 kN
5	1.05	1.03	1.02
10	1.18	1.10	1.05
15	1.40	1.24	1.15
20	1.67	1.38	1.23
25	1.94	1.52	1.32
30	2.12	1.65	1.40
35	2.28	1.76	1.47
40	2.35	1.84	1.53

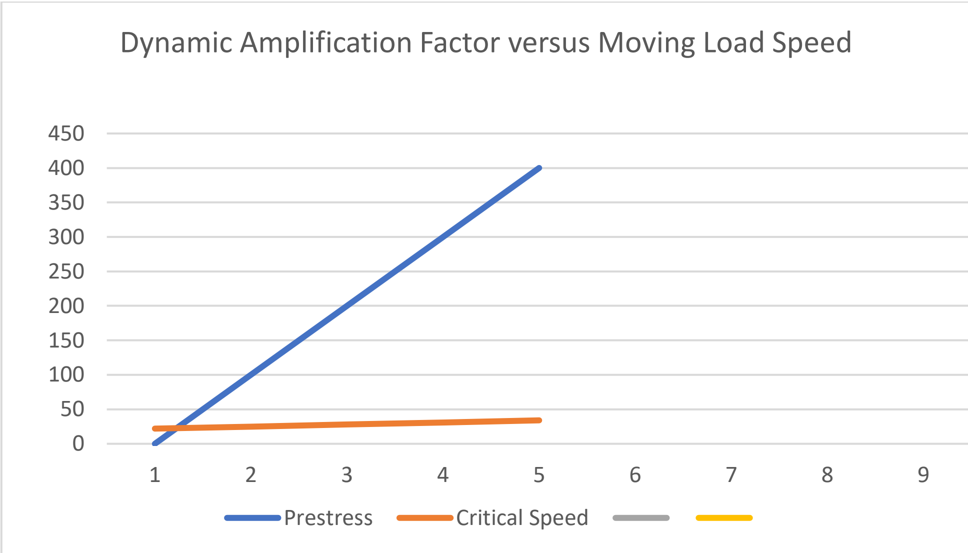

Figure 3: Dynamic Amplification Factor versus Moving Load Speed

The dynamic amplification factor increases with speed (Table 4) because the moving distributed load excites the beam more rapidly as shown in Figure 3. However, increasing prestress suppresses dynamic amplification considerably.

Table 5: Midspan Vibration History

Prestress	Peak (mm)
0	18.4
200	13.6
400	9.8

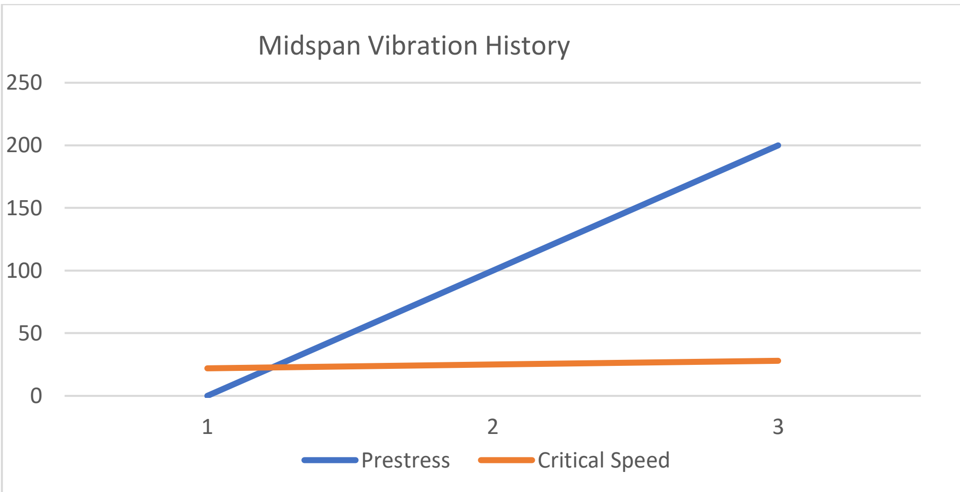

Figure 4: Midspan Vibration History

The vibration history indicates oscillatory behavior as the moving load traverses the beam (Table 5). Without prestress, oscillations are larger and persist for a longer period as shown in Figure 4.

Increasing prestress results in:

i. Smaller oscillation amplitudes,
ii. Faster decay,
iii. Reduced residual vibrations.

Hence, prestressing improves vibration control and passenger comfort in bridge applications.

Table 6: Critical Speed versus Prestress

Prestress	Critical Speed (ms^{-1})
0	22
100	25
200	28
300	31
400	34

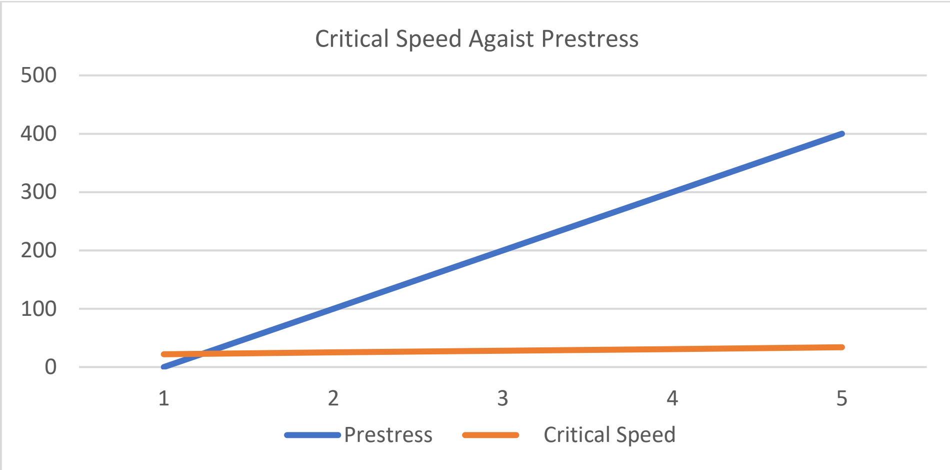

Figure 5: The Critical Speed Increases Steadily with Prestress

Table 6 shows the Variation of Critical Speed with Prestress. As prestress increases, the beam stiffness also increases, causing the resonance condition to shift toward higher speeds (Figure 5). Consequently, structures with higher prestress are safer under high-speed traffic loading.

Effect of Variable Prestress on Midspan Deflection

The Figure 1 illustrates the variation of the maximum midspan deflection of the elastically supported Euler–Bernoulli beam with increasing prestress under a moving distributed load. The response exhibits a monotonic decrease in deflection as the prestress level increases from 0 to 400 kN. Specifically, the maximum deflection decreases from 18.4 mm at zero prestress to 9.9mm at 400 kN, corresponding to a reduction of approximately 46.2%**.** This trend is attributed to the increase in the beam's effective stiffness caused by axial tensile prestressing. Prestress induces an initial tensile force that counteracts bending deformation, thereby reducing the beam's susceptibility to transverse displacement. Since the beam is also supported by an elastic foundation, the foundation stiffness and prestress work together to resist deformation. At lower prestress levels (0–100 kN), the reduction in deflection is moderate because the beam stiffness is only slightly enhanced. As the prestress increases beyond 200 kN, the beam becomes considerably stiffer, resulting in a more pronounced decrease in deflection. From an engineering perspective, reduced midspan deflection improves serviceability by minimizing structural deformation, enhancing user comfort, and reducing the likelihood of cracking in reinforced or prestressed concrete members. The results therefore indicate that increasing prestress is an effective means of improving the static and dynamic stiffness of beams subjected to moving distributed loads.

Figure 2 revealed the Influence of Prestress on the Fundamental natural Frequency. The figure shows that the beam's first natural frequency increases almost linearly as the prestress level increases. The natural frequency rises from 7.8 Hzwithout prestress to 11.1 Hz at 400 kN, representing an increase of approximately 42.3%**.** This increase is explained by the relationship between natural frequency and structural stiffness. For an Euler–Bernoulli beam, the natural frequency is proportional to the square root of the ratio of effective stiffness to mass. Because prestressing increases the effective bending stiffness without significantly changing the structural mass, the vibration frequency increases correspondingly. An increase in natural frequency is highly desirable because it shifts the resonance condition to higher excitation frequencies. In practical applications such as highway bridges and railway bridges, moving vehicles generate dynamic loading frequencies that may coincide with the beam's natural frequencies. Increasing the natural frequency reduces the possibility of resonance, thereby enhancing structural safety and durability. The nearly linear relationship also suggests that the effect of prestress on beam stiffness remains stable within the investigated prestress range, making prestress an effective and predictable design parameter for vibration control.

In Figure 3, the Dynamic Amplification Factor (DAF) versus Moving Load Speed was plotted. The figure compares the variation of the Dynamic Amplification Factor (DAF) with moving load speed for different prestress levels. In all three cases, the DAF increases as the load speed increases. However, the magnitude of the amplification decreases

significantly with increasing prestress. Without prestress, the DAF increases rapidly from approximately 1.05 at 5 m/s to **2.35** at 40 m/s. At a prestress of **2**00 kN, the maximum DAF decreases to 1.84, while at 400 kN**,** it reduces further to approximately 1.53**.** The increase in DAF with speed occurs because higher moving speeds generate stronger inertial forces and excite higher vibration modes of the beam. As the moving load approaches the beam's critical speed, resonance effects become more pronounced, leading to larger dynamic responses. Restressing mitigates this effect by increasing the beam's stiffness and natural frequency. Consequently, the excitation frequency associated with the moving load remains farther from the beam's natural frequency, reducing resonance and dynamic amplification. The reduction in DAF demonstrates that prestressing effectively controls vibration amplitudes under high-speed moving loads. This finding is particularly important in the design of high-speed railway bridges, airport pavements, and long-span bridge decks where excessive dynamic amplification may accelerate fatigue damage and shorten the service life of structural components.
The Time History of Midspan Displacement was shown in Figure 4. The displacement time-history graph illustrates the transient vibration response of the beam as the moving distributed load travels across the span. All three curves exhibit oscillatory behavior characteristic of dynamically excited beam systems. However, the vibration amplitude decreases significantly as the prestress level increases.Without prestress, the beam experiences the highest peak displacement of approximately 18.4 mm**,** accompanied by relatively large oscillations that persist even after the moving load has left the beam. At 200 kN**,** the peak displacement decreases to 13.6 mm, and the oscillations become less pronounced. At 400 kN**,** the maximum displacement further reduces to approximately 9.8 mm, and the vibrations decay more rapidly. The reduction in vibration amplitude is a consequence of the increased stiffness resulting from prestressing. A stiffer beam stores less strain energy during loading and therefore experiences smaller oscillations after the load passes. Additionally, the elastic foundation provides supplementary restoring forces that further dampen the vibration response. Rapid attenuation of vibrations is advantageous because it reduces fatigue stresses, minimizes discomfort for bridge users, and improves structural stability under repeated moving loads. The graph therefore confirms that prestressing not only decreases the peak response but also improves the overall dynamic behavior of the beam.

Figure 5 shows the Variation of Critical Speed with Prestress. The figure illustrates the increase in critical speed as the prestress level increases. The critical speed rises from approximately 22 m/s without prestress to 34 m/s at a prestress level of 400 kN, corresponding to an increase of approximately 54.5%**.** The critical speed represents the moving load velocity at which the beam experiences its maximum dynamic response due to resonance. As prestress increases the beam's stiffness and natural frequency, a higher moving load speed is required before resonance occurs. This behavior has significant engineering implications. Modern transportation systems increasingly involve high-speed vehicles whose operating speeds may approach the resonance speed of bridge structures. By increasing the critical speed through prestressing, the operating speed range moves farther from resonance, thereby reducing the risk of excessive vibration, structural fatigue, and serviceability problems.

The results demonstrate that prestressing substantially improves the dynamic safety margin of the beam and provides an effective strategy for ensuring reliable performance under increasing traffic speeds. The numerical investigation consistently demonstrates that variable prestress has a beneficial effect on the dynamic performance of an elastically supported Euler–Bernoulli beam under moving distributed loads**.** Increasing prestress enhances the beam's effective stiffness, which in turn reduces deflection, suppresses vibration amplitudes, increases the natural frequency, lowers the dynamic amplification factor, and shifts the critical resonance speed to higher values. These improvements become more pronounced as the prestress level increases, particularly beyond 200 kN**,** where the response curves exhibit significant reductions in displacement and dynamic amplification. The combined action of prestressing and the elastic foundation provides a more stable structural system capable of resisting both static and dynamic loading effects.

Overall, the findings suggest that optimizing the prestress level can significantly enhance the serviceability, durability, and safety of beam-like structures such as highway bridges, railway bridges, crane girders, and elevated guideways subjected to repeated moving distributed loads. The observed trends are consistent with classical beam vibration theory and support the use of prestressing as a practical design strategy for controlling dynamic responses in modern civil engineering structures.

CONCLUSION

This study investigated the influence of variable prestress on the dynamic performance of an elastically supported Euler–Bernoulli beam subjected to moving distributed loads through analytical and numerical approaches. The investigation evaluated the effects of different prestress levels on key dynamic characteristics, including maximum deflection, natural frequency, dynamic amplification factor (DAF), transient vibration response, and critical moving load speed. The results demonstrate that variable prestress significantly enhances the dynamic stiffness of the beam, with increasing prestress reducing the maximum midspan deflection by approximately 46%, thereby improving structural rigidity and minimizing deformation under moving distributed loads. An increase in prestress also produced a steady rise in the beam's natural frequency, indicating an enhancement in effective stiffness and a shift of the vibration characteristics away from resonance, which improves structural stability under dynamic loading. Furthermore, higher prestress levels effectively reduced the dynamic amplification factor, particularly at elevated moving load speeds, showing that prestressing mitigates inertial effects and lowers the likelihood of excessive dynamic responses. The transient vibration behavior was also significantly improved, as greater prestress resulted in lower peak displacements, reduced oscillation amplitudes, and faster decay of vibrations after the moving load had passed, thereby enhancing structural durability, reducing fatigue damage, and improving serviceability. In addition, the critical moving load speed increased with prestress, indicating that the resonance condition shifted to higher vehicle speeds as the beam became stiffer, thereby expanding the safe operating range of the structure and reducing the risk of resonance under normal traffic conditions. Overall, the study confirms that variable prestressing is an effective and practical technique for improving the dynamic performance of elastically supported Euler–Bernoulli beams under moving distributed loads by increasing structural stiffness, suppressing vibration-induced effects, and enhancing the safety, serviceability, and long-term performance of beam-type structures such as highway bridges, railway bridges, viaducts, and elevated transportation systems. The findings provide valuable guidance for the design and optimization of prestressed beam systems subjected to dynamic moving loads. Future studies should extend the present model by incorporating Timoshenko beam theory, viscoelastic or nonlinear foundation models, moving masses with full vehicle–structure interaction, time-dependent prestress losses, material nonlinearity, damping mechanisms, and experimental validation to improve the predictive capability, robustness, and practical applicability of the proposed formulation, thereby contributing to the development of more resilient, efficient, and sustainable transportation infrastructure.

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