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Deterministic Chaos in Precipitation and Atmospheric Temperature Time Series over North-Central Nigeria: A Recurrence Quantification Analysis Approach

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ABSTRACT

This work is focused at assessing the chaotic features in precipitation and atmospheric temperature records over North-Central Nigeria using recurrence quantification analysis. The mean daily precipitation and atmospheric temperature data for the seven states in North-Central Nigeria were collected from the Modern Era Retrospective Reanalysis (MERRA-2) spanning from 1982-2020. Recurrence plots (RP) were constructed based on the reconstitution of phase space via the method of delays while Lyapunov exponents and recurrence statistics were also computed. The results obtained show that the recurrence plots for precipitation comprise of points that are not arranged in shorter diagonal lines but form a congruent chequered pattern which is regular throughout the RP while the RPs for atmospheric temperature showed regular patterns with very short diagonal lines parallel to the line of identity (LOI) indicating chaos. The Lyapunov exponents were positive but low values (< 0.014) indication deterministic chaos while the recurrence statistics computed showed the recurrence rate of temperature has very low values (0.30-0.69 %) across all the sampled states but that of precipitation has higher values (09.83-28.24 %) as a result of its fixed seasonality. The determinism values for temperature also had lower values (0.099-0.182 %) while that of precipitation recorded higher values (75.48-88.85 %) implying more sensitive to distortions like greenhouse gases emissions from human anthropogenic activities. These results are a confirmation of deterministic chaos in the dynamics of the climate of North-Central Nigeria which is has manifested in floods, heat waves and droughts over the years.

Keywords: Chaos, Recurrence Plots, Recurrence Quantification Analysis, Recurrence Rate, Determinism

INTRODUCTION

One of the growing concerns of developing nations in the world today is knowing the vulnerability level of its socio-economic activities due to climate change and implementing measures that could be adopted in either coping or mitigating such impacts. Climatic variability is a term used to explain the changes in climatic conditions on various spatio-temporal scales which include fluctuation, trends and cycles which constitutes "noise" in the climatic series as man could easily adapt to such minor differentials. Chaos theory is another dimension of measuring climate vulnerability and fluctuation from the nonlinear dynamical perspective. Recurrence plots and recurrence quantification analysis is a useful tool in characterizing weather variables in order to unveil the salient nonlinear features present and the degree of chaos in the climate of an area over a period of time. Recurrence plot (RP) was first proposed by Eckmann *et al.* (1987) as a nonlinear technology to observe the features of a dynamical system in phase space. The recurrence plots (RP) show the time dependent behaviour of the dynamical systems which are pictured on the phase space. A recurrence plot RP is a visualization of state space dynamics that shows all those times at which a state of the dynamical systems recurs (Adediji *et al.*, 2018). The recurrence plot RP method may expose some future knowledge about time series visually through the graph, such as the similarity contained in time series and the predictability of information (Wen Zhang *et al.*, 2016). The graphical difficulty and the need to quantitatively described the physical significance exposed by the recurrence plot, Zhibilut and Webber (1992) proposed a tool called recurrence quantification analysis (RQA), which can measure complexity of time series effectively, assess, and classify the space recurrence graph of each time to determine differences and similarities of the dynamic structure, so as to detect the abrupt change of structure of the system dynamics by the evolution of recurrence graph. According to Miralles *et al.* (2015) recurrence quantification analysis (RQA) is a powerful nonlinear data analysis tool that can be used to measure complexity and chaos (among other characteristics) in any temporal series. This process involves computing the RP with a given recurrence threshold. But choosing the appropriate recurrence threshold is not an easy task to date there is no systemic study that help in choosing the right threshold (Marwan *et al.*, 2011). This method quantifies the density of recurrence points as well as the histograms of length of the diagonal and vertical lines in the RP. The measures to quantify the deterministic structure and complexity of recurrence plots (RPs) include the following: The recurrence rate (RR), determinism (DET), max line (Lmax), divergence (DIV), entropy (ENT), laminarity (LAM), trapping time (TT), largest vertical line (Vmax) (Wallot et al., 2017).

Numerous researchers have applied RPs and RQA to characterize climatic systems across the world; however, in Nigeria only very few have explored the technique in examining random climatic fluctuations. Ogunjo *et al.* (2015) used the recurrence plot (RP) and recurrence quantification analysis (RQA) to investigate nonlinearity in solar radiation data in a tropical station. The data used was obtained from Federal University of Technology, Akure (FUTA) South-Western Nigeria using an integrated sensor suit (vantage 2 pro). Half hourly and daily values were tested for each month of the year. Both were found to be nonlinear. The dry months of the year exhibits higher chaoticity compared to the wet months of the year. The values of the daily average were found to be mildly chaotic. Using RQA, features due to external effects such as harmattan and intertropical discontinuity (ITD) on solar radiation data were uniquely identified. Adeniji *et al.* (2018) used the RP and (RQA) techniques to investigate a nonlinear deterministic dynamical process and non-stationarity in hourly wind speed data from five different stations in the tropic zone. The wind speed data used were collected over a period of two years by National

Space Research and Development Agency (NASRDA) from five different stations which include; Abuja, Akungba, Nsukka, Port-Harcourt and Yola. The result obtained from different RQA measures (L_{max}, DET and ENT) revealed that Yola and Abuja which are far from the North-West, are better locations for citing wind power plants than the rest of the regions considered. The result also show that the dry season period is better for the harvesting of wind energy than the wet season, in all the regions. Bichi *et al.* (2026) applied a nonlinear approach to investigate the spatial and temporal variability of deterministic chaos in daily mean tropospheric radio refractivity across 12 locations in Nigeria using RP and RQA. The results from the recurrence plots using embedding dimension, m = 4, time lag, τ =10 and recurrence threshold, e = 0.02 using the fixed recurrence rate criterion, showed the appearance of short diagonal line structures indicating deterministic chaos in the underlying dynamics of tropospheric radio refractivity and comparable system complexity despite geographical differences. Results of the RQA showed recurrence rates (RR) ranging from 0.11 to 5.36%, Determinism (DET) ranging from 0.01 to 0.3, and Laminarity ranging from 0.01 – 0.51. Overall, the results show that tropospheric radio refractivity in the study region is deterministic and weakly chaotic but strongly controlled by climatic variability.

In the North-Central part of Nigeria, not much has been done in investigating the climate dynamics and complexity using recurrence quantification analysis. Hence, there is the need to use chaos theory in assessing chaotic features in precipitation and atmospheric temperature across North-central Nigeria using RPs and RQA so as to accurately model and forecast it despite its random fluctuations.

Theoretical Consideration

Phase space reconstruction

The basic method for the reconstruction of phase space used in this work is the method of delays (MOD), which was being developed by Packard *et al.* (1980) and Takens (1981). When dealing with a time series $\{s_1, s_2, s_3, \dots, s_N\}$ where N is the number of observations, the attractor can be reconstructed into an m-dimensional phase space of delay coordinates τ, by forming the vectors:

$$S_i = [s_i\,, s_{i+\tau}, s_{i+2\tau}, \dots, s_{i+(m-1)\tau}] \quad (1)$$

Where τ is the time delay or time lag for a digitized time series and m is the embedding dimension. In more quantitative terms, the time lag, τ is the shortest time over which there are clearly measurable variations in the observable signal. The embedding dimension (m) is the minimum dimension of the space in which you can reconstruct phase portraits from your measurements and in which the trajectory does not cross itself, or in other words, it is the minimum number of independent variables (degrees of freedom) required to describe the dynamics of the system in phase space. A good estimate of time delay and embedding dimension is critical because it helps in correctly unfolding the attractor (if one exist). In the phase space (Velickov, 2006). The phase space vector S_i is a $M \times m$ matrix, and the constant m, M, τ and N are related by:

$$M = N - (m - 1)\,\tau \quad (2)$$

M being the number of constructed phase space points (Rosenstein et al., 1992). The time delay τ is evaluated using the method of average mutual information (AMI) developed by Fraser and Swinney (1986). The average mutual information takes into account nonlinear correlation and is computed using the equation (Fraser and Swinney, 1986):

$$I(\tau) = -\sum_{ij}^{N} P_{ij}(\tau) ln \frac{P_{ij}(\tau)}{P_i(\tau)P_j(\tau)} \quad (3)$$

Where $P_i(\tau)$ and $P_j(\tau)$ are the probability of finding a time series value in the i^{th} and j^{th} intervals while $P_{ij}(\tau)$ is the joint probability that an observation falls in the i^{th} or j^{th} interval. From the plot of AMI vs increasing lag length, the time delay is the value of the lag length at the first local minimum. Meanwhile the optimum embedding dimension is computed using the method of false nearest neighbours (FNN) developed by Kennel *et al.* (1992). By computing and plotting the percentage of FNN against increasing embedding dimension, a monotonic decreasing curve is usually observed and the optimum embedding dimension can be evaluated from the point where the percentage of FNN drops to almost zero or a minimum value.

Recurrent Plots (RP)

Recurrence plots (RP) is a useful tool in the analysis of nonlinear, non-stationary time series in finding hidden correlations in highly complicated data (Marwan *et al.*, 2007). With RP one can graphically detect hidden patterns and structural changes in data or see similarities in patterns across time series under study (Bideli, et al., 2009). The RP is derived directly from the distance matrix:

$$D = D_{i,j} = x_i - x_j \quad (4)$$

Where; $i, j =$ 1, 2, 3, N (N being the length of the time series). Considering a recurrence thresholdε, then the matrix of recurrent points, $R_{i,j}$ is derived from the expression:

$$R_{i,j} = \Theta\left(\varepsilon - D_{i,j}\right) \quad (5)$$

Where Θ is the Heaviside function defined by:

$$\Theta(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases} \quad (6)$$

If $x_i \approx x_j$ then$R_{i,j} = 1$, else$R_{i,j} = 0$. Next, black dots are assigned the value one and white dots are assigned the value zero. The recurrence plot (RP) is a two-dimensional graphical representation of $R_{i,j}$. A close visual inspection of the recurrent plot reveals typical small-scale structures such as: single dots, diagonal lines with vertical and horizontal lines inclusive. In addition, even curved lines may be found (Marwan et al., 2007). Single isolated recurrence points can occur if states are rare i.e., if they do not persist for a time or fluctuate frequently. A diagonal line may occur when the trajectory visits the same region of phase space at different times (Marwan and Kurths, 2005). The recurrence plots of different dynamical systems are shown in Figure 1 (Zhang et al., 2016):

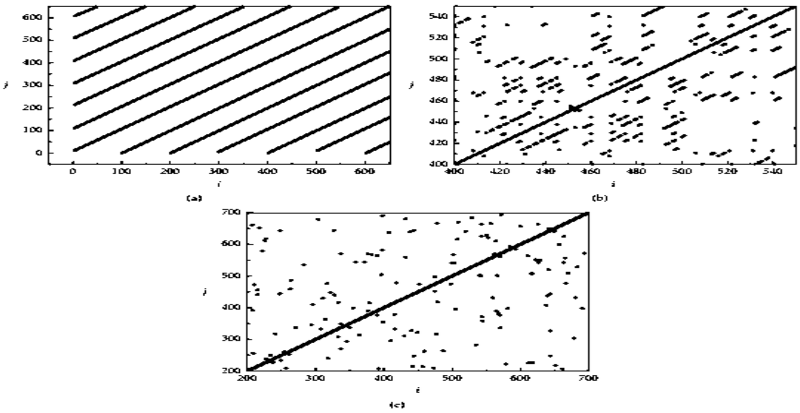


Figure 1: Recurrence Plots of Different Dynamic Systems: (A) Periodic, (B) Chaotic (C) Random (Stochastic)

The recurrence plot RP gives the reader a first impression of the pattern of recurrences which will allow studying dynamical systems based on the pattern of their trajectories, like periodic systems, stochastic random systems and chaotic ones. The dynamics of systems based on the structure of the diagonals in their recurrent plots is illustrated in Table 1 (Bigdeli *et al.*, 2009):

Table 1: Different Dynamics Systems and the Features of the Diagonal in Their Recurrence Plots

Dynamic system	Recurrence plot features
Periodic	long and non-interrupted diagonals, the period of oscillation is equal to the vertical distance between these lines
Quasi-periodic	Presence of several short diagonals
Chaotic	Predominance of much shorter diagonals
Uncorrelated/Stochastic (random noise)	Many black dots with erratic distribution

In summary, the shorter the diagonals are in the RP, the less predictable the system is. The threshold limit Ɛ is estimated using the expression (Lettellier *et al.*, 2007):

$$\varepsilon = \sqrt{m} \times 10\% \text{ of the fluctuations of the signal } \times \tau \quad (7)$$

Where m is the embedding dimension and τ is the time lag. The fluctuations of the signal here is a specific fraction of its phase space diameter or its standard deviation, δ (Schinkel *et al*., 2007).

Lyapunov Exponents

Lyapunov exponent is a measure of the sensitive dependence on the initial condition (SDIC). Also, it is the property of chaotic systems, which measures the average rate at which nearby trajectories of a dynamical system diverge or converge exponentially over time in phase space (Rosenstein *et al.*, 1993). In topological terms Lyapunov exponents describe the stretching (or divergence) in phase space needed to generate a strange attractor and is a fundamental property that characterizes the rate of separation of infinitesimally close trajectories (Hakki, 2006). Wolf *et al*., (1985) estimated the largest Lyapunov exponent of a time series using the model equation:

$$\lambda = \lim_{t\to\infty} \frac{1}{t} \ln \frac{\|\Delta x(t)\|}{\|\Delta x(0)\|} \quad (8)$$

A positive value of Lyapunov exponent indicates the orbit is unstable, deterministic and chaotic. The nearby points, no matter how closer or farther they may be, will diverge to any arbitrary separation. Future evolutions are completely correlated to past behaviour. A negative value of λ on the other hand infers that the orbit attracts to a stable fixed point, is dissipative and non-conservative. Such a system exhibits asymptotic stability and more the negative exponent, the more the stability (Echi *et al.*, 2025).

Recurrence Quantification Analysis (RQA)

Recurrence quantification analysis which was developed by Zbilut and Webber in 1992 as a technique for nonlinear analysis which quantifies the number and duration of recurrences of a dynamical system as presented in its phase space trajectory. RQA statistics can be used to characterize dynamical systems and also detect the bifurcation points in the chaos-order transitions. The RQA variables, corresponding to the vertical structures, enable the detection of the chaos-chaos transition. The following RQA parameters along with their potentials are used in the identification of the changes in the recurrence plot and are discussed in the next section as follows (Marwan et al., 2007):

Recurrence Rate (RR) or Percent Recurrences:

The recurrence rate (RR) is a measure of the density (percentage) of the recurrence points in the recurrence plot. It is the simplest variable of the RQA parameters and is mathematically defined as:

$$RR(\varepsilon) = \frac{1}{N^2} \sum_{i,j=1}^{N} R_{i,j}(\varepsilon) \quad (8)$$

RR is a function of ε and embedding dimension m, hence no fixed range of values of RR universally characterizes a time series as chaotic. However, RR decreases in value as the system becomes more chaotic as trajectories diverge more (Pánis *et al.*, 2020).

Determinism (DET)

This is the percentage of recurrence points which form diagonal lines. It is also expressed as the ratio of the number of recurrence points forming diagonal structures. It is given by:

$$DET = \frac{\sum_{l=l_{min}}^{N} l\,P(l)}{\sum_{l=1}^{N} l\,P(l)} \quad (9)$$

Where P(l) is the histogram of the lengths l of the diagonal lines. DET gives a number between 0 and 1 such that a periodic signal (e.g., a sinusoid) will have a value of 1 and a purely stochastic signal will result in a value extremely close to 0. Furthermore, the determinism, DET is lower for chaotic dynamics with aligned diagonal lines, lowest for typical stochastic systems and higher for periodic behaviour (Goswami, 2019, Pánis *et al.*, 2020).

Laminarity (LAM)

This is the percentage of recurrence points which form vertical lines. It is also the ratio of the number of recurrence points forming vertical lines to the total number of recurrence points in the recurrence plot. LAM provides valuable information about the occurrence of the laminar states in the system but does not describe the length of the laminar states. The value of LAM decreases if more number of single recurrence points are present in the recurrence plot than the vertical structures (Ye *et al.*, 2015). It is given by:

$$LAM = \frac{\sum_{v=v_{min}}^{N} vP(v)}{\sum_{v=1}^{N} vP(v)} \quad (10)$$

Where P (v) is the histogram of the length v of the vertical lines.

Longest Diagonal Line(L_{max}):

This is the length of the longest diagonal line in the plot, excluding the main diagonal line of identity and is related the Largest Lyapunov exponent of the system.

$$L_{max} = max(\{l_i\ ;\ i = 1, 2, \ldots, N\}) \quad (11)$$

Thus, the shorter the line max, the more chaotic (less stable) the signal.

Shannon Entropy (ENTR):

This is the probability distribution of the diagonal line lengths p(l). it is the probability p(l) of finding a diagonal line of length l in the recurrence plot, it indicates the complexity of the recurrence plot in respect of the diagonal lines and is expressed as (Krakrovska *et al.*, 2015):

$$\text{ENTR} = \sum_{l=lmin}^{N} p(l) \ln p\,(l) \quad (12)$$

Chaotic systems exhibit medium-to-high entropy and finite L_{max} while stochastic systems have very high entropy values and very short L_{max} (Pánis *et al.*, 2020).

MATERIALS AND METHODS

The Study Area

The study area in this work is North-Central Nigeria, which includes seven states: Benue, Kwara, Kogi, Plateau, Niger, Nasarawa and the Federal Capital Territory (FCT), Abuja. It spans from latitudes $7°00' - 11°30'N$ of the equator and longitude $4°00' - 11°00'E$ of the Greenwich Meridian. The climate of the region is characterized by two seasons: wet and dry, with a Guinea Savannah vegetation type. The wet season runs from April to October while the dry season runs from November to March. The states are purely Agrarian states with fishing also being a preoccupation as a result of the Rivers Niger and Benue and their tributaries which transverse the region. The region has about 20 million people, around 115 of the total population of the country (Wada, 2025). Table 2 shows the sample locations, their geographical coordinates and elevation above sea level (altitude) while Figure 2 shows a map of Nigeria showing the seven states sampled in this work.

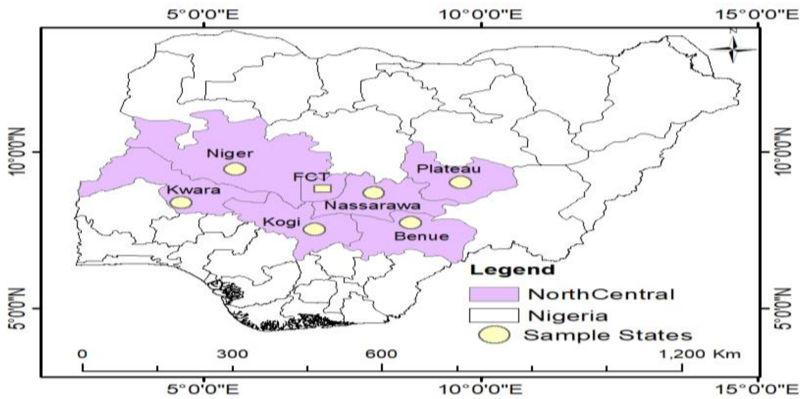


Figure 2: Map of Nigeria Showing The Seven States In North-Central Nigeria (Source: JOSTUM GIS lab.)

Table 2: Sampled Locations, their Geographical Coordinates and Elevation above Sea Level

S/No.	Location (States)	Mean Altitude (m)[a]	Geo (D/D) Coordinates[b]	
			Latitude (N)	Longitude (E)
1	Benue	95.72	7.71071	8.62661
2	FCT, Abuja	474.43	9.06161	7.47631
3	Plateau	1171.06	9.86371	8.88131
4	Kogi	87.83	7.80341	6.73441
5	Nasarawa	175.43	8.50651	8.51681
6	Niger	256.60	9.60521	6.54631
7	Kwara	326.00	8.51751	4.52121

Source: [a] GMAO-MERRA-2 (2025), [b] topographic-map.com

Data Source and Pre-Processing

The data source for this work is the Modern Era Retrospective Reanalysis (MERRA-2) from the National Aeronautical and Space Administration (NASA). The meteorological data contains daily averages of precipitation (mm) and atmospheric temperature (℃) at a height of 2 m above the ground for the seven states across north-central Nigeria from 1982 – 2020. The spatial resolution of the dataset is $0.25° \times 0.25°$. The descriptive statistics of precipitation and atmospheric temperature is presented in Table 3.

Table 3: Descriptive Statistics of Precipitation and Atmospheric Temperature across North-Central Nigeria

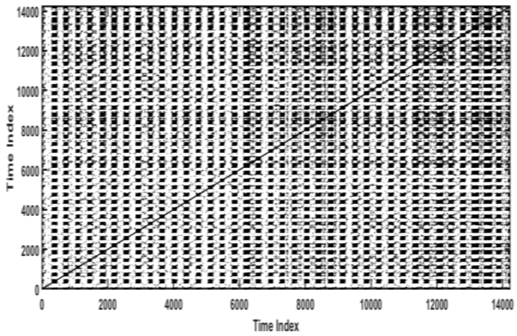
Locations	**Precipitation (mm)**				**Atmospheric temperature (℃)**			
	Mean	**Standard deviation**	**Signal-noise ratio (SNR)**	**Skewness**	**Mean**	**Standard deviation**	**Signal-noise ratio (SNR)**	**Skewness**
Benue	3.690	5.421	0.681	3.208	27.879	3.729	7.476	-0.104
FCT Abuja	3.959	6.046	0.655	3.114	25.025	2.027	12.348	0.344
Plateau	3.054	5.509	0.554	5.094	22.980	2.260	10.170	0.288
Kogi	3.574	5.537	0.645	3.682	26.045	1.847	14.105	0.114
Nasarawa	3.340	5.192	0.643	3.025	25.581	2.160	11.845	0.497
Niger	3.996	6.237	0.641	3.563	25.092	2.192	11.445	-0.097
Kwara	3.002	4.608	0.652	5.174	26.206	1.944	13.478	0.184

The data was pre-processed by filling in the missing data using moving average method and then it was normalized to remove trends and cyclic components that are responsible for the seasonal variations rendering the time series stationary (zero mean and constant variance). This reveals the true deterministic nature of the data for nonlinear analysis (Caesarendra *et al.*, 2013). The RPs and RQAs were implemented using MATLAB version R2020a while the spatial interpolation was executed using the spatial Analyst tool box of ArcGis 10.5.

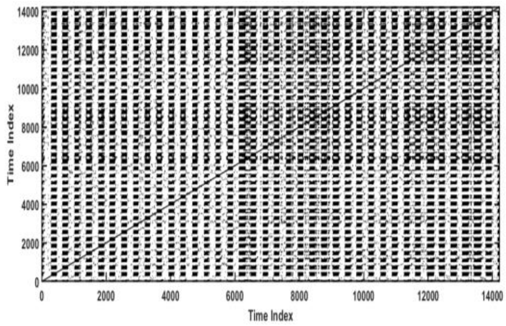
RESULTS AND DISCUSSION

Recurrence Plots for Precipitation and Atmospheric Temperature

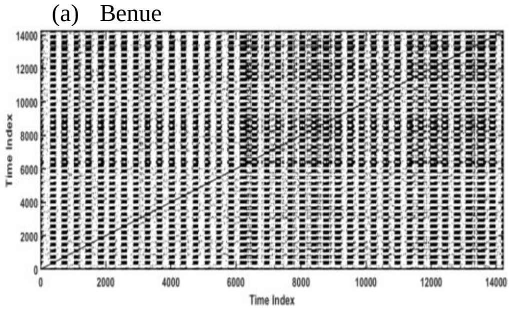
The recurrence plots for precipitation for the different locations in north-central Nigeria is presented in Figure 3(a-g). The plot shows block chequered patterns of regular sizes arranged in a periodic manner, with no visible diagonal lines but diagonal patterns. This is an indication of the low dimensional chaos in the precipitation time series.



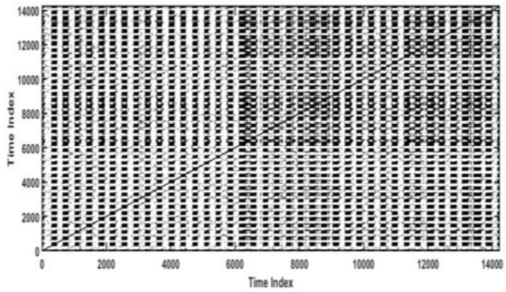
(a) Benue



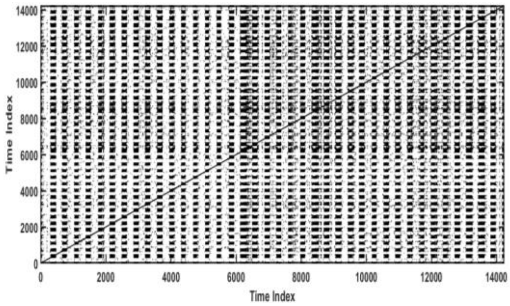
(b) FCT Abuja



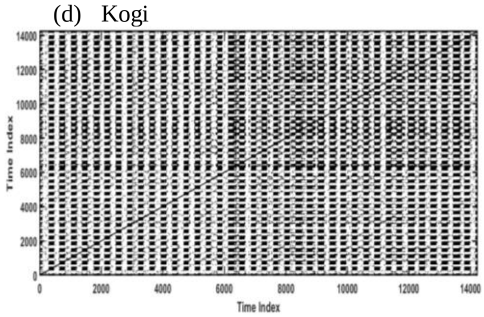
(c) Plateau



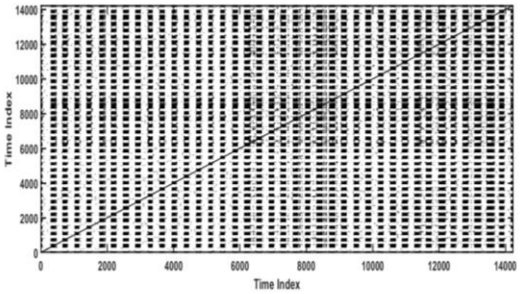
(d) Kogi



(e) Nasarawa



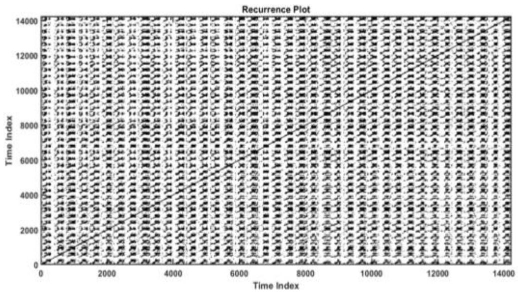
(f) Niger



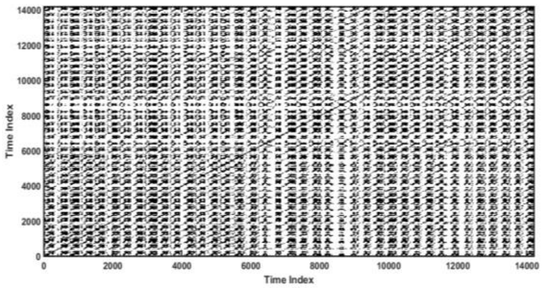
(g) Kwara

Figure 3: Recurrence plots of daily mean precipitation (mm) across north-central Nigeria

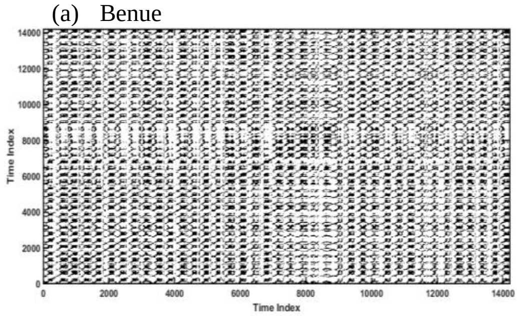
The recurrence plots for daily mean atmospheric temperature across north-central Nigeria are presented in Figure 4 (a-g). The plots are characterized by very short diagonal line patterns. This is an indication of low deterministic chaos in the atmospheric temperature time series.



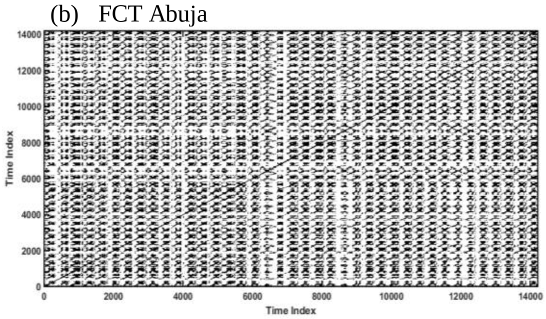
(a) Benue



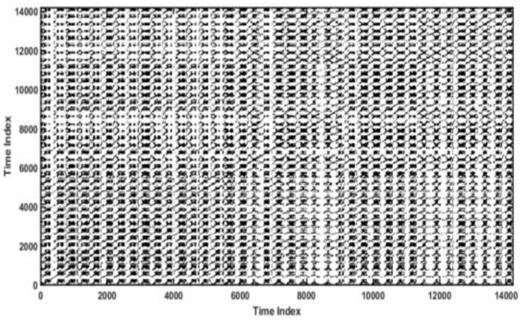
(b) FCT Abuja



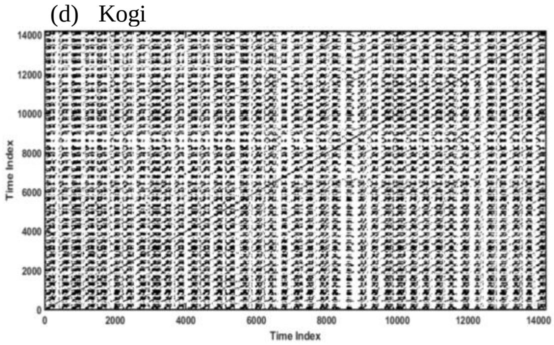
(c) Plateau



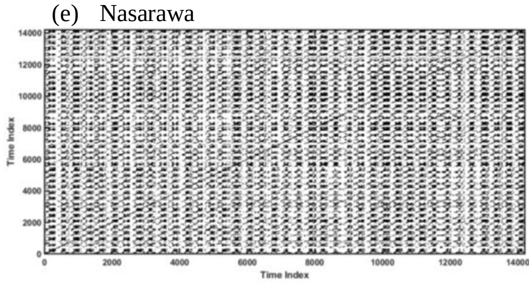
(d) Kogi



(e) Nasarawa



(f) Niger



(g) Kwara

Figure 4: Recurrence plots of daily mean atmospheric temperature (℃) across north-central Nigeria

Lyapunov Exponents of Precipitation and Atmospheric Temperature

The Lyapunov exponents of the daily mean values of precipitation and atmospheric temperature across north-central Nigeria from 1982-2020 were computed and the results presented in Tables 4 & 5 while the spatial distribution is presented in Figure 5. It is worthy to note that all the Lyapunov exponent values are positive and small indicating low degree of deterministic chaos.

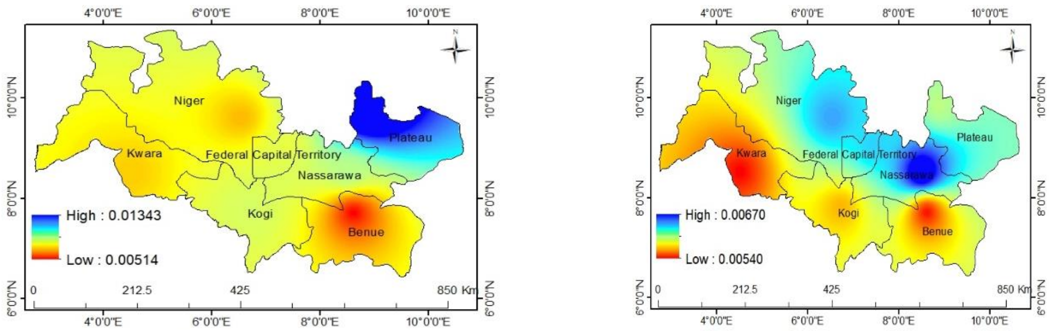
Table 4: Lyapunov Exponents of Precipitation in the Sampled States

Locations	Embedding Dimension, m	Time Delay, τ (days)	Lyapunov exponents, λ (per day)
Benue	4	9	0.00515
FCT Abuja	4	9	0.00712
Plateau	4	3	0.01340
Kogi	4	7	0.00735
Nasarawa	4	6	0.00765
Niger	4	8	0.00631
Kwara	4	2	0.00646

Table 5: Lyapunov Exponents of Atmospheric Temperature in the Sampled States

Locations	Embedding Dimension, m	Time Delay, τ (days)	Lyapunov exponents, λ (per day)
Benue	4	10	0.00540
FCT Abuja	4	10	0.00620
Plateau	4	10	0.00593
Kogi	4	10	0.00564
Nasarawa	4	10	0.00671
Niger	4	10	0.00627
Kwara	4	10	0.00539

The plot in Figure 5 shows the spatial variation of Lyapunov exponents (deterministic chaos) in daily mean values of precipitation and average temperature across North-central Nigeria.



(a) Precipitation (b) Atm. Temperature

Figure 5: Spatial variation of Lyapunov exponents in mean daily values of the meteorological parameters across North-central Nigeria from 1982-2020

The results of the Lyapunov exponents of precipitation shows that, Plateau recorded the highest value (0.0134/day) and Makurdi recording the lowest value (0.00515/day). This may be attributed to the difference in altitude and level of anthropogenic activities across the different locations as the higher altitude and cooler weather of Plateau coupled with the numerous manufacturing companies emitting greenhouse gases leads to more condensation of clouds with aerosols and gaseous pollutants causing more chaotic precipitation in the state. Furthermore, the Lyapunov exponents of atmospheric temperature indicates that Nasarawa recorded the highest value (0.00671/day) while Kwara recording the lowest value (0.00539/day). This too may be attributed to the difference in level of anthropogenic activities such as vehicular emissions, mining activities and manufacturing companies across the different locations emitting greenhouse gases leading to more aerosols and gaseous pollutants causing fluctuating/chaotic air temperature across the region (Echi *et al.*, 2025).

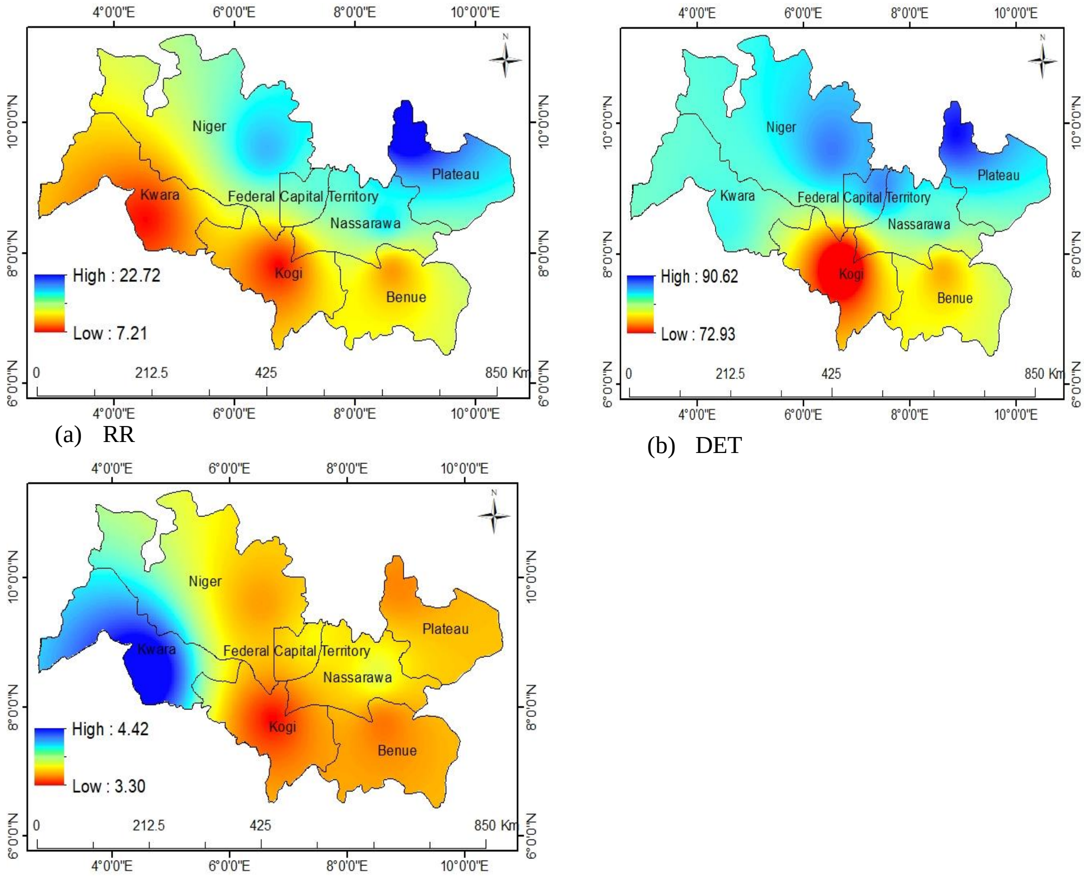
Recurrence Statistics of Precipitation and Atmospheric Temperature

The results from the computation of RQA Parameters, such as Recurrence Rate (RR), Determinism (DET), longest diagonal line (L_{max}), Shannon Entropy (ENTR), Laminarity (LAM) and Trapping Time (TT) for precipitation and atmospheric temperature across seven states are displayed Tables 6 – 9. The spatial interpolation and temporal trends are presented in Figures 6 – 9:

Table 6: RQA Statistics of Precipitation in the sampled locations

Locations	Embed. Dim. m	Time Delay τ (days)	Recurrence threshold, ε	Recurrence Rate (RR) (%)	Determinism (DET) (%)	Max. Diagonal (L_{max})	Shannon Entropy (ENTR)	Laminarity (LAM)
Benue	4	9	0.2160	15.422	83.549	108.00	3.513	91.489
FCT Abuja	4	9	0.2160	22.245	88.297	142.00	3.996	93.938

Locations	**Embed. Dim. m**	**Time Delay τ (days)**	**Recurrence threshold, ε**	**Recurrence Rate (RR) (%)**	**Determinism (DET) (%)**	**Max. Diagonal (L_{max})**	**Shannon Entropy (ENTR)**	**Laminarity (LAM)**
Plateau	4	3	0.0930	28.247	87.520	166.00	5.897	93.440
Kogi	4	7	0.1610	11.751	75.482	139.00	3.467	86.671
Nasarawa	4	6	0.1500	21.207	86.818	157.00	3.747	93.251
Niger	4	8	0.2400	25.743	88.854	161.00	3.925	94.103
Kwara	4	2	0.0444	9.833	87.147	103.00	4.718	93.039



(a) RR

(b) DET

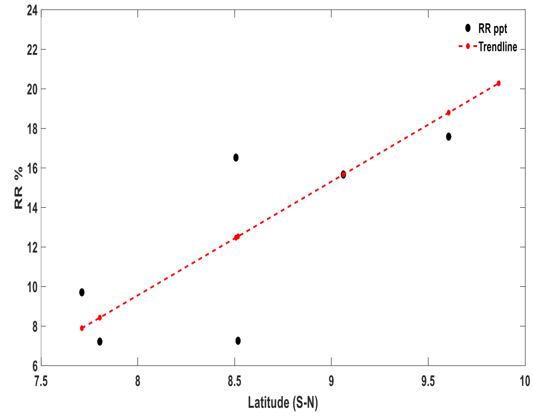
(c) ENTR

Figure 6: Spatial variation of RR, DET and ENTR for daily mean precipitation across North-central Nigeria

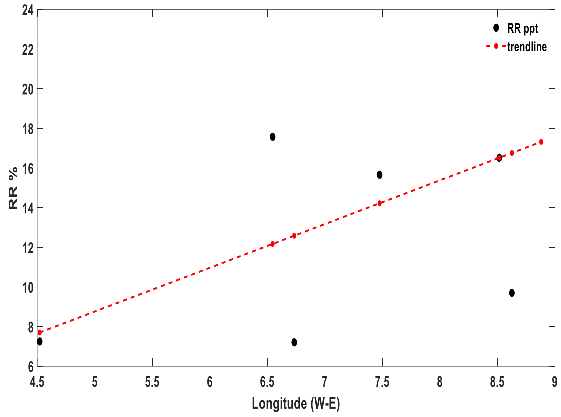
The latitudinal and longitudinal trends of RQA stats for precipitation using Pearson's correlation coefficient are presented in Table 7.

Table 7: Latitudinal and Longitudinal Trends of RQA Stats for Precipitation

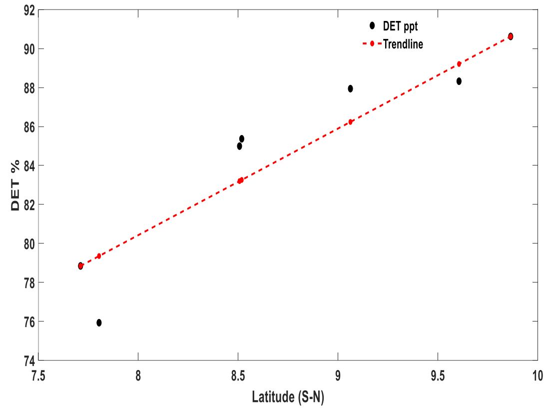
RQA Parameter	**Latitudinal**			**Longitudinal**		
	Pearson's Correlation coefficient, R	**p-value (α = 0.05)**	**Trend description (significance)**	**Pearson's Correlation coefficient, R**	**p-value (α = 0.05)**	**Trend description (significance)**
RR	0.8545	0.0143	increasing (significant)	0.5742	0.1777	increasing (not significant)
DET	0.9340	0.0021	increasing (significant)	0.0590	0.9000	No trend (not significant)
ENTR	-0.0046	0.9922	decreasing (not significant)	-0.7003	0.0797	decreasing (not significant)



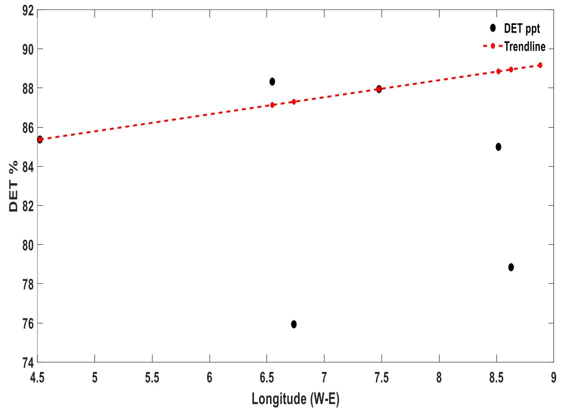
(a) Latitudinal trend of RR for precipitation



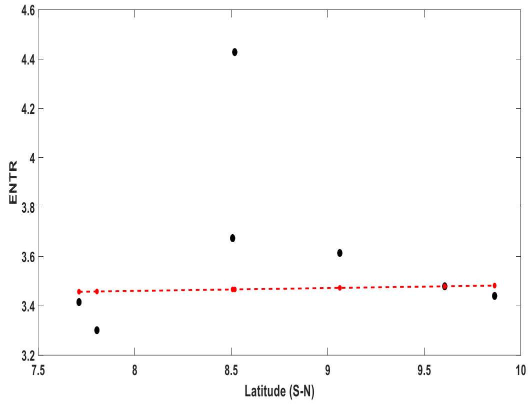
(b) Longitudinal trend of RR for precipitation



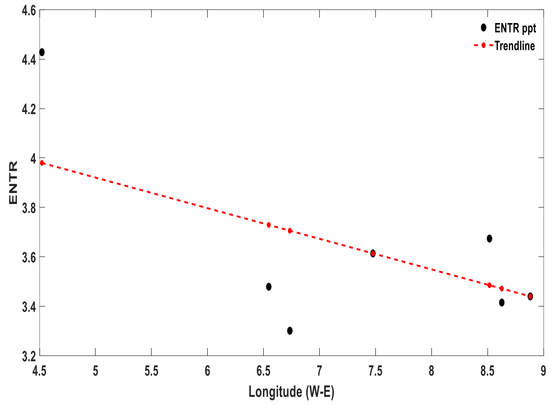
(c) Latitudinal trend of DET for precipitation



(d) Longitudinal trend of DET for precipitation



(e) Latitudinal trend of ENTR for precipitation

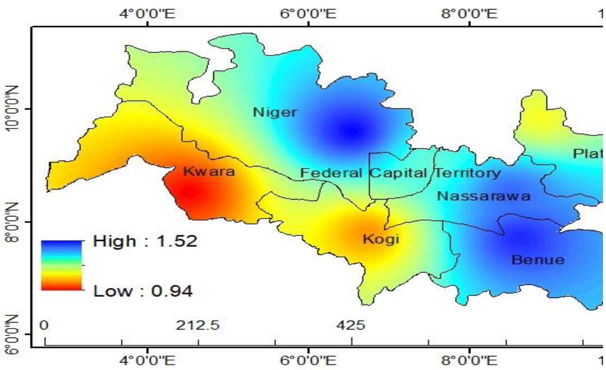


(f) Longitudinal trend of ENTR for precipitation

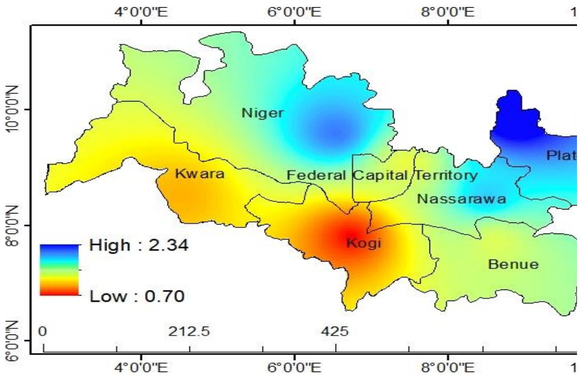
Figure 7: Temporal Trend of RQA Statistics for Precipitation across North-Central Nigeria

Table 8: RQA Statistics of Atmospheric Temperature in the Sampled Locations

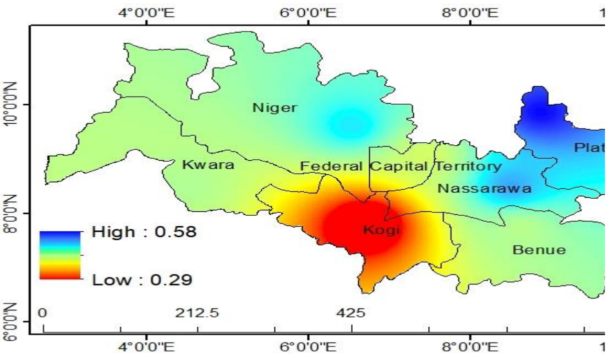
Locations	Embedding Dimension m	Time Delay τ (days)	Recurrence threshold, ε	Recurrence Rate (RR)	Determinism (DET)	Max. Diagonal (L_{max})	Shannon Entropy (ENTR)	Laminarity (LAM)
Benue	4	10	0.3000	0.5122	0.1612	5.0000	0.2523	1.1394
FCT Abuja	4	10	0.3000	0.4394	0.1820	4.0000	0.2311	1.1760
Plateau	4	10	0.3000	0.3905	0.4015	4.0000	0.1574	2.1048
Kogi	4	10	0.3600	0.6996	0.3192	4.0000	0.1640	1.7762
Nasarawa	4	10	0.2600	0.2837	0.0991	4.0000	0.1485	0.6478
Niger	4	10	0.2600	0.3019	0.1128	4.0000	0.1274	0.7926
Kwara	4	10	0.3000	0.3140	0.1816	4.0000	0.2022	1.1950



(a) RR



(b) DET



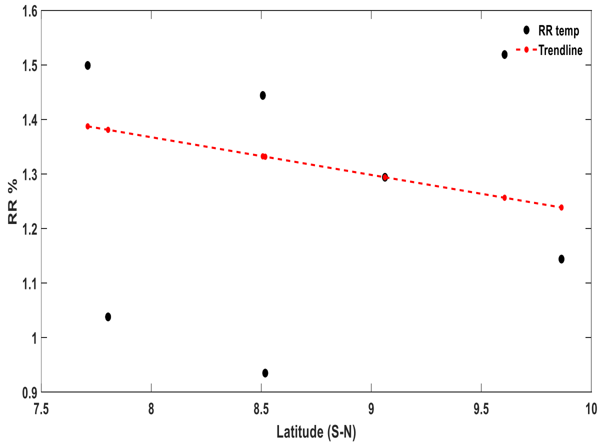
(c) ENTR

Figure 8: Spatial variation of RR, DET and ENTR for daily mean atmospheric temperature across North-central Nigeria

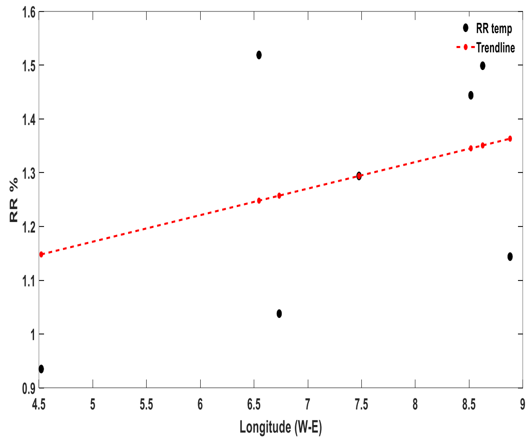
The latitudinal and longitudinal trends of RQA stats for atmospheric temperature using Pearson's correlation coefficient are presented in Table 9.

Table 9: Latitudinal and Longitudinal Trends of RQA Stats for Atmospheric Temperature

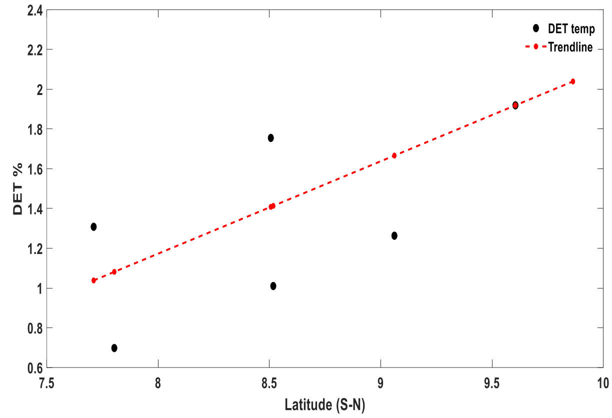
RQA Parameter	Latitudinal			Longitudinal		
	Pearson's Correlation coefficient, R	p-value (α = 0.05)	Trend description (significance)	Pearson's Correlation coefficient, R	p-value (α = 0.05)	Trend description (significance)
RR	0.0830	0.8595	No trend (not significant)	0.5512	0.1997	increasing (not significant)
DET	0.7885	0.0351	increasing (significant)	0.5423	0.2085	increasing (not significant)
ENTR	0.6905	0.0859	increasing (not significant)	0.3795	0.4012	increasing (not significant)



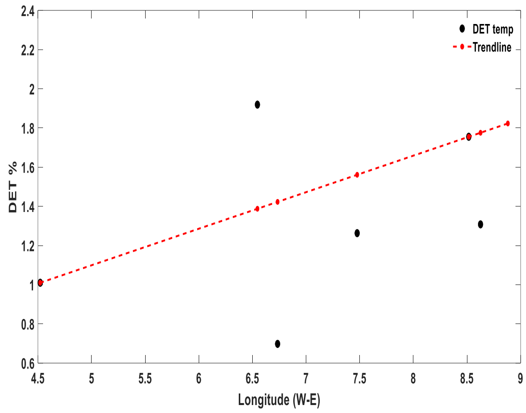
(a) Latitudinal trend of RR for temperature



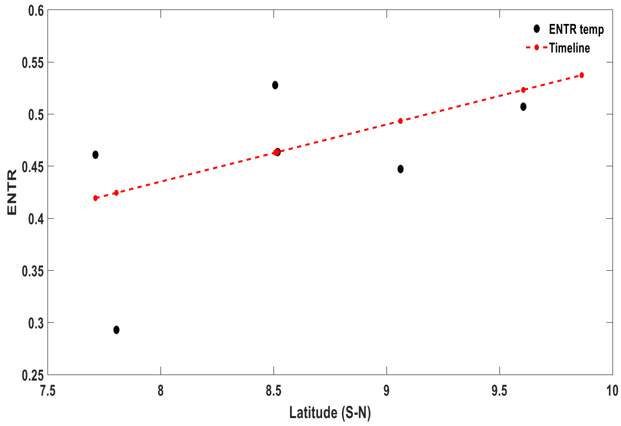
(b) Longitudinal trend of RR for temperature



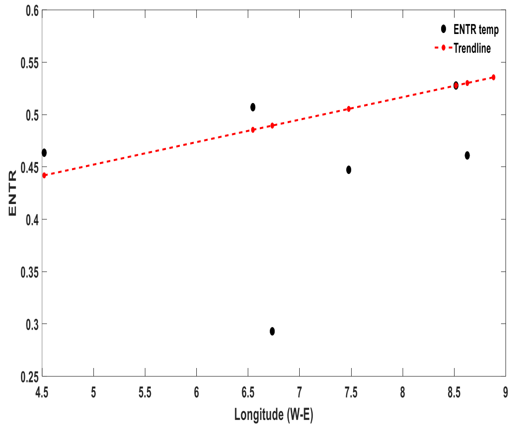
(c) Latitudinal trend of DET for temperature



(d) Longitudinal trend of DET for temperature



(e) Latitudinal trend of ENTR for temperature



(f) Longitudinal trend of ENTR for temperature

Figure 9: Temporal Trend of RQA Statistics for Atmospheric Temperature across North-Central Nigeria

The RQA statistics computed for precipitation and atmospheric temperature include: recurrence rate, determinism, maximum length of diagonal line, entropy and laminarity. The results in Tables 6 – 9 as well as the spatial maps (Figures 6 & 8) and temporal trends (Figures 7 & 9) show that, the recurrence rate which quantifies the similarity in cyclic/seasonal behaviour generally has very low values across all the sampled states in the North-Central Nigeria. The spatial variation of recurrence rate (RR %) for precipitation and atmospheric temperature increases latitudinally (Figure 6a & 8a) over the years as you move up north, having the highest value recorded in Plateau as 22.72 % followed by Kwara and Kogi with value as 7.21 %, whereas atmospheric temperature (Figures 7a & 9a) recorded low recurrence rate values of 0.3-0.7 %. The longitudinal variations (Figures 7b & 9b) also showed insignificant increasing trends (Table 7) from East –West. This may be attributed to the increase in altitude of the locations from East-West with peak at Jos-Plateau.

Spatial variation of Determinism (DET %) in mean daily values of the meteorological parameters across North-central Nigeria illustrated in Figures 6b & 8b indicates that the precipitation in Plateau has the highest value recorded as 90.62% while atmospheric temperature recorded the low values ranging from 0.03-0.70%. However, from the results of the latitudinal and longitudinal spatial trend analysis of the daily mean precipitation and atmospheric temperature in Figure 7(c & d) and 9(c & d), it was observed that the determinism (DET) for precipitation increases significantly from South to North and records no significant trend from W-E while that of atmospheric temperature also increases significantly from S-N but insignificantly from West – East as shown in Figure 9. The chaotic trends in atmospheric temperature can be described as quasi-stochastic, while that of precipitation which shows higher DET and RR values show stronger determinism (sensitivity to initial conditions), this is attributed to its seemingly fixed period of occurrence with its distribution within the season being irregular.

Shannon entropy (ENTR) in precipitation across North-central Nigeria presented in Table 6, show that the precipitation in Kwara has the highest value as 4.42 followed by Niger with the value as 1.38 while that of Kogi recorded the lowest value of 0.29. However, the results of the latitudinal temporal trends for daily mean precipitation in Figure 7(e & f) shows no significant trend while the longitudinal trend shows an insignificant decreasing trend (Table 7). For the atmospheric temperature (Figures 9e & 9f and Table 9) both the latitudinal and longitudinal trends were observed to be insignificant increasing trends ($\alpha < 0.05$) across the sampled states. These variations in trend may be attributed to turbulent nature of the winds powered by varying changes in sea surface temperature (El Nino/southern oscillation) from the Atlantic Ocean in the South to the Sahara Desert in the North, irregular shifting of the intertropical discontinuity and varying degrees of emissions from human anthropogenic activities and which causes random climatic fluctuations (Ogunjo *et al.*, 2015, Fuwape *et al.*, 2016).

CONCLUSION

This research has applied recurrence quantification analysis to investigate the chaotic features in mean daily precipitation and atmospheric temperature time series across the seven states of North Central Nigeria from 1982 – 2020. Recurrence plots were successfully constructed based on the reconstitution of state space via the method of delays in accordance with Takens' embedding theorem while quantitative tools like Lyapunov exponents and recurrence quantification statistics were accurately computed. The results obtained show that the mean daily precipitation and atmospheric temperature across the seven locations sampled show low degree of deterministic chaos recurrence plots showed correlated points in chequered patterns with short diagonal lines in an aligned manner parallel to the line of identity. Furthermore, the largest Lyapunov exponents are all positive values and finite inferring divergence of trajectories and system complexity. The RQA statistics computed show that the recurrence rate and determinism which quantifies the recurrence in cyclic behaviour generally has very low values across all the sampled states in the North-Central Nigeria for atmospheric temperature but has higher values for RR and DET for precipitation as a result of its seasonality and thus is more sensitive to distortions like human anthropogenic activities. These results are a confirmation of deterministic chaos in the dynamics of the climate of North-Central Nigeria which is responsible for the manifestation of floods and heat waves during the study period across North-Central Nigeria.

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