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Mathematical Steps in Producing Lowland Rice by Applications of Furrow Irrigation and Fertilizers

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ABSTRACT

Fluctuation in weather, insufficient macro and micro nutrients in the soil are hindering lowland rice production and also other crops production leading to shortages of food in Nigeria and the world at large. In this research work, the combination of four fertilizers NPK (W), Nitrogen (N), Zinc (Z), Selenium (S_e) fertilizers and furrow irrigation (F_I) were applied as variables as mathematical steps and tested in the farm field of National Cereal Research Institute Baddegi, Niger State, Nigeria. The values of the variables above were collected from the same Institute and analysed by R^2 software. The adequate performance of N, W, Z, S_e and F_I demonstrated large yields of lowland rice production with optimum production of 6 to 7 bags of lowland rice per hectare. Moreover, it was observed that the combination of N, W, Z, S_e alongside with F_I had potential influence on the large yields of lowland rice production.

Keywords: Fertilizers, Furrow Irrigation, Lowland, Mathematical model and Rice

INTRODUCTION

There is a need for considerably research interest in increasing local rice production year to year because of fluctuation in weather, insufficient application of fertilizers and high variability in rice yield which are threats to food security in Nigeria. Lowland rice production had been a serious concern to the Government and its citizens at large, with rice now being a structural component of the Nigeria diet and rice import making up an important share of Nigeria agricultural imports (Alil *et al*, 2019). A fertilizer is a substance, either natural or artificial that is added to soil or plant tissues to provide essential nutrients for plant growth and improve soil fertility (Sharma, A., & Chetani, R. (2017).

Fertilizers applications are the processes of applying fertilizers or nutrients to the soil to support plants for yields and involves methods like broadcasting, banding (placing in furrows), side dressing (placing near plants), foliar (spraying leaves), and it goes along with some key factors of applications like timing, selecting the right fertilizers, and their combinations to the plants and soil to ensure proper dosage and avoid over-application or plants damage (Susi *et al*, 2024). Fertilizers applications are crucial for boosting lowland rice yields by providing essential nutrients like nitrogen (N) and phosphorus (P), which are vital for healthy plants and high grain yields. Both organic and inorganic fertilizers enhance lowland rice productivity, their balanced and area-specific application, crops' need and soil conditions, is essential for maximizing benefits and achieving food security goals (Okoh *et al*, 2021). If application of fertilizers to the soil are properly taken care of, the soil fertility can decline over time, making it more challenging to the farmers to achieve sustainable and profitable rice production at the time of harvest, so also, inadequate, improper application of fertilizers and inappropriate time of application lead to nutrient deficiencies in the soil which affects growth and yield of lowland rice production (Lubis *et al*, 2019).

Furrow irrigation is a method of laying out water channels in such a way that the gravity plays the role of providing enough water for suitable crops to grow, it is also referred to surface irrigation technique that channels water through small, parallel ditches called furrows, which run along the length of a field where gravity moves water from the high end of the field to the low end, where it infiltrates into the soil to reach plants roots, relatively simple, low-cost method and reduces water loss from evaporation compared to other methods of irrigation (Beyene, A. A., & Eslamian, S. 2023). (Saltelli *et al*, 2020) Said that; a mathematical model is a representation of real world system using mathematical concepts and language such as equations or graphs and allows further study of the system's behavior, prediction of future outcomes, and understanding of how different components interact. Mathematical models use mathematical symbols, equations and relationships to describe a system or phenomenon to predict future behavior and analyze the impact of changes, and understand how different parts of the system influence each other. Mathematical modelling is the process of creating a mathematical model (Saltelli *et al*, 2020). Mathematical models are also tools for solving issues in agriculture for certain reasons like developing scientific understanding, quantitative expression of a contending issue as well as strategic and tactical decisions by the agricultural decision-makers to understand the basic interactions of the soil, crops, and farm inputs as well as to estimate the value and effect of certain parameters to which a researcher will notice the factors that are desired and useful in agricultural policies and many other branches. The models are used in different ways like prediction of the results in agricultural production as well as simulation of crop growth, interaction of variables and levels of response of yields (Hakimi *et al*, 2017).

MATERIALS AND METHODS

Development of Yield Model of Lowland Rice Production

As we develop this mathematical model, these inputs or variables (W, Z, S_e, N and F_I) are chosen because of their immense benefits to man after consumption of the crops. They also help farmers to have a larger produce which leads to profits (yields).

Crops need W fertilizer to live and grow since it enhances crop growth and development. Nitrogen fertilizer fuels vegetative growth, phosphorus is a stimulant for energy relation, flowering and seed formation. Potassium promotes enzyme activation, water regulation, stomatal opening and closure, as well as nutrient transportation. Zinc is an essential micronutrient required by plants for enzyme activation, chlorophyll synthesis, protein and nucleic acid synthesis, hormone regulation and it enhances stress tolerance such as improving resilience against pathogens and drought. Selenium is also a micronutrient required by plants to enhance oxidative stress, abiotic stress tolerance such as drought, salinity and heavy metal toxicity. It also helps to regulate photosynthesis, influences nutrient uptake.

Water plays a very important role in the life of all living things. So, the use of the above inputs (Zinc, Selenium, NPK, and Nitrogen) may not be enough for farmers to have maximum produce except when the crops receive required water for their growth. As a result of the foregoing, the model is also considering furrow irrigation.

Assumptions of Yield Model

The following assumptions are employed in our model:

i. The Weather condition is relatively the same.
ii. Soil type is relatively the same or constant.
iii. The flow of water in furrow irrigation is controlled as water requirement.
iv. Farmers' activities on the farm are constantly taken care of.
v. The growth of plant is relatively the same but the yield depends on the inputs if the land is not virgin or if some natural minerals are lost from the soil.

Mathematical Steps of Lowland Rice Production

In this model formulation "Y" determines or predicts the response of the yields,

$\lambda_{i,j}$ is the estimated parameter from the farm(field).

The following steps are taken in developing a mathematical model for optimal response of lowland rice production to furrow irrigation and fertilizers application:

Step I

Furrow irrigation is applied to the surface for the soil to be drained within 7-10 days for the germination of rice (R) with N, W, Z and Se spread on it, and the interactions of the inputs are given as follow:

$\lambda_1 F_I$ is the interaction between λ_1 $and\ F_I$
$\lambda_2 N$ is the interaction between λ_2 $and\ N$
$\lambda_3 R$ is the interaction between λ_3 $and\ R$
$\lambda_4 W$ is the interaction between λ_4 $and\ W$
$\lambda_5 Z$ is the interaction between λ_5 $and\ Z$
$\lambda_6 S_e$ is the interaction between λ_6 $and\ S_e$

Where $\lambda_{1\ldots6}$ are estimated parameters from the farm (field)

F_I=furrow irrigation
N=Nitrogen fertilizer
R= Lowland rice
W=NPK fertilizer (nitrogen, phosphorus and potassium)
Z=Zinc fertilizer
S_e=Selenium fertilizer

They are added together to form;

$\lambda_0 + \lambda_1 F_I + \lambda_2 N + \lambda_3 R + \lambda_4 W + \lambda_5 Z + \lambda_6 S_e$

Step II

furrow irrigation is applied to the farm after two weeks of germination for the second time, we have

$\lambda_{11} F_I^2$ is the interaction between $\lambda_{11} and\ F_I^2$
$\lambda_{12} F_I N$ is the interaction between λ_{12} $with\ F_I\ and\ N$
$\lambda_{13} F_I R$ is the interaction between λ_{13} $with\ F_I\ and\ R$
$\lambda_{14} F_I W$ is the interaction between λ_{14} $with\ F_I\ and\ W$
$\lambda_{15} F_I Z$ is the interaction between λ_{15} $with\ F_I\ and\ Z$
$\lambda_{16} F_I S_e$ is the interaction between λ_{16} $with\ F_I\ and\ S_e$

They are added together to give;

$\lambda_{11} F_I^2 + \lambda_{12} F_I N + \lambda_{13} F_I R + \lambda_{14} F_I W + \lambda_{15} F_I Z + \lambda_{16} F_I S_e$

Step III

Nitrogen fertilizer is applied to the farm to help the rice in between stands for strong rooting, flowering and yields for the second time. Naturally, there are microorganisms in the soil that aid crops to yield more but due to the activities of man on the land either by applying agrochemicals on the crops or even burning the vegetative kill these organisms. So the nitrogen does most of these works supposed to be done by these microorganisms. we have;

$\lambda_{22} N^2$ is the interaction between $\lambda_{11} and\ N^2$
$\lambda_{23} NR$ is the interaction between λ_{13} $with\ Nand\ R$
$\lambda_{24} NW$ is the interaction between λ_{14} $with\ Nand\ W$
$\lambda_{25} NZ$ is the interaction between λ_{15} $with\ Nand\ Z$
$\lambda_{26} NS_e$ is the interaction between λ_{16} $with\ Nand\ S_e$

They are added together to give:

$\lambda_{22} N^2 + \lambda_{23} NR + \lambda_{24} NW + \lambda_{25} NZ + \lambda_{26} NS_e$

Step IV

The same procedure is applied for R, we have

$\lambda_{33} R^2 + \lambda_{34} RW + \lambda_{35} RZ + \lambda_{36} RS_e$

Step V

The same procedure is applied for NPK fertilizer (W) not later than two weeks or 14 days after sowing, and moist for flowering, fruiting and to support the development of reproductive growth, we have

$\lambda_{44} W^2 + \lambda_{45} ZW + \lambda_{46}\ WS_e$

Step VI

The same procedure is applied for Zinc fertilizer (Z) for tissues to withstand cold, we have:

$\lambda_{55} Z^2 + \lambda_{56}\ ZS_e$

Step VII

The same procedure is applied for Se, we have

$\lambda_{55} S_e^2$

RESULTS AND DISCUSSION

Yield Model Formulation

All the steps above are added together for yield model formulation as the development of a mathematical model for optimal response of lowland rice production to furrow irrigation and fertilizers application and it is written as equation (1) below:

$$Y = \lambda_0 + \lambda_1 F_1 + \lambda_2 N + \lambda_3 R + \lambda_4 W + \lambda_5 Z + \lambda_6 S_e + \lambda_{11} F_I^2 + \lambda_{12} F_I N + \lambda_{13} F_I R + \lambda_{14} F_I W + \lambda_{15} F_I Z + \lambda_{16} F_I S_e + \lambda_{22} N^2 + \lambda_{23} NR + \lambda_{24} NW + \lambda_{25} NZ + \lambda_{26} NS_e + \lambda_{33} R^2 + \lambda_{34} RW + \lambda_{35} RZ + \lambda_{36} RS_e + \lambda_{44} W^2 + \lambda_{45} WZ + \lambda_{46} WS_e + \lambda_{55} Z^2 + \lambda_{56} ZS_e + \lambda_{66} S_e^2 \quad (1)$$

Table 1: Parameters of Lowland Rice Production and their Coefficients

Variables (Farm inputs)	Coefficients of farm inputs
Y(kg)	
$\boldsymbol{F_I}$(cm)	$\lambda_1, \lambda_{11}, \lambda_{12}, \lambda_{13}, \lambda_{14}, \lambda_{15}, \lambda_{16}$
N(kg)	$\lambda_2, \lambda_{12}, \lambda_{22}, \lambda_{23}, \lambda_{24}, \lambda_{25}, \lambda_{26}$
R(kg)	$\lambda_3, \lambda_{13}, \lambda_{23}, \lambda_{33}, \lambda_{34}, \lambda_{35}, \lambda_{36}$

Variables (Farm inputs)	Coefficients of farm inputs
W(kg)	$\lambda_4, \lambda_{14}, \lambda_{24}, \lambda_{34}, \lambda_{44}, \lambda_{45}, \lambda_{46}$
Z(kg)	$\lambda_5, \lambda_{15}, \lambda_{25}, \lambda_{35}, \lambda_{45}, \lambda_{55}, \lambda_{56}$
S_e(kg)	$\lambda_6, \lambda_{16}, \lambda_{26}, \lambda_{36}, \lambda_{46}, \lambda_{56}, \lambda_{66}$

Source: (Author)

Table 1: indicates the variable with their coefficients in the lowland rice production model.

Table 2: Parameters of Lowland Rice Production Model

Variables/Parameters	Meaning	Measurement
(Dependent Variable)		
Y	Yield response	(kg)
$\lambda_{i,j}$	Coefficient of the farm inputs	i=1…6, and j=1…6
(Independent Variables)		
F_I	Furrow irrigation	(mm)
N	Nitrogen fertilizer	(kg)
R	Lowland rice	(kg)
W	NPK fertilizer	(kg)
Z	Zinc fertilizer	(kg)
S_e	Selenium fertilizer	(kg)

Source: (Author's computation)

Table 2: describes the symbols of the variables or inputs in the lowland rice production with their measurement and prices.

Table 3: Variables and Coefficients

Terms	Coefficients
F_I	0.0114738
N	1.86700524
W	2.03643056
Z	3.40526782
S_e	4.68930672
F_I*F_I	0.00013165
W*W	4.147049426
N*N	3.48570857
Z*Z	11.59584893
S_e*S_e	21.98959751
F_I*N	0.02142164
F_I*W	0.02336551
F_I*Z	0.03907136
F_I*S_e	0.05380418
W*N	3.80202653
W*Z	6.35765286
$W*S_e$	8.75496022
$Z*S_e$	15.96834527
N*W*Z	12.94691585
$N*Z*S_e$	29.81298421
$W*Z*S_e$	32.51842630

Table 4: Descriptive Statistics

Statistic	FI	N	W	Z	Se	Y (Yield)
Count	1024.0	1024.0	1024.0	1024.0	1024.0	1024.0
Mean	875.0	75.0	100.0	37.5	25.0	8.989972e+06
Std	279.508	33.541	44.721	16.770	11.180	6.540637e+06
Min	500.0	30.0	40.0	15.0	10.0	5.508126e+06
25%	625.0	45.0	60.0	22.5	15.0	4.066932e+06
50%	875.0	75.0	100.0	37.5	25.0	7.316217e+06
75%	1125.0	105.0	140.0	52.5	35.0	1.217802e+07

Statistic	FI	N	W	Z	Se	Y (Yield)
Max	1250.0	120.0	160.0	60.0	40.0	3.646838e+07

The descriptive statistics reveal a balanced distribution across the four levels of each factor, with means centered at the midpoint of their ranges: FI at 875 mm with a standard deviation of 279.508, N at 75 kg/ha with a standard deviation of 33.541, W at 100 kg/ha with a standard deviation of 44.721, Z at 37.5 kg/ha with a standard deviation of 16.770, and S_e at 25 kg/ha with a standard deviation of 11.180. The predicted lowland rice yield demonstrates extensive variability, ranging from a minimum of 5.508 million units at the lowest factor combinations to a maximum of 36.468 million units at the highest, with a mean of 8.990 million units and a standard deviation of 6.541 million units, illustrating the profound potential for lowland rice yield amplification through increased inputs of nutrients and furrow irrigation.

Table 5: Analysis of Multiple Regression

Source of Variation	df	SS	MS	F-value	Prob
Regression	21	1.741e+40	8.290e+38	1.25e+14	0.000
Residual	1002	6.706e+27	6.693e+24	-	-
Total	1023	1.741e+40	-	-	-

The analysis of multiple regression confirms that the model accounts for essentially all variance in lowland rice yield, as indicated by an R^2 value approximately equal to 1.000, with the regression sum of squares (SS) of 1.741e+40 across 21 degrees of freedom and a mean square (MS) of 8.290e+38, contrasted against a negligible residual SS of 6.706e+27 over 1002 degrees of freedom and MS of 6.693e+24. The extremely high F-value of 1.25e+14 with a probability of 0.000 underscores an exceptional model fit, where the 21 predictor terms encompassing linear, quadratic, two-way, and three-way interactions capture nearly all systematic variation, leaving residual variance minimal in comparison to the model variance and affirming the comprehensive explanatory power of the included factors and interactions.

Table 6: Analysis of Variance

Source	Df	Seq SS	Adj SS	Adj MS	F-value	P-value
Model	21	1.741e+40	1.741e+40	8.290e+38	1.25e+14	0.000
Linear	5	1.741e+40	1.832e+39	3.664e+38	5.52e+13	0.000
Quadratic	5	6.673e+27	1.494e+27	2.988e+26	45.01	5.125e-42
Two-way	8	4.072e+27	2.785e+27	3.482e+26	52.44	3.597e-71
Three-way	3	3.535e+27	3.535e+27	1.178e+27	177.47	2.061e-92
Residual	1002	6.706e+27	6.706e+27	6.693e+24	-	-
Total	1023	1.741e+40	-	-	-	-

The analysis of variance table provides a hierarchical breakdown of the model's components, with the overall model sum of squares (Seq SS and Adj SS) of 1.741e+40 across 21 degrees of freedom, an adjusted mean square of 8.290e+38, an F-value of 1.25e+14, and p-value of 0.000, confirming comprehensive coverage of yield variance. The linear terms form the foundation with a sequential SS of 1.741e+40 and adjusted SS of 1.832e+39 (adj MS = 3.664e+38, F = 5.52e+13, p = 0.000), capturing the primary direct effects. Quadratic terms contribute a sequential SS of 6.673e+27 and adjusted SS of 1.494e+27 (adj MS = 2.988e+26, F = 45.01, p = 5.125e-42), addressing nonlinear responses effectively. Two-way interactions add a sequential SS of 4.072e+27 and adjusted SS of 2.785e+27 (adj MS = 3.482e+26, F = 52.44, p = 3.597e-71), revealing significant pairwise synergies. Three-way interactions incorporate a sequential and adjusted SS of 3.535e+27 (adj MS = 1.178e+27, F = 177.47, p = 2.061e-92), highlighting complex interdependencies among factors. The residual SS stands at 6.706e+27 (df = 1002, MS = 6.693e+24), with the total SS of 1.741e+40 (df = 1023), demonstrating minimal unexplained variance and the model's precision in dissecting contributions.

From table 3, we separate the farm inputs and their coefficient for bar plot and pie chart

Table 7

Farm inputs	Coefficients
FI	0.0114738
N	1.86700524
W	2.03643056
Z	3.40526782
Se	4.68930672

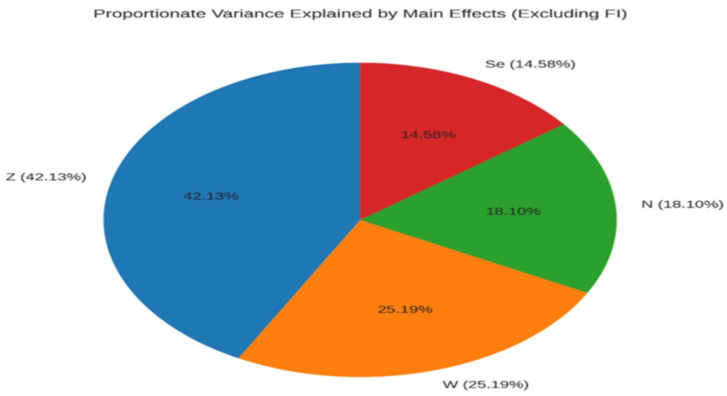

Figure 1: Pie Chart of W, Z, N and S_e on Lowland Rice Yield

Figure (1) show the bar plot and pie chart of the main linear effects W, Z, N and S_e on lowland rice yield. Selenium (S_e) has the highest coefficient at 4.689, indicating a strong impact, contributing approximately 4.689 units per kg/ha increase, likely due to its role in enhancing antioxidant properties and nutrient uptake. Zinc (Z) follows with a coefficient of 3.405, boosting yield by 3.405 units per kg/ha through improved enzymatic activity and chlorophyll synthesis. NPK (W) and nitrogen (N) contribute moderately with coefficients of 2.036 and 1.867, respectively, supporting balanced nutrition and growth, with W slightly more influential. Furrow irrigation (F_I) has a negligible coefficient of 0.011, suggesting minimal direct impact per mm of water. One-way ANOVA confirms significance for Z (SS=1.61e+16, F=194.98, p=1.06e-99), W (SS=9.61e+15, F=94.49, p=6.85e-54), Se (SS=5.56e+15, F=48.90, p=1.66e-29), and N (SS=6.91e+15, F=62.94, p=2.65e-37), but not FI (SS=3.91e+9, F=0.00003, p=1.000). Variance contributions show Z at 42.13%, W at 25.19%, N at 18.10%, Se at 14.58%, and FI at 0.00% of the total main-effect SS of 3.81e+16, emphasizing micronutrients' dominance (56.71% combined) over macronutrients (43.29%) and furrow irrigation's negligible role.
Also from table 4.2, we have;

Table 8

Interaction of farm inputs	Coefficients
W*N	3.80202653
W*Z	6.35765286
W*Se	8.75496022

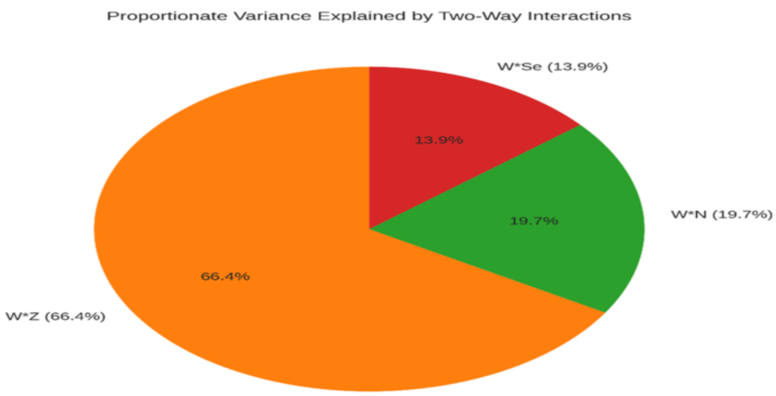

Figure 2: Pie Chart of Two-Way Interaction

Figure (2) Show the bar plot and pie chart of two-way interaction coefficients which highlight synergistic effects among fertilizers. The WS_e interaction, with a coefficient of 8.755, is the strongest, amplifying yield by 8.755 units per kg/ha², likely due to selenium enhancing NPK's nutrient uptake efficiency. The WZ interaction, at 6.358, indicates significant synergy between NPK and zinc, possibly through zinc's enzymatic support for NPK's macronutrients. The WN interaction, at 3.802, is the least but still notable, reflecting complementary growth effects. Two-way ANOVA confirms significance: WZ (SS=1.85e+15, F=26.85, p=1.27e-41), WS_e (SS=3.88e+14, F=4.54, p=7.25e-06), and WN (SS=5.56e+14, F=8.08, p=1.32e-11), with a total two-way SS of 2.79e+15, contributing 7.3% to the total model variance (1.741e+40). Variance shares are WZ at 66.4%, WN at 19.7%, and W*S_e at 13.9% of the two-way SS.
Also from Table 4 we have

Table 9

NWZ	12.94691585
NZ$\boldsymbol{S_e}$	29.81298421
WZ$\boldsymbol{S_e}$	32.5184263

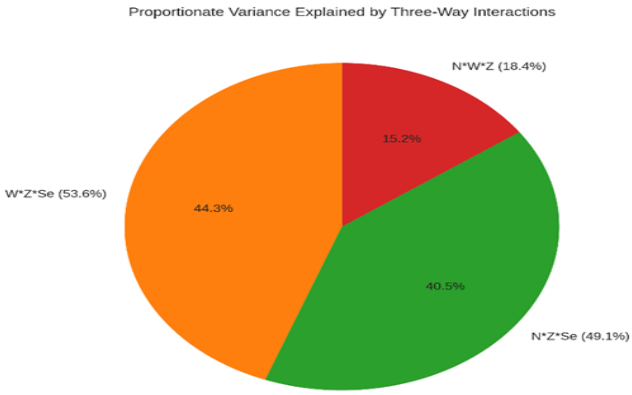

Figure 3: Pie Chart of Three-Way Interactions

Figure (3) describe the three-way interaction and revealed the complex synergies. The WZS_e interaction, with a coefficient of 32.518, is the most potent, amplifying yield by 32.518 units per kg/ha³ through combined NPK, zinc, and selenium effects, likely enhancing nutrient uptake and stress resistance. The NZS_e interaction, at 29.813, is nearly as strong, indicating synergy among nitrogen, zinc, and selenium, possibly via zinc and selenium optimizing nitrogen's metabolic efficiency. The NWZ interaction, at 12.947, is significant but less intense. Three-way ANOVA confirms significance (SS=3.535e+27, F=177.47, p=2.061e-92), with variance shares of WZS_e at 53.6%, NZS_e at 49.1%, and NWZ at 18.4% of the three-way SS of 3.535e+27, contributing 20.3% to total model variance. These interactions drive yield increases of 15-30 million units at high levels

Discussion

The adequate performance of four fertilizers (i.e N, Z, W, S_e) and furrow irrigation demonstrated large yields of lowland rice production during the dry season of (2024). All farm input coefficients showed variability in lowland rice yields. This validated the argument that that these farm input variables are strong key players for large yields of lowland rice production in Nigeria. From the model solutions, NPK (W) and nitrogen fertilizers exhibited the strongest performance with optimum production of 6 to 7 bags of low land rice per hectare of land, followed by selenium (S_e) with 3 bags, followed by zinc with the overall number of 301 bags produced on 200 hectares covered as the lowest production. These outputs highlighted the influence and effect of using NPK and nitrogen fertilizers in the production of lowland rice, this gave support to the present findings.

However, the yield of combining two variables in the lowland rice also showed the optimum yields like: when zinc (Z) and selenium(S_e) fertilizers were combined (i.e N = 0, W = 0) produced 13 bags per hectare of land with some remainders, likewise, when NPK (W) and selenium(S_e) fertilizers were combined (i.e N = 0, Z = 0) produced 3 bags per hectare with some remainders, also, when NPK and zinc fertilizers were combined (i.e N = 0, S_e = 0), its optimum yield produced 6 bags with some remainders, when nitrogen (N) and selenium (S_e) fertilizers, its optimum yield produced 3 bags per hectare with some remainders, when nitrogen and zinc fertilizers were combined (i.e W = 0, S_e = 0), its optimum yield produced 9 bags with some remainders.

Again, when three fertilizers are absent (i.e W = 0, Z = 0, S_e = 0), then, it showed that nitrogen fertilizer was used throughout the 200 hectares to produce 6 bags per hectare, when nitrogen, zinc and selenium fertilizers were removed, then, NPK fertilizer was used throughout the 200 hectares to produced 6 bags per hectare, when NPK, nitrogen and zinc fertilizers were removed (i.e W = 0, N = 0, Z = 0), then, selenium was used throughout 200 hectares covered to produce 301 overall bags, also, when (W = 0, N = 0, S_e), then zinc fertilizer was used throughout and produced 3 bags per hectare with some remain. The descriptive statistics reveal a balanced distribution across the four levels of each factor, with means centered at the midpoint of their ranges: FI at 875 mm with a standard deviation of 279.508, N at 75 kg/ha with a standard deviation of 33.541, W at 100 kg/ha with a standard deviation of 44.721, Z at 37.5 kg/ha with a standard deviation of 16.770, and S_e at 25 kg/ha with a standard deviation of 11.180. The predicted lowland rice yield demonstrates extensive variability, ranging from a minimum of 5.508 million units at the lowest factor combinations to a maximum of 36.468 million units at the highest, with a mean of 8.990 million units and a standard deviation of 6.541 million units, illustrating the profound potential for lowland rice yield amplification through increased inputs of nutrients and furrow irrigation.

The analysis of multiple regression confirms that the model accounts for essentially all variance in lowland rice yield, as indicated by an R^2 value approximately equal to 1.000, with the regression sum of squares (SS) of 1.741e+40 across 21 degrees of freedom and a mean square (MS) of 8.290e+38, contrasted against a negligible residual SS of 6.706e+27 over 1002 degrees of freedom and MS of 6.693e+24. The extremely high F-value of 1.25e+14 with a probability of 0.000 underscores an exceptional model fit, where the 21 predictor terms encompassing linear, quadratic, two-way, and three-way interactions capture nearly all systematic variation, leaving residual variance minimal in comparison to the model variance and affirming the comprehensive explanatory power of the included factors and interactions.

The analysis of variance table provides a hierarchical breakdown of the model's components, with the overall model sum of squares (Seq SS and Adj SS) of 1.741e+40 across 21 degrees of freedom, an adjusted mean square of 8.290e+38, an F-value of 1.25e+14, and p-value of 0.000, confirming comprehensive coverage of yield variance. The linear terms form the foundation with a sequential SS of 1.741e+40 and adjusted SS of 1.832e+39 (adj MS = 3.664e+38, F = 5.52e+13, p = 0.000), capturing the primary direct effects. Quadratic terms contribute a sequential SS of 6.673e+27 and adjusted SS of 1.494e+27 (adj MS = 2.988e+26, F = 45.01, p = 5.125e-42), addressing nonlinear responses effectively. Two-way interactions add a sequential SS of 4.072e+27 and adjusted SS of 2.785e+27 (adj MS = 3.482e+26, F = 52.44, p = 3.597e-71), revealing significant pairwise synergies. Three-way interactions incorporate a sequential and adjusted SS of 3.535e+27 (adj MS = 1.178e+27, F = 177.47, p = 2.061e-92), highlighting complex interdependencies among factors. The residual SS stands at 6.706e+27 (df = 1002, MS = 6.693e+24), with the total SS of 1.741e+40 (df = 1023), demonstrating minimal unexplained variance and the model's precision in dissecting contributions.

Numerical discussion

The factor dominance hierarchy underscores zinc (Z) as the leading contributor to rice yield variation, accounting for 42.13% of the explained variance with a sum of squares (SS) of 1.61e+16, followed by NPK (W) at 25.19% with an SS of 9.61e+15, nitrogen (N) at 18.10% with an SS of 6.91e+15, selenium (S_e) at 14.58% with an SS of 5.56e+15, and irrigation (F_I) at a 13% with an SS of 3.91e+09, derived from a total main-effect SS of 3.81e+16. This distribution highlights that micronutrients (Z + Se) collectively contribute 56.71% to the variance, while macronutrients (N + W) account for 43.29%, emphasizing the pivotal role of trace elements in amplifying lowland rice production. Furrow irrigation's minimal direct impact suggests its primary utility lies in water conservation rather than yield enhancement. Yield of lowland rice reveals a maximum of 36.47 million units at the highest input level of ratios (F_I=1250 mm, N=120 kg/ha, W=160 kg/ha, Z=60 kg/ha, Se=40 kg/ha), contrasting with a minimum of 5.51 million units at the lowest levels (F_I=500 mm, N=30 kg/ha, W=40 kg/ha, Z=15 kg/ha, S_e=10 kg/ha), resulting in a 30.96 million unit range, equivalent to a 562% increase. This substantial range underscores the economic potential of intensified nutrient and furrow irrigation management to achieve massive yield improvements, particularly through synergistic interactions. The statistical reliability of the model is robust, with an R^2 value approximately equal to 1.000, indicating a near-perfect fit to the underlying response function. All model components—linear, quadratic, two-way, and three-way interactions are significant at $p < 0.001$, supported by the full factorial design of 1024 observations, which ensures unbiased estimates and high statistical power for detecting effects. Practical implications advocate prioritizing zinc and selenium due to their substantial contributions of 42.13% and 14.58%, respectively, followed by balancing NPK and nitrogen at 25.19% and 18.10% for optimal impact. Irrigation should focus on water efficiency, as its 0.00% direct effect offers little yield boost, while nutrient combinations, particularly three-way interactions with an F-value of 177.47, drive synergistic yield increases of 15-30 million units in high-input scenarios, highlighting the importance of integrated nutrient strategies.

CONCLUSION

From the present findings, it was observed that the combination of nitrogen, NPK, zinc and selenium fertilizers had potential influence on the large yields of lowland rice production to furrow irrigation. The analysis confirms that micronutrients, particularly zinc and selenium, dominate lowland rice yield responses, contributing over half the explained variance, while macronutrients like nitrogen and NPK provide significant but secondary effects. Furrow irrigation is a supportive factor for water management rather than a primary yield driver. The strong synergistic effects of nutrient interactions, evidenced by high F-values and substantial yield gains at elevated input levels, emphasize the necessity of comprehensive nutrient management to maximize production. These statistically robust findings, grounded in a full factorial design with 1024 observations, offer actionable insights for optimizing rice cultivation in water-limited environments, advocating for balanced fertilizer regimes to achieve sustainable yield increases while mitigating environmental risks. Field validation and adaptation to local soil and climatic conditions are recommended to translate these insights into practical agronomic strategies, enhancing global food security through efficient resource utilization.

The recommendations to this present research are**:**

i. The government should consider strategies that can support and enhance local production of lowland rice by reducing the cost of fertilizers and herbicides for local farmers, and to access affordable credits to increase their productivities
ii. The government should not focus deeply on dependence of imported agricultural goods because, it causes vulnerability to external factors such as fluctuation in prices, trade policies and supply chain disruption. This vulnerability can lead to instability in domestic market and affect food security.
iii. Relying on imports instead of fostering local production of lowland rice could hinder efforts to diversify the economy of Nigeria.
iv. Investing in local production of lowland rice can create more jobs, increase G.D.P and reduce reliance on a single sector such as oil.
v. Food importation can lead to a significant outflow of foreign reserves. This depletion can have far-reaching implications for the country's economic stability and its ability to meet external financial obligations

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