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APPLICATION OF QUEUEING ANALYSIS IN INPATIENT WARD OF COTTAGE HOSPITAL GIREI, GIREI, ADAMAWA STATE NIGERIA

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ABSTRACT

This research work focuses on analyzing patients waiting time in Cottage hospital Girei. The research focused on three wards (male medical ward, female medical ward and pediatric ward) and data was collected for a period of three months (March, April and May 2024). M/M/1 and M/M/1/N models were used to compare the queue metrics across the wards. The result shows that M/M/1/N has a higher waiting time across the wards (MMW=63.04hrs, PW=82.12hrs and FMW=89.61) compared to M/M/1(MMW=29.31hrs, PW=25hrs). Furthermore, a simulation was carried out across all wards to obtain an optimal waiting time and system capacity. The result reveals that, a dual-ward system gives a least waiting time (MMW =23.87hrs, PW=31.24hrs and FWM=39.9hrs) compared to M/M/1 across all the wards with an increase in system capacity.

Keywords: Patient Waiting Time, Inpatient Ward Management, Hospital Queue System, Healthcare Operations Research

INTRODUCTION

Many organizations, such as banks, airlines, telecommunication companies and police departments routinely use queueing models to help manage and allocate resources in order to respond to demands in a timely and cost-efficient fashion (Green, 2011). Though queueing analysis has been used in hospitals and other healthcare settings, its use in this sector is not widespread. Yet, given the pervasiveness of delays in healthcare and the fact that many healthcare facilities are trying to meet increasing demands with constrained resources, queueing models can be very useful in developing more effective policies for allocating and managing resources in healthcare facilities. Queueing analysis is also a useful tool for estimating capacity requirements and managing the demand for any system in which the timing of service needs is random (Hall, 2013).

The need for application of queuing theory in healthcare settings is very important because the wellbeing and life of someone is concerned. The time spent by a patient while waiting to be attended to by a doctor is critical to the patient and to the image of the hospital before the public. Various attempts by the Federal Government of Nigeria to improve on Health Care services will not yield good results except the issue of time spent by patients at various hospitals (Ameh *et al.,* 2013)

MATERIALS AND METHODS

Materials

Habson (2014) analyzed patient waiting time for consulting a doctor in an outpatient Department of Cottage Hospital Hong, Adamawa State. The aim of the study was to investigate time patients waits for consultation at the outpatient department Cottage Hospital, Hong. A primary data was collected during the doctor's busy period (i.e. from 8am -1pm) from Monday to Friday. A M/M/1 queuing model was used to analyzed the patient waiting time. The study found the doctor to be busy for about 78% of the 35hours considered in collecting the data. The average number of patients waiting for the doctors consultation are 3, the average time spend in the queue to see the doctor is 35minutes, the inter-arrival time of patient waiting in the queue is 33minutes and the average time a patients spend waiting for the doctors consultation is 54 minutes. The reason for the high waiting time and long queue is due to lack of enough staff, Habson therefore recommend that one more doctor should be employed so that they can work hand in hand to reduce the waiting time of patients.

Alkalizakari (2022) analyzed the waiting time of patient in an outpatient Health facility considering patients behavior, a case study of Cottage Hospital Hong, Adamawa State. The aim of the study was to investigate the time patients wait for consultation when they are forced to balk or renege at the outpatient Health Facility Cottage Hospital Hong. The data was obtained from the work of Habson (2014) who obtain the data originally and analyzed patients waiting time in Cottage Hospital Hong, using M/M/1. The study found that Balking and Reneging has the least average waiting time in the system and in the queue with 20 minutes and 12 minutes respectively than normal, balking and reneging when the measure of effectiveness (n=1), has least average waiting time in the system and in the queue with 14 minutes and 7 minutes respectively than normal, balking and reneging when the measure of effectiveness (n=2), has the least average waiting time in the system and in the queue with 10 minutes and 3 minutes respectively than normal, balking and reneging when the measure of effectiveness (n=3), has the least average waiting time in the system and in the queue with 7 minutes and 2 minutes respectively than normal, balking and reneging when the measure of effectiveness (n=4), has the least average waiting time in the system and in the queue with 5 minutes and 1 minutes respectively than normal, balking and reneging when the measure of effectiveness (n=5). He recommended that another doctor is needed to reduce the waiting time of patient if they can balk and renege.

Abdus-salam et al. (2021), analyzed the Antenatal Clinic Waiting Time, Patient Satisfaction, and Preference for Staggered Appointment. A purposive sampling technique was employed to enroll a total of 122 pregnant women in the study. Data were collected using a pretested, semi-structured, interviewer-administered questionnaire—data collection form. The information collected included the sociodemographic and obstetric characteristics, the time of arrival at the clinic, the time of arrival at each service point, time of commencement, and completion of procedures at different service points, waiting time, transit time, and the queue size on arrival at each of the service points and departure time. The data were entered into the computer and analyzed using (SPSS) software version 23. Descriptive statistics were done using univariate analysis of mean, median, mode, standard deviation, and bivariate analysis using chi-square test. The level of significance was set at P < .05. One hundred and twenty-two participants were interviewed. Mean age was 30.52 (±4.65) years, they were mostly multi-gravid, married, and with tertiary education. Mean time spent in ANC and waiting time were 191 min and 143 min, respectively. Waiting time was longest at doctor's consultation (59 min), laboratory services (38 min), and the cash pay-point (18 min). About 68.9% were satisfied with services and highest at doctors' consultation. Satisfaction was associated with waiting time of <45 min. Dissatisfaction was high at the cash pay-point (28.7%), followed by the laboratory (16.4%). About 56.5% preferred staggered appointments. Time spent in ANC should be reduced and staggered appointments may be a useful strategy to reduce waiting time and patient load.

Mensur et al. (2022) studied, Waiting time and its associated factors in patients presenting to outpatient departments at Public Hospitals of Jimma Zone, Southwest Ethiopia. An institution-based cross-sectional study design was used from March 22 to June 3, 2020. A total of 422 study subjects were included in the study and systematic random sampling methods were used. The data were collected by observing the whole service points of each patient. The exit interview was made at the last point of the service unit. Descriptive statistics, bi-variable and multi-variable logistic regressions were used. The result show that patients spent in the hospitals before getting service was a minimum of 41 and a maximum of 185 min. Patients who came far from the hospitals were 1.93 times (AOR = 1.93; 95% CI, 1.16, 3.21) more likely to spend longer waiting time as compared to those who came from the hospital's area. Patients visited on Monday were 2.64 times (AOR = 2.64; 95% CI, 1.45, 4.79) more likely to spend longer waiting time than those who visited the hospital on Friday. Patients who arrived early in the morning were 3.22 times (AOR = 3.22; 95% CI, 1.32, 7.86) more likely to spend longer waiting time than those who arrived in the afternoon. He concluded that the mean waiting time was higher than the average recommended time by Business Process Reengineering (BPR) and more than five out of every ten clients spent long waiting time at outpatient departments Waiting time was affected by Educational status, residence, arrival time, and date of the visit.

Yahya *et al*. (2021) analyses patient's queuing system at Muhammad Abdullahi Wase Specialist Hospital Kano. This study essentially sourced for data primarily through direct observation. A pen, notebook, and a wristwatch were utilized as pre-requisite to getting relevant information(s) that includes: service time, arrival time of patients, number of patients, and waiting time. This data was collected for about four weeks barring weekends from (8:00 am - 12:00 pm). Kolmogorov-Smirnov one sample test which is famous for having no restrictions about the sample size to ascertain the distribution of the arrival and service patterns was employed to test the goodness of fit and a Gamma inter-arrival and service was detected. A G/G/1 and G/G/S model was used to analyze the queuing system of the Hospital. The study have been able to identify that the inter-arrival and service time at the Pediatrics department of Muhammad Abdullahi Wase Specialist Hospital, Kano follow the general distribution with the First-come, First-serve (FCFS) queuing discipline. It is also observed that the average number of patients in queue and system, average waiting time in queue and system improved when the number of servers were increased indicating that the multi-server G/G/s model performs better in comparison with the single-server G/G/1 model. In order to create a balance for the cost of providing an improved service by the management of the hospital, it was recommended that the multi-server G/G/4 model should be implemented since it would minimize patients waiting time, thereby improving service delivery and also considerably save cost for the management.

Method

The study assumes that hospital bed spaces are allocated on a first-come, first-served (FCFS) basis, using a single queue service pattern**,** and that patients arrive independently for admission at Cottage Hospital Girei (CHG). Secondary data were collected from CHG's monthly records for three months (March to May 2024**)**, focusing on three purposively selected wards: the Male Medical Ward (7 beds)**,** Female Medical Ward (8 beds), and Pediatric Ward (9 beds)**.** The collected data were analyzed using M/M/1 and M/M/1/N queuing models, which assume a Poisson arrival rate and a negative exponential service rate, to determine the steady-state probabilities and other system performance metrics.

In the M/M/1/N queue model**,** the system has a finite capacity N, and the utilization factor (traffic intensity)**,** The steady-state probability of having n customers in the system average number of customers in the system is given by

$$\rho = \frac{\lambda}{\mu} \quad (1)$$

Where;

$\lambda = Arrival\ rate$

$\mu = Service\ rate$

The Steady State Probability is given by;

$$P_n = \frac{\rho^n(1-\rho)}{1-\rho^{N+1}} \quad (2)$$

The length of the system is given by;

$$Ls = \frac{\rho}{1-\rho} - \frac{(N+1)\rho^{N+1}}{1-\rho^{N+1}} \quad (3)$$

In the M/M/1 queue model, which assumes an infinite system capacity, the steady-state probability and the average number of customers in the system is given by;

$$P_n = \rho^n(1-\rho) \quad (4)$$

The length of the system is given by;

$$L_s = \frac{\rho}{1-\rho} \quad (5)$$

Other metrics for both M/M/1/N and M/M/1 can be obtained using Little's formula. John (2011).

In the comparison of queue metrics, the waiting times in the queue for both M/M/1 and M/M/1/N models are analyzed, and the model with the shorter waiting time is considered the more efficient (better) model.

RESULTS AND DISCUSSION

Male Medical Ward

In the Male Medical Ward, the performance of the queuing system was analyzed using both M/M/1 and M/M/1/N models. The ward has a system capacity of seven (7) beds, with an average patient arrival rate (λ) of 1.2065 patients per day and an average service rate (μ) of 2.0253 patients per day. The computed traffic intensity (ρ**)**, representing the utilization level of the system, was 0**.**60, indicating that the ward operates at 60% capacity on average. The results of the computations and related discussions are presented in Tables 1 and 2.

Table 1: Computations of Queue Metrics for Male Medical Ward

	M/M/1		M/M/1/N	
Performance measure	**Value**	**Hours (based on 24 hr day)**	**Value**	**Hours (based on 24 hr day)**
Lq	0.88		4.001	
Ls	1.47		5.32	
Wq	0.73	17.43	2.14	51.22
Ws	1.22	29.31	2.63	63.04
λe			2.02	
Probability that customer waits			1	

Table 1compares the single-ward performance measure analysis for Male Medical Ward; for M/M/1, it was observed that the mean number of patients in the system (Ls) and the mean number of patients in the queue (Lq) are approximately 2 and 1 patients respectively, while the expected waiting time in the system (Ws) and expected waiting time in the queue (Wq) are 29.31 hours and 17.43 hours respectively. For the M/M/1/N, the mean number of patients in the system (Ls) and the mean number of patients in the queue (Lq) are approximately 6 and 4 patients respectively, while the expected waiting time in the system (Ws) and expected waiting time in the queue (Wq) are 63.04 and 51.22 hours respectively.

From the above table, M/M/1/N reveals a high waiting time in the system (63.04hrs) and hence causing queue in the ward. To show that the time patients spend in the ward can be reduced, a two-ward system was simulated for N=7, 8, 9, 10, 11, 12,13 and 14 to obtain an optimal waiting time and system capacity. The result is as shown in Table 2.

Table 2: Summary of Queue Simulation for a Two-Ward System with Finite Capacity in MMW

Ward=2	N=7	N=8	N=9	N=10	N=11	N=12	N=13	N=14
Parameter	**Value**	**Value**	**Value**	**Value**	**Value**	**Value**	**Value**	**Value**
λe	3.84	3.96	4.02	4.05	4.06	4.06	4.06	4.06
ρ	.95	.98	1	1	1	1	1	1
Lq	1.93	2.77	3.7	4.66	5.65	6.65	7.65	8.64
Ls	3.82	4.72	5.68	6.66	7.65	8.65	9.64	10.64
Wq	12.05	16.79	22.06	27.66	33.44	39.3	45.2	51.1
Ws	23.87	28.61	33.88	39.48	45.26	51.13	57.02	62.93

From Table 2, it was observed that an increase in ward(S=2) yielded significant reduction in the values of the expected waiting time in system (Ws). Also, the waiting time (Ws) when N=14 is smaller compared to the waiting time (Ws) in M/M/1/N when N=7 as seen in Table 1 above.

Pediatric Ward

In the Pediatric Ward, the performance of the queuing system was analyzed using both M/M/1 and M/M/1/N models. The ward has a system capacity of seven (9) beds, with an average patient arrival rate (λ) of 1.1304 patients per day and an average service rate (μ) of 2.0918 patients per day. The computed traffic intensity (ρ**)**, representing the utilization level of the system, is 0**.**5404, indicating that the ward operates at 54.04% capacity on average. The results of the computations and related discussions are presented in Tables 4 and 5.

Table 3: Computations of Queue Metrics for Pediatric Ward

	M/M/1		M/M/1/N	
Performance measure	**Value**	**Hours (based on 24 hr. day)**	**Value**	**Hours (based on 24 hr. day)**
Lq	.64		6.001	
Ls	1.18		7.15	
Wq	.56	13.52	2.94	70.64
Ws	1.04	25	3.42	82.12
λe			2.09	
Probability that customer waits			1	

Table 3 compares the single-ward performance measure analysis for Pediatric Ward. The results shows that for M/M/1, the mean number of patients in the system (Ls) and the mean number of patients in the queue (Lq) are approximately 2 and 1 patients respectively while the expected waiting time in the system (Ws) and expected waiting time in the queue (Wq) are 25 and 13.52 hours respectively. While for M/M/1/N, the mean number of patients in the system (Ls) and the mean number of patients in the queue (Lq) are approximately 8 and 6 patients respectively while the expected waiting time in the system (Ws) and expected waiting time in the queue (Wq) are 82.12 and 70.64 hours respectively.

From Table 3, M/M/1/N reveals a high waiting time in the system (82.12hrs) and hence causing queue in the ward. To show that the time patients spend in the ward can be reduced, a two-ward system was simulated for N=9, 10, 11, 12,13,14,15,16 and 17 to obtain an optimal waiting time and system capacity. The result is as shown in Table 4.

Table 4: Summary of Queue Simulation for a Two-ward System with Finite Capacity in PW

Ward=2	N=9	N=10	N=11	N=12	N=13	N=14	N=15	N=16	N=17
Parameter	**Value**	**Value**	**Value**	**Value**	**Value**	**Value**	**Value**	**Value**	**Value**
λe	4.12	4.15	4.17	4.18	4.18	4.18	4.18	4.18	4.18
ρ	.98	1	1	1	1	1	1	1	1
Lq	3.39	4.34	5.31	6.3	7.3	8.3	9.3	10.3	11.3
Ls	5.36	6.32	7.31	8.3	9.3	10.3	11.3	12.3	13.3
Wq	19.76	25.05	30.57	36.23	41.94	47.67	53.4	59.14	64.89
Ws	31.24	36.53	42.06	47.71	53.42	59.15	64.89	70.63	76.37

From Table 4, it was observed that an increase in $ward$ (S= 2) yielded significant reduction in the values of the expected waiting time in system (Ws). Also the waiting time (Ws) when $N = 17$ is smaller compared to the waiting time (Ws) in M/M/1/N when $N = 9$ as seen in Table 2 above.

Female Medical Ward

In the Female Medical Ward, the analysis of the queuing system using M/M/1 and M/M/1/N models was conducted to compute the traffic intensity (ρ) and other performance metrics. The ward has a system capacity of eight (8) beds), with an average arrival rate (λ) of 2.4348 patients per day and an average service rate (μ) of 1.9282 patients per day. The resulting traffic intensity (ρ = 1.2627) indicates that the arrival rate exceeds the service rate (ρ > 1), suggesting that the system is overloaded and cannot accommodate all incoming patients. The computed results and detailed discussions are presented in Tables 5 and 6.

Table 5: Computations of Queue Metrics for Female Medical Ward

METRICS MMW	M/M/1/N value	Hours (based on 24 hr day)
Lq	6.002	
Ls	7.21	
Wq	3.22	77.17
Ws	3.73	89.61
λe	1.93	
Probability that customer waits	1	

Table 5 compares the single-ward performance measure analysis for Female Medical Ward. for M/M/1, the results shows that the traffic intensity is greater than 1 which violet the assumption of M/M/1 model as such the arrival is faster than the ward can accommodate. While for M/M/1/N, the mean number of patients in the system (Ls) and the mean number of patients in the queue (Lq) are approximately 8 and 6 patients respectively while the expected waiting time in the system (Ws) and expected waiting time in the queue (Wq) are 89.61 and 77.17 hours respectively.

From Table 5, M/M/1/N reveals a high waiting time in the system (89.61hrs) and hence causing queue in the ward. To show that the time patients spend in the ward can be reduced or minimized, a two-ward system was simulated for N= 8, 9, 10, 11, 12,13,14 and 15 to obtain an optimal waiting time and system capacity. The result is as shown in Table 6.

Table 6: Summary of Queue Simulation for a Two-ward System with Finite Capacity in FMW

S=2	N=8	N=9	N=10	N=11	N=12	N=13	N=14	N=15
Parameter	**Value**	**Value**	**Value**	**Value**	**Value**	**Value**	**Value**	**Value**
λe	3.86	3.86	3.86	3.86	3.86	3.86	3.86	3.86
ρ	1	1	1	1	1	1	1	1
Lq	4.41	5.41	6.41	7.41	8.41	9.41	10.41	11.41
Ls	6.41	7.41	8.41	9.41	10.41	11.41	12.41	13.41
Wq	27.46	33.65	39.87	46.08	52.3	58.52	2.7	70.95
Ws	39.9	46.09	52.3	58.52	64.73	70.95	3.22	83.39

Table 6 revealed that an increase in ward(S=2) yielded significant reduction in the values of the expected waiting time in system (Ws). Also, the waiting time (Ws) when N=15 is smaller compared to the waiting time (Ws) in M/M/1/N when N=8 as seen in Table 5 above.

CONCLUSION

In conclusion, the study reveals that the M/M/1/N model in each of the select wards has a higher waiting time in the system (MMW= 63.04hrs, PW=82.12hrs, FMW=89.61hrs) compared to M/M/1(MMW=29.31hrs, PW=25hrs).

Since the M/M/1/N has higher waiting time compare to M/M/1. A two-ward system was used to simulate the performance measures with different system capacity. The result reveals that the two-ward system yields a minimum waiting time in each of the selected wards. This position was reached after computing the values of the waiting time (Ws). The result suggests that to reduce the waiting time in the selected wards, a two-ward system has to be introduced with a capacity of at most 13 beds for MMW, 17 beds for PW and 15 beds for FMW.

Based on the research findings in order to reduce the waiting time in the system, we recommend the following;

i. A dual ward system should be adopted in Male Medical ward with a capacity of 8-14 beds.
ii. A dual ward system should be adopted in Female Medical ward with a capacity of 9-15 beds.
iii. A dual ward system should be adopted in Pediatric ward with a capacity of 9-17 beds.
iv. The above recommendation was made without considering cost analysis. We therefore recommend

a further study that will incorporate cost model of the system.

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