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A Comparative Analysis of Gaussian, Triangular, and Trapezoidal Membership Functions in a Mamdani Fuzzy Inference System for Evaluating Mathematics Learning Achievement

***Audu Abubakar, Musa Adeku Ibrahim and Rotimi Kehinde**

Department of Mathematics, Faculty of Science, Federal University Lokoja, Kogi State, Nigeria.

*Corresponding authors' email: auduabubakar572@gmail.com

ABSTRACT
Educational assessment systems conventionally rely on crisp numerical scores that inadequately capture the gradual and uncertain nature of student learning, and while fuzzy inference systems (FIS) have been applied to address this limitation, prior studies rarely compare alternative membership function designs under identical experimental conditions, leaving open the practical question of which membership function type best supports fuzzy-based educational assessment. This study addresses that gap by presenting a comparative evaluation of Gaussian, Triangular, and Trapezoidal membership functions, which were selected because they are among the most widely used membership functions in fuzzy educational assessment and differ in their smoothness, computational simplicity, and plateau characteristics within a Mamdani Fuzzy Inference System (FIS) for assessing student learning achievement. Three fuzzy models were developed using examination score, assignment completion, and class participation as input variables, while learning achievement served as the output variable. A dataset comprising 360 secondary school students was used for model development and validation. Teacher evaluation scores were employed as benchmark values for assessing model performance. The three models were implemented using identical rule bases, inference mechanisms, and defuzzification procedures, differing only in the type of membership function employed. Model performance was evaluated using the Pearson correlation coefficient, root mean square error (RMSE), and mean absolute error (MAE). The results revealed that the Gaussian model achieved the highest correlation with teacher evaluations ($r = 0.9623$), indicating superior agreement with teacher assessment patterns, whereas the Triangular model produced the lowest RMSE (7.194) and MAE (4.867), demonstrating greater numerical prediction accuracy. The Trapezoidal model exhibited comparatively poor performance across all evaluation metrics. By isolating membership function type as the sole experimental variable, this study provides a controlled empirical comparison of the three functions for educational assessment and offers practical guidance for selecting membership functions in fuzzy-based educational and decision-support systems, with implications extending to fuzzy inference system design more broadly.

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INTRODUCTION

Mathematics is widely recognized as a fundamental discipline that underpins scientific advancement, technological innovation, and logical reasoning. Despite its importance, mathematics remains one of the most challenging subjects for secondary school students worldwide, with many learners experiencing difficulties in achieving satisfactory performance levels (Turker & Turanli, 2008). Traditional educational assessment systems typically rely on rigid numerical grading schemes that reduce complex learning processes to single numerical scores. Such approaches often fail to represent students' actual learning progress.

One major limitation of conventional grading systems is the presence of sharp classification boundaries. For example, a student obtaining a score of 49% may be classified as having failed, whereas another student scoring 50% is considered successful, despite the negligible difference in their actual performance (Ertugrul, 2006). Such binary distinctions do not adequately reflect the continuous nature of learning and may result in assessment outcomes that differ from the nuanced judgments typically made by experienced educators. Consequently, there is a growing need for assessment methodologies capable of handling uncertainty, vagueness, and overlapping performance categories.

Fuzzy logic, introduced by Zadeh (1965), provides an effective framework for representing imprecision and uncertainty. Unlike classical binary logic, fuzzy logic allows partial membership in linguistic categories such as *Poor*, *Average*, *Good*, and *Excellent*. This capability enables educational performance to be modeled more realistically by reflecting the gradual progression of student achievement. Over the years, fuzzy logic has been successfully applied to numerous educational domains including student grading, academic performance prediction, teaching quality evaluation, learning analytics, and educational decision-support systems (Biswas, 1995; Chen & Lee, 1999; Bai & Chen, 2008; Saleh & Kim, 2009). Karma et al. (2024) proposed a fuzzy logic approach to support decision-making regarding students' programme changes. The model utilized academic indicators such as SSCE results, UTME scores, and course grades as inputs to a fuzzy inference system implemented in MATLAB. The study demonstrated that fuzzy logic can effectively handle uncertainty and subjectivity in educational decision processes, providing reliable recommendations. These findings further demonstrate the suitability of fuzzy logic for educational assessment and decision support application.

The Mamdani Fuzzy Inference System (FIS) is among the most widely adopted fuzzy reasoning frameworks because of its interpretability, linguistic transparency, and ability to emulate human reasoning through rule-based inference (Mamdani, 1974). Mamdani systems allow multiple assessment criteria to be integrated simultaneously while preserving expert knowledge in the form of linguistic rules. Previous studies have demonstrated the effectiveness of Mamdani fuzzy systems in educational assessment, student performance evaluation, and intelligent tutoring environments (Voskoglou, 2013; Doz et al., 2022). Their ability to handle uncertainty and model qualitative judgments makes them particularly suitable for educational applications where precise boundaries between performance categories rarely exist.

A critical component of any fuzzy inference system is the membership function, which determines how crisp input values are transformed into fuzzy linguistic representations. Membership functions directly influence system sensitivity, decision boundaries, and overall predictive performance. Among the most frequently used membership function types are Triangular, Trapezoidal, and Gaussian functions. Triangular membership functions are popular because of their simplicity, ease of implementation, and low computational cost (Subbotin & Bilotski, 2014). Trapezoidal membership functions provide broader regions of complete membership and are often employed when stable performance intervals must be represented (Nguyen & Walker, 2018). Gaussian membership functions offer smooth and continuously differentiable transitions between linguistic categories, making them theoretically suitable for modeling gradual learning progression and reducing abrupt changes in inference outputs.

Several studies have investigated the application of fuzzy membership functions in decision-support and classification systems. For example, Subbotin and Bilotski (2014) reported that triangular functions provide efficient computational performance in classification problems, while Nguyen and Walker (2018) demonstrated the robustness of trapezoidal functions in uncertainty modelling. Similarly, Gaussian membership functions have been widely employed in intelligent systems because of their smooth representation of uncertainty and their ability to reduce discontinuities in decision surfaces. However, most existing studies adopt a single membership function type and rarely evaluate alternative representations under identical experimental conditions.

Beyond these general applications, several studies have applied fuzzy logic directly to grading and assessment, but each has evaluated only a single membership function configuration rather than comparing alternatives under controlled conditions, which limits the generalisability of their conclusions. Echauz and Vachtsevanos (1995) and Fourali (1997) demonstrated the feasibility of fuzzy grading and portfolio assessment, while Gedela and Bodanki (2021) and Mitra and Gog (2015) developed fuzzy evaluation systems for general and technical-skills assessment, respectively. Subbotin and Voskoglou (2016) reviewed a range of fuzzy assessment methods without empirically contrasting membership function types. Nor et al. (2021) applied fuzzy logic to compare mathematics performance across rural and urban students but, like the studies above, relied on a single, pre-selected membership function design. Turkmen and Killik (2024) developed a MATLAB-based fuzzy grading system for secondary school mathematics using triangular membership functions, however, their study did not examine whether alternative membership function types would have produced different or superior results. Taken together, this body of work establishes that fuzzy logic is a viable tool for educational assessment but leaves open the more fundamental question of which membership function design is best suited to that task and under what conditions, a question that requires holding the dataset, rule base, inference mechanism, and defuzzification procedure constant while varying only the membership function, which to the best of our knowledge, none of the studies reviewed here has done.

Therefore, a significant knowledge gap exists regarding the influence of membership function selection on the performance of fuzzy-based educational assessment models. To address this gap, the present study develops three Mamdani fuzzy inference system variants employing Gaussian, Triangular, and Trapezoidal membership functions for the assessment of student learning achievement. The three models employ identical input variables, output variables, rule bases, inference mechanisms, and defuzzification procedures. The only difference among the models is the type of membership function employed during fuzzification. This controlled design enables a direct investigation of the effect of membership function selection on model performance.

The models were developed and validated using data obtained from 360 secondary school students. Examination score, assignment completion, and class participation were used as input variables, while learning achievement served as the output variable. Teacher evaluation scores were employed as benchmark values for validation. Model performance was assessed using the Pearson correlation coefficient, Root Mean Square Error (RMSE), and Mean Absolute Error (MAE). Its novelty lies not in the use of fuzzy logic for assessment as such, but in providing a controlled, empirical basis for comparing these three membership functions rather than assuming or asserting the suitability of any single design, as prior studies have done. The study addresses the following research questions:

i. How does membership function type affect the correlation between fuzzy model outputs and teacher evaluations?
ii. How does membership function type affect the numerical prediction accuracy (RMSE and MAE) of fuzzy model outputs?
iii. What practical guidance can be derived for selecting membership functions in fuzzy-based educational assessment and decision-support systems?

By answering these questions, the study is expected to contribute a controlled empirical benchmark for membership function selection that can inform the design of future fuzzy inference systems in education and related decision-support domains, drawing on established fuzzy set theoretic foundations (Ross, 2010; Zimmermann, 2011).

MATERIALS AND METHODS

Study Design and Dataset

This study adopted a quantitative comparative research design to investigate the influence of membership function selection on the performance of Mamdani fuzzy inference systems (FISs) for educational assessment. A dataset comprising 360 secondary school students was collected from six public secondary schools in Lokoja, Kogi State, Nigeria, during the 2024–2025 academic session. Stratified random sampling was employed to ensure balanced representation across schools, with 60 students selected from each institution.

Three academic performance indicators were used as input variables:

i. Examination Score
ii. Assignment Completion
iii. Class Participation

The output variable was Learning Achievement, while Teacher evaluation scores served as benchmark values for model validation. Examination scores were obtained from formal school examinations, assignment completion records were extracted from continuous assessment records, and class participation scores were derived from classroom observation reports maintained by subject teachers.

A total of three Mamdani fuzzy inference system variants were developed using the same dataset. The models differed only in the type of membership function employed during fuzzification:

i. Gaussian Membership Functions
ii. Triangular Membership Functions
iii. Trapezoidal Membership Functions

This experimental design enabled a controlled comparison of membership function effects while maintaining identical datasets, rule bases, inference mechanisms, and defuzzification procedures across all models.

Fuzzy Inference System Architecture

Three Mamdani fuzzy inference systems were independently developed under identical experimental conditions. Each model employed identical input variables, output variables, rule bases, inference mechanisms, and defuzzification strategies. The only difference among the models was the membership function type used during fuzzification.

Each model consisted of:

i. Three input variables: Examination Score, Assignment Completion, and Class Participation.
ii. One output variable: Learning Achievement.
iii. Three linguistic terms for each input: Low, Medium, and High.
iv. Five linguistic terms for the output: Very Poor, Poor, Average, Good, and Excellent.
v. Twenty-seven expert-defined fuzzy IF–THEN rules.
vi. Mamdani min–max inference.
vii. Centre of Gravity (COG) defuzzification method.

The Mamdani inference procedure consists of **four stages:** fuzzification, rule evaluation, aggregation, and defuzzification.

For each input variable, crisp values are transformed into fuzzy membership degrees. The firing strength of rule *r* is determined using the minimum operator:

$$\alpha_r = \min \{\mu_A1 (x_1), \mu_A2 (x_2), \ldots, \mu_An (x_n)\} \quad (1)$$

Where $\mu_Ai (x_i)$ represents the membership degree of input x_i in its corresponding fuzzy set. The final crisp output is obtained using Centre of Gravity defuzzification:

$$y^* = \frac{\int y\mu_B(y)dy}{\int \mu_B(y)dy} \quad (2)$$

Where y is the crisp value and $\mu_B(y)$ is the aggregated output function.

Membership Function Definitions

The three membership function types investigated in this study are defined as follows.

Triangular Membership Function

The triangular membership function is defined by three parameters [*a*, *b*, *c*]:

$$\mu(x; a, b, c) = \begin{cases} 0, & x \leq a \text{ or } x \geq c \\ \frac{x-a}{b-a}, & a < x \leq b \\ \frac{c-x}{c-b}, & b < x < c \end{cases} \quad (3)$$

Trapezoidal Membership Function

The trapezoidal membership function is defined by four parameters [*a*, *b*, *c*, *d*]:

$$\mu(x; a, b, c, d) = \begin{cases} 0, & x \leq a \text{ or } x \geq d \\ \frac{x-a}{b-a}, & a < x \leq b \\ 1, & b < x \leq c \\ \frac{d-x}{d-c}, & c < x \leq d \end{cases} \quad (4)$$

Gaussian Membership Function

The Gaussian membership function is represented by:

$$\mu(x; \mu, \sigma) = e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (5)$$

Where *c* is the centre and σ controls the spread of the curve.

Membership Function Parameters

Model 1: Triangular Membership Functions
Model 2: Trapezoidal Membership Functions
Model 3: Gaussian Membership Functions

The Gaussian membership functions were designed using evenly distributed centres to ensure smooth transitions among neighbouring linguistic categories.

Fuzzy Rule Base

A total of 27 fuzzy IF–THEN rules were formulated to represent expert knowledge regarding student learning achievement.

The general form of each rule is:

If x1 is A_1 AND x_2 is A_2 AND AND x_n is A_n then y is B (6)

Where A_1, A_2, and A_3 denote fuzzy input conditions and B represents the output linguistic term.

MATLAB Implementation

All fuzzy inference systems were implemented using MATLAB R2024a and the Fuzzy Logic Toolbox (MathWorks, 2023). The implementation procedure consisted of:

i. Creating a Mamdani FIS.
ii. Defining input and output variables.
iii. Specifying membership functions.
iv. Constructing the fuzzy rule base.
v. Applying Centre of Gravity defuzzification.
vi. Evaluating student records using evalfis().

The generated outputs represent predicted learning achievement scores for all 360 students.

RESULTS AND DISCUSSION

Descriptive Statistics of the Dataset

A total of 360 student records were used to develop and evaluate the three fuzzy inference system models. The dataset consisted of examination score, assignment completion, class participation, and teacher evaluation scores. Descriptive statistics were computed to provide an overview of the dataset characteristics.

Table 1 presents the descriptive statistics of the variables used in the study.

Table 1: Descriptive Statistics of the Study Variables

Variable	Mean	Std. Dev.	Minimum	Maximum
Examination Score	65.053	14.347	30.000	100.000
Assignment Completion	67.731	13.668	40.000	100.000
Class Participation	62.475	14.725	30.000	100.000
Teacher Evaluation	65.086	12.565	34.000	98.667

The descriptive statistics indicate substantial variability among students, suggesting that the dataset is appropriate for evaluating the discriminative performance of the fuzzy inference models.

Fuzzy Inference System Architecture

Figures 1, 2 and 3 illustrate the structures of the Gaussian, Triangular, and Trapezoidal Mamdani fuzzy inference systems developed in this study.

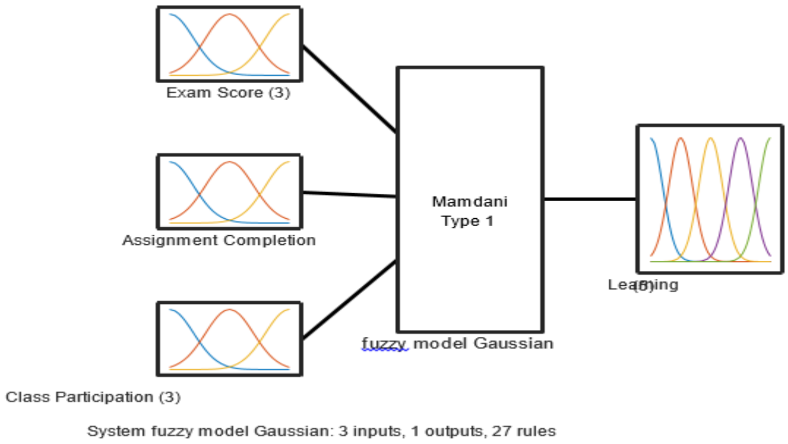

Figure 1: Gaussian Fuzzy Inference System Architecture

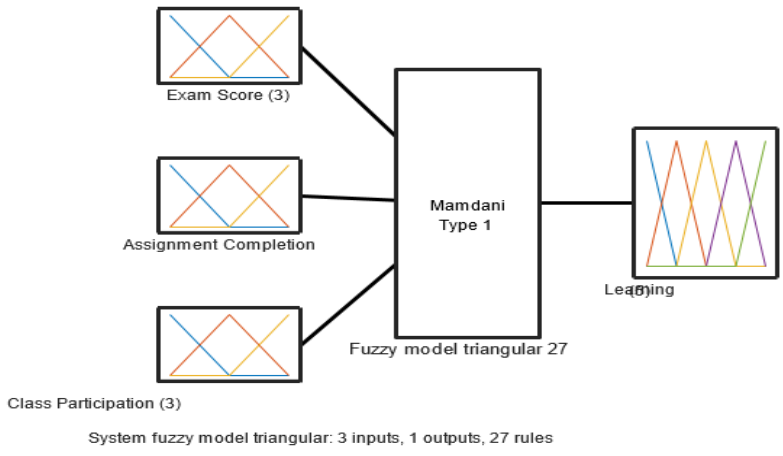

Figure 2: Triangular Fuzzy Inference System Architecture

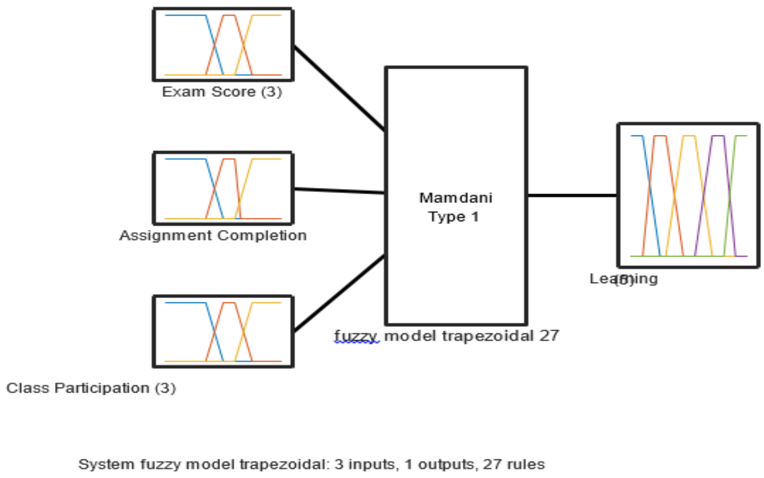

Figure 3: Trapezoidal Fuzzy Inference System Architecture

All three models share the same input variables, output variable, rule base, inference mechanism, and defuzzification method. The only difference lies in the type of membership function employed.

Membership Function Analysis

Examination Score Membership Functions

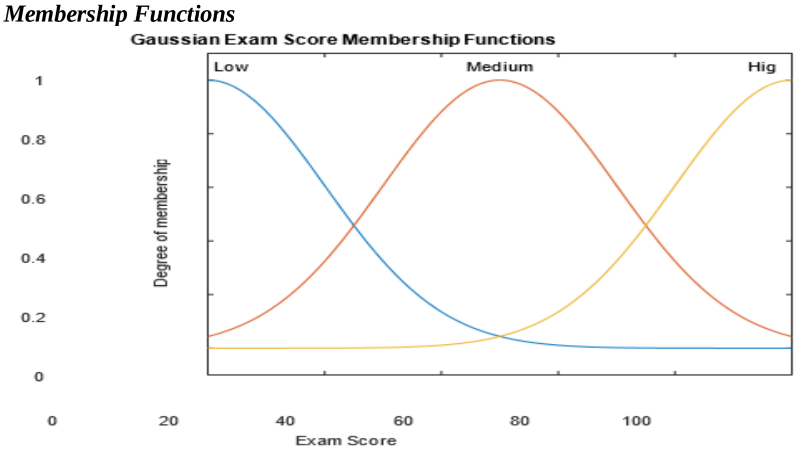

Figure 4: Gaussian Membership Functions for Examination Score

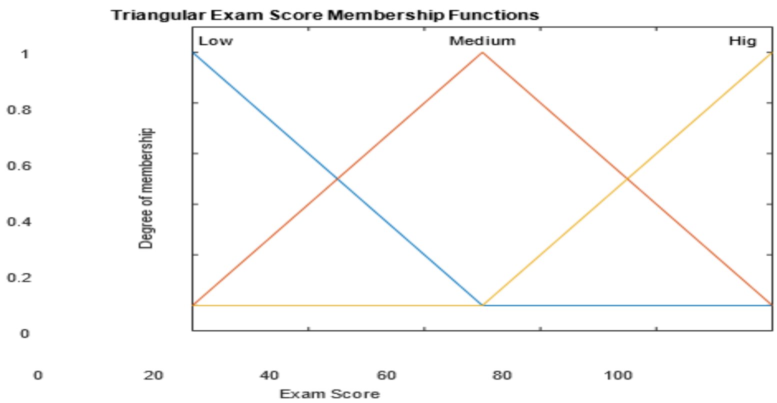

Figure 5: Triangular Membership Functions for Examination Score

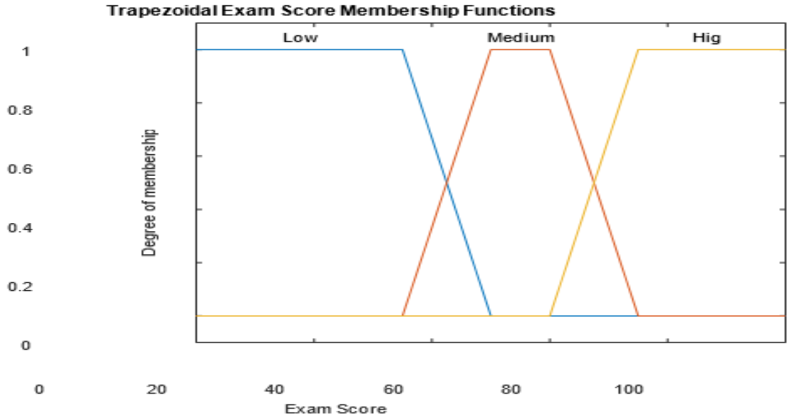

Figure 6: Trapezoidal Membership Functions for Examination Score

The Gaussian model provides smooth transitions between linguistic categories, whereas the Triangular and Trapezoidal models exhibit piecewise linear transitions.

Assignment Completion Membership Functions

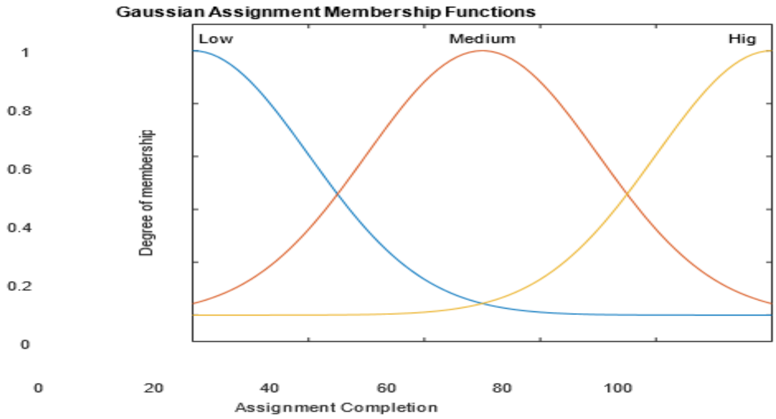

Figure 7: Gaussian Membership Functions for Assignment Completion

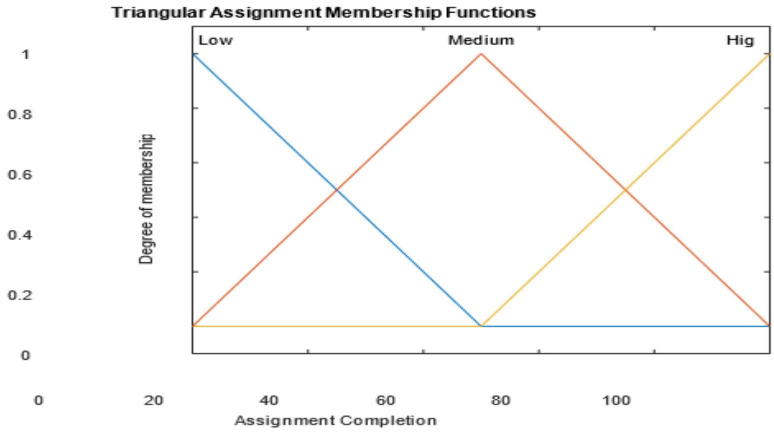

Figure 8: Triangular Membership Functions for Assignment Completion

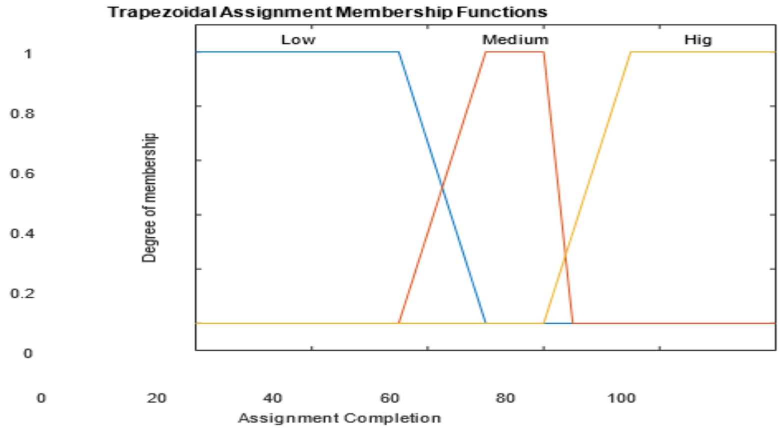

Figure 9: Trapezoidal Membership Functions for Assignment Completion

Class Participation Membership Functions

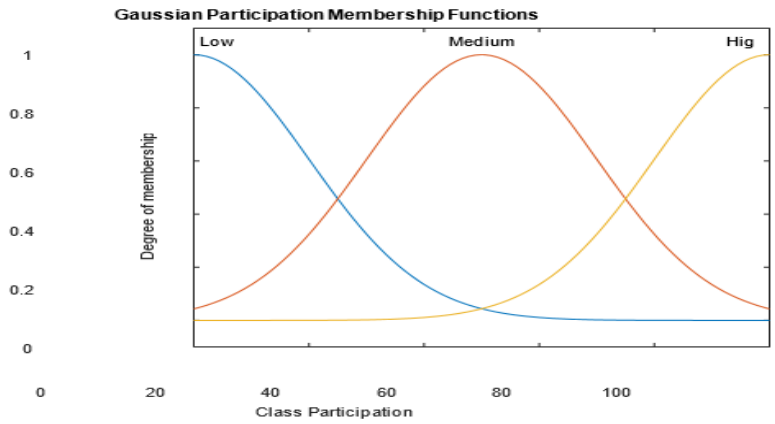

Figure 10: Gaussian Membership Functions for Class Participation

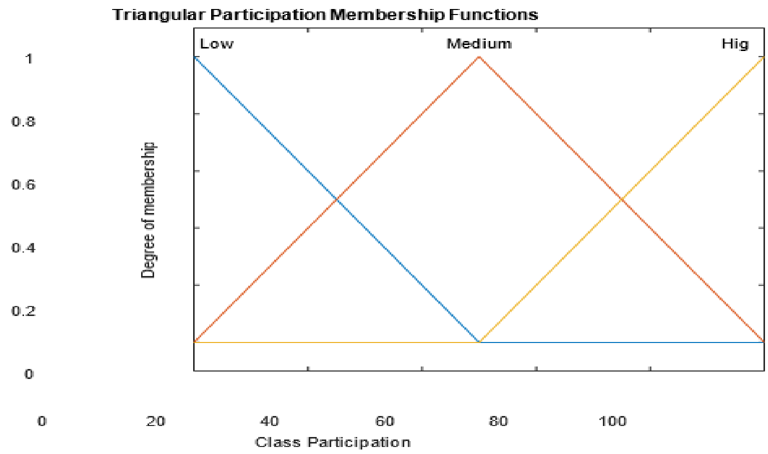

Figure 11: Triangular Membership Functions for Class Participation

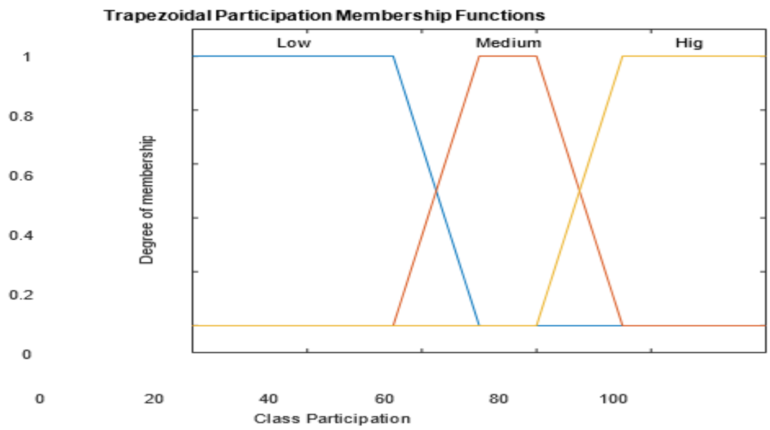

Figure 12: Trapezoidal Membership Functions for Class Participation

Learning Achievement Output Membership Functions

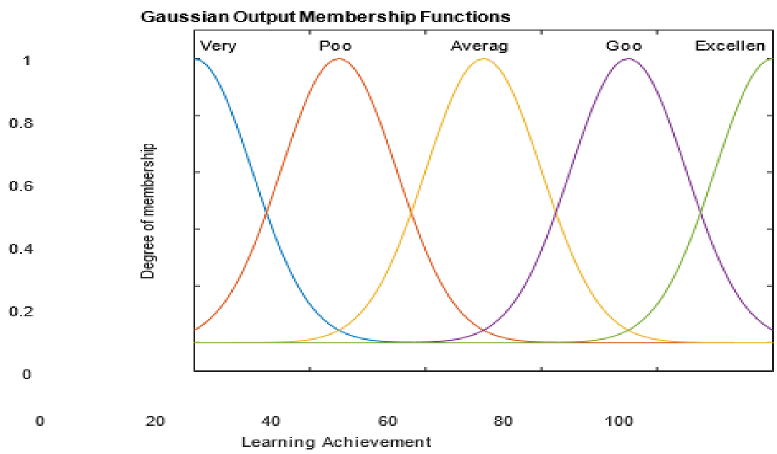

Figure 13: Gaussian Output Membership Functions

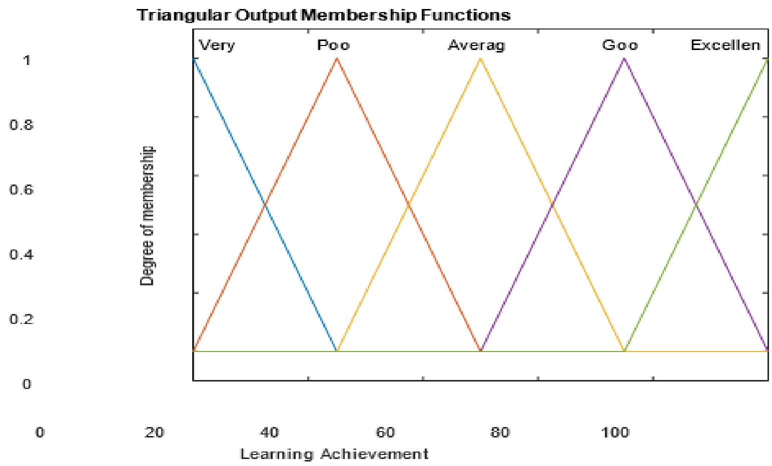

Figure 14: Triangular Output Membership Functions

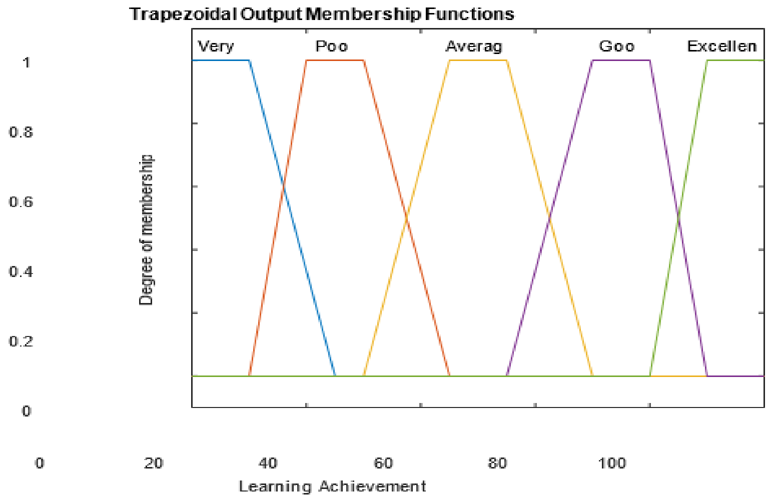

Figure 15: Trapezoidal Output Membership Functions

Surface Analysis

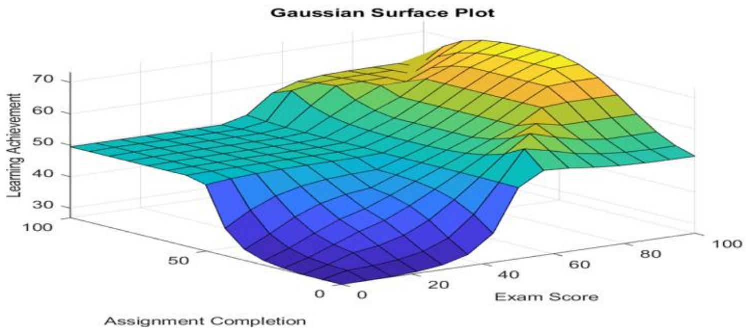

Figure 16: Gaussian Surface Plot

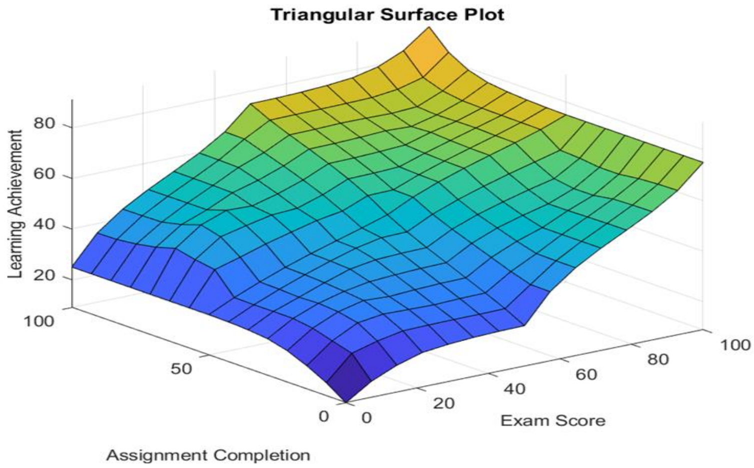

Figure 17: Triangular Surface Plot

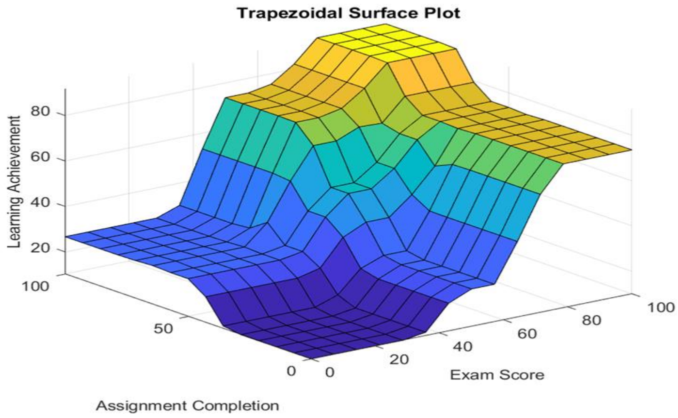

Figure 18: Trapezoidal Surface Plot

The Gaussian model exhibits the smoothest response surface, indicating gradual transitions across linguistic categories. The Triangular model produces sharper local transitions, while the Trapezoidal model exhibits flat plateau regions corresponding to complete membership intervals.

Comparison with Teacher Evaluation

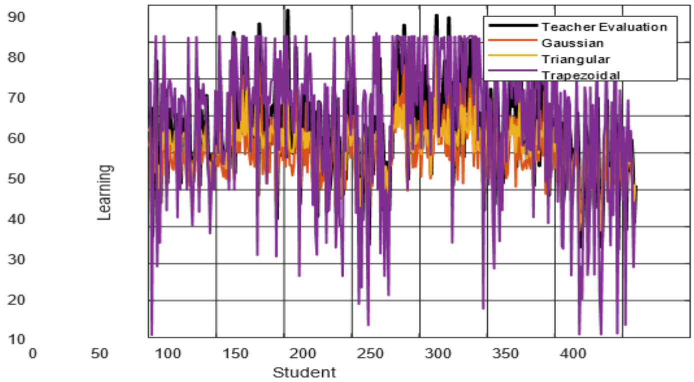

Figure 19: Comparison of Teacher Evaluation and Fuzzy Model Outputs

Visual inspection shows closest agreement between fuzzy model outputs and teacher evaluations, with the Gaussian model exhibiting the strongest clustering around the expected trend.

Quantitative Performance Evaluation

Table 2: Performance Comparison of Membership Function Models

Model	Correlation	RMSE	MAE
Gaussian	0.9623	7.263	6.561
Triangular	0.9280	7.194	4.867
Trapezoidal	0.9158	10.654	8.896

Correlation with Teacher Evaluation

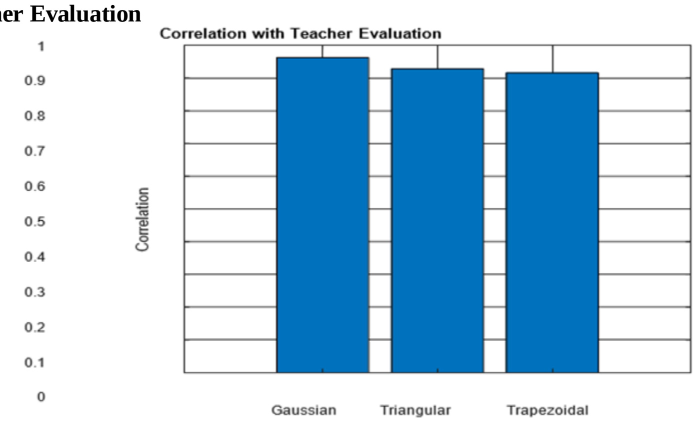

Figure 20: Correlation Comparison

RMSE Comparison

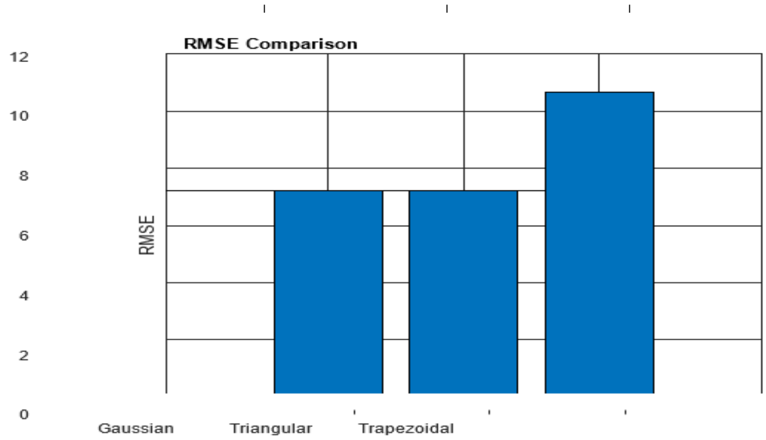

Figure 21: RMSE Comparison

MAE Comparison

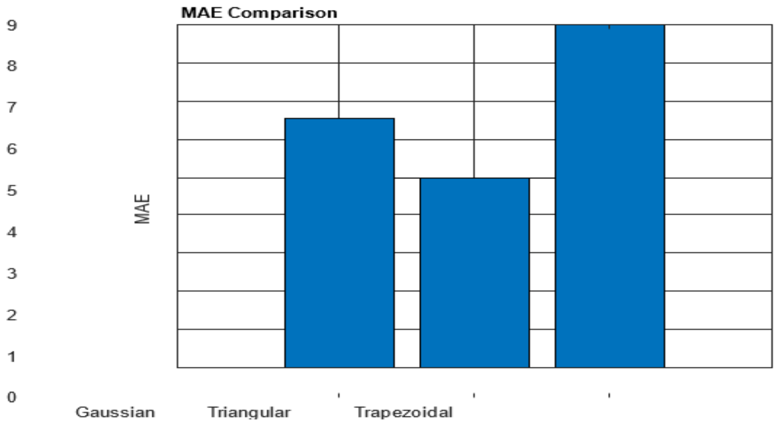

Figure 22: MAE Comparison

The Gaussian model achieved the highest correlation coefficient, indicating superior agreement with teacher-assessment trends. The Triangular model achieved the lowest RMSE and MAE values, indicating highest numerical prediction accuracy. The Trapezoidal model produced the weakest overall performance.

Discussion

The results demonstrate that the selection of membership functions significantly affects fuzzy system behaviour and prediction accuracy. The Gaussian model achieved the strongest correlation with teacher evaluations because its smooth bell-shaped membership functions preserve ranking consistency among students. In contrast, the Triangular model achieved the lowest RMSE and MAE values, suggesting superior numerical prediction accuracy. The Trapezoidal model exhibited lower sensitivity to moderate changes in input variables due to its extended plateau regions, resulting in comparatively weaker performance.

These findings suggest that Gaussian membership functions are preferable when maintaining consistency with teacher evaluation patterns is the primary objective, whereas Triangular membership functions are advantageous when minimizing prediction error is more important. Overall, the results confirm that membership function design plays a critical role in educational fuzzy inference systems.

These findings are broadly consistent with, and extend, prior work on fuzzy logic in educational assessment. Turkmen and Killik (2024), who evaluated only triangular membership functions for secondary school mathematics assessment, reported that triangular functions performed adequately for grading purposes. The present results confirm this, showing that the Triangular model achieved the lowest prediction errors, but also show that this advantage in numerical accuracy comes at the cost of weaker correlation with teacher judgements than the Gaussian model. Similarly, Nor et al. (2021) and Gedela and Bodanki (2021) demonstrated that fuzzy models can track human-assigned scores reasonably well, which aligns with the strong overall agreement observed here between the fuzzy model outputs and teacher evaluations across all three membership function types, while Mitra and Gog (2015) and Subbotin and Voskoglou (2016) fuzzy evaluation frameworks did not report comparable membership function contrasts, underscoring the relative scarcity of controlled comparative evidence to which this study contributes.

From a theoretical perspective, these results are consistent with the mathematical properties of the three membership function families (Ross, 2010; Zimmermann, 2011). The Gaussian function's smooth, infinitely differentiable shape produces gradual changes in firing strength as input values change, which likely explains its closer alignment with the continuous, holistic judgements that teachers make. The Triangular function's piecewise linear form concentrates membership more sharply around a single point, which appears to sharpen numerical discrimination and reduce averaging error even though it departs more readily from teachers' assessments. The Trapezoidal function's flat plateau regions, intended to represent stable performance intervals, instead reduced sensitivity to moderate within category variation in this dataset, which is a plausible explanation for its comparatively weaker performance across all three metrics.

Practically, these findings offer guidance for developers of fuzzy-based educational assessment and decision-support systems, including intelligent tutoring applications, where the priority is producing scores similar to teacher's judgement, Gaussian membership functions are preferable, whereas Triangular membership functions are preferable where minimising numerical prediction error is the priority, for example in automated grading that must closely match verifiable benchmark scores.

This study has several limitations. The dataset was drawn from a single geographical location (Lokoja, Kogi State, Nigeria) and a single educational level (secondary school), which may limit the generalisability of the findings to other regions, education systems, or age groups. In addition, only three membership function types and a single Mamdani inference configuration were examined, other function types (e.g., bell-shaped, sigmoidal) and inference frameworks (e.g., Sugeno, Tsukamoto) were outside the scope of the present study.

Future research could extend this comparative approach to Sugeno and Tsukamoto fuzzy inference systems, examine additional membership function types, apply optimisation techniques such as genetic algorithms, particle swarm optimisation, or neuro-fuzzy learning (Jang et al., 1997), and evaluate larger and more geographically diverse datasets to test the robustness of the present findings.

CONCLUSION

This study investigates how membership function selection affects the performance of Mamdani fuzzy inference systems for student learning achievement assessment. It compares Gaussian, Triangular, and Trapezoidal membership functions under identical dataset, rule-base, inference, and defuzzification conditions. The Gaussian model achieved the highest correlation with teacher evaluations (r = 0.9623), while the Triangular model achieved the lowest prediction errors (RMSE = 7.194, MAE = 4.867), and the Trapezoidal model demonstrated comparatively weaker performance across all evaluation metrics. The study's main contribution is this controlled empirical comparison itself. Unlike prior fuzzy assessment studies that adopt a single membership function design without justification, the present findings provide direct, quantitative evidence that membership function selection affects both the qualitative alignment of fuzzy assessment outputs with expert judgement and their numerical accuracy.

For educators, researchers, and developers of intelligent educational systems, these findings suggest a practical decision rule. Gaussian membership functions are preferable when the goal is to preserve consistency with teacher assessment patterns, whereas Triangular membership functions are preferable when minimising numerical prediction error is the priority, for instance in automated grading.

The findings should be interpreted in light of the study's reliance on data from a single geographical location and a single educational level, which limits the extent to which the results can be generalised beyond similar contexts. Future work should test the robustness of these findings using data from other regions and educational levels, investigate additional membership function types and Sugeno or Tsukamoto fuzzy inference systems, and explore adaptive optimisation approaches such as genetic algorithms, particle swarm optimisation, and neuro-fuzzy systems (Jang et al., 1997) to further enhance fuzzy-based educational assessment models. More broadly, the study underscores that membership function design is not a neutral implementation detail but a substantive choice that shapes the behaviour and trustworthiness of fuzzy inference systems in educational decision support applications.

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