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Legendre Least Squares Approach with Perturbation for the Solution of Fractional Order Differential Equations

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ABSTRACT
In this article, we present the Perturbed Shifted Legendre based approach for the solution of fractional order differential equations using Least Squares Method. Here, an assumed solution is substituted into the slightly perturbed fractional order differential equation and the residual equation is minimized to yield a system of equations which are then solved to obtain the constants involved. The required approximate solution is obtained when the constant values are substituted into the assumed solution.

Keywords: Fractional Calculus, Least Squares Method, Legendre Polynomial, Fractional Differential Equation, Basis Function

INTRODUCTION

In numerical analysis, the method of Least Squares is often used to find approximate solutions to equations or to fit data to mathematical models. It minimizes the sum of the squares of the residuals to find the best fitting parameters (Sastry (2010)). Least squares method is a very useful tool for solving differential and integro-differential equations. The idea of fractional is a concept of non-integer order in calculus which has gained popularity over the past four to five decades and has impacted all fields such as mathematics, physics, chemistry, engineering, economics, etc. Several authors have used different methods to solve differential and integro-differential equations involving fractional derivatives and integrals equations. Mahdy et al. (2024) presented the use of shifted Legendre polynomials for the solution of fractional integro-differential equations. In the presentation, the properties of the Legendre and shifted Legendre as well as fractional calculus were examined and at the end, using some truncated shifted Legendre polynomials series, the authors obtained good results on some examples based on the suggested approach.

In the application of perturbation method, Oyedepo et al. (2021) and Uwaheren et al. (2020) presented the Solution of fractional integro-differential equation using modified homotopy perturbation technique and constructed orthogonal polynomials as basic functions for the so- lution of multi-order fractional differential equations of Lane-Emden type respectively. The authors use trial function which was substituted into a perturbed general class of the problem.

Their results converged to the exact solution rapidly. Chuanhua and Ziqiang (2024) proposed a high-precision numerical algorithm for fractional integro-differential equation based on the shifted Legendre polynomials. The idea of Gauss-Legendre quadrature rule and a new spectral collocation methods were used to solve some examples of fractional integro-differential equation and the error analysis of the problems considered yield highly accurate results. Umar and Bichi (2024) studied the problem of multi-term fractional order Volterra integro- differential equation with the Schauder's fixed point theorem. The authors converted the problem to an equivalent integral equation and established the existence of solutions for the problem under some mild conditions with the aid of Riemann-Liouville fractional integral. Examples were given to test the applicability of the proposed method. Mohammedi-Nejab, et al. (2022) worked on numerical solution for a class of variable-order fractional differential equations with Atangana-Beleanu-Caputo Fractional Derivative. The authors introduce Chebyshev polynomials to determine the numerical solution of the variable-order fractional mobile-immobile advection dispersion model using collocation method. The equation was transformed into the products of several dependent matrixes by operational matrixes in form of system of algebraic equations. Solving the linear equations yielded the approximate solution. Sadabad, et al. (2020) studied numerical Eigenvalues and eigen functions of fractional sturm-Liouville problems via Laplace transform where they constructed numerical schemes based on the Lagrange polynomial interpolation to solve fractional Sturm's Liouville problems. After converting differential equation into integral form and generating a system of algebraic equations, the eigenvalues and corresponding eigenvectors were calculated and the order convergence of the numerical method was obtained. Yang, et al. (2020) investigated the numerical analysis of intermediate value problems and the well-posedness. Two numerical methods, Convergence and sensitivity analysis for solving the problems were proposed in the work and the estimated order of convergence from some examples was sharp. Akrami, and Owolabi (2023) studied the solution of fractional differential equations using Atangana's beta derivative and its applications in chaotic systems. Zayed, et al. (2020) worked on fractional order of Legendre-type matrix polynomials with very good results that converged well with the exact solution. Kayode, et al. (2026) applied Variational Iteration Method (VIM) to fourth-order Volterra Integro-Differential Equations (VIDEs) an obtained approximate analytical solution with negligible errors compared to the exact solutions.

In this article, we are concerned with fractional order differential equations and their solutions using Least squares method with perturbed shifted Legendre polynomial as the basis functions.

The general form of fractional order differential equation is given as

$$D^{\alpha} y(x) + y(x) = f(x) \qquad (1)$$

Subject to the conditions; $y(0) = 0$, $y(1) = 1$, $0 < \alpha \le 1$

Legendre Polynomials

Legendre polynomials are a family of orthogonal polynomials that have various applications in mathematics and physics. There are different kinds of Legendre polynomials which are often denoted as $P_n(x)$ where n is a non-negative integer. The definition of the first kind of Legendre polynomials in the interval [-1,1], highlights distinct features as well as make suggestions for generalizations to various mathematical structures. Generating functions, differential equations, and orthogonal systems can be used to define Legendre polynomials. The first few terms of Legendre polynomials are generated here using Rodriquez formula

$$p_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n, \quad n = 0, 1 \tag{2}$$

The recurrence Legendre polynomials is given as

$$P_n(x) = \frac{2n-1}{n} x P_{n-1}(x) - \frac{n-1}{n} P_{n-2}; \quad n \geq 2 \tag{3}$$

when n=2,3,4,5. we have

$$\begin{array}{c|c} p_0(x)=1 & p_3(x)=\frac{1}{2}(5x^3-3x) \\ p_1(x)=x & p_4(x)=\frac{1}{8}(35x^4-30x^2+3) \\ p_2(x)=\frac{1}{2}(3x^2-1) & p_5(x)=\frac{1}{8}(63x^5-70x^3+15x) \end{array} \tag{4}$$

To transform (4) from interval [-1,1] to [a,b]; (shifted), we let $X = mx + n$ (5)

Substituting $x = -1$ and $x = 1$ respectively, we have

$a = m + n$ $(x = 1)$ and $b = -m + n$ $(x = -1)$ (6)

Solving the above equations simultaneously, we have

$m = \frac{1}{2}(b-a)$ and $n = \frac{1}{2}(b+a)$

substituting the value of m and n into (5), we have

$$x = \frac{2X-(b+a)}{b-a} \Longrightarrow x = 2X - 1 \tag{7}$$

If [a,b] = [0,1], then Substituting (6) into (4), we have the first few Legendre polynomials valid in the interval [0,1] as follows$p_0(x) = 1$

$p_1(x) = 2x - 1$

$p_2(x) = 6x^2 - 6x + 1$

$p_3(x) = 20x^3 - 30x^2 + 12x - 1$

$p_4(x) = 70x^4 - 140x^3 + 90x^2 - 20x + 1$

$p_5(x) = 252x^5 - 630x^4 + 560x^3 - 210x^2 + 30x - 1$

Fractional Calculus

Fractional Calculus is a generalization of ordinary differentiation and integration to an arbi- trary order. Weilbeer (2005) and Podlubny , agreed that fractional calculus does not mean the calculus of fractions or fractions of any calculus (differential or integral) but the theory of integrals and derivatives of arbitrary order, which unify and generalize the notions of integer- order differentiation and n-fold integration. Two commonly used are that of Riemann liouville and Caputo definitions.

Riemann-Liouville Fractional Integral

The Riemann-Liouville integral operator of order α denoted by j_a^α defined on $L_1[a,b]$ is given as

$$J_a^\alpha f(x) = \frac{1}{\Gamma(\alpha)}\int_a^x (x-t)^{\alpha-1} f(t)dt, \text{ for } a \leq x \leq b \tag{8}$$

Riemann-Liouvelle Fractional Derivative

The Riemann-Liouville derivative operator of order α denoted by D_a^α is given as:

$$D_a^\alpha f(x) = D^n J_a^{n-\alpha} f(x)$$

$$= \frac{1}{\Gamma(n-\alpha)}\left(\frac{d}{dx}\right)^n \int_a^x (x-t)^{n-\alpha-1} f(t)dt, \tag{9}$$

$a \leq x \leq b$

Caputo Fractional Derivative

The Caputo fractional derivative of order α denoted by D_{*a}^α is given as: $D_{*a}^\alpha f(x) = J_a^{n-\alpha} D^n f(x)$

$$= \frac{1}{\Gamma(n-\alpha)}\int_a^x (x-t)^{n-\alpha-1}\left(\frac{d}{dt}\right)^n f(t)\, dt, \quad a \leq x \leq b \tag{10}$$

(Constant Rule) Let $\alpha > 0$ and cbe constants. then,

$$D_{*a}^\alpha (c) = 0 \tag{11}$$

(Power Rule) Let $n - 1 < \alpha < n$, $n \in N$ and $x > 0$. then

$$D_{*a}^\alpha (x^\beta) = \begin{cases} \frac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)}, & \beta > \alpha - 1, \\ 0, & \beta \leq \alpha - 1 \end{cases} \tag{12}$$

MATERIALS AND METHODS

Methodology

Description of Least Squares

Consider the fractional order differential equation of the form

$$D^\alpha y(x) + y(x) = f(x) \tag{13}$$

Subject to the conditions;

$$y(0) = 0, \quad y(1) = 1, \quad 0 < \alpha \leq 1 \tag{14}$$

This approach relies on an approximating function of the form

$$y_N(x) = \sum_{i=0}^{N} C_i \phi_i(x) \quad 0 \leq x \leq 1 \tag{15}$$

In order to solve equations (13) and (14), we substitute equation (15) into (13) to get

$$D^\alpha\left(\sum_{i=0}^{N} C_i\phi_i(x)\right) + \sum_{i=0}^{N} C_i\phi_i(x) = f(x) \tag{16}$$

Hence, the residual equation is defined as

$$R(C_0, C_1, C_2, C_3, C_4) = D^\alpha\left(\sum_{i=0}^{N} C_i\phi_i(x)\right) + \sum_{i=0}^{N} C_i\phi_i(x) - f(x) = 0 \tag{17}$$

We will define the functional integral of (17) multiplied by weight function $w(x) = 1$ in the interval [0,1] as

$$S(C_0, C_1, C_2, C_3, C_4) = \int_0^1 \left[D\left(\sum^{N} C_i\phi_i(x)\right) + \sum^{N} C_i\phi_i(x) - f(x)\right]^2 dx \tag{18}$$

The general and necessary condition for a minimum to be obtained at $C_0, C_1, C_2 \cdots, C_n$ is that

$$\frac{\partial S}{\partial C_0} = \frac{\partial S}{\partial C_1} = \frac{\partial S}{\partial C_2} = \frac{\partial S}{\partial C_3} = \cdots = \frac{\partial S}{\partial C_n} = 0 \tag{19}$$

So, we minimize (18) by finding the partial derivative with respect to C_i $i = 0, 1, 2, 3$ So, taking the partial derivatives of equation (18) yields an $(n + 1)$ equations as follows

$$\left.\begin{aligned} \frac{\partial S}{\partial C_0} &= -2\int_a^b [w(x)\,[y(x) - (C_0 + C_1x + C_2x^2 + \cdots + C_nx^n)]\,dx = 0 \\ \frac{\partial S}{\partial C_1} &= -2\int_a^b [w(x)\,[y(x) - (C_0 + C_1x + C_2x^2 + \cdots + C_nx^n)]\,x dx = 0 \\ &\vdots \\ \frac{\partial S}{\partial C_n} &= -2\int_a^b [w(x)\,[y(x) - (C_0 + C_1x + C_2x^2 + \cdots + C_nx^n)]\,x^n dx = 0 \end{aligned}\right\} \tag{20}$$

Thus, (N+1) systems of algebraic equations with (N+1) unknown constants ($C_0, C_1, C_2, \ldots C_N$) are now solved by Gaussian elimination method or using some mathematical package to obtain the unknown constants. Substituting the values into (15), we obtained the required approximate solution.

Description of Least Squares with Perturbation

Consider the fractional order differential problem (13) with conditions (14) and using the approximating function (15). Putting (15) in (13), we have

$$D^{\alpha}\left(\sum_{i=0}^{N} C_i\phi_i(x)\right) + \sum_{i=0}^{N} C_i\phi_i(x) = f(x) \tag{21}$$

Now, (21) is perturbed by adding the perturbation term

$$H_N(x) = \sum_{n=0}^{N} T^*_{(N-n)}(x)q_n(x) \tag{22}$$

$$D^{\alpha}\left(\sum_{i=0}^{N} C_i\phi_i(x)\right) + \sum_{i=0}^{N} C_i\phi_i(x) + \sum_{n=0}^{N} T^*_{(N-n)}(x)q_n(x) = f(x) \tag{23}$$

where $T^*(N-n)(x)$ is Chebyshev polynomial, N is degree of the assumed trial solution and n is taken as the $\lceil\alpha\rceil$

Hence, the residual equation is defined as

$$D^{\alpha}\left(\sum_{i=0}^{N} C_i\phi_i(x)\right) + \sum_{i=0}^{N} C_i \phi_i(x) + \sum_{n=0}^{N} T^*_{(N-n)}(x)q_n(x) - f(x) = 0 \tag{24}$$

We will define the functional integral of (24) taking the weight function $w(x) = 1$ in the interval [0,1], we have

$$S(C_0, C_1, C_2, C_3, C_4) = \int_0^1 \left[D^{\alpha}\left(\sum_{i=0}^{N} C_i\phi_i(x)\right) + \sum_{i=0}^{N} C_i\phi_i(x) + \sum_{n=0}^{N} T^*_{(N-n)}(x)q_n(x) - f(x)\right]^2 dx$$

25) The general and necessary condition for a minimum to be obtained at $C_0, C_1, C_2 \cdots, C_n$ is

$$\frac{\partial S}{\partial C_0} = \frac{\partial S}{\partial C_1} = \frac{\partial S}{\partial C_2} = \frac{\partial S}{\partial C_3} = \frac{\partial S}{\partial C_n} = \frac{\partial S}{\partial q_1} = \frac{\partial S}{\partial q_n} = 0 \tag{26}$$

We minimize (25) by finding the partial derivative with respect to C_i $i = 0, 1, 2, 3$

So, taking the partial derivatives of equation (25) yields an $(n + 1)$ equations as follows

$$\left.\begin{aligned}
\frac{\partial S}{\partial C_0} &= -2\int_a^b [w(x)\,[y(x) - (C_0 + \cdots + C_n x^n + T_0 q_0(x) + \cdots + T_n q_n(x))]\,dx = 0 \\
\frac{\partial S}{\partial C_0} &= -2\int_a^b [w(x)\,[y(x) - (C_0 + \cdots + C_n x^n + T_0 q_0(x) + \cdots + T_n q_n(x))]\,x dx = 0 \\
\frac{\partial S}{\partial C_0} &= -2\int_a^b [w(x)\,[y(x) - (C_0 + \cdots + C_n x^n + T_0 q_0(x) + \cdots + T_n q_n(x))]\,x^n dx = 0 \\
&\vdots \qquad\qquad \vdots \qquad\qquad \vdots \\
\frac{\partial S}{\partial C_0} &= -2\int_a^b [w(x)\,[y(x) - (C_0 + \cdots + C_n x^n + T_0 q_0(x) + \cdots + T_n q_n(x))]\,q^n dx = 0
\end{aligned}\right\} \tag{27}$$

Thus, $(N + q + 1)$ systems of algebraic equations with $(N + q + 1)$ unknown constants $(C_0, C_1, C_2, \ldots C_N, q_0, \ldots q_n)$ are now solved by Gaussian elimination method or using some mathematical package to obtain the unknown constants. Substituting these values into (15), we obtained the required approximate solution.

Numerical Example

Problem 1

Using the Least squares method, solve the fractional order differential equation

$$D^{\alpha}y(x) = x^2 + \frac{2}{\Gamma(3-\alpha)}x^{2-\alpha} - y(x), \tag{28}$$

with the conditions:

$$y(0) = 0, \quad 0 < \alpha \le 1. \tag{29}$$

whose exact solution is given by $y(x) = x^2$ and $\alpha = \frac{1}{2}$

We solve problem 1 by Least squares method and following the same procedure we obtain the unknown constants

$C_0 = 0.3333333237$; $C_1 = 0.5000000033$; $C_2 = 0.1666666650$;

$C_3 = 1.356991645 \times 10^{-9}$; $C_4 = -8.799392025 \times 10^{-10}$

substituting the values into (25), we have

$$y_4(x) = -0.912e^{-6}x^4 - 0.5687e^{-6}x^3 + .925436x^2 - 0.98956e^{-3} \tag{30}$$

We solve problem 1 by Perturbed Least squares method, we obtain the unknown constants

$C_0 = 0.3333333237$; $C_1 = 0.5000000033$; $C_2 = 0.1666666650$; $C_3 = 1.356991645 \times 10^{-9}$; $C_4 = -9.599292025 \times 10^{-10}$, $q_0 = 0.000056923$ $q_1 = 0.000926473$

substituting the values into (25), we have

$$y_4(x) = -0.9113e^{-6}x^4 - 0.9668e^{-6}x^3 + 0.925436x^2 - 0.98956e^{-3} \tag{31}$$

Problem 2

Using the Least squares method, solve the fractional order differential equation

$$D^{\frac{3}{2}}y(t) - t^{\frac{3}{2}}y(t) = 4\frac{\sqrt{t}}{\sqrt{\pi}} - t^{\frac{7}{2}}, \tag{32}$$

with the conditions:

$$y(0) = 0, \quad y'(0) = 0, \tag{33}$$

whose exact solution is given by $y(t) = -t^3 + 5t^2$ and $\alpha = \frac{5}{2}$

We solve problem 2 by Least squares method and following the same procedure we obtain the unknown constants

$C_0 = 0.3333649116$; $C_1 = 0.5000031359$; $C_2 = 0.1666684633$;

$C_3 = 3.922513306 \times 10^{-7}$;

$C_4 = 3.561155042 \times 10^{-8}$

Substituting the values into (25), we have

$$y_4(t) = -0.6654e^{-4}t^4 - 1.0009657t^3 + 4.5799899353t^2 - 0.99976e^{-3} \tag{34}$$

We solve problem 2 by Perturbed Least squares method, we obtain the unknown constants

$C_0 = 0.3333649116$; $C_1 = 0.5000031359$; $C_2 = 0.1666684633$; $C_3 = 4.1111334563 \times 10^{-7}$; $C_4 = 3.561155234 \times 10^{-8}$, $q_0 = 0.9087231$, $q_1 = 0.087634534$, $q_2 = 0.0000865402$

Substituting the values into (25), we have

$$y_4(t) = -0.568235e^{-2}t^4 - 1.0987126t^3 + 4.99935306t^2 - 0.89865e^{-3} \tag{35}$$

Problem 3

Using the Least squares method, solve the fractional order differential equation

$$D^{\alpha}y(t) + y(t) = \frac{2t^{2-\alpha}}{\Gamma(3-\alpha)} - \frac{t^{1-\alpha}}{\Gamma(2-\alpha)} + t^2 - t \tag{36}$$

with the conditions:

$$y(0) = 0, \quad 0 < \alpha \le 1. \tag{37}$$

whose exact solution is given as $y(t) = t^2 - t$ and $\alpha = \frac{7}{2}$

We solve problem 3 by Least squares method and following the same procedure we obtain the unknown constants $C_0 = -0.1666666695$; $C_1 = 0.0000000009871753844$; $C_2 = 0.1666666662$; $C_3 = 0.0000000004005401662$; $C_4 = -0.0000000003180223119$ Substituting these values into (25), we have

$$y_4(t) = -0.889568e^{-4}t^4 - 0.987e^{-5}t^3 + 0.99935306t^2 - 1.000435699t - 0.756342e^{-2} \tag{38}$$

We solve problem 3 by Least squares method, we obtain the unknown constant

$C_0 = -0.1666666695$; $C_1 = 0.0009871753844$; $C_2 = 0.1666666662$; $C_3 = 0.000004005401$; $C_4 = -0.0000003180223119$, $q_0 = 0.000987612$, $q_1 = 0.00000564531$, $q_2 = 0342895400$, $q_3 = 0.0000006667654$, $q_4 = 0.00000056457588$ (39)

Substituting these values into (25), we have

$$y_4(t) = -0.9680003e^{-5}t^4 - 0.98732e^{-3} + 1.00009935306t^2 - 0.99001109t - 0.908723e^{-3} \quad (40).$$

Table 1: Absolute Errors for Problem 1

x	Exact	LSM	Error	PLSM	Error
0.0	0.0000000000	-1.69169×10^{-8}	1.6917×10^{-8}	-1.69169×10^{-8}	1.6917×10^{-8}
0.2	0.0400000000	0.0399999890	1.0816×10^{-8}	0.039999989	1.0818×10^{-8}
0.3	0.0900000000	0.0899999900	9.8389×10^{-9}	0.089999990	9.8369×10^{-9}
0.4	0.1600000000	0.1599999906	9.4352×10^{-9}	0.1599999906	9.5352×10^{-9}
0.6	0.3600000000	0.3599999911	8.8909×10^{-9}	0.3599999911	7.8909×10^{-9}
0.8	0.6400000000	0.6399999921	7.8807×10^{-9}	0.6399999921	6.9807×10^{-9}
0.9	0.8100000000	0.8099999925	7.4993×10^{-9}	0.8099999925	7.5093×10^{-9}
1.0	1.0000000000	0.9999999923	7.6820×10^{-9}	0.9999999923	6.6820×10^{-9}

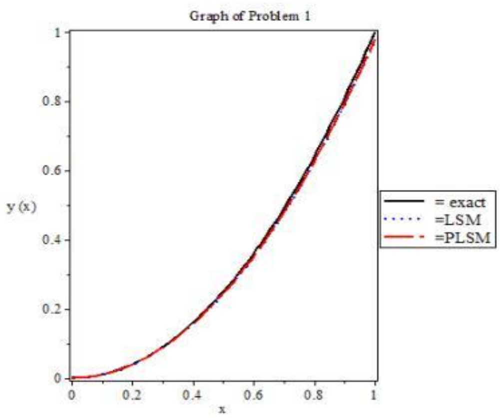

Figure 1: Graph Representation of Problem 1

Table 2: Absolute Errors for Problem 2

t	Exact	LSM	Error	PLSM	Error
0.0	0.0000000000	0.000029853	2.8853×10^{-5}	0.000029853	2.9852×10^{-5}
0.1	−0.05950000000	0.0029854100	1.9854×10^{-5}	0.0029854100	2.9754×10^{-5}
0.3	−0.4230000000	0.0900300361	3.1003×10^{-5}	0.0900300361	3.0203×10^{-5}
0.4	−0.7360000000	0.1600302872	3.0287×10^{-5}	0.1600302872	2.9028×10^{-5}
0.6	−0.58400000000	0.3600313057	3.1306×10^{-5}	0.3600313057	3.1306×10^{-5}
0.7	−2.10700000000	0.4900321761	3.2576×10^{-5}	0.4900321761	3.2177×10^{-5}
0.9	−3.32100000000	0.8100349395	3.4840×10^{-5}	0.8100349395	3.4946×10^{-5}
1.0	−4.00000000000	1.00003697151	3.6971×10^{-5}	1.00003697151	3.6975×10^{-5}

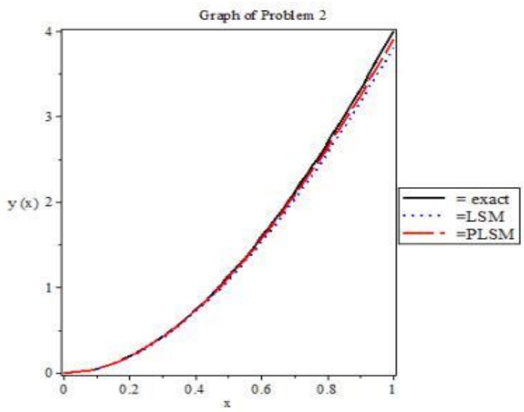

Figure 2: Graph Representation of Problem 2

Table 3: Absolute Errors for Problem 3

t	Exact	LSM	Error	PLSM	Error
0.0	0.0000000000	0.0000000000	5.1006×10^{-9}	-5.01856×10^{-9}	5.0186×10^{-9}
0.1	−0.0900000000	−0.0900000000	3.7123×10^{-9}	−0.0900000000	3.8123×10^{-9}
0.3	−0.2100000000	−0.2100000029	2.9165×10^{-9}	−0.2100000029	2.9565×10^{-9}
0.4	−0.2400000000	−0.2400000028	2.8163×10^{-9}	−0.2400000028	2.8103×10^{-9}
0.6	−0.2400000000	−0.2400000026	1.9403×10^{-9}	−0.2400000026	2.6403×10^{-9}
0.8	−0.1600000000	−0.1600000023	2.3955×10^{-9}	−0.1600000023	3.2955×10^{-9}
0.9	−0.0900000000	−0.0900000021	2.3711×10^{-9}	−0.0900000021	2.1711×10^{-9}
1.0	0.00000000000	-2.2461×10^{-9}	2.2462×10^{-9}	-2.1461×10^{-9}	2.2462×10^{-9}

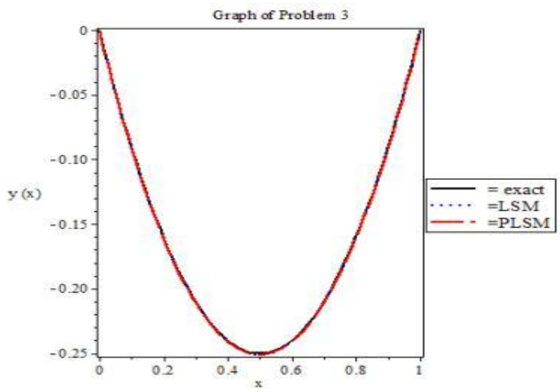

Figure 3: Graph Representation of Problem 3

Convergence and Stability

Convergence of the solution $y_n(x)$ is seen from the results as presented in the graphs which follows closely with the pattern of the exact solution Stability of the method can be analyzed by examining the convergence of the polynomial used for the approximation of the solution. For the polynomial $P_n(x)$ to converge stably, showing that it was well-defined and bounded within the interval of consideration. Its application did not yield irregular fluctuations in the approximate solutions.

Discussion

It can be seen from the table of results that there is a high convergence between our approximate results and the exact solutions. This is an evidence that the proposed method was reliable and accurate for the numerical solution the problems under consideration. The computed errors from the SCM and PSCM are both so small to the extent that is negligible in the three problems considered. In problems 1, 2, and 3, we obtained errors as small as to between 10^{-05} and 10^{-09} with comparison to the exact solution which shows that the results were accurate and the methods are stable.

CONCLUSION

It is clear from Table 1 above that the result comes very near to the correct answer. The Graph of problem 1 demonstrates that the approximate and exact solutions coincide especially at lower values of horizontal axis. From Table 2, it can seen that the result comes very close to the exact result. The Graph of problem 2 also shows that the approximate and the exact solutions coincide. Table 3 above that the result comes very near to being the correct answer. The The Graph of problem 3 demonstrates that the approximate and exact solutions coincide. We can therefore conclude that the method presented is efficient and accurate for solving the class of problems considered.

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