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Markov Chain Monte Carlo Modelling of the Impact of Climate Change on the Growth of Millet and Soybean in North Central, Nigeria

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ABSTRACT

This study investigates the spatio-temporal dynamics of rainfall and temperature suitability for soybean and millet production across states in North Central Nigeria using a joint binary Markov model. Monthly rainfall and temperature data obtained from the National Aeronautics and Space Administration (NASA) for the period 1984–2022 were classified into favourable (1) and unfavourable (0) crop-growth conditions based on crop-specific requirements. Four joint climatic states, (0,0), (0,1), (1,0), and (1,1), were defined to represent combined rainfall-temperature suitability. Transition probability matrices were estimated to assess climatic persistence and state transitions, while MCMC methods were used for parameter estimation. Convergence and reliability of the posterior estimates were confirmed using the Gelman–Rubin diagnostic.The results revealed marked spatial differences in climatic suitability across the study area. Benue and Kwara exhibited the highest persistence of favourable climatic conditions for millet production, with probabilities of remaining in the favourable state (1,1) of 0.6843 and 0.6661, respectively, while Kogi also showed strong climatic stability (0.5675). In contrast, Plateau displayed the highest persistence of unfavourable conditions, remaining in state (0,0) with probability 0.6668. Niger and the FCT showed greater climatic variability, whereas Nasarawa exhibited mixed persistence in both favourable (0.5483) and unfavourable (0.5345) states. These findings highlight significant regional variations in rainfall–temperature suitability and provide useful information for agricultural planning, climate risk management, and adaptation strategies. The joint binary Markov model estimated using MCMC offers a robust framework for assessing climate variability and its implications for soybean and millet production.

Keywords: Joint Binary Markov Process, Rainfall–Temperature Dynamics, MCMC Estimation, Gelman–Rubin Diagnostic, Climate Variability, Soybean Production, Millet Production

INTRODUCTION

Binary Markov processes are foundational tools for modeling systems with two discrete state dynamics, with applications spanning ecology, epidemiology, reliability theory, and Bayesian computation (Norris, 1997; Ross, 2014). When multiple binary Markov chains arise within a single system, a central modeling decision concerns how to combine these components into a unified stochastic framework. A standard construction is to define an independent joint Markov chain on the product state space, while alternative approaches often rely on logical combinations of individual chains through operators such as AND, OR, or XOR. Related formulations appear in probabilistic logic and stochastic Boolean systems, where logical rules are embedded within probabilistic models to describe interactions among binary variables (Domingos & Richardson, 2007; Shmulevich et al., 2002; Murrugarra et al., 2012).

Several recent studies have explored alternative strategies for combining multiple Markov chains beyond independent joint constructions. In particular, Adah et al. (2025) proposed using logical connectives, including AND, and OR-based rules, to aggregate multiple binary Markov chains into a single derived process. While this approach offers intuitive interpretability and dimensional reduction, it implicitly collapses the underlying joint state space and obscures latent dependence structures. Similar aggregation mechanisms arise in stochastic Boolean networks and probabilistic rule-based systems, where logical update functions define observable states while underlying dependencies remain partially hidden (Shmulevich et al., 2002; Abou-Jaoudé et al., 2016). As a consequence, the resulting process generally fails to retain the Markov property, since its transition dynamics cannot be expressed solely as a function of the observed logical state but instead depend on unobserved joint configurations. This phenomenon is closely related to classical results on functions of Markov chains, where non-invertible transformations typically destroy the Markov property (Doob, 1953; Kemeny & Snell, 1976).

The dependent (interacting) coupling of two binary Markov chains provides a realistic framework for modeling climate variables by capturing the dependence between rainfall and temperature processes. Unlike independent coupling, this approach allows the transition probabilities of one variable to depend on the state of the other, thereby representing persistence, feedback mechanisms, and joint climate events more effectively. Consequently, dependent coupling improves predictive accuracy and enhances the analysis of climate variability and uncertainty in environmental applications (Norris, 1997; Wilks, 2011; Zucchini et al., 2016). Furthermore, the coupled process preserves the Markov property, which is essential for analytical tractability, since it allows the transition dynamics, stationary distributions, and long-run behavior of the system to be rigorously characterized. In contrast, stochastic processes obtained through logical transformations of component chains often fail to retain the Markov property because their transition probabilities may depend on hidden joint configurations rather than solely on the observed state (Doob, 1953; Ross, 2014). Similar findings have been reported in probabilistic logic frameworks, where marginalization over latent variables introduces dependencies that cannot be adequately represented within a simple Markovian structure (Domingos & Richardson, 2007).

Within Bayesian statistical modeling, Markov chain Monte Carlo (MCMC) methods provide an indispensable computational framework for posterior inference when closed-form solutions are unavailable. MCMC algorithms construct Markov chains whose stationary distributions coincide with target posterior distributions, allowing inference to proceed without explicit evaluation of normalizing constants (Robert & Casella, 2004; Brooks et al., 2011). This paradigm has become central to modern Bayesian analysis, particularly for high-dimensional hierarchical models, state-space systems, and latent variable formulations where analytical integration is infeasible.

Influential contributions by Carlin and Chib (1995) demonstrated how MCMC enables flexible Bayesian model fitting and model comparison by traversing complex posterior landscapes. Subsequent methodological advances have focused on improving convergence, mixing, and computational efficiency, including coupling strategies, parallel and interacting chains, and sophisticated diagnostic tools (Gelman et al., 2013; Jacob et al., 2020). These developments underscore the intimate relationship between Markov process structure and Bayesian computational performance. In particular, poorly specified state representations or hidden dependencies can significantly degrade convergence behavior and lead to misleading posterior Despite the extensive literature on Markov Chain Monte Carlo methodology and the widespread application of Markov chain models in Bayesian inference, existing studies have largely focused on improving MCMC algorithms, convergence diagnostics, and sampling efficiency, while giving limited consideration to the implications of model construction itself for posterior inference (Brooks et al., 2011). In particular, several applied studies employ joint binary Markov formulations to represent interacting binary processes, yet these models are often treated primarily as latent-state or coupled systems without explicitly examining how alternative state representations affect the preservation of the Markov property and subsequent Bayesian estimation (Norris, 1997; Gilks, et al 1996).

This limitation is especially relevant in binary stochastic systems, where researchers frequently aggregate or combine component processes via logical or rule-based mechanisms for interpretability. Such transformations may introduce hidden dependence structures and potentially violate assumptions required for standard Markovian modeling and Bayesian inference, as observed in stochastic Boolean and logical-probabilistic systems (Murrugarra et al., 2012; Abou-Jaoudé et al., 2016

In this paper, we address this methodological gap by investigating binary Markov systems constructed through joint formulations and examining their inferential properties. We show that independent joint binary Markov constructions preserve essential Markovian properties required for rigorous statistical inference and facilitate effective posterior estimation through MCMC methods, even in complex high-dimensional settings where analytical solutions are infeasible. By clarifying the relationship between state representation, dependence structure, and Bayesian computation, this study contributes to a deeper understanding of model specification choices in stochastic process modeling and Bayesian inference. Specifically, this study develops a joint binary Markov framework for rainfall and temperature dynamics and evaluates its implications for millet and soybean production in North Central Nigeria

In many environmental and climatic applications, multiple interacting processes evolve simultaneously and can be naturally represented as binary Markov chains. In this study, we consider two such components: a rainfall state process and a temperature state process, each evolving on a binary state space. Each process is modeled independently as a first-order Markov chain, reflecting temporal dependence driven by persistence and short-memory dynamics commonly observed in climate variables (Wilks, 2011).

Kamara et al. (2023) showed that location-specific management strategies are necessary for optimal soybean production. To extend the environmental relevance of the framework, we also incorporate agricultural response variables, specifically the growth dynamics of millet and soybean in North Central Nigeria. Crop growth in this region is strongly influenced by climatic variability, particularly by fluctuations in rainfall and temperature. We represent the growth states of millet and soybean as binary processes (e.g., favorable vs. unfavorable growth conditions), allowing them to be modeled within the same Markovian structure. These crop processes may be linked to the underlying rainfall and temperature chains through conditional transition probabilities, capturing how climatic conditions influence crop performance over time. North Central Nigeria was selected because agriculture is predominantly rain-fed and highly vulnerable to climatic variability.

Millet (*Pennisetum glaucum*) is adapted to dry savanna conditions, performing well under 300–600 mm of rainfall and tolerating high temperatures of 25–35°C, with good resilience to heat but sensitivity to waterlogging (FAO, 2019). Soybean (*Glycine max*) requires higher moisture, about 500–900 mm of well-distributed rainfall, and grows optimally at 20–25°C, with yield reduction occurring when temperatures exceed 30°C, especially under moisture stress (Dugje et al., 2009; Adah and Agada 2019). Millet (*Pennisetum glaucum*) and soybean (*Glycine max*) were selected for this study since they are economically important crops in the region and possess contrasting climatic requirements.

This joint modeling approach provides a simplified yet powerful way to study the interaction between climate variability and agricultural productivity. In North Central Nigeria, where rain-fed agriculture dominates, such a framework can help quantify the persistence of favourable or adverse conditions for millet and soybean growth, assess the impact of short-term climate variability, and support decision-making in crop management and food security planning.

The joint evolution of rainfall and temperature can then be modeled in a principled manner by constructing an independent joint Markov chain on the product state space formed by the two binary processes. This formulation preserves the Markov property and explicitly captures the coupled dynamics of rainfall and temperature without collapsing or masking latent state information. By contrast, combining rainfall and temperature chains through logical connectives, such as defining a single aggregated state via AND/OR rules, induces an observable process whose transition structure generally depends on the unobserved joint states rather than the observed logical outcome alone. Similar issues arise in logical-probabilistic and Boolean network models, where aggregation obscures underlying dependencies (Shmulevich et al., 2002; Domingos and Richardson, 2007). As a result, such logically combined processes typically violate the Markov assumption and complicate both probabilistic interpretation and Bayesian inference

The study aims to model the joint dynamics of rainfall and temperature suitability for millet and soybean production in North Central Nigeria using a binary Markov framework using MCMC. Specifically, it examines the persistence and transition of favourable and unfavourable climatic states across the region. The study seeks to determine whether

climatic suitability conditions persist over time and whether transition patterns vary spatially across the region.

The novelty of this study lies in the application of a joint rainfall–temperature binary Markov model estimated using Markov Chain Monte Carlo (MCMC) methods. By modelling rainfall and temperature simultaneously rather than separately, the framework captures their combined influence on crop suitability and provides a probabilistic basis for assessing climate variability and supporting climate-smart agricultural planning.

MATERIALS AND METHODS

This study develops a Joint Binary Markov Model for rainfall and temperature using a Bayesian hierarchical framework. Rainfall and temperature are modeled as binary stochastic processes evolving through time according to first-order Markov dependence. Bayesian inference is employed to estimate transition probabilities, while hierarchical priors are introduced to account for uncertainty and heterogeneity across transition states. Parameter estimation is carried out using Markov Chain Monte Carlo (MCMC) methods.

Data Preprocessing and MCMC Implementation

Prior to analysis, the dataset was screened for completeness, consistency, and outliers. The dataset contained no missing observations; therefore, no imputation or missing-data treatment was required.The data were subsequently transformed into the required format for Markov Chain Monte Carlo (MCMC) technique.

Data transformation and MCMC estimation were implemented using Python (version 3.12) with the NumPy, SciPy, and PyMC packages

Data Preparation and Binary State Classification

Monthly rainfall and temperature observations are transformed into binary states for Markov chain analysis. The classification thresholds were based on the climatic suitability requirements of millet and soybean crops.

Let

$R_t \in \{0,1\}$

Represent the rainfall state at time t, where

$$R_t = \begin{cases} 1, & \text{if rainfallis suitable for crop growth at time } t, \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

Similarly, let

$T_t \in \{0,1\}$

denote the temperature state at time t, where

$$T_t = \begin{cases} 1, & \text{if temperature is suitable for crop growth at time, t,} \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

Table 1: Criteria for Classification of Climatic Suitability States

Variable	Crop	Favourable State (1)	Unfavourable State (0)
Rainfall (mm)	Millet	300–600 mm	<300 mm or >600 mm
Temperature (°C)	Millet	25–35°C	<25°C or >35°C
Rainfall (mm)	Soybean	500–900 mm	<500 mm or >900 mm
Temperature (°C)	Soybean	20–25°C	<20°C or >25°C

The resulting binary sequences form the basis for the Markov modeling procedure.

Construction of the Joint Binary Markov Process

To jointly model rainfall and temperature dynamics, define the bivariate process

$X_t = (R_t, T_t)$.

The joint state space is

$S = \{(0,0), (0,1), (1,0), (1,1)\}$.

where the first component denotes rainfall condition and the second component denotes temperature condition. A value of 0 represents an unfavourable condition for crop growth, while a value of 1 represents a favourable condi

tion. Hence, the states correspond respectively to:

(0,0): unfavourable rainfall and unfavourable temperature,
(0,1): unfavourable rainfall and favourable temperature,
(1,0): favourable rainfall and unfavourable temperature,
(1,1): favourable rainfall and favourable temperature.

The process satisfies the Markov property:

$$\Pr(X_{t+1} = x' \mid X_t = x, X_{t-1}, \dots, X_0) = \Pr(X_{t+1} = x' \mid X_t = x). \quad (3)$$

Rainfall and temperature are assumed conditionally independent given their current states. Thus, for all states in the joint state space (see Kemeny & Snell, 1976; Norris, 1997).

The joint transition probability matrix is given by

Thus,

$$P = \begin{pmatrix} p_{00,00} & p_{00,01} & p_{00,10} & p_{00,11} \\ p_{01,00} & p_{01,01} & p_{01,10} & p_{01,11} \\ p_{10,00} & p_{10,01} & p_{10,10} & p_{10,11} \\ p_{11,00} & p_{11,01} & p_{11,10} & p_{11,11} \end{pmatrix}$$

Where

$$p_{ij} = P(X_t = j \mid X_{t-1} = i) \quad (4)$$

And

$$\sum_{j=1}^{4} p_{ij} = 1. \quad (5)$$

Transition Count Matrix

Let

N_{ij}

Denote the observed number of transitions from state i to state j.

The transition counts are arranged as

$$N = \begin{pmatrix} N_{00,00} & N_{00,01} & N_{00,10} & N_{00,11} \\ N_{01,00} & N_{01,01} & N_{01,10} & N_{01,11} \\ N_{10,00} & N_{10,01} & N_{10,10} & N_{10,11} \\ N_{11,00} & N_{11,01} & N_{11,10} & N_{11,11} \end{pmatrix}.$$

For each state i,

$$n_i = \sum_{j=1}^{4} N_{ij}. \quad (6)$$

Hierarchical Bayesian Specification

A hierarchical Bayesian framework is adopted to model uncertainty in the transition probabilities and to account for parameter variability across transition states. (Robert & Casella, 2004; Gelman et al., 2013).

Likelihood Function

For each row$i$, the observed transitions follow a multinomial distribution:

$(N_{i1}, N_{i2}, N_{i3}, N_{i4}) \sim \text{Multinomial}(n_i, p_{i1}, p_{i2}, p_{i3}, p_{i4})$

Where

$\mathrm{p}_i = (p_{i1}, p_{i2}, p_{i3}, p_{i4})$

Is the vector of transition probabilities for row i.

Prior Distribution

Each transition probability vector is assigned a Dirichlet prior:

$\mathrm{p}_i \sim \text{Dirichlet}(\alpha_{i1}, \alpha_{i2}, \alpha_{i3}, \alpha_{i4})$

The density function is

$$f(x_1, \ldots, x_k) = \frac{1}{B(\alpha)} \prod_{i=1}^{k} x_i^{\alpha_i - 1} \quad (7)$$

Where

$\alpha_{ij} > 0$

Are prior concentration parameters.

Hyperprior Distribution

Instead of fixing the Dirichlet hyperparameters, hierarchical priors are assigned:

$\alpha_{ij} \sim \text{Gamma}(a, b)$

The Gamma density function is

$$f(x) = \frac{b^a}{\Gamma(a)} x^{a-1} e^{-bx}, \qquad x > 0. \quad (8)$$

Where:

a = shape parameter and b = rate parameter.

The Gamma distribution is selected because it is a flexible prior for positive-valued parameters and is therefore appropriate for the Dirichlet concentration parameters, which must be strictly greater than zero. This hierarchical structure allows the data to influence both the transition probabilities and the prior uncertainty. Consequently, the model becomes more flexible and less sensitive to subjective prior specification

Posterior Distribution

Combining the likelihood and prior using Bayes' theorem gives the posterior distribution:

$$P(\mathrm{p}, \alpha \mid \text{Data}) \propto P(\text{Data} \mid \mathrm{p}) P(\mathrm{p} \mid \alpha) P(\alpha) \quad (9)$$

Can be expressed explicitly as

$$P(\mathrm{p}, \alpha \mid \text{Data}) \propto \prod_{i=1}^{4} \left[\left(\prod_{j=1}^{4} p_{ij}^{N_{ij}} \right) \frac{\Gamma\left(\sum_{j=1}^{4} \alpha_{ij}\right)}{\prod_{j=1}^{4} \Gamma(\alpha_{ij})} \prod_{j=1}^{4} p_{ij}^{\alpha_{ij}-1} \right] \prod_{i=1}^{4} \prod_{j=1}^{4} \alpha_{ij}^{a-1} e^{-b\alpha_{ij}} \quad (10)$$

Where:

N_{ij} = observed transition count from state i to state$j$,p_{ij} = transition probability, α_{ij} = Dirichlet hyperparameter, a, b = Gamma hyperprior parameters, and $\Gamma(\cdot)$ = Gamma function.

Combining the powers ofp_{ij},

$$P(\mathrm{p}, \alpha \mid \text{Data}) \propto \prod_{i=1}^{4} \left[\frac{\Gamma\left(\sum_{j=1}^{4} \alpha_{ij}\right)}{\prod_{j=1}^{4} \Gamma(\alpha_{ij})} \prod_{j=1}^{4} p_{ij}^{N_{ij} + \alpha_{ij} - 1} \right] \prod_{i=1}^{4} \prod_{j=1}^{4} \alpha_{ij}^{a-1} e^{-b\alpha_{ij}} \quad (11)$$

This is the full hierarchical posterior distribution used in MCMC estimation.

The corresponding conditional posterior for each transition probability row is

$(p_{i1}, p_{i2}, p_{i3}, p_{i4} \mid \text{Data}, \alpha) \sim \text{Dirichlet}(\alpha_{i1} + N_{i1}, \alpha_{i2} + N_{i2}, \alpha_{i3} + N_{i3}, \alpha_{i4} + N_{i4})$

Which is sampled iteratively during the MCMC procedure (Gelman et al., 2013; Robert & Casella, 2004).

MCMC Estimation Procedure

Since the joint posterior distribution has no closed-form analytical solution, Markov Chain Monte Carlo (MCMC) methods are employed.

The MCMC algorithm proceeds as follows:

i. Specify initial values for transition probabilities and hyperparameters.
ii. Draw samples iteratively from the posterior distributions.
iii. Discard early iterations as burn-in.
iv. Retain converged samples for posterior inference.
v. Compute posterior summaries such as means, standard deviations, and credible intervals.

The posterior mean estimator for each transition probability is

$$\hat{p}_{ij} = \frac{1}{M} \sum_{m=1}^{M} p_{ij}^{(m)} \quad (12)$$

Where:

M Is the number of retained posterior samples, and $p_{ij}^{(m)}$ is the m-Th MCMC draw.

The model was estimated using 4 MCMC chains with 5,000 iterations each, of which the first 2,000 iterations were discarded as burn-in. The remaining 3,000 iterations per chain were retained for posterior inference, yielding a total of 12,000 posterior samples

Convergence Diagnostics and Model Assessment

To assess convergence and sampling efficiency, standard MCMC diagnostics are employed (Gelman et al., 2013; Brooks et al., 2011).

Gelman–Rubin Diagnostic

The Gelman–Rubin statistic is defined as

$$\hat{R} = \sqrt{\frac{\hat{V}}{W}}, \quad (13)$$

Where

$$W = \frac{1}{m} \sum_{i=1}^{m} S_i^2 \quad (14)$$

Is the within-chain variance,

$$B = \frac{n}{m-1} \sum_{i=1}^{m} (\bar{\theta}_i - \bar{\theta})^2 \quad (15)$$

Is the between-chain variance, and

$$\hat{V} = \frac{n-1}{n} W + \frac{1}{n} B. \quad (16)$$

Convergence is achieved when

$\hat{R} \approx 1.$

Effective Sample Size (ESS)

The effective sample size is computed as

$$ESS = \frac{N}{1 + 2\sum_{k=1}^{\infty} \rho_k}, \quad (17)$$

Where $N$the total number of is retained samples and ρ_k is the autocorrelation at lag$k$.

Autocorrelation Analysis

Autocorrelation decay patterns were assessed using lag-based autocorrelation plots of posterior samples. A rapid decay toward zero indicates efficient mixing and weak serial dependence among MCMC draws, while slow decay suggests poor chain mixing and potential inefficiency in posterior exploration

The lag-k Autocorrelation is defined as

$$\rho_k = \frac{\text{Cov}(\theta_t, \theta_{t+k})}{\text{Var}(\theta_t)} \quad (18)$$

Rapid decay of autocorrelation indicates good mixing and efficient sampling.

Data and Study Area Description

Rainfall (mm) and temperature (℃) Data for North Central, Nigeria, were obtained from National Aeronautics and Space Administration (NASA) for the period of thirty-nine (39) years (1984-2022). The study area is North Central Nigeria, a region comprising of Benue State, Kogi State, Kwara State, Nasarawa State, Niger State, Plateau State, and the Federal Capital Territory. The region lies within the Guinea savanna and derived savanna ecological zones and represents one of Nigeria's major agricultural production areas

Model Specification

The model specification is based on the climatic requirements of millet and soybean crops, where rainfall and temperature conditions are transformed into binary suitability states to evaluate crop suitability under changing climatic conditions. Millet (is adapted to dry savanna environments and performs well under rainfall between 300–600 mm and temperatures ranging from $25°C$ and $35°C$ Soybean require higher moisture availability, with optimal rainfall between 500–900 mm and moderate temperatures between $20°C$ and$25°C$.

Based on these thresholds, rainfall and temperature observations are classified into favourable (1) and unfavourable (0) states. Let

$R_t \in \{0,1\}$M
Represent the rainfall suitability state at time t, and
$T_t \in \{0,1\}$M
Represent the temperature suitability state at time t, where a value of 1 indicates that the climatic condition falls within the crop suitability range, while 0 indicates otherwise.

RESULTS AND DISCUSSION

In this section, the analysis of the joint model corresponding to the methodology is presented

Matrix of Transition of Probability for Millet Crop

$$P^{Niger} = \begin{pmatrix} 0.0991 & 0.4546 & 0.0997 & 0.3466 \\ 0.2513 & 0.4562 & 0.0816 & 0.2109 \\ 0.1945 & 0.1945 & 0.1958 & 0.4153 \\ 0.0414 & 0.3806 & 0.0406 & 0.5374 \end{pmatrix}, P^{Benue} = \begin{pmatrix} 0.1951 & 0.1941 & 0.1933 & 0.4176 \\ 0.0510 & 0.3801 & 0.1271 & 0.4418 \\ 0.1968 & 0.1949 & 0.1977 & 0.4105 \\ 0.0623 & 0.2310 & 0.0224 & 0.6843 \end{pmatrix}$$

$$P^{Plateau} = \begin{pmatrix} 0.6668 & 0.0270 & 0.2785 & 0.0277 \\ 0.1969 & 0.1937 & 0.4143 & 0.1951 \\ 0.3746 & 0.0334 & 0.5573 & 0.0347 \\ 0.2436 & 0.2531 & 0.2540 & 0.2493 \end{pmatrix}, P^{Kogi} = \begin{pmatrix} 0.1940 & 0.1964 & 0.1942 & 0.4154 \\ 0.0989 & 0.4015 & 0.1002 & 0.3993 \\ 0.1929 & 0.1923 & 0.1979 & 0.4169 \\ 0.0257 & 0.3804 & 0.0264 & 0.5675 \end{pmatrix}$$

$$P^{FCT} = \begin{pmatrix} 0.4965 & 0.1889 & 0.1244 & 0.1901 \\ 0.2016 & 0.2687 & 0.1333 & 0.3964 \\ 0.3447 & 0.1576 & 0.1562 & 0.3416 \\ 0.1527 & 0.3057 & 0.0384 & 0.5032 \end{pmatrix}, P^{Kwara} = \begin{pmatrix} 0.1933 & 0.1956 & 0.1963 & 0.4148 \\ 0.1001 & 0.1000 & 0.3436 & 0.2621 \\ 0.0501 & 0.0509 & 0.6369 & 0.2621 \\ 0.0729 & 0.1889 & 0.0721 & 0.6661 \end{pmatrix}$$

$$P^{Nasarawa} = \begin{pmatrix} 0.5345 & 0.1561 & 0.1531 & 0.1563 \\ 0.0612 & 0.2891 & 0.2205 & 0.4292 \\ 0.2533 & 0.1157 & 0.2536 & 0.3774 \\ 0.1397 & 0.2787 & 0.0333 & 0.5483 \end{pmatrix}$$

Niger state has the highest probability in most rows, which is associated with state (1,1), especially from the current favourable state (1,1) to (1,1) with probability 0.5374. This indicates moderate persistence of favourable rainfall and temperature conditions for millet growth. However, transitions to (0,1) are also relatively high, implying that temperature tends to remain favourable even when rainfall becomes unfavourable. Benue exhibits strong persistence of the favourable climate state. The probability of remaining in (1,1) is very high at 0.6843. Transitions from other states into (1,1) are also large, suggesting that Benue has highly suitable climatic conditions for millet cultivation. This implies relatively stable rainfall and temperature conditions that are favourable for millet growth. Plateau shows strong persistence of unfavourable climate conditions. The probability of staying in (0,0) is very high at 0.6668. The persistence of (1,0) with probability 0.5573 indicates rainfall may remain favourable while temperature becomes limiting. The favourable state (1,1) has relatively low persistence (0.2493), suggesting unstable climatic suitability for millet growth. Kogi demonstrates strong movement toward the favourable state (1,1). The persistence probability of (1,1) is 0.5675, indicating good climatic stability for millet production. There is also substantial persistence in (0,1), meaning temperature conditions are often favourable even when rainfall fluctuates. The FCT shows moderate persistence in both unfavourable and favourable states. The probability of remaining in 0,0) is relatively high (0.4965), suggesting frequent unfavourable climate conditions. However, the favourable state (1,1) also has moderate persistence (0.5032). This indicates climatic variability and alternating favourable and unfavourable conditions for millet growth. Kwara has very strong persistence in favourable states. The probability of remaining in 1,1) is 0.6661 indicating highly stable, favourable climatic conditions. Persistence of (1,0) is also high (0.6369), suggesting rainfall conditions are particularly stable. Kwara appears highly suitable for millet cultivation. Nasarawa shows strong persistence of the unfavourable state (0,0) with probability 0.5345. Nevertheless, the favourable state (1,1) also has moderate persistence (0.5483). This suggests mixed climatic behaviour with alternating favourable and unfavourable periods for millet growth. This agrees with the findings of Biasutti (2019), who reported that increasing variability in West African rainfall patterns has important implications for agricultural productivity. The high persistence of unfavourable climatic conditions in Benue, Kogi, Kwara, and Niger suggests the need for drought- and heat-tolerant millet varieties, improved water management, and climate-smart farming practices. Conversely, Plateau, Nasarawa, and FCT should focus on sustaining their relatively stable favourable conditions through improved agronomic and soil conservation practices.

Table 2: Effective Sample Size (ESS) for the Posterior Estimates (Millet)

STATE	NIGER		BENUE		PLATEAU		KOGI		FCT		KWARA		NASARAWA	
$P[i,j]$	ESS Bulk	ESS Tail	ESS Bulk	ESS Tail	ESS Bulk	ESS Tail	ESS Bulk	ESS Tail	ESS Bulk	ESS Tail	ESS Bulk	ESS Tail	ESS Bulk	ESS Tail
P[0,0]	4233	1913.4	5932.8	2769	12013.3	7487.9	5325.2	2414.2	12666.6	10098.8	2016.5	740	11866.6	9310.8
P[0,1]	12606.9	9321.7	6453.6	2967.1	2436.2	1035.3	9261.4	5054	10627.1	7130.9	5268.6	2662.2	7379.2	5100.3
P[0,2]	3980.8	1742.2	4686.6	2254.1	12199.6	8433.2	5556.3	2519	9214.3	5779.9	5701.8	2864.4	7470.8	5351.6
P[0,3]	12922.9	8976.8	11354.3	9095.9	3239.8	1502.8	13825.7	9624.7	14027	8566.1	10493	8586	12326.1	7573.3
P[1,0]	15127.9	8547.2	4720.4	2608.4	2130	746.7	10965.1	6386.5	10689	8295.5	4381.6	2323	4424.9	2047.2
P[1,1]	14389.3	9733.5	13592.1	10397.5	3736.9	1641.4	14338.5	9248.3	10930.3	8661.4	2900.8	1391.7	13598.6	9778.8
P[1,2]	8149.3	5063.8	13850	7850.2	8052.1	8175.4	10442.8	6546.5	9060.8	6493.1	12573.4	10108.9	14518.1	10110.5
P[1,3]	14210.5	7713.7	11934.1	9068.8	5184.8	2605.7	14549.3	8411.4	13729.9	9670.7	13222.3	9064.1	12408.5	8824.6
P[2,0]	4816.1	2306	3081.6	1362	13077.9	8398.5	5738.2	2912.5	12250.2	9286.2	3371.9	1472	11010.1	8614.2

P[2,1]	5068.1	2309.8	2379.1	979.6	1375.7	528.6	3054	1217	6465.3	3030.5	2941.6	1197.6	3337	1548.6
P[2,2]	4273.7	1664.5	4421.6	2464.7	13755.3	8360.1	6559.6	3982.2	3249.9	1453.8	12530.8	9962.4	10775.4	8220.3
P[2,3]	13784.5	9744	12160.8	8886.8	3639.1	2086.6	12349	9152.2	12644	9230.1	13030.3	8674.2	13217.1	9270.8
P[3,0]	3038.6	1208.9	11773.5	7223.2	2324.6	963.6	1450.1	515.2	14646.8	8901.8	7977.2	6579.9	12457.9	8780
P[3,1]	13738.9	9332.7	12926.7	9824.3	4791.5	2778.3	13189.4	9509.2	15211.9	11125	13141.4	9742.3	13308.8	10241.4
P[3,2]	3205.3	1438	2724.2	1409.3	3549.2	1787.8	1728.4	612.9	1751.7	773.7	8276.3	6134	465.4	145.9
P[3,3]	12691.5	8497.7	12270.4	7919.9	4453.1	2478.3	12712.7	8254.1	13025.1	9119.4	12959.2	9004.4	12164.5	9645.8

Table 2 presents the Effective Sample Size (ESS) for the posterior estimates of transition probabilities, $P[i,j]$ for the joint rainfall–temperature Markov chain across the seven states. ESS Bulk measures sampling efficiency for the central part of the posterior distribution, while ESS Tail measures efficiency in the tails of the distribution. The estimated parameters have large ESS values for Bulk and Tail, indicating good MCMC convergence and efficient posterior sampling. This suggests that the Bayesian estimates of the transition probabilities are reliable. Most ESS Bulk and ESS Tail values are well above 1000, indicating good sampling efficiency and reliable posterior estimation.

Table 3: Gelman–Rubin Convergence Diagnostic (Millet)

$P[i,j]$	NIGER	BENUE	PLATEAU	KOGI	FCT	KWARA	NASARAWA
P[0,0]	1.0014	1.0007	1.0007	1.0009	1.0002	1.0007	0.9999
P[0,1]	1.0001	1.0004	1.0028	1.0002	1.0005	1.0004	0.9999
P[0,2]	1.0005	1.0005	1.0003	1.0003	1.0006	1.0009	1.0003
P[0,3]	1	1	1.0008	0.9999	1.0001	1.0001	1.0004
P[1,0]	1.0001	1.0012	1.0017	1.0003	1.0003	1.0024	1.0009
P[1,1]	1.0002	1	1.0022	0.9998	1.0002	1.0016	1
P[1,2]	1.0001	1.0001	1.0003	1.0002	1.0003	1.0001	0.9999
P[1,3]	1.0002	1.0008	1.0001	1.0001	1.0007	1.0011	1.0003
P[2,0]	1.0003	1.0015	0.9999	1.0004	1	1.0006	1.0003
P[2,1]	1	1.0015	1.002	1.0007	1	1.0021	1.0016
P[2,2]	1.001	1.0012	1	1.0006	1.001	1.0001	1.0001
P[2,3]	1.0004	1.0004	1.0005	1.0017	0.9999	1.0001	1.0001
P[3,0]	1.0015	1.0004	1.0004	1.0021	1.0001	1.0004	1
P[3,1]	1.0002	1.0001	1.0006	1.0003	1.0001	1	1
P[3,2]	1.0007	1.0001	1.002	1.0012	1.0012	1.0004	1.0108
P[3,3]	1.0003	1.0004	1.0021	1.0005	1	1.0002	1.0002

Table 3 presents the Gelman–Rubin convergence diagnostic, $\hat{R}$ for the posterior transition probabilities $P[i,j]$ across the seven states. The $\hat{R}$ statistic is used to assess whether the MCMC chains have converged to the target posterior distribution. From the table, nearly all $\hat{R}$ values are extremely close to 1.000, indicating excellent convergence of the MCMC chains for all transition probabilities. The $\hat{R}$ diagnostics confirm that the MCMC simulations converged successfully, implying that the posterior estimates of the transition probability matrices are reliable and suitable for inference on climatic conditions affecting millet growth.

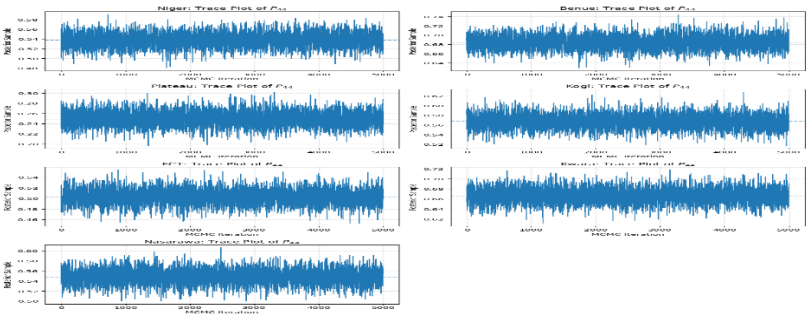

Figure 1: Trace Plots of Posterior Samples for the Fourth Transition Probability (P_{44}) of Millet Climatic Suitability

Trace plots of posterior samples for P_{44} probability of millet climatic suitability across North Central Nigeria. The chains exhibit stable mixing around the posterior means, indicating satisfactory MCMC convergence.

Matrix of Transition of Probability for Soybean Crop

$$P^{Niger} = \begin{pmatrix} 0.4096 & 0.2355 & 0.2381 & 0.1168 \\ 0.2018 & 0.4951 & 0.1509 & 0.1522 \\ 0.3875 & 0.2358 & 0.3114 & 0.0653 \\ 0.2909 & 0.2863 & 0.2914 & 0.1314 \end{pmatrix} P^{Benue} = \begin{pmatrix} 0.4980 & 0.1107 & 0.2805 & 0.1108 \\ 0.3465 & 0.1554 & 0.3432 & 0.1550 \\ 0.1524 & 0.0724 & 0.7481 & 0.0271 \\ 0.3423 & 0.1578 & 0.1572 & 0.3426 \end{pmatrix}$$

$$P^{Plateau} = \begin{pmatrix} 0.2491 & 0.2528 & 0.2485 & 0.2495 \\ 0.0320 & 0.4886 & 0.0321 & 0.4473 \\ 0.2520 & 0.2481 & 0.2458 & 0.2541 \\ 0.0273 & 0.4347 & 0.0273 & 0.5107 \end{pmatrix} P^{Kogi} = \begin{pmatrix} 0.3145 & 0.1579 & 0.4627 & 0.0648 \\ 0.3063 & 0.4028 & 0.2048 & 0.0861 \\ 0.1447 & 0.1087 & 0.6767 & 0.0700 \\ 0.3413 & 0.1555 & 0.1596 & 0.3437 \end{pmatrix}$$

$$P^{FCT} = \begin{pmatrix} 0.0997 & 0.5654 & 0.1015 & 0.2334 \\ 0.1220 & 0.5239 & 0.0291 & 0.3250 \\ 0.4145 & 0.1954 & 0.1970 & 0.1932 \\ 0.1112 & 0.3910 & 0.1092 & 0.3886 \end{pmatrix} P^{Kwara} = \begin{pmatrix} 0.0713 & 0.0699 & 0.6838 & 0.1749 \\ 0.1618 & 0.3409 & 0.3412 & 0.1562 \\ 0.2706 & 0.0636 & 0.5310 & 0.1348 \\ 0.2277 & 0.0980 & 0.4458 & 0.2285 \end{pmatrix}$$

$$P^{Nasarawa} = \begin{pmatrix} 0.1155 & 0.2602 & 0.5082 & 0.1161 \\ 0.1804 & 0.5317 & 0.0465 & 0.2413 \\ 0.1540 & 0.1546 & 0.3074 & 0.3840 \\ 0.1041 & 0.2657 & 0.2131 & 0.4170 \end{pmatrix}$$

The transition probability matrices for soybean crop conditions across the seven states describe the likelihood of moving from one climatic state to another between successive periods.

For Niger State, the matrix shows moderate persistence across all states, particularly in state 01 with probability 0.4951. The transitions are fairly distributed, indicating variability in soybean climatic conditions without a single overwhelmingly dominant state.

In Benue State, state 10 has a very high persistence probability of 0.7481, suggesting that once the soybean system enters this climatic condition, it is highly likely to remain there. This indicates relatively stable environmental behaviour favorable to recurring climatic patterns.

For Plateau State, the probabilities are more balanced, although states 01 and 11 show stronger persistence with probabilities 0.4886 and 0.5107, respectively. This suggests alternating but stable climatic regimes for soybean production.

In Kogi State, state 10 dominates with a persistence probability of 0.6767, indicating strong continuity in that climatic condition. The relatively lower probabilities in other transitions imply less variability once this state is attained.

For the Federal Capital Territory, the matrix indicates strong transitions toward state 01, especially from state 0 with probability 0.5654. This suggests a tendency for climatic conditions to shift toward a wetter or warmer joint state rather than remain stable.

In Kwara State, state 10 is highly dominant with a persistence probability of 0.5310 and strong transitions from state 00 to 10 at 0.6838. This indicates that soybean climatic conditions in Kwara are strongly influenced by the persistence of this combined state. Finally, Nasarawa State exhibits moderate persistence in states 01 and 11, with probabilities 0.5317 and 0.4170, respectively. The relatively high transition probability from state 10 to 11(0.3840) suggests gradual shifts between climatic regimes rather than abrupt changes. According to Sultan et al. (2023), variations in temperature extremes and rainfall variability have a significant impact on climate change's effects on African agriculture, which can shift the likelihood of favorable crop-growing seasons.

In General, the transition matrices reveal that soybean crop climatic conditions differ substantially across states, with some states exhibiting strong persistence in specific climatic regimes while others display greater variability and transitions between states. These findings are important for understanding climate dynamics and improving soybean crop planning and adaptation strategies. The dominance of the (0,1) state (unfavourable rainfall, favourable temperature) across several states indicates that rainfall deficiency is the primary constraint to soybean production, highlighting the need for irrigation, rainwater harvesting, and drought-tolerant varieties. Areas with persistent (0,0) conditions require climate-smart practices, including heat-tolerant cultivars, adjusted planting dates, and improved climate information services to reduce production risks.

Table 4: Effective Sample Size (ESS) for the Posterior Estimates (Soybean)

P[i,j]	NIGER		BENUE		PLATEAU		KOGI		FCT		KWARA		NASARAWA	
	ESS Bulk	ESS Tail	ESS Bulk	ESS Tail	ESS Bulk	ESS Tail	ESS Bulk	ESS Tail	ESS Bulk	ESS Tail	ESS Bulk	ESS Tail	ESS Bulk	ESS Tail
P[0,0]	12158.2	9356	18854.6	9912.6	3177.8	1403.4	16453.4	9887.8	3953	1434.9	2236	832.2	3418.7	1527.3
P[0,1]	11380.1	7227.3	11592.1	7646.3	1705	601.9	12548.8	8568.5	12687	9208	2445.1	1211.8	15102.9	8543.3
P[0,2]	9640.6	7162.1	17852.5	9288.6	3737	1833.8	17108.8	9806.9	3562.1	1467.9	11290.8	8401.7	14423.5	9913.8
P[0,3]	11354.1	6908.7	14463.4	8485.6	4598.8	2082.2	2642.7	1078.3	12495	7658	12316.1	7084.3	6074.9	2850.4
P[1,0]	8927.8	6771.3	12585.4	8755	3372.8	1758.4	14658	9544.9	16478.8	8807.6	6660.4	3784.9	15872.7	8781.7
P[1,1]	10919.8	9434.6	1655	620.7	12468.5	10733.7	13446.2	9067.2	14844.2	10543.4	12109	8544.5	15208.8	10588.2
P[1,2]	7576.7	6090.2	13939.2	9070.4	3415.3	1568.2	12823.6	7959.2	2542.5	1241	14594.9	9040.1	1218.3	438.3
P[1,3]	15842.9	9117.9	6607.6	3470	12019.1	9009.1	1848.5	612.2	13277.6	8727.1	5826.7	2568.2	12892.8	8627.5
P[2,0]	14481.8	10804.2	14621.6	9660.4	4368.4	2279.6	13629.9	8171.7	9202.7	8796.5	15576.6	8211.5	11095.4	7069.8
P[2,1]	15329.5	9286.9	14434.7	8213.1	1446.8	493.6	12345.2	7979.8	4884.2	2658.9	6584.7	4944.2	11649.6	7078.7
P[2,2]	15126.2	9099.1	14524.4	9470	1862.2	764.7	14430.2	8461.2	4417.6	2244.6	16734	9311.5	13858	9509.5
P[2,3]	1376.8	621.3	4044.6	2076.5	4073.6	2076.5	12250	7106	5923.8	3105.6	16117.2	9203.1	14167.6	9685.2
P[3,0]	10611.4	7955.5	12463.9	9661.5	2111.5	942.5	11109.9	8461.4	6627	4851.3	12895	7555.5	10272.7	6891.2
P[3,1]	10418.7	7582.1	3694.9	1594.3	13125.7	9853.8	3177.2	1238.8	11716.6	10253	2777.4	1279.7	15222.3	9460.8
P[3,2]	9843.8	7189.6	4382.9	2056.3	1624.3	675.9	2813.9	1182.2	7166.4	5152.1	13445.2	9785.2	14394.2	8862.7
P[3,3]	3937.1	1955.4	12796.9	9310.9	12883	8602.7	12830.9	9233.3	13542.3	8652.8	14005.1	8409.8	14440	9861.8

The Effective Sample Size (ESS) results for the posterior transition probabilities indicate that the MCMC estimation achieved good sampling efficiency across most parameters and states. Both ESS Bulk and ESS Tail values are generally large, suggesting that the Markov chains mixed well and produced reliable posterior estimates for the soybean crop

transition probabilities. ESS diagnostics confirm that the MCMC simulations for the soybean crop transition probability matrices converged satisfactorily and generated sufficient independent posterior samples for robust statistical inference. Parameters with relatively lower ESS values may require longer chains or additional tuning to further reduce autocorrelation and improve sampling efficiency.

Table 5: Gelman–Rubin Convergence Diagnostic (Soybean)

P[i, j]	NIGER	BENUE	PLATEAU	KOGI	FCT	KWARA	NASARAWA
P[0,0]	1.0004	1.0003	1.0015	1	1.0011	1.0022	1.0014
P[0,1]	1	0.9999	1.0032	1.0003	1.0002	1.002	1
P[0,2]	1.001	1.0002	1.0008	1.0007	1.0004	1.0001	1.0002
P[0,3]	0.9999	1.0002	1.0001	1.0008	1.0003	1.0003	1.0002
P[1,0]	1.0002	1.0002	1.0012	1	0.9999	1.0003	1.0001
P[1,1]	1	1.0027	1.0005	1.0001	1.0001	1	1
P[1,2]	1.0008	1.0003	1.0019	1.0001	1.0017	1	1.003
P[1,3]	1	0.9999	1.0008	1.0054	1.0009	1.0006	1
P[2,0]	1.0001	1.001	1.0005	1.0003	1.0005	1.0001	1.0002
P[2,1]	1.0005	1	1.0032	1	1.0002	1.0006	1.0003
P[2,2]	1	1.0004	1.001	0.9999	1.0008	1.0001	1.0002
P[2,3]	1.0037	1.0009	1.0004	0.9999	1	1.0004	1.0002
P[3,0]	1.0007	0.9999	1.0017	1.0003	1.0003	1.0002	1
P[3,1]	1	1.0012	1.0002	1.0002	1	1.0023	1.0002
P[3,2]	1.0001	1.0005	1.0023	1.0013	1.0001	1.0001	1.0003
P[3,3]	1.0002	0.9999	1.0006	1.0002	1.0002	1	1.0006

Table 5 shows the Gelman–Rubin convergence diagnostic for an MCMC model applied to soybean data across different states (Niger, Benue, Plateau, Kogi, FCT, Kwara, Nasarawa. All values are approximately 1. This implies that the Markov chains are very stable and consistent and there is no evidence of non-convergence. It also means that the between-chain and within-chain variances are nearly identical

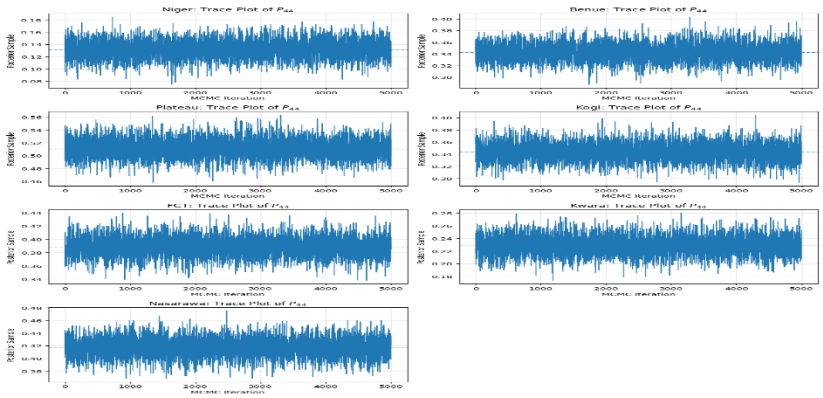

Figure 2: Trace Plots of Posterior Samples for the Fourth Transition Probability (P_{44}) of Soybean Climatic Suitability

The MCMC trace plots of P_{44} for soybean climatic suitability showed stable oscillation around the posterior mean values for all regions, indicating satisfactory chain mixing and convergence. Plateau recorded the highest persistence probability (P_{44} =0.5107), followed by Nasarawa (0.4170) and FCT (0.3886), suggesting stronger persistence of the fourth climatic suitability state. In contrast, Niger (0.1314) and Kwara (0.2285) exhibited lower persistence, indicating greater temporal variability in soybean climatic suitability conditions.

CONCLUSION

This study successfully developed and applied a joint binary Markov process to model the spatio-temporal dynamics of rainfall and temperature conditions influencing soybean and millet production across seven Nigerian states: Niger, Benue, Plateau, Kogi, the FCT, Kwara, and Nasarawa. The study contributes to knowledge by providing a probabilistic approach that integrates rainfall and temperature interactions with Bayesian uncertainty estimation, extending beyond conventional climate trend analysis. The estimated transition probability matrices revealed clear spatial differences in climatic persistence and variability. Benue and Kwara states exhibited strong persistence in favourable climatic conditions, indicating relatively stable environments that are highly suitable for crop production. Kogi also showed moderate stability in favourable regimes, suggesting generally supportive but less consistent conditions. In contrast, Plateau and Nasarawa were characterised by stronger persistence in unfavourable or mixed climatic states, reflecting more challenging agricultural environments. Niger State and the Federal Capital Territory (FCT) displayed higher levels of climatic variability, with frequent transitions between states, indicating less stable conditions for sustained crop productivity. The results therefore highlight substantial regional differences in climatic suitability for millet and soybean production.The Markov chain Monte Carlo (MCMC) estimation procedure produced reliable parameter estimates,

and the Gelman–Rubin diagnostic confirmed convergence of all model parameters, with values close to one, indicating stable posterior distributions and robust inference. Rainfall and temperature interactions exhibit substantial spatial heterogeneity across the study area, with important implications for agricultural planning and climate risk management. The joint binary Markov framework provides a useful tool for understanding climate dynamics and supports evidence-based strategies for improving soybean and millet production under variable and uncertain climatic conditions**.** The model simplifies climate suitability into binary rainfall and temperature states and assumes that future conditions depend only on the current state, which may not capture all climatic complexities. Additionally, other factors affecting crop growth, such as soil properties, management practices, and extreme events, were not considered. Future studies could incorporate additional environmental and agronomic variables and explore higher-order or non-homogeneous Markov models to improve predictive accuracy.

DECLARATION OF CONFLICTING INTERESTS

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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