*FUDMA Journal of Sciences (FJS)*
*ISSN online: 2616-1370*
*ISSN print: 2645 - 2944*
*Vol. 5 No. 2, June, 2021, pp 1 - 6*
*DOI: 516https://doi.org/10.33003/fjs-2021-0502-516*

# ESTIMATION OF INTERIOR TEMPERATURE OF AN ELECTRIC OVEN

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**ABSTRACT**

This paper is aimed at estimating interior temperature of an electric oven with respect to the jacket temperature. A discrete dynamic model of first order difference equation is described for the system. Kalman filtering technique is applied to the discrete dynamic model for estimation of the interior temperature. A computer program is written to simulate the system. It was observed that the estimates of the interior temperatures are directly proportional to estimates of the Jacket temperatures with proportionality constant of 0.0009. With this method it is therefore possible to obtain the interior temperature of the electric oven at any given time.

**Keywords**: Electric oven, Kalman filter, Modelling, Estimation.

## INTRODUCTION

Foods are cooked in the oven based on the interior temperature of the oven. This interior temperature of the oven is directly propositional to the jacket temperature. The higher the temperature of the interior of the oven the quicker the food is cooked. It is therefore necessary to estimate the temperature of the interior of the oven at any given time in order to determine whether the oven has the required temperature to cook the food.

In this research, a methodology is presented for estimating the interior temperature of an electric oven through Kalman filter techniques which involves modelling and simulating of the system.

According to Purlis (2012), Baking and roasting are generalized cooking methods consisting in heating the food inside an oven at a uniform temperature. In these processes heat is transferred to the load mainly by means of radiation and convection. Although these are widely-known phenomena, complex ad combined thermal, chemical, and mass transfer processes occur within the product and change its properties during the cooking. This complexity often requires the process to be supervised or even controlled by an 'expert', which usually leads to suboptimal and highly variable results in terms of food quality and energy consumption. It is then necessary to improve the understanding of the system dynamics in order to make progress in the automation and optimization of those cooking processes.

Burlon et al., (2017) presented a dynamic model that defines the energy conservation equations of a professional oven, where a high temperature thermal source positioned inside its cavity produces thermal power radiated and modulated over time, according to a suitable control strategy.

A modelling approach based on simple algebraic models is presented in Abraham and Sparrow (2004). The authors predict the heat transfer to a load positioned in an electric oven, considering the contributions of natural convection and radiation. To allow an analytical solution of the model equation, the radiative transfer term was linearized considering the temperature differences between the oven walls and the surface of the thermal load, instead of being driven by the fourth-power. The analysis takes into account changes of size, shape, materials, radiation surface properties and oven set point temperatures, showing discrepancies of about 1% between predicted and experimental data.

## MATERIALS AND METHODS

### Kalman filter

In system analysis, a fundamental problem is to provide values for the unknown states or parameters of a system given noisy measurements which are some functions of these states and parameters.

Kalman filter is defined as a set of mathematical equations that provides an efficient computational (recursive) means to estimate the state of a process, in a way that minimizes the mean of the square error.

Kalman (1960) published the discrete-time filter in a Mechanical Engineering Journal and Kalman and Bucy (1961), the continuous-time filter. In the meantime, Swerling (1959) had derived an equivalent formulation of the Kalman filter and applied it to the problem of estimating the trajectories of satellites using ground-based sensor. His results were published in an Astronomy journal the year before Kalman (1960) appeared as given in Grewal and Andrews (2010).

The Kalman filter model assumes the true state at time k is evolved from the state at (k-1) as stated below.

$$X_{k+1} = \Phi X_k + BU_k + \xi_k \quad (1)$$

Where,

$\Phi$ is the state transition model which is applied to the previous state $X_k$;

$B$ is the control-input model which is applied to the control vector $U_k$

$\xi_k$ is the process noise which is assumed to be drawn from a zero mean multivariate normal distribution with covariance Q.

$\xi_k \sim N(0,Q)$

At time k an observation (or measurement) $Y_k$ of the true state $X_k$ is made according to

$$Y_k = HX_k + \eta \quad (2)$$

Where H is the observation model which maps the true state space into the observed space and $\eta_k$ is the observation noise which is assumed to be zero mean Gaussian white noise with covariance R, $\eta_k \sim N(0,R)$

The initial state, and the noise vectors at each step $\{X_o, \eta_1, ....., \eta_k, \xi, .......\xi_k\}$ are all assumed to be mutually independent.

The Kalman filter is a recursive estimator. This means that only the estimated state from the previous time step and the current measurement are needed to compute the estimate for the current state and as such no history of observations and /or estimates is required. In what follows, the notation $\hat{X}_{k|k-1}$ which is the 1 step prediction represents the estimates of $x_k$ at time k given observations up to and including at time k-1.

$$\hat{X}_{k|k-1} = \Phi \hat{X}_{k-1|k-1} + BU_k \quad (3)$$

The covariance matrix for the one step prediction error is given by

$$p_{k|k-1} = \Phi p_{k-1|k-1} \Phi^T + Q \quad (4)$$

$\hat{X}_{k|k-1}$ and $P_{k|k-1}$ are the Predicted(a priori) state estimate and Predicted(a priori) estimate of covariance respectively and represent the initial values for the Kalman filter.

The state of the filter is represented by two variables

$\hat{X}_{k|k}$, the updated (a posterior) state estimate at time k given observations up to and including at time k and given by

$$\hat{X}_{k|k} = \hat{X}_{k|k-1} + K_k(Y_k - H\hat{X}_{k|k-1}) \quad (5)$$

$P_{k|k}$, the updated( a posterior) error covariance matrix (variance of the estimation error) given by

$$P_{k|k} = (I - K_k H)P_{k|k-1} \quad (6)$$

Where $K_k$ is the Kalman (Filter) gain and given by

$$K_k = P_{k|k-1}H^T(H P_{k|k-1}H^T + R)^{-1} \quad (7)$$

**The Kalman filter loop**

The Kalman filter loop given below summarizes what is known as the Kalman filter.

Enter prior estimate $\hat{X}_{k|k-1}$ and its covariance $\mathbf{P}_{k|k-1}$

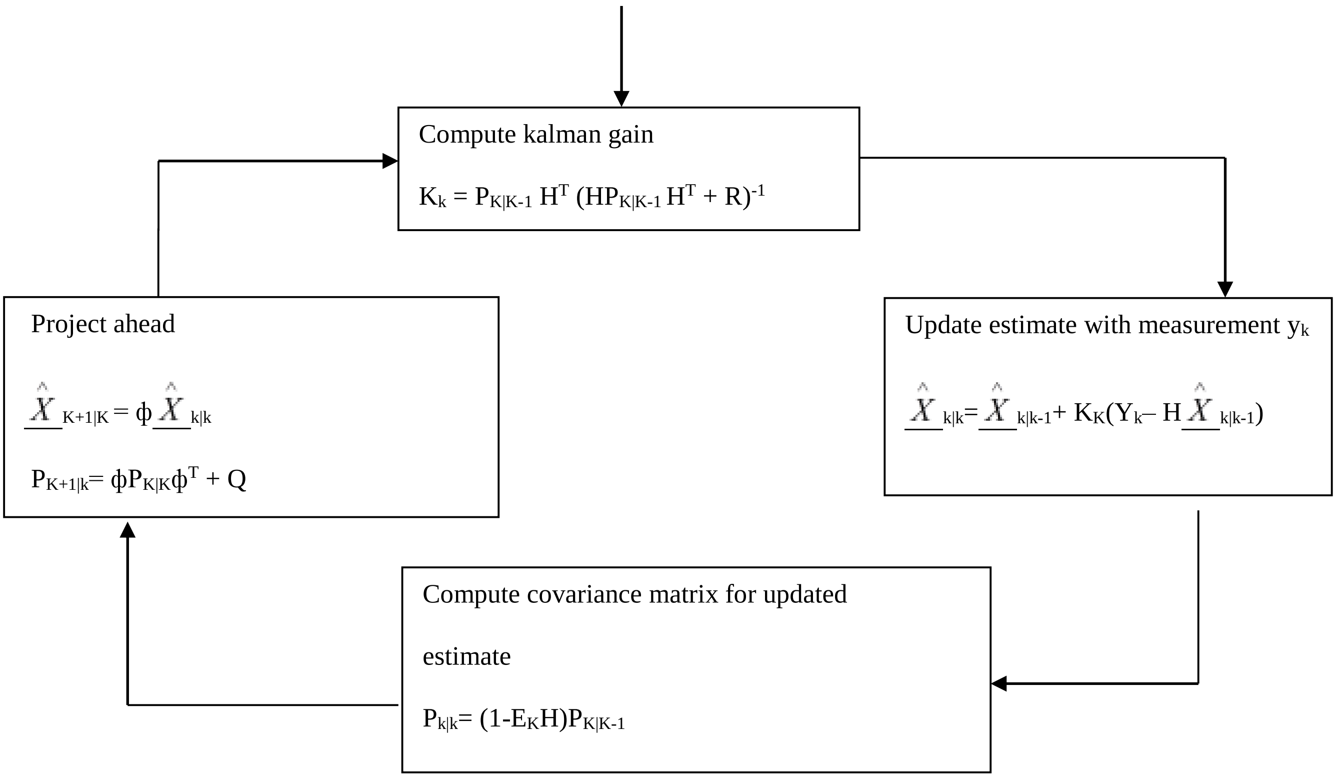


Fig1:The Kalman Filter Loop as in Robert and Patrick (1992)

Once the loop is entered it can be continued for any N ($N \geq 1$) iterations, k = 0,1,……..,N-1.

**Modelling and identification**

The interior temperature of an electrically heated oven is to be controlled by varying the heat input u to the jacket, as shown in fig.2 below.

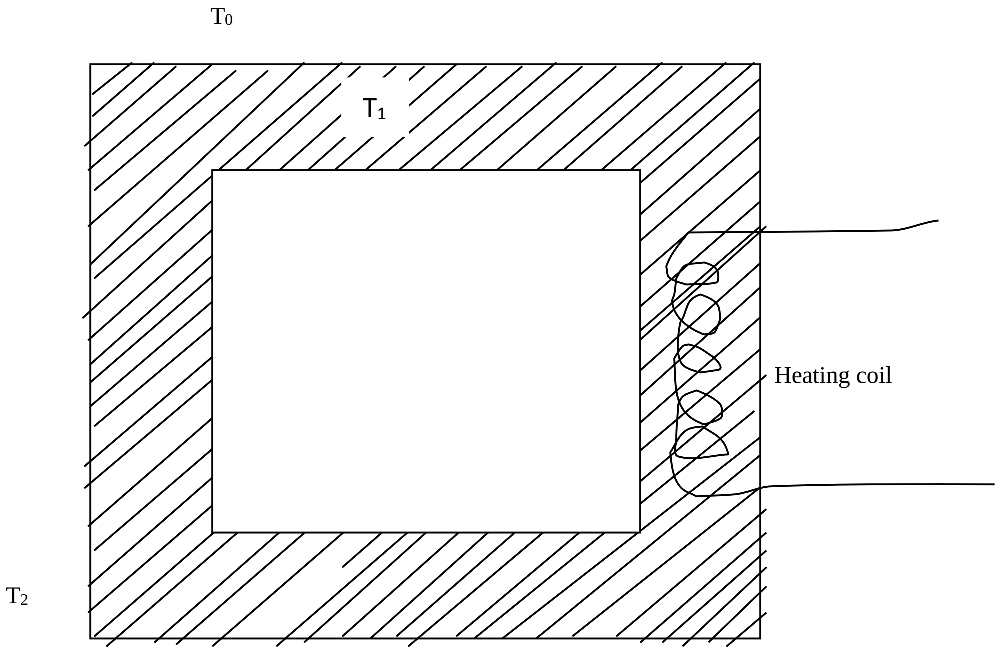


Fig.2: Sketch diagram of Electric oven.

Let the heat capacities of the oven interior and of the jacket be $c_1$ and $c_2$ respectively; let the interior and exterior jacket surface areas be $a_1$ and $a_2$; and let the radiation coefficients of the interior and exterior jacket surfaces be $r_1$ and $r_2$. Assume that there is uniform and instantaneous distribution of temperature throughout and that rate of loss of heat is proportional to area and the excess of temperature over that of the surroundings. Ignoring other effects, if the external temperature is $T_0$, the jacket temperature is $T_1$ and the interior temperature is $T_2$, then as in Barnett (1975) we have:

For the jacket:

$$c_1 \dot{T}_1 = -a_2 r_2 (T_1 - T_0) - a_1 r_1 (T_1 - T_2) + u \qquad (8)$$

For the oven interior:

$$c_2 \dot{T}_2 = a_1 r_1 (T_1 - T_2) \qquad (9)$$

Equation (8) and equation (9) can be written in matrix form as in equation (10) below

$$\begin{pmatrix} \dot{T}_1 \\ \dot{T}_2 \end{pmatrix} = \begin{pmatrix} -(a_2 r_2 + a_1 r_1)/c_1 & a_1 r_1 / c_1 \\ a_1 r_1 / c_2 & -a_1 r_1 / c_2 \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \end{pmatrix} + \begin{pmatrix} a_2 r_2 / c_1 \\ 0 \end{pmatrix} T_0 + \begin{pmatrix} 1/c_1 \\ 0 \end{pmatrix} u \qquad (10)$$

Equation (10) can be written in the form of equation (11) below to conform with the Kalman filter equation(1).

$$X_{k+1} = \Phi X_k + BU_k + C + \xi_k \qquad (11)$$

Where,

$$\Phi = \begin{pmatrix} -(a_2 r_2 + a_1 r_1)/c_1 & a_1 r_1 / c_1 \\ a_1 r_1 / c_2 & -a_1 r_1 / c_2 \end{pmatrix} \qquad (12)$$

$$B = \begin{pmatrix} 1/c_1 \\ 0 \end{pmatrix} \qquad (13)$$

$$C = \begin{pmatrix} a_2 r_2 / c_1 \\ 0 \end{pmatrix} \qquad (14)$$

$$X_k = \begin{pmatrix} T_1 \\ T_2 \end{pmatrix} \qquad (15)$$

$\xi_k =$ process noise.

The estimation problem is then stated as follows:
Given,

$$\left.\begin{aligned} \mathbf{X}_{k+1} &= \Phi X_k + BU_k + C + \xi_k \\ Y_k &= HX_k + \eta_k \end{aligned}\right\} \qquad (16)$$

From the observed values of $Y_0, Y_1, \ldots, Y_k$, find an estimate $\hat{X}_{k|k}$ of $X_k$ which minimizes the expected loss.

Where:
$\Phi$ = (2× 2) constant matrix obtained from the transition model
B = ( 2× 1) control input matrix which is applied to the control vector $\underline{U}_k$ .
$\underline{Y}_k$ =(1× 1) output vector (vector measurement at time $t_k$ ), itis the measured value of the temperature of the exterior jacket at $t_k$.
H = ( 1× 2) constant matrix giving the ideal connection between the measurement and the state vector at time $t_k$

$\underline{X}_k$= (2× 1) process state vector at time $t_k$ , i.e.,$\underline{X}_k = \underline{X}(t_k) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$ , $x_1$ and $x_2$ are the estimated jacket temperature and the oven interior temperature at time $t_k$ respectively.
$\underline{U}_k$ = (1× 1) control vector
C = (1x1) constant matrix
$\underline{\eta}_k$ = ( 1× 1) measurement error –assumed to be white noise sequence with known co-Variance R and having zero cross correlation with $\underline{\xi}_k$ sequence.

$\underline{\xi}_k$= ( 2× 1) vector-assumed to be white noise sequences with known co-variance Q

**Table 1:Parameter Values for the Model of an electric oven at $304^0$K**

| Symbol | Value | Explanation |
|---|---|---|
| L | 0.482 m | length of the interior jacket |
| B | 0.329 m | breath of the interior jacket |
| H | 0.387 m | height of the interior jacket |
| L | 0.582 m | length of the exterior jacket |
| B | 0.429 m | breath of the exterior jacket |
| H | 0.487 m | height of the exterior jacket |
| $c_1$ | 376 J/kg.k | heat capacity of the interior jacket |
| $c_2$ | 460.6 J/kg.k | heat capacity of the exterior jacket |
| $a_1$ | 0.946 $m^2$ | interior jacket surface area |
| $a_2$ | 1.484 $m^2$ | exterior jacket surface area |
| $r_1$ | 0.32 | radiation coefficient of interior jacket surface |
| $r_2$ | 0.87 | radiation coefficient of exterior jacket surface |

Values of $\Phi$ and B were obtained from equation (12) and equation (13) respectively

$$\Phi = \begin{pmatrix} -0.004 & 0.001 \\ 0.001 & -0.001 \end{pmatrix}$$

$$B = \begin{pmatrix} 0.003 \\ 0 \end{pmatrix}$$

The prior estimate $X_{0|-1}$ and its covariance$P_{0|-1}$ are given by $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ respectively.

$H = \begin{pmatrix} 1 & 0 \end{pmatrix}$

The values of Q and R were tuned to 0.1 and $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ respectively.

**Table 2: Measured values of the jacket temperature($Y_k$) at time $t_k$ of the Oven in degree Kelvin**

| K(step) | $t_k$(seconds) | $Y_k$(degree Kelvin) |
|---|---|---|
| 0 | $t_0$ | 473 |
| 1 | $t_1$ | 490 |
| 2 | $t_2$ | 530 |
| 3 | $t_3$ | 550 |
| 4 | $t_4$ | 600 |
| 5 | $t_5$ | 650 |

The table above shows measured values of the jacket temperature ($Y_k$) which is not real value of the jacket temperature($T_1$). K is the step of the process and $t_k$ are the different times of the process. If we have N ($N \geq 1$) iterations, k= 0,1,2,..........,N-1. The first line gives the temperature of 473°K at time $t_0$ in step 0.

**Recursive processing of the noisy measurements through Kalman filter**.

The algorithm for processing the noisy measurements as in Ogwola (2017) is as follows:

Given the initial values $\underline{\hat{X}}_{k|k-1}$ and its co-variance, $p_{k|k-1}$

(i)$K_k = P_{k|k-1}H^T(HP_{k|k-1}H^T + R)^{-1}$

(ii)$\underline{\hat{X}}_{k|k} = \hat{X}_{k|k-1} + K_K(Y_k - H\hat{X}_{k|k-1})$

(iii) $P_{k|k} = P_{k|k-1} - K_kHP_{k|k-1}$

(iv) $\underline{\hat{X}}_{k+1|k} = \Phi\underline{\hat{X}}_{k|k}$

$$p_{k+1|k} = \Phi p_{k|k}\Phi^T + Q$$

(v)

The computer programme developed in Ogwola (2017) was used to simulate the electricoven. The above values of $\Phi$,B, Q,R, H , $\hat{X}_{0|-1}$ ,$P_{0|-1}$ were used to run the computer programme in which the following results were obtained:

**RESULTS AND DISCUSSION**

From the result of the research in table 3 below, the following were observed:

(i) At the $t_1$ the temperature of the jacket was 474.2$^0$K and that of interior temperature was 0$^0$K, this was because the temperature of the jacket has to raise up to a desire level before the interior temperature could rise from 0$^0$K.

(ii) The temperatures of the jacket at every interval of time were far greater than that of the interior temperature, this was because part of temperature were loss from the jacket by radiation due to the excess of temperature over the surroundings and there was a uniform and instantaneous distribution of temperature throughout and that rate of loss of heat is proportional area of the exterior jacket.

(iii) As the jacket temperatures of the Oven increases the interior temperatures also increases. The interior temperatures are directly proportional to the jacket temperatures with proportionality constant of 0.0009 (0.474/512.2=0.00092, 0.512/532.2=0.00096, 0.531/580.6=0.00091, 0.579/629.0=0.00092)

**Table 3: Simulation Results for the system**

| K(step) | $t_k$(seconds) | $x_1$(°Kelvin) | $x_2$(°Kelvin) |
|---|---|---|---|
| 0 | $t_0$ | 0 | 0 |
| 1 | $t_1$ | 474.2 | 0 |
| 2 | $t_2$ | 512.8 | 0.474 |
| 3 | $t_3$ | 532.2 | 0.512 |
| 4 | $t_4$ | 580.6 | 0.531 |
| 5 | $t_5$ | 629.0 | 0.579 |

The time taken ($t_0$ to $t_5$) for the simulation is 1 hour (60 seconds). Heat was inputted into the jacket at interval of 12 seconds, i.e $t_0$ = 0s, $t_1$ = 12s, $t_2$ = 24s, $t_3$ = 36s, $t_4$ = 48s, $t_5$= 60s.

Where,

$x_1$ is the estimate of $T_1$ (the measured jacket temperature).

$x_2$ is the estimate of $T_2$ (interior temperature).

**CONCLUSION**

We need to estimate the interior temperature of the electric oven in order to determine the type of food to be cooked in the oven. In this research, Kalman Filter was applied to estimate the interior temperature of the electric oven as it is difficulty through any other method. The measured jacket temperatures were used for the estimation of the interior temperatures of the electric oventhrough Kalman Filter Technique.

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