

Generalized Transmuted Lindley-Inverse Exponential Distribution: Properties and Applications to Bladder Cancer Data

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ABSTRACT

The Generalized Transmuted Lindley-Inverse Exponential Distribution (GTLIED) is introduced as a new flexible lifetime distribution developed by combining the generalized transmuted family of distributions with the Lindley-Inverse Exponential Distribution (LIED). The model is proposed to overcome the limitations of the classical exponential and inverse exponential distributions, particularly their inability to adequately model highly skewed lifetime data and varying hazard rate behaviors. The cumulative distribution function and probability density function of the GTLIED are derived, and its statistical properties, including moments, moment generating function, characteristic function, survival function, and hazard function, are obtained. Maximum likelihood estimation is employed for parameter estimation. The flexibility and practical usefulness of the proposed model are demonstrated through an application to remission times of 128 bladder cancer patients. The performance of the GTLIED is compared with several competing distributions, including the LIED, EIED, OLIED, OLomIED, IED, ExD, and Lindley distributions. Based on model selection criteria such as AIC, CAIC, BIC, HQIC, Anderson-Darling, Cramér–Von Mises, and Kolmogorov–Smirnov statistics, the GTLIED provides the best fit to the bladder cancer dataset. The findings indicate that the proposed distribution is highly flexible and effective for modeling positively skewed lifetime data encountered in reliability and survival studies.

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INTRODUCTION

The Exponential distribution is used in Poisson processes and gives a description of the time between events. It is applied mostly in life testing experiments. It has memoryless property with a constant failure rate which makes the distribution unsuitable for real life problems and hence creating a vital problem in statistical modeling and applications. Besides the applications of Exponential distribution and its attractive properties, its usage has been limited in modeling real life situations due to the fact that it has a constant failure rate (Lemonte, 2013). One other problem with exponential distribution is in its memoryless property, this is because the memoryless assumption is hardly obtained in real life situations. To make up for these limitations, Keller and Kamath (1982) proposed a modified version of the Exponential distribution, called the inverse exponential distribution and it has also been studied in some details by Lin *et al.*, (1989). Its modified version known as inverse exponential distribution was found adequate for modeling datasets with inverted bathtub failure rates (Keller and Kamath, 1982) but this modified version also has a limitation which is its inability to efficiently capture datasets that are highly skewed (either positively or negatively) (Abouammoh and Alshingiti (2009)). This therefore gives room for introducing skewness and flexibility into the Inverse Exponential distribution to enable it adequately model heavily skewed datasets.

To address these limitations, researchers have developed many families of distributions to enhance flexibility by incorporating additional shape parameters while retaining analytical tractability. Notable among these families include

the quadratic rank transmutation map by Shaw and Buckley (2007), the exponentiated T-X family by Alzaghal *et al.* (2013), the Weibull-X family by Alzaatreh *et al.* (2013), the Weibull-G family by Bourguignon *et al.* (2014), the Lindley-G family by Cakmakyapan and Ozel (2016), the generalized transmuted family of distributions by Alizadeh *et al.* (2016), the odd Lindley-G family by Gomes-Silva *et al.* (2017), the odd Lomax-G family by Cordeiro *et al.* (2019), the odd Chen-G family of distributions by Anzagra *et al.* (2022), a novel bivariate Lomax-G family of distributions Fayomi *et al.* (2023), the X-exponential-G family of distributions by Mohammad (2024) and the new Frechet-G family by Ieren *et al.* (2024).

These families of distributions have been used by many researchers to develop flexible compound probability distributions including extended forms of the exponential and inverse exponential distributions such as the Weibull-exponential distribution by Oguntunde *et al.* (2015), the transmuted Weibull-exponential distribution by Yahaya and Ieren (2017), the exponential inverse exponential distribution by Oguntunde *et al.* (2017), the transmuted exponential-inverse exponential distribution by Ieren *et al.* (2020), transmuted Lindley-exponential distribution by Umar *et al.*, (2019), odd Lindley-inverse exponential distribution by Ieren and Abdullahi (2020), the Lomax-exponential distribution by Ieren and Kuhe (2018), the transmuted odd generalized exponential-exponential distribution by Abdullahi *et al.* (2018), the transmuted exponential distribution by Owoloko *et al.* (2015), the Beta Kumaraswamy distribution by Makama *et al.* (2018), the odd Lomax-inverse exponential distribution by Ieren *et al.* (2021), the Lomax- inverse exponential

distribution by Abdulkadir *et al.* (2020), the exponentiated exponential-inverse exponential distribution by Asongo *et al.* (2020), the Weibull-exponential inverse exponential distribution by Chama *et al.* (2021), the sine Lomax-exponential distribution by Joel *et al.* (2024) and the Lindley-inverse exponential distribution by Makama *et al.* (2025). These extended distributions have been found to be better than the conventional or classical ones.

According to Makama *et al.*, (2025), the probability density function (pdf) and cumulative distribution function (cdf) of the Lindley-inverse exponential distribution are defined respectively as;

$$G(x) = 1 - \left[1 + \frac{\alpha\theta}{(\alpha+1)x} \right] \left[1 - e^{-\frac{\theta}{x}} \right]^\alpha \quad (1)$$

and

$$g(x) = \frac{\alpha^2\theta x^{-2}e^{-\frac{\theta}{x}}}{(\alpha+1)} \left[1 + \frac{\theta}{x} \right] \left[1 - e^{-\frac{\theta}{x}} \right]^{\alpha-1} \quad (2)$$

where $x > 0, \alpha > 0, \theta > 0$; α is a shape parameter and θ is the scale parameter. This distribution was found to perform better than the exponential-inverse exponential distribution, conventional exponential distribution and the inverse exponential distribution

This article is organized in sections as follows: the new model with its simplification and graphical representation is given in section 2. Section 3 derived some properties of the new distribution. The estimation of unknown parameters of the distribution using maximum likelihood estimation is provided

$$F(x) = (1+\lambda) \left[1 - \left(\left(1 + \frac{\alpha\theta}{(\alpha+1)x} \right) \left(1 - e^{-\frac{\theta}{x}} \right)^\alpha \right)^\gamma \right] - \lambda \left[1 - \left(\left(1 + \frac{\alpha\theta}{(\alpha+1)x} \right) \left(1 - e^{-\frac{\theta}{x}} \right)^\alpha \right)^\gamma \right]^2 \quad (5)$$

And

$$f(x) = \frac{\alpha^2\gamma\theta x^{-2}e^{-\frac{\theta}{x}}}{(\alpha+1)} \left[1 + \frac{\theta}{x} \right] \left(1 + \frac{\alpha\theta}{(\alpha+1)x} \right)^{\gamma-1} \left[1 - e^{-\frac{\theta}{x}} \right]^{\alpha\gamma-1} \left\{ 1 - \lambda + 2\lambda \left(\left(1 + \frac{\alpha\theta}{(\alpha+1)x} \right) \left(1 - e^{-\frac{\theta}{x}} \right)^\alpha \right)^\gamma \right\} \quad (6)$$

where $-1 \leq \lambda \leq 1$, and $\alpha, \theta, \gamma > 0$ are the parameters of the GTrLIED. Also, $\gamma > 0$, $-1 \leq \lambda \leq 1$, and $x > 0$, where γ and λ

in section 4. An application of the new model to a real-life dataset is done in section 5 and a summary and conclusion is presented in section 6.

MATERIALS AND METHODS

The Generalized Transmuted Lindley-Inverse Exponential Distribution (GTLIED)

According to Alizadeh *et al.* (2016), if $g(x)$ and $G(x)$ are the PDF and CDF of a random variable X respectively, then the CDF and PDF of the generalized transmuted family of distributions are defined as:

$$F(x) = (1+\lambda) \left[1 - (1-G(x))^\gamma \right] - \lambda \left[1 - (1-G(x))^\gamma \right]^2 \quad (3)$$

and

$$f(x) = \gamma g(x) (1-G(x))^{\gamma-1} \left\{ 1 + \lambda - 2\lambda \left[1 - (1-G(x))^\gamma \right] \right\} \quad (4)$$

respectively,

where for $\gamma > 0$, $-1 \leq \lambda \leq 1$, and $x > 0$, where γ and λ are the two new shape parameters that will enhance the flexibility and shape of the new distribution. Note that $g(x)$ and $G(x)$ represent the pdf and the cdf of the continuous distribution to be modified, extended or generalized respectively.

Substituting the pdf and cdf of the Lindley-inverse exponential distribution into the generalized transmuted family in Equation (3) and (4) accordingly and simplifying, the cdf and pdf of the generalized transmuted Lindley-inverse exponential distribution (GTrLIED) are obtained as given in Equation (5) and (6) respectively:

are the two new shape parameters that will enhance the flexibility and shape of the GTrLIED.

The Figures 1 present the PDF and CDF of the GTLIED using arbitrary parameter values.

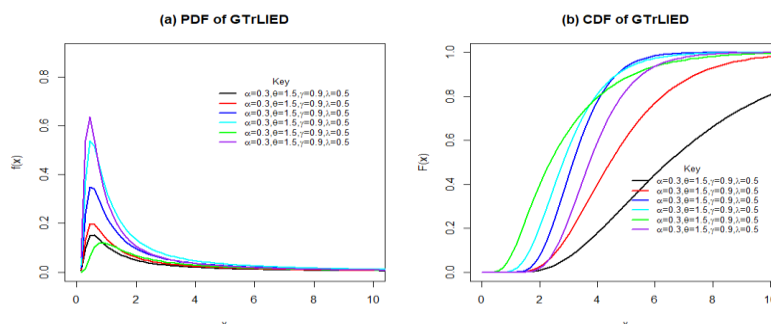


Figure 1: PDF and CDF of GTLIED for Selected Parameter Values

The plots of the PDF and CDF of the GTrLIED show that the proposed distribution is skewed and flexible, and that its shape depends on the values of the parameters. Also, the shapes can be classified as unimodal, right-skewed, nearly symmetrical, and asymmetrical shapes, as shown in Figures 1(a). The plot of CDF all converges to one as expected which also confirms the validity of the GTrLIED.

Simplification of the PDF of the GTrLIED

This section presents a useful expansion and mixture or linear representation of the PDF of the GTrLIED. Recall that the pdf of the GTrLIED is given as:

$$f(x) = \frac{\alpha^2 \gamma \theta x^{-2} e^{-\frac{\theta}{x}}}{(\alpha+1)} \left[1 + \frac{\theta}{x} \right] \left(1 + \frac{\alpha \theta}{(\alpha+1)x} \right)^{\gamma-1} \left[1 - e^{-\frac{\theta}{x}} \right]^{\alpha \gamma - 1} \left\{ 1 - \lambda + 2\lambda \left(\left(1 + \frac{\alpha \theta}{(\alpha+1)x} \right) \left(1 - e^{-\frac{\theta}{x}} \right)^\alpha \right)^\gamma \right\}$$

$$f(x) = \frac{\alpha^2 (1-\lambda) \gamma \theta x^{-2} e^{-\frac{\theta}{x}}}{(\alpha+1)} \left[1 + \frac{\theta}{x} \right] \left(1 + \frac{\alpha \theta}{(\alpha+1)x} \right)^{\gamma-1} \left[1 - e^{-\frac{\theta}{x}} \right]^{\alpha \gamma - 1} + \frac{2\lambda \alpha^2 \gamma \theta x^{-2} e^{-\frac{\theta}{x}}}{(\alpha+1)} \left[1 + \frac{\theta}{x} \right] \left(1 + \frac{\alpha \theta}{(\alpha+1)x} \right)^{2(\gamma-1)} \left[1 - e^{-\frac{\theta}{x}} \right]^{\gamma(\alpha+1)-1} \quad (7)$$

Using binomial expansion on the last exponential term in (7) gives:

$$\left[1 - e^{-\frac{\theta}{x}} \right]^{\gamma(\alpha+1)-1} = \sum_{k=0}^{\infty} (-1)^k \binom{\gamma(\alpha+1)-1}{k} e^{-\frac{k\theta}{x}} \quad (8)$$

By substituting the result above and simplifying, the pdf becomes:

$$f(x) = \sum_{k=0}^{\infty} \binom{\alpha \gamma - 1}{k} \frac{\alpha^2 (1-\lambda) \gamma \theta x^{-2} e^{-\frac{\theta}{x}}}{(-1)^k (\alpha+1)} \left[1 + \frac{\theta}{x} \right] \left(1 + \frac{\alpha \theta}{(\alpha+1)x} \right)^{\gamma-1} + \sum_{k=0}^{\infty} \binom{\gamma(\alpha+1)-1}{k} \frac{2\lambda \alpha^2 \gamma \theta x^{-2} e^{-\frac{\theta}{x}}}{(-1)^k (\alpha+1)} \left[1 + \frac{\theta}{x} \right] \left(1 + \frac{\alpha \theta}{(\alpha+1)x} \right)^{2(\gamma-1)} \quad (9)$$

Again, using binomial expansion on the last term in (9) gives:

$$\left(1 + \frac{\alpha \theta}{(\alpha+1)x} \right)^{2(\gamma-1)} = \sum_{l=0}^{\infty} \binom{2(\gamma-1)}{l} \left(\frac{\alpha \theta}{(\alpha+1)x} \right)^l \quad (10)$$

By substituting the result above and simplifying, the pdf becomes:

$$f(x) = \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \binom{\alpha \gamma - 1}{k} \binom{\gamma - 1}{l} \frac{\alpha^{l+2} (1-\lambda) \gamma \theta^{l+1} x^{-l-2} e^{-\frac{\theta}{x}}}{(-1)^k (\alpha+1)^{l+1}} \left[1 + \frac{\theta}{x} \right] + \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \binom{\gamma(\alpha+1)-1}{k} \binom{2(\gamma-1)}{l} \frac{2\lambda \alpha^{l+2} \gamma \theta^{l+1} x^{-l-2} e^{-\frac{\theta}{x}}}{(-1)^k (\alpha+1)^{l+1}} \left[1 + \frac{\theta}{x} \right] \quad (11)$$

Let

$$\eta_A = \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \binom{\alpha \gamma - 1}{k} \binom{\gamma - 1}{l} \frac{\alpha^{l+2} (1-\lambda) \gamma \theta^{l+1}}{(-1)^k (\alpha+1)^{l+1}} \quad \text{and} \quad \eta_B = \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \binom{\gamma(\alpha+1)-1}{k} \binom{2(\gamma-1)}{l} \frac{2\lambda \alpha^{l+2} \gamma \theta^{l+1}}{(-1)^k (\alpha+1)^{l+1}} \quad \text{such that the}$$

simplified form of the pdf of the GTLIED can be expressed as:

$$f(x) = \eta_A x^{-l-2} e^{-\frac{\theta}{x}} \left[1 + \frac{\theta}{x} \right] + \eta_B x^{-l-2} e^{-\frac{\theta}{x}} \left[1 + \frac{\theta}{x} \right] \quad (12)$$

Hence, equation (12) is a simplified form or linear representation of the pdf and will be used for the derivation of some properties of the GTLIED.

$$\mu'_n = E(X^n) = -\eta_A \left[\frac{\Gamma(l-n+1)}{(\theta(k+1))^{l=n+1}} + \frac{\theta \Gamma(l-n+2)}{(\theta(k+1))^{l=n+2}} \right] - \eta_B \left[\frac{\Gamma(l-n+1)}{(\theta(k+1))^{l=n+1}} + \frac{\theta \Gamma(l-n+2)}{(\theta(k+1))^{l=n+2}} \right]$$

$$\mu'_n = E(X^n) = \left[\frac{\Gamma(l-n+1)}{(\theta(k+1))^{l=n+1}} + \frac{\theta \Gamma(l-n+2)}{(\theta(k+1))^{l=n+2}} \right] [-\eta_A - \eta_B] \quad (17)$$

The formula in equation (17) can be used to obtain the mean (μ'_1), variance (σ^2), coefficient of variation (CV), coefficient of skewness (CS) and coefficient of kurtosis (CK) based on some well-known relationships.

Moment Generating Function

The moment generating function of a continuous random variable X can be obtained as

Statistical Properties of the Generalized Transmuted Lindley-inverse exponential Distribution (GTLIED)

This section presents various statistical properties of the GTLIED. These are presented as follows:

Moments

Let X denote a continuous random variable the n^{th} moment of X is given by;

$$\mu'_n = E(X^n) = \int_0^{\infty} x^n f(x) dx \quad (13)$$

where f(x), the simplified PDF of the GTLIED is as given in Equation (12) as:

$$f(x) = \eta_A x^{-l-2} e^{-\frac{\theta}{x}} \left[1 + \frac{\theta}{x} \right] + \eta_B x^{-l-2} e^{-\frac{\theta}{x}} \left[1 + \frac{\theta}{x} \right] \quad (14)$$

Substituting Equation (14) in (13) and simplifying the results yields;

$$\mu'_n = E(X^n) = \int_0^{\infty} x^n f(x) dx = \eta_A \int_0^{\infty} x^{n-l-2} e^{-\frac{\theta}{x}} dx + \theta \int_0^{\infty} x^{n-l-3} e^{-\frac{\theta}{x}} dx + \eta_B \int_0^{\infty} x^{n-l-2} e^{-\frac{\theta}{x}} dx + \theta \int_0^{\infty} x^{n-l-3} e^{-\frac{\theta}{x}} dx \quad (15)$$

Making use of integration by substitution method in Equation (15) gives the following results:

$$u = \theta(k+1)x^{-1} \Rightarrow x = \left(\frac{u}{\theta(k+1)} \right)^{-1} \quad \text{such that} \quad \frac{du}{dx} = -\theta(k+1)x^{-2} \Rightarrow dx = -\frac{du}{\theta(k+1)x^{-2}}$$

Substituting for x , u and dx in Equation (15) and simplifying the result yields:

$$\mu'_n = E(X^n) = \eta_A \left(-\frac{1}{\theta(k+1)} \right) \left(\frac{1}{\theta(k+1)} \right)^{l-n} \int_0^{\infty} u^{l-n+1} e^{-u} du + \theta \left(\frac{1}{\theta(k+1)} \right)^{l-n} \int_0^{\infty} u^{l-n+1} e^{-u} du + \eta_B \left(-\frac{1}{\theta(k+1)} \right) \left(\frac{1}{\theta(k+1)} \right)^{l-n} \int_0^{\infty} u^{l-n+1} e^{-u} du + \theta \left(\frac{1}{\theta(k+1)} \right)^{l-n} \int_0^{\infty} u^{l-n+1} e^{-u} du \quad (16)$$

$$\int_0^{\infty} t^{k-1} e^{-t} dt = \Gamma(k) \quad \text{and that} \quad \int_0^{\infty} t^k e^{-t} dt = \int_0^{\infty} t^{k+1-1} e^{-t} dt = \Gamma(k+1)$$

Using the statement above, the n^{th} ordinary moment of X for the GTLIED is obtained as:

$$M_x(t) = E[e^{tx}] = \int_{-\infty}^{\infty} e^{tx} f(x) dx \quad (18)$$

Recall that by power series expansion,

$$e^{tx} = \sum_{n=0}^{\infty} \frac{(tx)^n}{n!} = \sum_{n=0}^{\infty} \frac{t^n}{n!} x^n \quad (19)$$

Therefore, the moment generating function can also be expressed as:

$$M_x(t) = \sum_{n=0}^{\infty} \frac{t^n}{n!} \int_0^{\infty} x^n f(x) dx = \sum_{n=0}^{\infty} \frac{t^n}{n!} E(X^n) \quad (20)$$

Using the result in Equation (20) and simplifying the integral in (18) therefore yields:

$$M_X(t) = \sum_{n=0}^{\infty} \frac{t^n}{n!} \left[\frac{\Gamma(l-n+1)}{(\theta(k+1))^{l-n+1}} + \frac{\theta(l-n+2)}{(\theta(k+1))^{l-n+2}} \right] [-\eta_A - \eta_B] \quad (21)$$

Characteristics Function

A representation for the characteristics function of a continuous random variable is given by

$$\phi_X(t) = E(e^{itx}) = \int_0^{\infty} e^{itx} f(x) dx \quad (22)$$

Recall that by power series expansion,

$$e^{itx} = \sum_{n=0}^{\infty} \frac{(itx)^n}{n!} = \sum_{n=0}^{\infty} \frac{(it)^n}{n!} x^n \quad (23)$$

Hence, simple algebra and use of Equation (22) produce the following results:

$$\begin{aligned} \phi_X(t) &= E(e^{itx}) = \sum_{n=0}^{\infty} \frac{(it)^n}{n!} \int_0^{\infty} x^n f(x) dx = \sum_{n=0}^{\infty} \frac{(it)^n}{n!} E(X^n) = \sum_{n=0}^{\infty} \frac{(it)^n}{n!} [\mu_n'] \\ \phi_X(t) &= E(e^{itx}) = \sum_{n=0}^{\infty} \frac{(it)^n}{n!} \left[\frac{\Gamma(l-n+1)}{(\theta(k+1))^{l-n+1}} + \frac{\theta(l-n+2)}{(\theta(k+1))^{l-n+2}} \right] [-\eta_A - \eta_B] \end{aligned} \quad (24)$$

$$h(x) = \frac{\frac{\alpha^2 \gamma \theta x^{-2} e^{-\frac{\theta}{x}}}{(\alpha+1)} \left[1 + \frac{\theta}{x} \right] \left(1 + \frac{\alpha \theta}{(\alpha+1)x} \right)^{\gamma-1} \left[1 - e^{-\frac{\theta}{x}} \right]^{\alpha \gamma - 1} \left\{ 1 - \lambda + 2\lambda \left(\left(1 + \frac{\alpha \theta}{(\alpha+1)x} \right) \left(1 - e^{-\frac{\theta}{x}} \right)^{\alpha} \right)^{\gamma} \right\}}{(1+\lambda) \left(\left(1 + \frac{\alpha \theta}{(\alpha+1)x} \right) \left(1 - e^{-\frac{\theta}{x}} \right)^{\alpha} \right)^{\gamma} - \lambda + \lambda \left[1 - \left(\left(1 + \frac{\alpha \theta}{(\alpha+1)x} \right) \left(1 - e^{-\frac{\theta}{x}} \right)^{\alpha} \right)^{\gamma} \right]^2} \quad (28)$$

Below is a graphical representation of the survival and hazard functions at chosen parameter values. The plots are as given in the figure 2 below:

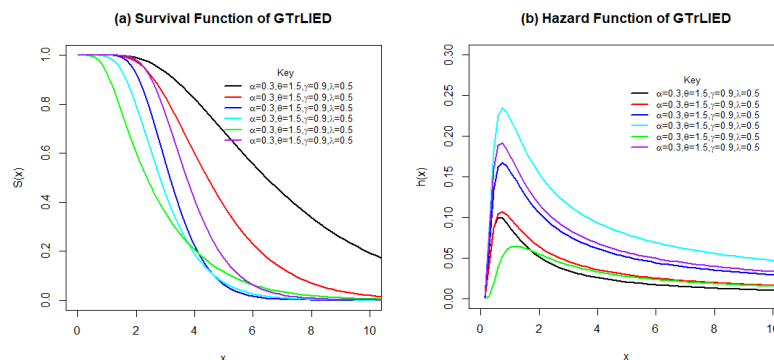


Figure 2: The Survival and Hazard Functions of the GTLIED

Figure 2(a) above revealed that the probability of survival for any random variable following a GTLIED decreases with time, that is, as age grows, probability of life decreases. This implies that the GTLIED can be used to model random variables whose survival rate decreases as their age grows.

Maximum Likelihood Estimation of the Unknown Parameters of the GTLIED

Let $X_1, X_2, \dots, X_n$ be a sample of size 'n' independently and identically distributed random variables from the

$$\begin{aligned} L(\underline{X} | \alpha, \theta, \gamma, \lambda) &= \left(\frac{\alpha^2 \gamma \theta}{(\alpha+1)} \right)^n \prod_{i=1}^n \left\{ x_i^{-2} e^{-\frac{\theta}{x_i}} \left[1 + \frac{\theta}{x_i} \right] \left(1 + \frac{\alpha \theta}{(\alpha+1)x_i} \right)^{\gamma-1} \left[1 - e^{-\frac{\theta}{x_i}} \right]^{\alpha \gamma - 1} \right\} \\ &\quad \prod_{i=1}^n \left\{ \left\{ 1 - \lambda + 2\lambda \left(\left(1 + \frac{\alpha \theta}{(\alpha+1)x_i} \right) \left(1 - e^{-\frac{\theta}{x_i}} \right)^{\alpha} \right)^{\gamma} \right\} \right\} \end{aligned} \quad (29)$$

Reliability Analysis

Survival Function

Survival function (SF) is the likelihood that a system or an individual will not fail after a given time. Mathematically, the survival function is given by:

$$S(x) = 1 - F(x) \quad (25)$$

Now, taking $F(x)$ to be the cdf of the proposed GTLIED and substituting, we have;

$$S(x) = (1+\lambda) \left(\left(1 + \frac{\alpha \theta}{(\alpha+1)x} \right) \left(1 - e^{-\frac{\theta}{x}} \right)^{\alpha} \right)^{\gamma} - \lambda + \lambda \left[1 - \left(\left(1 + \frac{\alpha \theta}{(\alpha+1)x} \right) \left(1 - e^{-\frac{\theta}{x}} \right)^{\alpha} \right)^{\gamma} \right]^2 \quad (26)$$

Hazard Function

Hazard function (HF) is the probability that a component will fail or die for an interval of time. The hazard function is defined as;

$$h(x) = \frac{f(x)}{1 - F(x)} = \frac{f(x)}{S(x)} \quad (27)$$

Substituting for $f(x)$ and $F(x)$, and simplifying gives the hazard function of GTLIED as follows:

GTLIED with unknown parameters, α , θ , γ , and λ defined previously.

The likelihood function of the GTLIED using the pdf in Equation (6) is given by:

Let the natural logarithm of the likelihood function be, $l = \log L(X | \alpha, \theta, \gamma, \lambda)$, therefore, taking the natural logarithm of the function Equation (29) above gives:

$$l = 2n \log \alpha + n \log \theta + n \log \gamma - n \log(\alpha + 1) - 2 \sum_{i=1}^n \log x_i - \theta \sum_{i=1}^n x_i + (\gamma - 1) \sum_{i=1}^n \log \left(1 + \frac{\alpha \theta}{(\alpha + 1) x_i} \right) + \sum_{i=1}^n \log \left[1 + \frac{\theta}{x_i} \right] + (\alpha \gamma - 1) \sum_{i=1}^n \log \left(1 - e^{-\frac{\theta}{x_i}} \right) + \sum_{i=1}^n \log \left\{ 1 - \lambda + 2\lambda \left(\left(1 + \frac{\alpha \theta}{(\alpha + 1) x_i} \right) \left(1 - e^{-\frac{\theta}{x_i}} \right)^\alpha \right)^\gamma \right\} \quad (30)$$

Differentiating l partially with respect to α , θ , γ , and λ respectively gives the following results:

$$\frac{\partial l}{\partial \alpha} = \frac{2n}{\alpha} - \frac{n}{(\alpha + 1)} + \theta(\gamma - 1) \sum_{i=1}^n \left\{ \frac{x_i [(\alpha + 1) x_i]^{-2}}{\left(1 + \frac{\alpha \theta}{(\alpha + 1) x_i} \right)} \right\} + \gamma \sum_{i=1}^n \log \left(1 - e^{-\frac{\theta}{x_i}} \right) + 2\lambda \gamma \sum_{i=1}^n \left\{ \frac{\left(1 - e^{-\frac{\theta}{x_i}} \right)^\alpha \left[\frac{\theta x_i}{((\alpha + 1) x_i)^2} + \left(1 + \frac{\alpha \theta}{(\alpha + 1) x_i} \right) \ln \left(1 - e^{-\frac{\theta}{x_i}} \right) \right]}{\left[1 - \lambda + 2\lambda \left(\left(1 + \frac{\alpha \theta}{(\alpha + 1) x_i} \right) \left(1 - e^{-\frac{\theta}{x_i}} \right)^\alpha \right)^\gamma \right] \left(\left(1 + \frac{\alpha \theta}{(\alpha + 1) x_i} \right) \left(1 - e^{-\frac{\theta}{x_i}} \right)^\alpha \right)^{-(\gamma-1)}} \right\} \quad (31)$$

$$\frac{\partial l}{\partial \theta} = \frac{n}{\theta} + \sum_{i=1}^n x_i + \sum_{i=1}^n \left\{ \frac{x_i^{-1}}{\left(1 + \frac{\theta}{x_i} \right)} \right\} + \frac{(\gamma - 1)}{(\alpha + 1)} \sum_{i=1}^n \left\{ \frac{\alpha}{\left(1 + \frac{\alpha \theta x_i^{-1}}{(\alpha + 1)} \right)} \right\} - (\alpha \gamma - 1) \sum_{i=1}^n \left\{ \frac{x_i^{-1} e^{-\frac{\theta}{x_i}}}{\left(1 - e^{-\frac{\theta}{x_i}} \right)} \right\} - \frac{2\alpha \lambda \gamma}{(\alpha + 1)} \sum_{i=1}^n \left\{ \frac{\left[1 + \frac{((\alpha + 1) x_i + \alpha \theta) e^{-\frac{\theta}{x_i}}}{\left(1 - e^{-\frac{\theta}{x_i}} \right) x_i} \right] \left(\left(1 + \frac{\alpha \theta}{(\alpha + 1) x_i} \right) \left(1 - e^{-\frac{\theta}{x_i}} \right)^\alpha \right)^{(\gamma-1)}}{\left[1 - \lambda + 2\lambda \left(\left(1 + \frac{\alpha \theta}{(\alpha + 1) x_i} \right) \left(1 - e^{-\frac{\theta}{x_i}} \right)^\alpha \right)^\gamma \right] x_i \left(1 - e^{-\frac{\theta}{x_i}} \right)^{-\alpha}} \right\} \quad (32)$$

$$\frac{\partial l}{\partial \gamma} = \frac{n}{\gamma} + \sum_{i=1}^n \log \left(1 + \frac{\alpha \theta}{(\alpha + 1) x_i} \right) + \alpha \sum_{i=1}^n \log \left(1 - e^{-\frac{\theta}{x_i}} \right) + 2\lambda \sum_{i=1}^n \left\{ \frac{\left(\left(1 + \frac{\alpha \theta}{(\alpha + 1) x_i} \right) \left(1 - e^{-\frac{\theta}{x_i}} \right)^\alpha \right)^\gamma \ln \left(\left(1 + \frac{\alpha \theta}{(\alpha + 1) x_i} \right) \left(1 - e^{-\frac{\theta}{x_i}} \right)^\alpha \right)}{\left[1 - \lambda + 2\lambda \left(\left(1 + \frac{\alpha \theta}{(\alpha + 1) x_i} \right) \left(1 - e^{-\frac{\theta}{x_i}} \right)^\alpha \right)^\gamma \right]} \right\} \quad (33)$$

$$\frac{\partial l}{\partial \lambda} = 2 \sum_{i=1}^n \left\{ \frac{\left(\left(1 + \frac{\alpha \theta}{(\alpha + 1) x_i} \right) \left(1 - e^{-\frac{\theta}{x_i}} \right)^\alpha \right)^\gamma - 1}{\left[1 - \lambda + 2\lambda \left(\left(1 + \frac{\alpha \theta}{(\alpha + 1) x_i} \right) \left(1 - e^{-\frac{\theta}{x_i}} \right)^\alpha \right)^\gamma \right]} \right\} \quad (34)$$

Making Equation (31), (32), (33) and (34) equal to zero (0) and solving for the solution of the non-linear system of equations produce the maximum likelihood estimates of parameters α , θ , γ , and λ . Note that it is difficult to solve the above equations analytically and therefore the Newton-Raphson's iteration method is applied using computer applications such as Python, R, Mat lab or any other suitable software.

RESULTS AND DISCUSSION

Application to Bladder Cancer Data

The aim of this section is to provide an application and evaluation of the performance of the generalized transmuted

Lindley-inverse exponential distribution (GTLIED) compared to some other extensions of the inverse exponential distribution which include the Lindley-inverse exponential distribution (LIED) by makama et al., (2025), the exponential inverse exponential distribution (EIED) by Oguntunde et al. (2017), odd Lindley-inverse exponential distribution (OLIED) by Ieren and Abdullahi (2020), the odd Lomax-inverse exponential distribution (OLOmIED) by Ieren et al. (2021), the inverse exponential distribution (IED), Lindley distribution (LnD) and exponential distribution (ExD).

The MLEs of the model parameters are determined and some goodness-of-fit statistics for these distributions are compared

using a dataset on the remission times of a random sample of 128 bladder cancer patients.

The model selection is carried out based upon the value of the log-likelihood function evaluated at the MLEs, ℓ , Akaike Information Criterion, AIC , Consistent Akaike Information Criterion, $CAIC$, Bayesian Information Criterion, BIC , Hannan Quin Information Criterion, $HQIC$, Anderson-Darling (A^*), Cramér-Von Mises (W^*) and Kolmogorov-smirnov (K-S) statistics. The details about the statistics A^* , W^* and K-S are discussed in Chen and Balakrishnan (1995). Meanwhile, the smaller these statistics are, the better the fit of the distribution is. The required computations are carried out using the R package “Adequacy Model” which is freely available from <http://cran.r-project.org/web/packages/AdequacyModel/AdequacyModel.pdf>.

Dataset

This data represents the remission times (in months) of a random sample of 128 bladder cancer patients. It has

previously been used by Lee and Wang (2003), Rady *et al.* (2016), Ieren and Chukwu (2018), Abdullahi *et al.* (2018) and Ieren and Balogun (2021). It is given and summarized as follows:

0.080, 0.200, 0.400, 0.500, 0.510, 0.810, 0.900, 1.050, 1.190, 1.260, 1.350, 1.400, 1.460, 1.760, 2.020, 2.020, 2.070, 2.090, 2.230, 2.260, 2.460, 2.540, 2.620, 2.640, 2.690, 2.690, 2.750, 2.830, 2.870, 3.020, 3.250, 3.310, 3.360, 3.360, 3.480, 3.520, 3.570, 3.640, 3.700, 3.820, 3.880, 4.180, 4.230, 4.260, 4.330, 4.340, 4.400, 4.500, 4.510, 4.870, 4.980, 5.060, 5.090, 5.170, 5.320, 5.320, 5.340, 5.410, 5.410, 5.490, 5.620, 5.710, 5.850, 6.250, 6.540, 6.760, 6.930, 6.940, 6.970, 7.090, 7.260, 7.280, 7.320, 7.390, 7.590, 7.620, 7.630, 7.660, 7.870, 7.930, 8.260, 8.370, 8.530, 8.650, 8.660, 9.020, 9.220, 9.470, 9.740, 10.06, 10.34, 10.66, 10.75, 11.25, 11.64, 11.79, 11.98, 12.02, 12.03, 12.07, 12.63, 13.11, 13.29, 13.80, 14.24, 14.76, 14.77, 14.83, 15.96, 16.62, 17.12, 17.14, 17.36, 18.10, 19.13, 20.28, 21.73, 22.69, 23.63, 25.74, 25.82, 26.31, 32.15, 34.26, 36.66, 43.01, 46.12, 79.05.

Table 1: Summary Statistics for the Bladder Cancer Dataset

parameters	n	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis
Values	128	0.0800	3.348	6.395	11.840	9.366	79.05	110.425	3.3257	19.1537

Based on the descriptive statistics in table 1, it can be seen that the dataset on the remission times of a random sample of 128 bladder cancer patients is heavily skewed to the right or positively skewed which could be good with skewed models.

Table 2: Maximum Likelihood Parameter Estimates for the Bladder Cancer Dataset

(1) Distribution	(2) Parameter Estimates				
(3) GTLIED	(4) $\hat{\theta} = 2.0559227$	(5) $\hat{\alpha} = 1.1528563$	(6) $\hat{\gamma} = 9.287719$	(7) $\hat{\lambda} = -0.9237027$	
(8) LIED	(9) $\hat{\theta} = 1.7089933$	(10) $\hat{\alpha} = 1.0149521$	(11) -	(12) -	
(13) OLIED	(14) $\hat{\theta} = 1.2565151$	(15) $\hat{\alpha} = 0.2779616$	(16) -	(17) -	
(18) EIED	(19) $\hat{\theta} = 0.9799743$	(20) $\hat{\alpha} = 0.1018989$	(21) -	(22) -	
(23) IED	(24) $\hat{\theta} = 2.4847288$	(25) -	(26) -	(27) -	
(28) ExD	(29) $\hat{\theta} = 0.1076452$	(30) -	(31) -	(32) -	
(33) LnD	(34) $\hat{\theta} = 0.1960113$	(35) -	(36) -	(37) -	
(38) OLomIED	(39) $\hat{\theta} = 0.9961264$	(40) $\hat{\alpha} = 1.7080106$	(41) $\hat{\beta} = 9.167488$	(42) -	

Table 3: The Statistics ℓ , AIC, CAIC, BIC and HQIC for the Bladder Cancer Dataset

(43) Distribution	(44) $\hat{\ell}$	(45) AIC	(46) CAIC	(47) BIC	(48) HQIC	(49) Ranks
(50) GTLIED	(51) 411.2404	(52) 830.4809	(53) 830.8061	(54) 841.8890	(55) 835.1160	(56) 1 st
(57) LIED	(58) 448.7825	(59) 901.5649	(60) 901.6609	(61) 907.2690	(62) 903.8825	(63) 7 th
(64) OLIED	(65) 433.5512	(66) 871.1024	(67) 871.1984	(68) 876.8065	(69) 873.4200	(70) 6 th
(71) EIED	(72) 419.2041	(73) 842.4082	(74) 842.5042	(75) 848.1123	(76) 844.7258	(77) 4 th
(78) IED	(79) 460.3823	(80) 922.7646	(81) 922.7963	(82) 925.6166	(83) 923.9234	(84) 8 th
(85) ExD	(86) 418.5461	(87) 839.0922	(88) 839.1239	(89) 841.9442	(90) 840.251	(91) 2 nd
(92) LnD	(93) 419.5299	(94) 841.0598	(95) 841.0916	(96) 843.9118	(97) 842.2186	(98) 3 rd
(99) OLomIED	(100) 425.3138	(101) 856.6277	(102) 856.8212	(103) 865.1838	(104) 860.1041	(105) 5 th

Table 4: The A^* , W^* , K-S Statistic and P-Values for the Bladder Cancer Dataset

(106) Distribution	(107) A^*	(108) W^*	(109) K-S	(110) P-Value (K-S)	(111) Ranks
(112) GTLIED	(113) 0.2526352	(114) 0.03507922	(115) 0.07628378	(116) 0.4457104	(117) 1 st
(118) LIED	(119) 5.488345	(120) 0.90915	(121) 0.1803676	(122) 0.0004831567	(123) 7 th
(124) OLIED	(125) 1.083013	(126) 0.1696767	(127) 0.09505	(128) 0.1977689	(129) 6 th
(130) EIED	(131) 1.140581	(132) 0.175374	(133) 0.09126863	(134) 0.2366932	(135) 4 th
(136) IED	(137) 6.605157	(138) 1.113517	(139) 0.231563	(140) 2.184898e-06	(141) 8 th

(106) Distribution	(107) A*	(108) W*	(109) K-S	(110) P-Value (K-S)	(111) Ranks
(142) ExD	(143) 0.7181843	(144) 0.1196739	(145) 0.08605178	(146) 0.299442	(147) 2 nd
(148) LnD	(149) 1.02567	(150) 0.171706	(151) 0.1164766	(152) 0.06203969	(153) 3 rd
(154) OLomIED	(155) 1.704334	(156) 0.2570522	(157) 0.131587	(158) 0.02376532	(159) 5 th

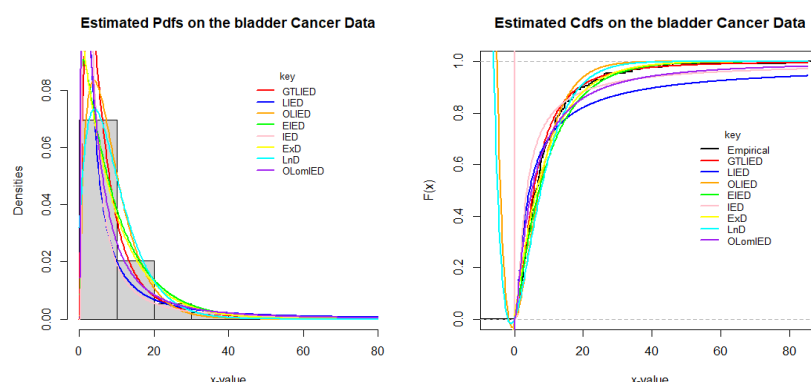


Figure 3: Histogram and Plots of the Estimated Densities and cdfs of the GTLIED and Other Fitted Distributions to the Blader Cancer Dataset

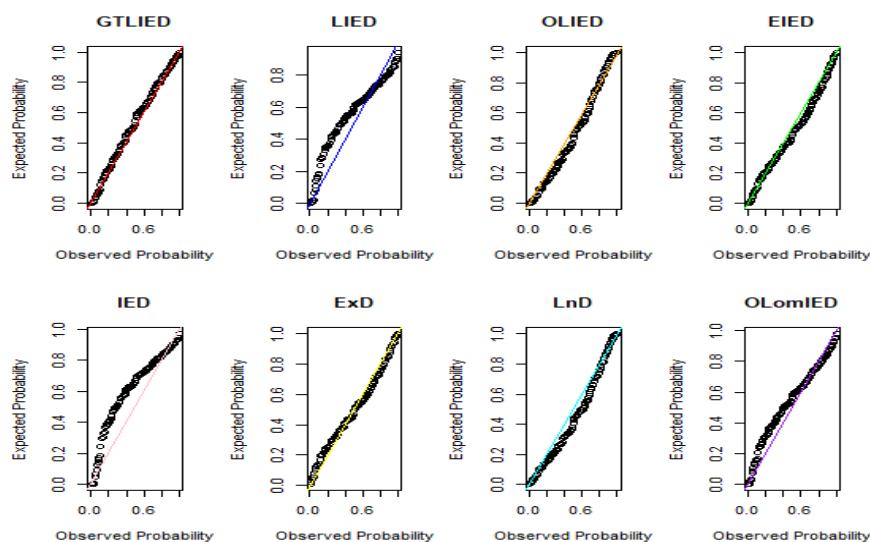


Figure 4: Probability Plots for the Fit of the GTLIED and Other Fitted Models Based on the Bladder Cancer Dataset

Table 2 list the values of the MLEs of the model parameters based on the bladder cancer dataset, whereas the values of AIC, CAIC, BIC and HQIC are listed in Table 3 for the same dataset. Also, the values of A*, W* and K-S for the same dataset are provided in Table 4.

The plots of the fitted GTLIED density and cumulative distribution with those of competing distributions based on the bladder cancer dataset are displayed in Figure 3. The PP-plots of the fitted distributions are given in Figure 4 for bladder cancer dataset. Based on the results, all the measures listed above indicate that the GTLIED provides the best fit and can be considered as a very good and more flexible model compared to the other seven distributions applied to this same dataset in this study. This indicates that the proposed GTLIED is a very flexible model due to its ability to provide a better fit for dataset as demonstrated in this research.

CONCLUSION

The study successfully developed the Generalized Transmuted Lindley-Inverse Exponential Distribution (GTLIED) as a flexible extension of the Lindley-Inverse Exponential Distribution. The proposed model incorporates

additional shape parameters that enhance its ability to model a wide range of lifetime data characteristics, including strong skewness and diverse hazard rate behaviors. Mathematical expressions for important statistical and reliability measures were derived, and parameter estimation was achieved through the maximum likelihood method. Application to bladder cancer remission-time data demonstrated the effectiveness of the GTLIED in real-world situations. Compared with several established lifetime distributions, the GTLIED produced the best goodness-of-fit results across multiple model selection criteria and statistical tests. Therefore, the distribution provides a valuable addition to the family of lifetime and reliability models and can be recommended for analyzing positively skewed survival and reliability datasets. Its flexibility and superior fitting performance suggest strong potential for future applications in medical, engineering, and actuarial research.

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CONFLICTS OF INTEREST

The authors declare no conflicts of interest

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