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# Fixed Point and Common Fixed Point for Modified Enriched Hardy-Rogers Contractions in Banach Spaces

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**ABSTRACT**

The celebrated Banach contraction principle and its numerous extensions continue to play a central role in nonlinear analysis, fixed point theory, optimization, and differential equations. Among its notable generalizations are the Hardy–Rogers contraction, enriched contractions, and Jungck-type contractive mappings, each of which has significantly broadened the scope of applicability of fixed point methods. Motivated by recent developments on enriched Hardy–Rogers contractions and the growing interest in enriched contractive frameworks, we introduce a new class of operators called *modified enriched Hardy–Rogers contractions* in arbitrary real Banach spaces. The proposed contractive conditions are shown to be structurally different from the existing enriched Hardy–Rogers-type formulations and establish a new enriched framework. By employing direct iterative techniques and a careful analysis of the associated contractive inequalities, we establish existence and uniqueness results for fixed points of modified enriched Hardy–Rogers contractions in Banach spaces. Furthermore, we introduce the notion of *modified enriched Jungck–Hardy–Rogers contractions* and prove a unique common fixed point theorem for commuting pairs of mappings satisfying the proposed condition. The developed theory extends and unifies several classical results including those of Banach, Reich, Hardy–Rogers, Berinde–Păcurar, and Jungck. In addition, illustrative examples are presented to demonstrate the independence and properness of the newly introduced classes of mappings. The results obtained herein contribute to the ongoing development of enriched fixed point theory and provide a flexible framework for studying nonlinear operators beyond the reach of existing contraction principles.

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## INTRODUCTION

Let $B$ be a nonempty set and $f$ a self-map on$B$. Any element $x \in B$ is a fixed point of $f$, if $f(x) = x$ and we denote the set of all fixed points of $f$ by $F_f = \{x \in B: f(x) = x\}$

If $(B, d)$ is a complete metric space and $f: B \to B$ an a-contraction, that is, an operator satisfying:

$$d(fx, fy) \leq ad(x, y), \quad \forall\, x, y \in B \tag{1}$$

With $a \in [0,1)$ fixed. Then, $f$ has a unique fixed point in $B$ as established by Banach (1922).

Motivated by Banach's work, Rakotch (1962) generalized the above using a monotone decreasing function $\alpha: (0, \infty) \to [0,1)$ such that, for each$x, y \in B, x \neq y$,

$$d(fx, fy) \leq \alpha(d(x, y)). \tag{2}$$

In generalizing Banach's theorem, Kannan (1968) used the inequality below: there exists $a \in [0, \frac{1}{2})$ such that

$$d(fx, fy) \leq a[d(x, fx) + d(y, fy)], \forall\, x, y \in X. \tag{3}$$

Hardy and Rogers (1973) generalized Banach's contraction principle and many other related contractive conditions involving all six possible distances from any two distinct points $x, y$ of a metric space $B$ with respect to a self-map $f$ on $B$.

**Definition 1.1**

Hardy and Rogers (1973)**:** A self-map $f$ on $B$ is a Hardy-Rogers contraction if there is$c_i \in \{\mathbb{R}_+ \cup 0\}, (i = 1,2, \cdots, 5)$ with $\sum_{i=1}^{5} \quad c_i < 1$, such that $\forall\, x, y \in B$ we have

$$d(fx, fy) \leq c_1 d(x, y) + c_2 d(x, fx) + c_3 d(y, fy) + c_4 d(x, fy) + c_5 d(y, fx). \tag{4}$$

The literature contains several generalizations and extensions of classical Banach's fixed point theorem. Interested readers can consult Omidire (2024), Ishola et al. (2026) and references therein.

Meanwhile, Jungck (1976), using the idea of commuting mappings, established the notion of common fixed point of mappings. He proved that the pair of commuting self-mappings $S, f$ on a complete metric space $(B, d)$ satisfying

$$d(fx, fy) \leq c \cdot d(Sx, Sy), \; \forall\; x, y \in B, c \in (0,1), \tag{5}$$

has a unique common fixed point in $B$.

**Definition 1.2 Omidire (2024)**

Let $X$ be a non-empty set. Two mappings $S, T: B \to B$ are said to commute iff $ST = TS$.

Example 1.3 Omidire (2024): Consider $S, T: B \to B$ such that, $Tx = x, Sx = 1 - x, \; \forall\; x \in B$.

$$T(S(x)) = T(1 - x) = 1 - x$$

$S(T(x)) = Sx = 1 - x$. Thus, $S$ and $T$ commute.

**Definition 1.4 Jungck (19976), Olatinwo and Omidire (2021), Omidire (2024)**

Let $B$ be a normed linear space, and a pair of operators $U, G: B \to B$. for any $x_0 \in B$, the sequence $\{Ux_n\}_{n=0}^{\infty}$, defined by

$$Ux_{n+1} = Gx_n, \quad n \geq 0, \tag{6}$$

Is called Jungck iteration.
Remark: If $U = I$ (the identity operator), the equation (6) becomes Picard iteration.

Interested readers can consult Jungck (1976), Olatinwo (2012), Olatinwo and Omidire (2016), Olatinwo and Omidire (2020), Olatinwo and Omidire (2021), Olatinwo and Omidire (2023), Singh et al. (2005), Singh et al. (2009), Sumitra et al. (2010) and Dharsini et al. (2020) for more insight on Jungck-type contractions and their generalizations.
However, just recently Berinde and Pacurar (2020) introduced a larger class of mappings, called enriched contractions which includes, among many others an a-contraction. Also worthy of note is the recently published work of Anjali et al. (2025), in which the Hardy-Rogers contraction was enriched as follows:

**Definition 1.5 Anjali and Batra (2025)**
Let $(B, ||.||)$ be a normed linear space. A mapping $f: B \to B$ is said to be an enriched Hardy-Rogers contraction if there exists $\delta \in [0, \infty)$ and non-negative $c_i \in \mathbb{R}_+, (i = 1,2,\cdots,5) \geq 0$ with $\sum_{i=1}^{5} \quad c_i < 1$, such that $\forall\, u, v \in N$ we have

$$||\delta(u-v) + fu - fv|| \leq c_1||u-v|| + c_2||u-fu|| + c_3||v - fv|| + c_4||(\delta+1)(u-v) + v - fv|| + c_5||(\delta+1)(v-u) + u - fu||.$$

In this paper, we employ a novel approach to proving the convergence of the Picard iteration to a unique fixed point of *modified enriched Hardy-Rogers contraction* which is independent of Definition (1.5) above. Furthermore, we establish a unique common fixed point of *modified enriched Jungck-type Hardy-Rogers contractions* using the Jungck iterative technique. Examples are also presented to demonstrate the validity of our contractive conditions.

## MATERIALS AND METHODS

**Definition 2.1**
Let $(B, \|\cdot\|)$ be a Banach space. A mapping $f: B \to B$ is called a *Modified Enriched Hardy–Rogers Contraction* if there exist constants

$\delta \geq 0, \qquad c_i \geq 0, \quad i = 1, \ldots, 5,$

Such that

$c_1 + c_2 + c_3 + c_4 + c_5 < 1$

And

$$\| \delta(x-y) + fx - fy \| \leq c_1 \| x-y \| + c_2 \| x - fx \| + c_3 \| y - fy \| + c_4 \| x - fy \| + c_5 \| y - fx \| \tag{7}$$

For all $x, y \in B$.

**Remark 2.2**
The present work extends several known results in the literature.
• If $\delta = 0$, then Definition 2.1 reduces to the classical Hardy–Rogers contraction.
• If $c_2 = c_3 = c_4 = c_5 = 0$, then Definition 2.1 becomes
$\| \delta(x-y) + fx - fy \| \leq c_1 \| x-y \|$,
which is precisely the enriched contraction introduced by Berinde and Păcurar (2020).

If additionally $\delta = 0$, then the above reduces to the classical Banach contraction principle.

The enriched Hardy–Rogers contraction introduced by Anjali and Batra (2025) contains terms of the form
$\| (\delta+1)(x-y) + y - fy \|$
and
$\| (\delta+1)(y-x) + x - fx \|$.
Our contractive condition replaces these terms by the simpler cross-distance expressions
$\| x - fy \|, \qquad \| y - fx \|$.
Hence the present contractive condition is structurally different from that of Anjali et al.
• The common fixed point theorem established herein extends Jungck's theorem by incorporating all five Hardy–Rogers distances together with the enrichment parameter $\delta$.

**Example 2.3**
Let $B = \mathbb{R}$ equipped with the usual norm and define
$f(x) = 2 - 3x$.
Then
$|f(x) - f(y)| = 3|x-y|$.
Hence $f$ is not a Banach contraction.
Choose $\delta = 3$.
Then
$\delta(x-y) + f(x) - f(y) = 3(x-y) - 3(x-y) = 0$.
Therefore
$|\delta(x-y) + f(x) - f(y)| = 0$
for all $x, y \in \mathbb{R}$.
Consequently,

$$0 \leq c_1|x-y| + c_2|x-fx| + c_3|y-fy| + c_4|x-fy| + c_5|y-fx|$$

for every choice of nonnegative constants $c_i$ satisfying

$$\sum_{i=1}^{5} \quad c_i < 1.$$

Thus $f$ is a modified enriched Hardy–Rogers contraction but not a Banach contraction.

**Example 2.4**
Consider

$$f(x) = \frac{x}{1+x}, \qquad x \geq 0.$$

Then

$$|f(x) - f(y)| = \left|\frac{x}{1+x} - \frac{y}{1+y}\right| = \frac{|x-y|}{(1+x)(1+y)}.$$

The map satisfies a Hardy–Rogers condition with suitable coefficients.
However,

$$\sup_{x\geq 0} |\, f'(x)| = 1,$$

and therefore $f$ is not a Banach contraction.
Hence Hardy–Rogers contractions properly contain Banach contractions.

**Example 2.5**
Consider
$f(x) = 1 - 2x$
on $\mathbb{R}$. Then
$|f(x) - f(y)| = 2|x-y|$.
Hence Banach and Hardy–Rogers conditions fail.
Choosing $\delta = 2$ gives
$\delta(x-y) + f(x) - f(y) = 2(x-y) - 2(x-y) = 0$.
Thus the enriched condition holds immediately.
Hence enriched Hardy–Rogers contractions properly extend Hardy–Rogers contractions.

**Definition 2.6**
Let$S, f: B \to B$. The pair $(S, f)$ is called a *Modified Enriched Jungck–Hardy–Rogers's pair* if there exist
$\delta \geq 0, \quad c_i \geq 0$ With $\sum_{i=1}^{5} \quad c_i < 1$
Such that
$$\| \delta(Sx - Sy) + fx - fy \| \leq c_1 \| Sx - Sy \| + c_2 \| Sx - fx \| + c_3 \| Sy - fy \| + c_4 \| Sx - fy \| + c_5 \| Sy - fx \| \quad (8)$$
for all $x, y \in B$.
The following will be required in the sequel:

**Lemma 2.7: [Analogue of Jungck's common fixed point theorem in Banach Spaces]**
Suppose that $S$ is a continuous self-map of a Banach space $B$. Then, $S$ has a fixed point in $B$ if and only if there exists $k \in (0,1)$ and a self-mapping $f$ on $B$ which commutes with $S$, satisfies $f(B) \subset S(B)$ and
$$||fx - fy|| \leq k||Sx - Sy||, \forall \; x, y \in B. \quad (9)$$
Then, $f$ and $S$ have a unique common fixed point in $B$.

**Definition 2.8 [Bector et al. (2005), Omidire (2024)]**
A mapping $S: \mathbb{R}^n \to \mathbb{R}$ is said to be convex if for any $x, y \in \mathbb{R}^n$ and $\aleph \in [0,1]$ we have that
$S(x + \aleph(y - x)) \leq \aleph S(x) + (1 - \aleph)Sy$.

## RESULTS AND DISCUSSION

**Theorem 3.1**
Let $(B, \|\cdot\|)$ be a Banach space and suppose that $f: B \to B$ satisfies inequality (7). Assume additionally that
$\delta + 1 > c_1 + c_2 + c_4$.
Then $f$ possesses a unique fixed point.

**Proof**
Let $\quad x_{n+1} = f(x_n), \quad n \geq 0.$
Applying inequality (7) with
$x = x_n, \quad y = x_{n-1},$
Gives
$$\| \delta(x_n - x_{n-1}) + x_{n+1} - x_n \| \leq c_1 \| x_n - x_{n-1} \| + c_2 \| x_n - x_{n+1} \| + c_3 \| x_{n-1} - x_n \| + c_4 \| x_n - x_n \| + c_5 \| x_{n-1} - x_{n+1} \|.$$
Using
$\| x_{n-1} - x_{n+1} \| \leq \| x_{n-1} - x_n \| + \| x_n - x_{n+1} \|,$
we obtain
$$\| \delta(x_n - x_{n-1}) + x_{n+1} - x_n \| \leq (c_1 + c_3 + c_5) \| x_n - x_{n-1} \| + (c_2 + c_5) \| x_{n+1} - x_n \|.$$
By the reverse triangle inequality,
$$\| \delta(x_n - x_{n-1}) + x_{n+1} - x_n \| \geq \delta \| x_n - x_{n-1} \| - \| x_{n+1} - x_n \|.$$
Hence
$$\delta \| x_n - x_{n-1} \| - \| x_{n+1} - x_n \| \leq (c_1 + c_3 + c_5) \| x_n - x_{n-1} \| + (c_2 + c_5) \| x_{n+1} - x_n \|.$$
Collecting terms gives
$$(1 - c_2 - c_5) \| x_{n+1} - x_n \| \leq (c_1 + c_3 + c_5 - \delta) \| x_n - x_{n-1} \|.$$
Therefore
$\| x_{n+1} - x_n \| \leq q \| x_n - x_{n-1} \|,$
where
$$q = \frac{c_1 + c_3 + c_5}{\delta + 1 - c_2 - c_5}.$$
Since
$c_1 + c_2 + c_3 + c_4 + c_5 < 1,$
and
$\delta + 1 > c_1 + c_2 + c_4,$
it follows that
$0 < q < 1.$
Iterating,
$\| x_{n+1} - x_n \| \leq q^n \| x_1 - x_0 \|.$
Hence
$$\sum_{n=0}^{\infty} \quad \| x_{n+1} - x_n \| < \infty.$$
Thus $\{x_n\}$ is Cauchy. Since $B$ is complete, there exists $u^* \in B$ such that
$x_n \to u^*.$
Passing to the limit in inequality (7) with $x = u^*$ and $y = x_n$ shows that
$f(u^*) = u^*.$
Therefore $u^*$ is a fixed point. Now suppose
$u^*, v^*$
are fixed points. Substituting them into inequality (7) yields
$(\delta + 1) \| u^* - v^* \| \leq (c_1 + c_4) \| u^* - v^* \|.$
Hence
$(\delta + 1 - c_1 - c_4) \| u^* - v^* \| \leq 0.$
Since
$\delta + 1 > c_1 + c_4,$
we obtain
$u^* = v^*.$
Therefore the fixed point is unique.

**Theorem 3.2**
Let $(B, \|\cdot\|)$ be a Banach space and let $f, S: B \to B$ be mappings satisfying
$f(B) \subseteq S(B),$
And assume that $S(B)$ is complete.
Suppose there exist constants
$\delta \geq 0, \quad c_i \geq 0, \quad i = 1, \ldots, 5,$
Such that
$$\sum_{i=1}^{5} \quad c_i < 1$$
and
$$\| \delta(Sx - Sy) + fx - fy \| \leq c_1 \| Sx - Sy \| + c_2 \| Sx - fx \| + c_3 \| Sy - fy \| + c_4 \| Sx - fy \| + c_5 \| Sy - fx \| \quad (10)$$
For all$x, y \in B$.
Assume further that
$\delta + 1 > c_1 + c_2 + c_4$
And that
$Sf = fS.$
Then $f$ and $S$ possess a unique common fixed point.

**Proof**
Let $x_0 \in B$ be arbitrary.
Since
$f(B) \subseteq S(B),$
We can inductively choose a sequence $\{x_n\}$ satisfying
$Sx_{n+1} = fx_n, \quad n \geq 0.$
We first show that $\{Sx_n\}$ is a Cauchy sequence.
Applying inequality (10) with
$x = x_n, \quad y = x_{n-1},$
We obtain
$$\| \delta(Sx_n - Sx_{n-1}) + fx_n - fx_{n-1} \| \leq c_1 \| Sx_n - Sx_{n-1} \| + c_2 \| Sx_n - fx_n \| + c_3 \| Sx_{n-1} - fx_{n-1} \| + c_4 \| Sx_n - fx_{n-1} \| + c_5 \| Sx_{n-1} - fx_n \|. \quad (11)$$
Using
$fx_n = Sx_{n+1}, \quad fx_{n-1} = Sx_n,$
Gives
$$\| \delta(Sx_n - Sx_{n-1}) + Sx_{n+1} - Sx_n \| \leq c_1 \| Sx_n - Sx_{n-1} \| + c_2 \| Sx_n - Sx_{n+1} \| + c_3 \| Sx_{n-1} - Sx_n \| + c_4 \| Sx_n - Sx_n \| + c_5 \| Sx_{n-1} - Sx_{n+1} \|. \quad (12)$$
Hence

$\| \delta(Sx_n - Sx_{n-1}) + Sx_{n+1} - Sx_n \| \leq (c_1 + c_3) \| Sx_n - Sx_{n-1} \| + c_2 \| Sx_{n+1} - Sx_n \| + c_5 \| Sx_{n-1} - Sx_{n+1} \|$. (13)

By the triangle inequality

$\| Sx_{n-1} - Sx_{n+1} \| \leq \| Sx_{n-1} - Sx_n \| + \| Sx_n - Sx_{n+1} \|$.

Substituting into inequality (13),

$\| \delta(Sx_n - Sx_{n-1}) + Sx_{n+1} - Sx_n \| \leq (c_1 + c_3 + c_5) \| Sx_n - Sx_{n-1} \| + (c_2 + c_5) \| Sx_{n+1} - Sx_n \|$.

Using the reverse triangle inequality,

$\| \delta(Sx_n - Sx_{n-1}) + Sx_{n+1} - Sx_n \| \geq \delta \| Sx_n - Sx_{n-1} \| - \| Sx_{n+1} - Sx_n \|$.

Therefore

$\delta \| Sx_n - Sx_{n-1} \| - \| Sx_{n+1} - Sx_n \| \leq (c_1 + c_3 + c_5) \| Sx_n - Sx_{n-1} \| + (c_2 + c_5) \| Sx_{n+1} - Sx_n \|$.

Rearranging,

$(1 - c_2 - c_5) \| Sx_{n+1} - Sx_n \| \leq (c_1 + c_3 + c_5 - \delta) \| Sx_n - Sx_{n-1} \|$.

Hence

$\| Sx_{n+1} - Sx_n \| \leq q \| Sx_n - Sx_{n-1} \|$, (14)

Where $q = \frac{c_1+c_3+c_5}{\delta+1-c_2-c_5}$.

Since $c_1 + c_2 + c_3 + c_4 + c_5 < 1$

And $\delta + 1 > c_1 + c_2 + c_4$,

It follows that $0 < q < 1$.

Iterating $\| Sx_{n+1} - Sx_n \| \leq q^n \| Sx_1 - Sx_0 \|$.

For integers $m > n$,

$$\| Sx_m - Sx_n \| \leq \sum_{k=n}^{m-1} \| Sx_{k+1} - Sx_k \| \leq \sum_{k=n}^{m-1} q^k \| Sx_1 - Sx_0 \|.$$

Therefore

$$\| Sx_m - Sx_n \| \leq \frac{q^n}{1-q} \| Sx_1 - Sx_0 \|.$$

Since $q \in (0,1)$,

$\| Sx_m - Sx_n \| \to 0$ as $m, n \to \infty$.

Thus $\{Sx_n\}$ is Cauchy.

Because $S(B)$ is complete, there exists $z \in S(B)$ such that

$Sx_n \to z$.

Since

$fx_n = Sx_{n+1}$,

we also obtain

$fx_n \to z$.

Since $z \in S(B)$, there exists $u \in B$ satisfying

$Su = z$.

We now show that

$fu = z$.

Applying inequality (10) with $x = u$ and $y = x_n$ gives

$\| \delta(Su - Sx_n) + fu - fx_n \| \leq c_1 \| Su - Sx_n \| + c_2 \| Su - fu \| + c_3 \| Sx_n - fx_n \| + c_4 \| Su - fx_n \| + c_5 \| Sx_n - fu \|$. (15)

Passing to the limit as $n \to \infty$ and using

$Su = z, \quad Sx_n \to z, \quad fx_n \to z,$

yields $\| fu - z \| \leq (c_2 + c_5) \| fu - z \|$.

Hence

$(1 - c_2 - c_5) \| fu - z \| \leq 0$.

Since $c_2 + c_5 < 1$,

We conclude

$fu = z = Su$.

Thus $u$ is a coincidence point of $f$ and $S$.

Since $Sf = fS$,

$fz = f(Su) = S(fu) = Sz$.

Using $fu = z$ gives

$fz = Sz$.

Hence $z$ is also a coincidence point.

Applying

$\| \delta(Sz - Su) + fz - fu \| = \| \delta(Sz - z) + fz - z \|$.

Using $fz = Sz$,

$= (\delta + 1) \| Sz - z \|$.

The right-hand side becomes

$(c_1 + c_4) \| Sz - z \|$.

Therefore

$(\delta + 1) \| Sz - z \| \leq (c_1 + c_4) \| Sz - z \|$.

Since

$\delta + 1 > c_1 + c_4$,

It follows that

$Sz = z$.

Because $fz = Sz$,

We obtain $fz = z$.

Thus

$z = fz = Sz$,

So $z$ is a common fixed point.

Finally, suppose that $p$ and $q$ are common fixed points.

Substituting these into inequality (10) gives

$(\delta + 1) \| p - q \| \leq (c_1 + c_4) \| p - q \|$.

Hence

$(\delta + 1 - c_1 - c_4) \| p - q \| \leq 0$.

Since

$\delta + 1 > c_1 + c_4$,

we obtain

$p = q$.

Therefore the common fixed point is unique.

## CONCLUSION

In this paper, we introduced a new class of nonlinear operators, termed modified enriched Hardy–Rogers contractions, within the framework of real Banach spaces. The proposed contractive condition extends the classical Hardy–Rogers contraction by incorporating an enrichment parameter together with a modified arrangement of the associated distance terms. This formulation yields a broader class of mappings and provides a unified framework that includes several well-known contractive conditions as special cases.

A fundamental contribution of this work is the establishment of existence and uniqueness results for fixed points of modified enriched Hardy–Rogers contractions. The convergence analysis was carried out by means of direct iterative arguments without relying on restrictive assumptions frequently encountered in related studies. Furthermore, a Jungck-type extension of the proposed contraction was introduced, leading to new common fixed point theorems for commuting pairs of mappings satisfying the modified enriched Hardy–Rogers condition.

Theoretical comparisons with existing literature reveal that the present framework generalizes and extends numerous classical and contemporary fixed point results, including those of Banach, Reich, Hardy–Rogers, Jungck, Berinde–Păcurar, and recently developed enriched Hardy–Rogers contractions. Several examples were constructed to demonstrate the properness and independence of the newly introduced classes of mappings, thereby establishing that the obtained results are genuine extensions rather than mere reformulations of known theorems.

The results developed in this work open several directions for future investigation. In particular, it would be of interest to study modified enriched Hardy–Rogers contractions in more general settings such as uniformly convex Banach spaces, CAT(0) spaces, modular function spaces, cone metric spaces, partial metric spaces, $b$ -metric spaces, and hyperbolic metric spaces. Another promising direction is the application of the proposed framework to nonlinear integral equations, variational inequalities, equilibrium problems, optimization models, and split feasibility problems. We believe that the ideas introduced herein will stimulate further developments in enriched fixed point theory and its applications to nonlinear mathematical analysis.

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