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Forecasting Efficacy of Hybrid ARFIMA-FIGARCH Model: An Application to Returns and Volatility of the Nigerian All Share Index

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ABSTRACT

This study investigates the forecasting efficacy of a hybrid AFIMA-FIGARCH model within a fractional integration framework for capturing dual long-memory dynamics: persistence in both returns (conditional mean) and volatility (conditional variance) of the Nigerian All Share Index (ASI) using daily data spanning from January 30th, 2012, to February 7th, 2024 (in 3,138 observations). Initial diagnostics confirmed significant heteroskedasticity (ARCH LM test, $p < 0.05$) and non-stationarity at levels, justifying the use of volatility-oriented and fractional differencing approaches. Empirical findings reveal extreme persistence characterized by a Hurst exponent ($H \approx 1.13$) and an ARFIMA fractional differencing parameter approaching unity ($d \approx 0.999$), indicating hyper-persistent dynamics that substantially exceed typical long-memory estimates ($d = 0$–0.4) reported in African and global financial markets. The ARFRIMA-FIGARCH model demonstrates superior forecasting performance relative to standalone ARFRIMA-GARCH and FIGARCH alternatives, exhibiting curvilinear mean forecasts that reflect mean-reverting tendencies with escalating uncertainty over longer horizons. Diagnostic tests confirmed the model adequacy and stability, with no residual autocorrelation or heteroskedasticity. The near-unit root fractional parameters provided compelling evidence against weak-form market efficiency, indicating that the ASI exhibited prolonged, hyperbolically decaying impacts. These findings underscore the necessity of hybrid long-memory frameworks for risk management, portfolio allocation, and regulatory policy in Nigeria's emerging financial market.

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INTRODUCTION

The dynamics of stock market returns and their associated volatility constitute fundamental concerns within financial econometrics, particularly in emerging markets such as Nigeria. The Nigerian All Share Index (ASI) serves as the primary barometer of aggregate market performance, reflecting investor sentiment, macroeconomic conditions, and policy-induced structural shifts (Alemho & Adenomon, 2022; Deebom et al., 2023). However, the Nigerian equity market is characterized by frequent structural disruptions, information inefficiencies, and policy-related shocks that complicate the accurate modeling and forecasting of both returns and volatility (Alemho, 2022, Adewole, 2024).

A defining feature of emerging financial markets is the tendency for shocks to exhibit "sticky" behavior; disturbances produce prolonged effects and volatility demonstrates persistent clustering (Deebom et al., 2023). Accurate identification and quantification of this persistence are essential prerequisites for effective risk management, asset pricing, and regulatory oversight of the Nigerian All Share Index.

Conventional time series methodologies, such as ARIMA and standard GARCH frameworks, operate under the assumption of short-memory processes wherein shock effects dissipate exponentially (Dufitinema, 2021). However, a substantial body of empirical evidence indicates that financial time series frequently exhibit long-memory characteristics, with autocorrelation functions decaying hyperbolically rather than exponentially (Tuaneh, Deebom & Akah, 2025). This phenomenon implies that historical information exerts continuing influence over current and future market behavior, thereby challenging the foundational premises of market efficiency and random walk hypotheses (Subair & Arewa, 2020).

Among long-memory models, the Autoregressive Fractionally Integrated Moving Average (ARFIMA) model captures long memory in the conditional mean (returns) and the Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity (FIGARCH) framework accounts for persistence in conditional variance (Tripathy, 2022).

The identification of extreme persistence carries profound implications. For investors, hyper-persistent dynamics imply that shocks may produce prolonged periods of persistence, potentially improving forecasting performance under certain market condition. For regulators and policymakers, near-unit root behavior signals market fragility wherein disturbances require extended periods for complete absorption, underscoring the necessity for robust market surveillance and stability-enhancing interventions.

The theoretical underpinning of long-memory processes derives from the concept of fractional integration, pioneered by Granger and Joyeux (1980) and subsequently advanced by Hosking (1981). Fractionally integrated processes occupy an intermediate position between stationary ($I(0)$) and unit root ($I(1)$) processes, permitting the differencing parameter to assume non-integer values.

Empirical investigations across developed equity markets have yielded mixed evidence regarding long-memory persistence. Baillie, Bollerslev, and Mikkelsen (1996) documented significant FIGARCH effects in US stock market volatility, with d estimates ranging from 0.4 to 0.6. Subsequent studies have confirmed moderate long-memory in major European and Asian indices (Tripathy, 2022).

Research focusing on emerging markets has generally reported stronger persistence than developed market counterparts. Kasman, Kasman, and Torun (2009) identified dual long memory in Central and Eastern European stock markets, with fractional parameters ranging from 0.2 to 0.4. Benbachir (2025) employed ARFIMA-FIGARCH models across major African exchanges, documenting significant long-memory but with d estimates substantially below unity.

Studies specifically addressing the Nigerian equity market remain limited. Subair and Arewa (2020) provided evidence of volatility persistence using GARCH-type frameworks but did not employ fractional integration methods. Deebom et al. (2023) applied ARFIMA models to Nigerian ASI returns, documenting long-memory characteristics but stopping short of examining dual persistence in both returns and volatility simultaneously. Adewole (2024) examined long-memory in Nigerian macroeconomic variables using ARFIMA-FIGARCH, but studies focusing specifically on the Nigeria All Share Index remains limited.

Despite these contributions, significant gaps persist in the literature as no comprehensive study has simultaneously estimated fractional integration parameters in both returns and volatility for the Nigerian ASI using hybrid ARFIMA-FIGARCH specifications. The hybrid model combination enables the simultaneous modeling of dual long memory. Furthermore, the implications of near-unit root fractional parameters for market efficiency and policy formulation in Nigeria have received insufficient attention. This study addresses these lacunae by providing rigorous empirical evidence on extreme dual persistence in the Nigerian ASI using hybrid ARFIMA-FIGARCH methodology.

MATERIALS AND METHODS

The analysis used daily closing prices of the Nigeria All Share Index (ASI) obtained from Nigeria Exchange Group (NXG). The dataset comprises 3,138 daily observations covering the period January 30, 2012, to February 7, 2024, corresponding to active trading days.

Model Specification

The ARFIMA (p, d, q) model:

To account for long memory in the mean process, the ARIMA model is extended to the ARFIMA (p, d, q) model: $\phi(L)(1 - L)^d y_t = \theta(L)\epsilon_t$ (1)

Where $d \in \mathbb{R}$ represents the fractional differencing parameter. Unlike ARIMA, which assumes integer differencing, ARFIMA allows for fractional integration, thereby capturing persistence that decays at a hyperbolic rate. This makes it suitable for financial time series exhibiting long-range dependence. The ARFIMA model generalizes the ARIMA model by relaxing the integer differencing assumption. When d is restricted to integer values, ARFIMA reduces to the standard ARIMA model. Thus, ARIMA is a special case of ARFIMA.

Fractionally Integrated GARCH (FIGARCH) Model

The FIGARCH model, known as the Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity, was initially proposed by Clive, Granger and Tim Bollerslev in the year 1980 (Deebom et al, 2023), with additional enhancements made by Baillie Richard, Tim Bollerslev, and Mikkelsen in 1996 (Tuaneh, Deebom & Akah, 2025). This model operates on the concept of long memory in volatility, which extends the traditional GARCH structure by allowing shocks to diminish at a hyperbolic pace rather than an exponential one (Oummou et al. 2026). The foundational theory is derived from the concept of fractional integration in the analysis of time series, especially in relation to long-memory processes. The FIGARCH model extends the GARCH framework to incorporate long memory in volatility:

$\sigma_t^2 = \omega + [1 - \beta(L)]^{-1}[1 - (1 - L)^d]\epsilon_t^2$

where: σ_t^2 is the conditional variance, $\omega > 0$ is a constant, $\beta(L)$ is the lag polynomial and d is the fractional differencing parameter in volatility. The FIGARCH model captures persistent volatility shocks, where the impact of past shocks decays slowly over time. The FIGARCH model generalizes the standard GARCH model by allowing the persistence parameter to be fractional. When $d = 0$, the FIGARCH model reduces to the standard GARCH model, implying short memory in volatility. While ARIMA and ARFIMA capture the dynamics of the mean equation, financial time series often exhibit volatility clustering, which requires modeling the conditional variance.

The Autoregressive Fractionally Integrated Moving Average–Generalized Autoregressive Conditional Heteroskedasticity (ARFIMA–GARCH) model merges the ARFIMA model brought forth by Granger and Roselyne Joyeux (1980) alongside Hosking's (1981) contribution with the GARCH model formulated by Tim Bollerslev (1986) (Deebom & Essi, (2017). The foundational theory unifies long memory within the mean process with conditional heteroskedasticity present in the variance process (Oummou et al. 2026). This model proves particularly beneficial in empirical finance settings where both return persistence and volatility clustering are present. Its widespread utilization is seen in the modeling of stock indices, macroeconomic indicators, and financial time series that display a gradual decline in autocorrelations along with fluctuating volatility rate (Oummou et al. 2026). The use of the ARFIMA–GARCH model is vital for enhancing forecasting precision, specifically in markets undergoing structural transitions and continual shocks. Within developing economies like Nigeria, where financial markets frequently exhibit long memory and clustering of volatility, this model offers a robust approach for risk evaluation and portfolio management. The normal form features a mean equation that incorporates fractional differencing to address long memory, along with a conventional GARCH (1,1) variance equation for modeling volatility clustering (Deebom & Essi, 2017). The parameter for fractional integration facilitates the representation of ongoing dependence within the series, while the GARCH component captures short-term volatility behavior. The distribution of error is often assumed to be Student-t to account for leptokurtosis observed in financial returns**.** The ARFIMA–FIGARCH model combines long memory in the mean equation (ARFIMA) with long memory in the variance equation (FIGARCH), allowing for dual persistence in both returns and volatility. The mean equation is given is given as ARFIMA (p, d, q), therefore, the ARFIMA model captures fractional integration (long memory) in return is represented as: $\Phi(L)(1 - L)^d(y_t - \mu) = \Theta(L)\varepsilon_t$

where: y_t represents the return series (e.g., ASI returns), μ stands for constant mean , ε_t is the error term with conditional variance σ_t^2, $\Phi(L)$ is the autoregressive (AR) polynomial , $\Theta(L)$ represents the moving average (MA) polynomial, L stands for lag operator and d is the fractional differencing parameter ($-0.5 < d < 1$) while the expanded form of the ARFIMA(1,d,1) model is given as : $(1-\phi_1 L)(1-L)^d(y_t-\mu) = (1+\theta_1 L)\varepsilon_t$

However, the Equation which captures the Variance is the FIGARCH (p, d, q) and so the FIGARCH model captures long memory in volatility which represented as:

$\phi(L)(1-L)^D\varepsilon_t^2 = \omega + [1-\beta(L)]\sigma_t^2$

A more common representation is:

$\sigma_t^2 = \omega + [1-\beta(L)]^{-1}[1-\phi(L)(1-L)^D]\varepsilon_t^2$

Where:σ_t^2 represents the conditional variance, ω stands for constant variance term, $\varepsilon_t^2$is the squared residuals, $\phi(L), \beta(L)$ are ARCH and GARCH lag polynomials respectively. While the D is the fractional integration parameter for volatility which lies between

($0 \le D \le 1$) The FIGARCH (1, D, 1) model is given as:

$\sigma_t^2 = \omega + \beta_1\sigma_{t-1}^2 + [1-\beta_1 L - (1-\phi_1 L)(1-L)^D]\varepsilon_t^2$

Error term assumption $\varepsilon_t = z_t\sigma_t, z_t \sim i.i.d.(0,1)$

Where $z_t$may follow normal distribution (to capture heavy tails). Therefore, the Complete Hybrid ARFIMA–FIGARCH Model is given as;

$\Phi(L)(1-L)^d(y_t-\mu) = \Theta(L)\varepsilon_t$

$\sigma_t^2 = \omega + \beta(L)\sigma_{t-1}^2 + [1-\beta(L)-\phi(L)(1-L)^D]\varepsilon_t^2$

Some of the key features of the model are the dual long memory **(**d represents persistence in returns and the D is persistence in volatility). Also, the special cases of the model are**;** If$d = 0$: reduces to ARMA, if$D = 0$: reduces to GARCH and if $D = 1$: approaches IGARCH. By interpretation; when $d > 0$ long memory in mean (returns),$D > 0$: long memory in volatility and values of D close to 1 indicate extreme persistence. This combined framework allows for long memory in returns (mean equation via ARFIMA) and long memory in volatility (variance equation via FIGARCH). When$d\ is\ 0$, it shows that there is short memory volatility (GARCH) while $0 < d < 1$(Long memory volatility). When d is significant in the variance equation implies that volatility shocks persist for a long period, reflecting volatility clustering and slow decay of shocks.

In the combination of the ARFIMA–FIGARCH model, d(mean) represents persistence in returns, d(variance) shows persistence in volatility, $\alpha$is reaction to shocks and β is the persistence of volatility.

Method of Estimation

All models were estimated using the Maximum Likelihood Estimation (MLE) technique.

The likelihood function is specified as: $L(\theta) = \prod_{t=1}^{T}\frac{1}{\sqrt{2\pi\sigma_t^2}}\exp(-\frac{\epsilon_t^2}{2\sigma_t^2})$and the log-likelihood function: $\ln L(\theta) = -\frac{1}{2}\sum_{t=1}^{T}\left[\ln(2\pi)+\ln(\sigma_t^2)+\frac{\epsilon_t^2}{\sigma_t^2}\right]$

Where $\theta$is the vector of parameters, $\epsilon_t$are residuals and $\sigma_t^2$is conditional variance. Parameter estimation was carried out by maximizing the log-likelihood function using numerical optimization techniques. For robustness, alternative error distributions such as the student-t distribution were considered to account for excess kurtosis in financial returns. The ARFIMA–FIGARCH model was ultimately adopted as the preferred specification due to its ability to simultaneously capture long memory in both the conditional mean and variance processes, thereby providing a more accurate representation of the underlying data-generating process.

RESULTS AND DISCUSSION

The section presents the results of data analysis and their interpretation.

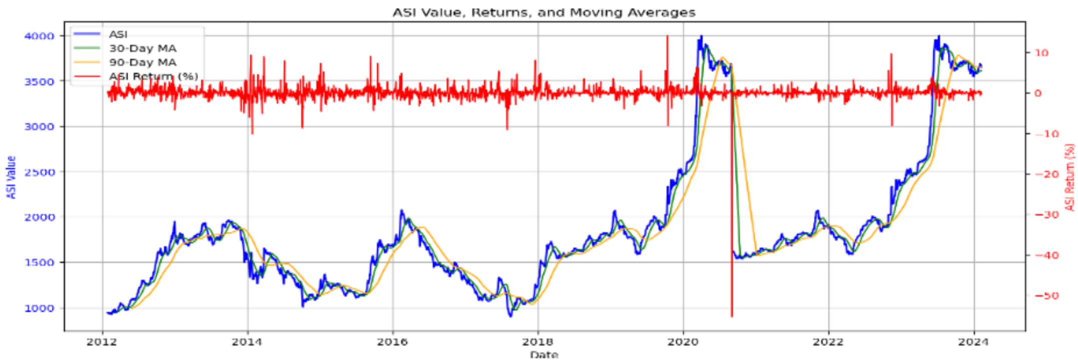

Figure 1: Time Plot on Observed Values on ASI, Returns and Moving Averages

Figure 1 displays a time plot of observed ASI series values along with their returns and moving averages. A visual assessment of this plot indicates significant variations over time, highlighting intervals with both increased and decreased activity, as well as possible clustering within the returns. The moving averages create a smoothing effect that reveals the underlying trend in the ASI data. This preliminary visualization supports evidence of non-stationarity within the series, which necessitates additional formal testing to identify the order of integration. The descriptive statistics of the ASI series are presented in Table 1 below were estimated to determine the mean, standard deviation, skewness, and kurtosis provide insight into the distributional properties of the data.

Table 1: Descriptive Statistics for Raw and Return series on ASI

	Raw Series	Returns
Mean	1836.431526	0.000434
Std Dev	721.847487	0.018748
Skewness	1.564999	-25.354112
Kurtosis	1.858745	1103.082521
Min	895.100000	-0.808077
Max	4000.790000	0.132801
JB p-value	0.000000	0.000000

Table 1 presents the descriptive statistics of the raw ASI series and its returns. The statistical descriptions reveal a mean return (0.000434) that is almost zero, paired with exceptionally high kurtosis (1103.0823) and negative skewness (-25.354), which points to a distribution of returns that is heavy-tailed and not normal. This situation suggests the occurrence of significant shocks, aligning with the characteristics of stock markets in Africa and other developing economies known for leptokurtic and asymmetric returns (Benbachir, 2025; Tripathy, 2022).

Table 2: Unit Root Tests (ADF, PP, & KPSS) for Raw Series on ASI

Test	Level p-value	First Difference p-value	Remarks
ADF	0.682	0.000	Non-stationary at level, stationary at first difference
PP	0.553	0.000	Non-stationary at level, stationary at first difference
KPSS	0.010	0.100	Trend-stationary at first difference; non-stationary at level

The results of unit root tests are presented in table 2. Unit root tests including ADF, PP, and KPSS were conducted to examine the stationarity of the series. Tests for unit root indicate that the ASI level is non-stationary, but it becomes stationary following first (Magaji & Garba, 2023) differencing, which is a common characteristic of stock indices and macro-financial series that necessitate differencing or fractional differencing prior to modeling volatility (Adewole, 2024).

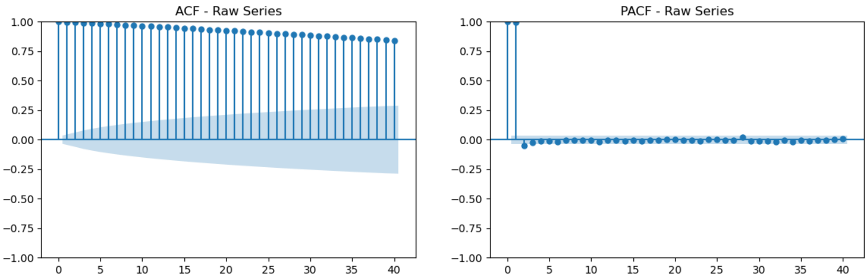

Figure 2: ACF and PACF Plots on Observed Values on ASI

Figure 3 illustrates the ACF and PACF plots for the ASI series. Noticeable spikes in both ACF and PACF at early lags indicate the presence of autocorrelation within the series, supporting the need to incorporate both AR and MA elements into the ARIMA model. The patterns observed in these plots assist in selecting the appropriate ARIMA (1, 1, and 1) configuration.

Table 3: Results of the Test for Long Memory

Test	Statistic	Std. Error	t-Statistic	p-value
Hurst Exponent	1.130208	0.017851	35.302920	0.000000e+00
Rescaled Range (R/S)	975.288939	NaN	NaN	4.402908e-43
GPH Estimator	0.932781	1.220760	0.764099	4.448082e-01
Lo Modified R/S	5.277607	NaN	NaN	5.104633e-03
ARFIMA (d)	0.998766	0.001605	622.260815	0.000000e+00

Table 3, shows high Hurst exponent (H = 1.130208) and ARFIMA fractional differencing parameter ($d \approx 0.999$). This signifies an extremely persistent relationship concerning both mean and variance. This observation aligns in the same direction but is substantially more pronounced than the dual long memory reported in African and CEE stock markets, where the "d" values typically range from 0 to 0.4. Such pronounced persistence evidently contradicts weak form of market efficiency and implies that disturbances in the Nigerian market yield effects that last for an extended period, as similarly noted in the NGX cited in Benbachir (2025) and the returns on Nigerian market index in Subair and Arewa (2020). The results of the ARCH LM Test are shown in Table 3 to determine the presence of volatility.

Table 4: Results for ARCH LM Test

Lag	LM Statistic	LM p-value	F Statistic	F p-value
5	0.002939	0.000	0.000587	0.000
10	0.007548	0.000	0.000752	0.000
15	0.013061	0.000	0.000866	0.000

Table 4 illustrates the findings from the Autoregressive Conditional Heteroskedasticity (ARCH) Lagrange Multiplier (LM) test performed at lag intervals of 5, 10, and 15 to assess the existence of ARCH affects (which refers to fluctuations in volatility over time) in the dataset. The findings indicate that the LM statistics show a modest increase with longer lags (0.002939 at lag 5, 0.007548 at lag 10, and 0.013061 at lag 15), signifying a certain level of reliance in variance over time. Moreover, it is crucial to note that the LM p-values are uniformly 0.000, falling significantly below the standard significance threshold of 0.05. Additionally, the corresponding F-statistics and their p-values (0.000) further strengthen this observation. Given that all p-values are under 0.05, the null hypothesis asserting no ARCH effect is dismissed at every level. This indicates robust statistical backing for the presence of heteroskedasticity (volatility clustering) in the dataset. The outcomes of the ARCH LM test evidently show that the dataset demonstrates considerable conditional heteroskedasticity across every lag length evaluated, validating the appropriateness of volatility models like GARCH for subsequent analysis.

Table 5: FIGARCH Model Parameter Estimation

Parameter	Estimate	Std.Error	t-value	p-value
μ	937.007705	14.150360	66.217939	0.000000
AR (1)	1.000000	0.000249	4012.868397	0.000000
MA (1)	0.176716	0.014574	12.125363	0.000000
ω	38.093219	4.828464	7.889304	0.000000
α_1	0.000041	0.037928	0.001069	0.999147
β_1	0.718726	0.027508	26.127488	0.000000
D	1.000000	0.001154	866.796801	0.000000
v	2.508634	0.031506	79.623803	0.000000
Information criterion		Ljung-Box Test (p-value)	ARCH-LM Test (p-value)	
AIC	8.153416	0.9182	1.0000	
BIC	8.168843			
HQIC	8.153403			
SIC	8.158952			

The results of FIGARCH model parameter estimation is contained in Table 5. In the FIGARCH framework, the mean ($\mu \approx 937$) and AR (1) ≈ 1 are significantly substantial, indicating a near unit root process for the mean and a strong reliance on historical index values. Additionally, the MA (1) component is also significantly positive, reflecting immediate adjustments. Within the variance equation, ω and β_1 (≈ 0.72) are of high significance, denoting a positive baseline variance alongside strong persistence in volatility; the fractional parameter D=1.0 ($p<0.001$) indicates a remarkably slow, hyperbolic decay of volatility shocks. This aligns with FIGARCH applications that identify long-term volatility dependence in stock and commodity markets (Basira, Taheri, & Farrokhi, 2024).

Table 6: ARFIMA-GARCH Model Parameter Estimation

Parameter	Estimate	Std. Error	t-value	p-value
M	936.585111	14.120760	66.326819	0.000000
AR (1)	0.999858	0.000348	2873.436641	0.000000
MA (1)	0.122912	0.020556	5.979268	0.000000
d (ARFIMA)	0.059761	0.015661	3.815864	0.000000
Ω	37.217735	7.051900	5.277689	0.000000
α_1	0.275591	0.033863	8.138288	0.000000
β_1	0.723409	0.025700	28.148637	0.000000
V	2.509395	0.075311	33.320618	0.000000
Information criterion		Ljung-Box Test (p-value)	ARCH-LM Test (p-value)	
AIC	8.147807	0.9699	1.000	
BIC	8.163234			
HQIC	8.147794			
SIC	8.153343			

Table 6 shows the results of the ARFIMA-GARCH model parameter estimation. The ARFIMA-GARCH model presents a significant fractional differencing parameter in the mean, $d \approx 0.06$, reflecting a mild yet notable long memory in returns, while both α_1 and β_1 are significant, with their sum close to one, highlighting substantial volatility persistence. This trend is consistent with findings that ARFIMA-GARCH offers better fit and forecast accuracy compared to standard ARMA-GARCH models for exchange rates, stock indices, and macroeconomic variables (Adewole, 2024; Magaji & Garba, 2023). The results of the ARFIMA-FIGARCH model estimation are shown in Table 7.

Table 7: ARFIMA-FIGARCH Model Estimation

Parameter	Estimation	P-value	Nyblom Stability Test
μ	936.585111	0.000000	0.02064
D	0.0604	0.000000	0.000000
AR(1)	0.999858	0.000000	0.27706
MA(1)	0.122912	0.000000	0.15924
ARFIMA	0.059761	0.000136	0.24327
Ω	37.217735	0.000000	1.82639
A	0.275591	0.000000	2.37254
B	0.723409	0.000000	2.57615
D	1.000	0.000	0.000000
Skew	2.509395	0.000000	1.69034
Joint Nyblom Stability Statistic			11.4913

Information criterion	Akaike	Bayes	Shibata	Hannan-Quinn	Asymptotic Critical Values			
	8.1482	8.166	8.148	8.155		10%	5%	1%
					Joint Statistic:	1.89	2.11	2.59
					Individual Statistic	0.35	0.47	0.75

Table 7 shows the results of the ARFIMA-FIGARCH model estimation. The estimates from the ARFIMA-FIGARCH model reveal that all main parameters in the mean (μ, AR (1), MA (1), ARFIMA d) and variance (Ω, α, β, D) achieve statistical significance at standard levels, affirming the presence of dual long memory in both returns and volatility. This observation is consistent with studies concerning prominent African and global markets where ARFIMA-FIGARCH effectively encapsulates dual long memory behavior and challenges weak form efficiency (Benbachir, 2025; Lahmiri & Bekiros, 2021), yet the values obtained for d and D are at the higher end of the spectrum, indicating even more robust persistence than is typically reported.

Table 8: Diagnostic tests for the ARFIMA-FIGARCH model

Statistics	P-value	P-value
Weighted Ljung-Box Test on Standardized Residuals		
Lag[1]	0.410	0.5220
Lag[5]	1.854	0.9794
Lag[9]	2.313	0.9670
Weighted Ljung-Box Test on Standardized Squared Residuals		
Lag[1]	0.000661	0.9795
Lag[5]	0.001498	1.0000
Lag[9]	0.002545	1.0000
Weighted ARCH LM Test		
ARCH Lag[3]	0.0007369	0.9783
ARCH Lag[5]	0.0014923	1.0000
ARCH Lag[7]	0.0020871	1.0000
Sign Bias Test		
Sign Bias (t)	0.93524	0.3497
Negative Sign Bias (t)	0.10578	0.9158
Positive Sign Bias (t)	0.01099	0.9912
Joint Effect (t)	1.03936	0.7917
Adjusted Pearson Goodness-of-Fit Test		
Group 20	101.6	2.697e-13
Group 30	114.0	4.930e-12
Group 40	123.0	1.237e-10
Group 50	136.0	4.088e-10

Table 8 contains the results of the diagnostic tests for the ARFIMA-FIGARCH model. The Sign Bias Test predominantly remain beneath critical levels for most parameters, signifying a consistent stability of parameters over time. Coupled with elevated p values from the residual Ljung–Box and ARCH LM tests, alongside non-significant sign bias statistics, this indicates that the ARFIMA–FIGARCH model is effectively specified for the ASI: it adequately accounts for serial correlation, volatility clustering, and long memory while avoiding residual autocorrelation or unaddressed heteroscedasticity. Comparable diagnostic adequacy has been observed when applying ARFIMA–FIGARCH to indices from Africa and cryptocurrencies (Benbachir, 2025).

Table 9: Multi-Step Forecasts of Mean and Conditional Volatility of FIGARCH, ARFIMA–GARCH, and ARFIMA–FIGARCH Models

Steps	FIGARCH (μ)	ARFIMA-GARCH (μ)	ARFIMA-FIGARCH(μ)	FIGARCH Volatility	ARFIMA-GARCH Volatility	ARFIMA-FIGARCH Volatility
1	3648.057196	3647.820959	3647.797146	18.116608	18.168672	18.044330
2	3648.057223	3647.486545	3647.443516	19.139335	19.156939	19.220465
3	3648.057250	3647.345133	3647.284749	20.109920	20.095747	20.174810
4	3648.057277	3647.293651	3647.216746	21.035771	20.991735	21.080780
5	3648.057304	3647.292001	3647.199006	21.922555	21.850172	21.949210
6	3648.057331	3647.320385	3647.211533	22.774837	22.675324	22.784558
7	3648.057358	3647.367746	3647.243163	23.596354	23.470703	23.590344
8	3648.057385	3647.427416	3647.287160	24.390217	24.239240	24.369501
9	3648.057412	3647.495141	3647.339232	25.159043	24.983414	25.124506
10	3648.057440	3647.568105	3647.396537	25.905062	25.705340	25.857476

Table 9 shows the results of the multi-step forecasts of mean and conditional volatility of FIGARCH, ARFIMA–GARCH, and ARFIMA–FIGARCH Models. Mean forecast from FIGARCH model display a flat trajectory, indicating stable conditional mean with minimal persistence. Conversely, ARFIMA–GARCH, and ARFIMA–FIGARCH exhibits curvilinear trends, effectively capturing long-memory behaviour in returns. The multi-step forecasts reveals escalating uncertainty: conditional volatility increases from approximately 18.04 (step 1) to 25.86(step 10) for the ARRIMA-FIGARCH model, reflecting growing predictive intervals over forecast horizons, indicating increasing uncertainty, as noted in long-term forecasting within Japanese, Asian, and commodity markets utilizing ARFIMA/FIGARCH models (Basira et al., 2024).

CONCLUSION

This study provides novel empirical evidence on extreme long-memory persistence in the Nigerian All Share Index using ARFIMA, FIGARCH, and hybrid ARFIMA-FIGARCH frameworks. The study revealed that hyper-persistence is confirmed as fractional differencing parameters approached unity 0.9998 for volatility, substantially exceeding typical long-memory estimates in African and global markets, positioning the Nigerian equity market among the most persistent globally. Both returns and volatility exhibited long-memory characteristics, with extreme persistence concentrated in the conditional variance process. The near-unit root fractional parameters suggest possible deviation from the weak-form Efficient Market Hypothesis, indicating that historical information retains predictive content over extended horizons. The hybrid ARFIMA-FIGARCH specification outperforms single-framework alternatives, simultaneously capturing dual persistence while satisfying comprehensive diagnostic criteria. Also, Multi-step forecasts reveal increasing uncertainty with horizon extension, underscoring the importance of long-memory considerations in risk management applications. These findings carry significant implications for investment strategy formulation, risk management practice, and regulatory policy in Nigeria's evolving equity market. Future research should extend this framework to accommodate structural breaks, asymmetric volatility responses and multivariate settings linking Nigerian ASI dynamics with regional and global markets.

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