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DYNAMICS OF SUTTERBY NANOFLUID FLOW PAST A LINEARLY STRETCHING SHEET WITH SORET AND DUFOUR EFFECTS IN THE PRESENCE OF HOMOGENEOUS–HETEROGENEOUS REACTIONS

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ABSTRACT

This paper investigates the effect of Dufour and Soret in the presence of homogeneous-heterogeneous reaction on the Sutterby nanofluid model over a stretching sheet. The problem of heat and mass transfer behavior of sutter by nanofluid a non-Newtonian fluid model represents dilute polymer solution which previously has not been treated together. The boundary layer partial differential equations are converted using appropriate transformations into ordinary differential equations. The homotopy analysis technique is used to solve nonlinear ordinary differential equations. The effects of various relevant parameters on velocity, temperature, and concentration profiles are graphically discussed. The behavior of the skin friction coefficient and the Nusselt number are also investigated.

Keywords: Boundary Layer, Convection, Sutterby Nanofluid, Homogeneous–Heterogeneous Reactions, Soret And Dufour

INTRODUCTION

Many chemically reactive processes, including combustion, catalysis, and biological systems, frequently involve homogeneous and heterogeneous reactions. In such processes, the heterogeneous reaction takes place on the catalyst surface, while the homogeneous reaction occurs in the bulk. According to a standard presumption for this flow problem, the response solely occurs in the boundary-layer region of a body surface, and an external flow is imposed on the boundary layer's outer edge chaundary and Merkin(1995). Williams et al.(1991). have provided several experimental experiments on methane/ammonia and propane oxidation flowing over platinum, which unmistakably demonstrate the occurrence of such homogeneous-heterogeneous processes. Researchers have conducted some theoretical studies to evaluate the development of this paradigm (2024).

Non-Newtonian fluids are useful in a variety of industries, including food processing, medicines, and construction. Understanding the dynamics of non-Newtonian fluids is critical for building and operating equipment that uses them. The contrast between Newtonian and non-Newtonian fluids is crucial for mechanism selection because the viscosity of a fluid directly impacts the pump's ability and effectiveness. The Sutterby fluid rheological model is one of several that describe aqueous solutions with significant polymer dispersion. Numerous experts have studied the flow of Sutterby based fluid and nanofluid extensively up to this point (2021). Due to their superior heat transmission abilities, nanofluids have recently attracted a lot of attention, Sutterby nanofluid is used in this study. When heat and mass transfer occur simultaneously in a moving fluid then the relations between the fluxes and the driving potentials are of a more intricate nature. It has been observed that an energy flux can be generated not only due to temperature gradients but also by concentration gradients. The heat transfer due to the concentration gradient is known as the diffusion-thermo (Dufour) effect. On the other hand, the mass transfer due to temperature gradient is known as the thermal-diffusion (Soret) effect. Generally, the Soret and Dufour effects are of a smaller order of magnitude than the effects described by Fourier's and Fick's laws and are often neglected in heat and mass transfer processes. Such effects are important in hydrology, nuclear waste disposal, petrology, geothermal energy etc. Furthermore, the Soret and Dufour effects in the presence of magnetic fields and thermal radiation are practical in magnetic resonance imaging (MRI), nuclear reactors, high-temperature processes, electrical power generation, solar power technology, and many others. Pal and Mondal (2022) studied the mass and heat transfer of a micropolar fluid in a porous material with variable thermal conductivity and Ohmic dissipation using the Magneto-Soret-Dufour thermoradiative double-diffusive convection method. Turkyilmazoglu and Pop (2012) studied the Soret effect in natural convection flow that is unstable and radiates heat. They took into account the flow that was created by the vertical plate's impulsive motion.Hayat, etal.(2012)commented about the Dufour and Soret effects in magnetohydrodynamic Casson fluid flow across a stretching surface. However, through the analysis and various papers within (2017), Auwal et'al.(2023). Investigate the numerical Approach for The Study Heat Generation in the presence of thermal Boundary layer for a flat Plate additional recent research on Soret and Dufour effects can be cited.

Motivated by the applications of Soret and Dufour effects, the underlying motive of this paper is to construct a mathematical model for the magnetohydrodynamic (MHD) two-dimensional flow of Sutterby nanofluid with Soret and Dufour effects in the presence of homogeneous-heterogeneous reactions. Flow is induced due to a linearly stretching sheet. The thermal radiation aspect is also considered. Such research work is not carried out in the past even in the absence of Soret and Dufour effects. Soret and Dufour effects are preferred here in order to account for both heat and mass transfer mechanisms.

MATERIALS AND METHODS

The incompressible, steady, two-dimensional, and stagnation-point flow of Sutterby nanofluid over a linearly stretched sheet is considered. The sheet is assumed to be along the x-axis with the limited flow to y>=0 and corresponding to the plane y=0 above the wall. The magnetic field strength is applied perpendicular to the sheet, ignoring the effect of the induced magnetic field. Examine a basic homogeneous-heterogeneous reactions model where a homogenous isothermal cubic autocatalytic reaction takes place within the boundary layer flow. However, there is only one first-order reaction heterogeneous occurring on the catalyst surface. See Fig. 1 for the sketch diagram. The governing equations for the model are provided in the setting of Sajid etal (2017) and Hayat(2018). The series solutions through the homotopy analysis method (HAM) Abdallah(2009). are obtained and the results discussed through the plotted graphs.

Considering the simple model for interaction between the homogeneous (bulk) reaction and a heterogeneous (or surface) reaction involving the two chemical species A and B in the boundary layer flow proposed by Chandhary and Merkin (1995) in the form

$A + 2B \rightarrow 3B,\ \text{rate} = k_c ab^2, \qquad A \rightarrow B,\ \text{rate} = k_s a$

Where a and b are concentrations of the chemical species A and B, and k_c and k_s are the constant rate. However the process is assumed to be isothermal.

$$\frac{\partial v}{\partial s} + \frac{\partial u}{\partial x} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial s} = u_e\frac{du_e}{dx} + \nu_f\frac{\partial^2 u}{\partial s^2} + \nu_f\frac{mE^2}{2}\left(\frac{\partial u}{\partial s}\right)^2\frac{\partial^2 u}{\partial s^2} - \frac{\sigma B_o^2}{\rho_f}(u_e - u) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial s} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial s^2}\right) + \tau\frac{D_T}{T_\infty}\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial s}\right)^2\right] + \frac{D_{E_1}K_T}{C_s c_p}\frac{\partial^2 C_1}{\partial s^2} + \frac{\sigma B_0^2}{\rho_f c_p}u^2$$
$$+\tau D_{E_2}\left(\frac{\partial T}{\partial x}\frac{\partial C_2}{\partial x} + \frac{\partial T}{\partial s}\frac{\partial C_2}{\partial s}\right) - \frac{1}{\rho_f c_p}\frac{\partial q_r}{\partial s} \tag{3}$$

$$u\frac{\partial C_1}{\partial x} + v\frac{\partial C_1}{\partial s} = D_{E_1}\left(\frac{\partial^2 C_1}{\partial x^2} + \frac{\partial^2 C_1}{\partial s^2}\right) + \frac{D_T K_T}{T_\infty}\frac{\partial^2 T}{\partial s^2} - k_c C_1 {C_2}^2 \tag{4}$$

$$u\frac{\partial C_2}{\partial x} + v\frac{\partial C_2}{\partial s} = D_{E_2}\left(\frac{\partial^2 C_2}{\partial x^2} + \frac{\partial^2 C_2}{\partial s^2}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial s^2}\right) + k_c C_1 {C_2}^2 \tag{5}$$

$$u = U_w(x) = ax, v = V_w, T = T_w, D_{E_1}\frac{\partial C_1}{\partial s} = k_r C_1, D_{E_2}\frac{\partial C_2}{\partial s} = -k_r C_1 \text{ at } s = 0$$
$$u \rightarrow u_e(x) = cx(c > 0), v \rightarrow 0, T \rightarrow T_\infty, \mathrm{C}_1 \rightarrow C_\infty, \mathrm{C}_2 \rightarrow 0 \quad \text{as } \mathrm{s} \rightarrow \infty. \tag{6}$$

Agreeing with the Rosseland estimation and Ozisik(2018) allows the acceptance of the radiation heat flux as

$$q_r = -\frac{4\sigma^*}{3\beta_R}\frac{\partial T^4}{\partial y} = -\frac{4\sigma^* T_2^3}{3\beta_R}\frac{\partial T}{\partial y} \tag{7}$$

Table 1: The Numerical Values of Nusselt Number

Pr	***Nr***	$R_{e_x}^{-1/2}\, Nu_x$
1.72	0.05	0.94913
1.72	0.10	0.95781
1.72	0.15	0.95950
1.92	0.20	0.99825
2.12	0.20	0.00531
2.32	0.20	1.09164

The system of equations $(2 - 6)$ will be transformed from the coordinate system (x, y) to the dimensionless coordinate system by introducing an appropriate transformation system$(u, v), T, C_1, C_2$ to f, θ, ϕ, φ. The following non-similarity transformations were retained to reduce the PDEs into ODEs, variables are defined as follows:$\xi = \sqrt{\frac{c}{\nu_f}}s, u = cxf'(\xi), v = -\sqrt{c\nu_f}f(\xi), \theta(\xi) = \frac{T - T_\infty}{T_w - T_\infty}, C_1 = C_\infty\phi(\xi), C_2 = C_\infty\varphi(\xi)$ (8)

The dimensionless equations are

$$f''' - f'^2 + ff'' + \frac{1}{2}Def''^2 f''' + M(1 - f') = 0, \tag{9}$$

$$\frac{1}{Pr\left(1 + \frac{4}{3}Nr\right)_{bt}^{\ 2} Pr\, D_f\frac{1}{2}Pr\, f^2}, \tag{10}$$

$$\frac{1}{Sc_{E_1}}\phi'' + f\phi' + Sr\theta'' - K\phi\varphi^2 = 0, \tag{11}$$

$$\frac{1}{\varepsilon Sc_{E_1}}\left(\varphi'' + \frac{N_t}{N_b}\theta''\right) + f\varphi' + K\phi\varphi^2 = 0 \tag{12}$$

With boundary conditions

$$f(\xi) = d = \frac{-v_w}{\sqrt{cv_f}}, f'(\xi) = \frac{a}{c}, \theta(\xi) = 1, \phi'(\xi) = K_c\phi(\xi), \varphi'(\xi) = -\varepsilon K_c\phi(\xi) \text{ at } \xi = 0$$
$$f'(\xi) \rightarrow 1, \theta(\xi) \rightarrow 0, \phi(\xi) \rightarrow 1, \varphi(\xi) \rightarrow 0 \text{ as } \xi \rightarrow \infty \tag{13}$$

$$M = \frac{\sigma B_0^2}{c\rho_f}, De = \frac{mE^2{u_e}^2 c}{v_f}, Pr = \frac{v_f}{\alpha}, Ec = \frac{u_e^2}{c_p(T_w - T_\infty)}, Du = \frac{D_{E_1}K_T C_\infty}{v_f C_s c_p(T_w - T_\infty)},$$
$$N_b = \frac{\tau C_\infty D_{E_2}}{\nu_f}, N_t = \frac{\tau D_T(T_w - T_\infty)}{\nu_f T_\infty}, Sr = \frac{D_{E_1}K_T(T_w - T_\infty)}{v_f T_\infty C_\infty}, Nr = \frac{4\sigma^* T_\infty^3}{\alpha k^*},$$
$$Sc_{E_1} = \frac{v_f}{D_{E_1}}, \varepsilon = \frac{D_{E_1}}{D_{E_2}}, K = \frac{k_r C_\infty^2}{c},\ K_c = \frac{k_c}{D_{E_1}}\left(\frac{c}{v_f}\right)^{-\frac{1}{2}}. \tag{14}$$

$$C_f = \frac{1}{Re_x^{\frac{1}{2}}}\left(f''(0) + \frac{De}{6}f'(0)\right) \text{ and } Nu = -Re_x^{\frac{1}{2}}\left(1 + \frac{4}{3}Nr\right)\frac{\theta\prime(0)}{\theta(0)} \qquad (15)$$

RESULTS AND DISCUSSION

The velocity profile receive a significant reduction for elevated values of parameters that increase resistance to follow such as the fluid non-newtonian index an any magnetor porosity parameters introduce, the sutterby fluid parameter notably alter the boundary layer tthcknessrelative to a newtonian baseline. The influence of different physical factors on the velocity $\phi(\xi)$, concentration $\phi(\xi)$ and temperature $\Theta(\xi)$ profiles are shown graphically. The parameters include the Dufour and Soret effects, Prandtl number Dufour numbers and Nusselts number. The homotopy analysis technique is used to get the solution of the system (9)–(11) with different parameter values. Effects on temperature profiles and velocity distributions. In the case of mass concentration, this tendency represented the opposite.

Figure 1-5 illustrate our comprehensive analysis of the impact of the porous medium factor (K) on various aspects of the system under study. As the porous medium factor increases, we observe concurrent decreases in both velocity and concentration profiles but increases in temperature, as depicted in these figures. Specifically, Our analysis reveals that while spherical nanoparticles exhibit a notable influence on velocity distribution in the presence of a magnetic field, their impact on mass velocity and concentration is less significant. This discrepancy underscores the nuanced role of nanoparticle shape in affecting different aspects of fluid flow. Moreover, we observe an increase in the temperature field due to the heightened resistance imposed by the applied magnetic field, leading to a corresponding decrease in the function $f(\eta)$. Interestingly, despite this decrease in velocity, the temperature profile exhibit contrasting behavior, These findings highlight the intricate interplay between various factors, such as nanoparticle shape and magnetic field strength, in shaping the overall dynamics of the system. we observe a simultaneous increase in both temperature $\theta(\eta)$ and velocity distribution $\phi(\xi)$. However, the mass concentration distribution exhibits an opposite trend, decreasing as velocity increases.

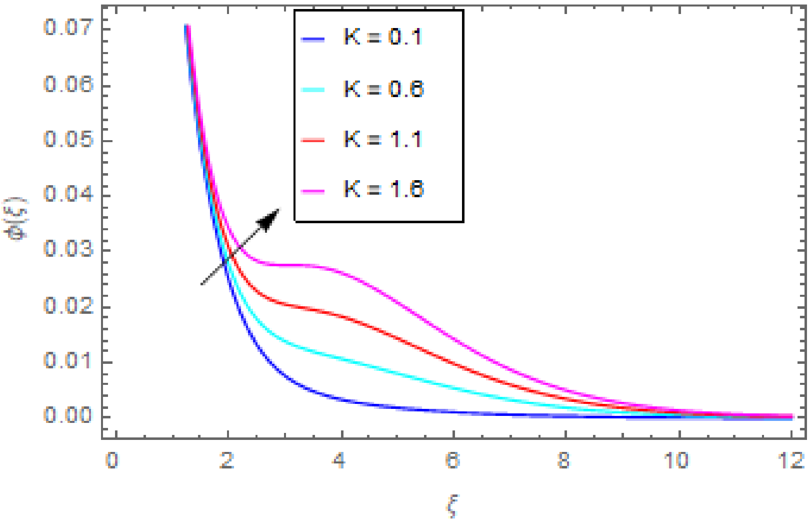

Figure 1: Effect of Concentration (ξ) vs Kθ

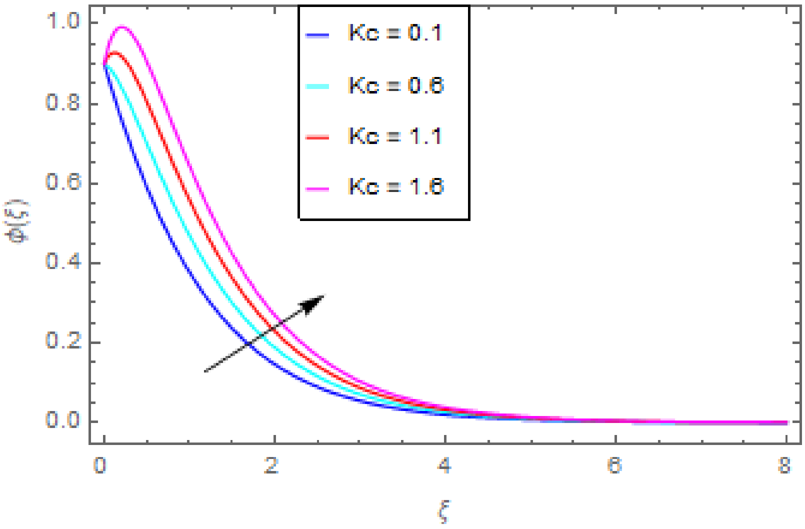

Figure 2: The Influence of Velocity (ξ) vs Kc

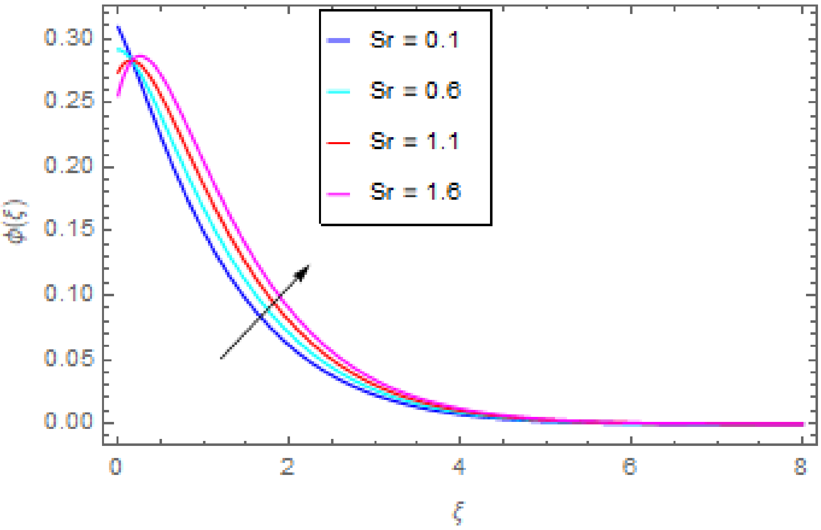

Figure 3: The Impacts of Velocity (ξ) vs Sr

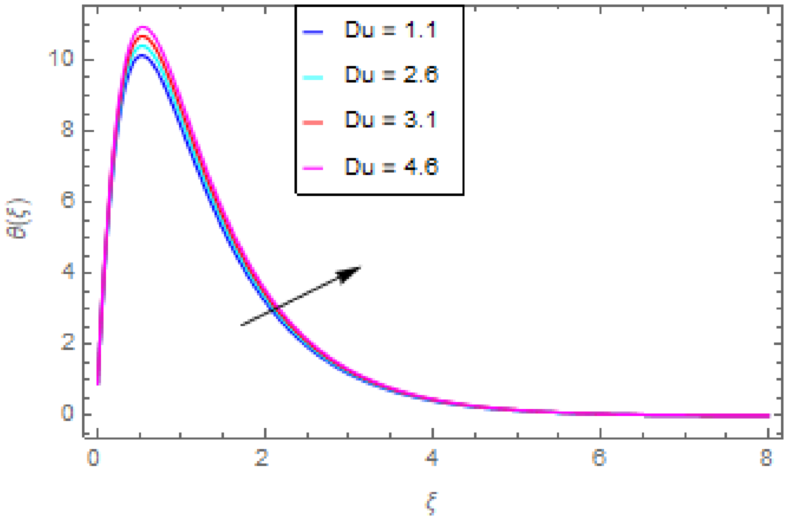

Figure 4: The Influence of Temperature (ξ) vs Du

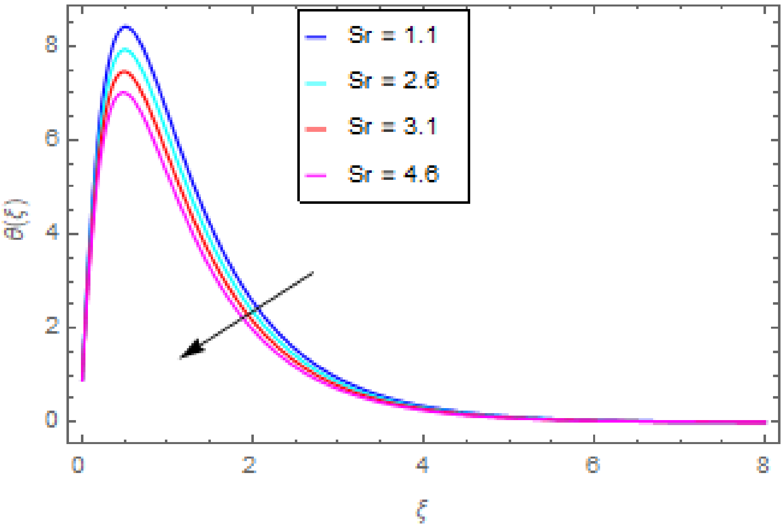

Figure 5: The Influence of Temperature (ξ) vs Sr

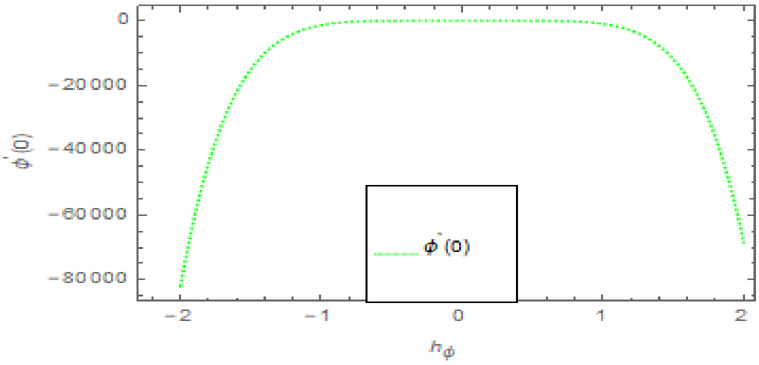

Figure 6: The Impact of Concentration ϕ'(0) against h(0)

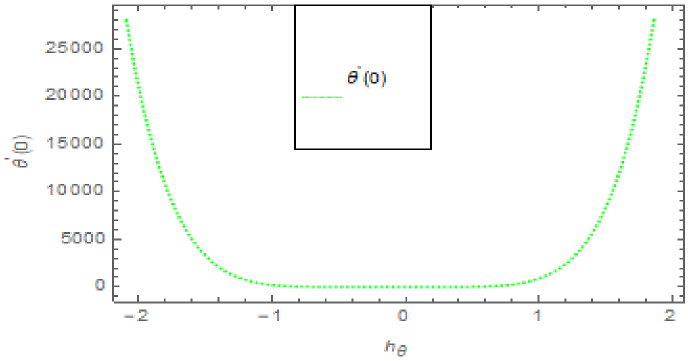

Figure 7: The Impact of Temperature θ'(0) vs h(0)

CONCLUSION

In this paper, we examined the effects of mixed convection, Dafour and Soret numbers in max well hybrid nanofluid flow through a porous medium a linearly stretched plate. The sutterby fluid model is appropriate for capturing the non-Newtonian shear thinning behavior of polymer base nanofluid and its rheological parameters play an important role in controlling he velocity and momentum boundary layer. He study validate he use of numerical similarities transformation methods as rubust tools for solving a resulting couple of nonlinear ODE system with results showing agreement with existing cases. We observed the changes in concentration and temperature effect of the flow over the porous surface, we use homotopy analysis technique and graph, we analyzed the system and found that our method and results are new to the field, this study provides an insights to the fluid flow dynamics in porous media, filling an important gap in research. We found various outcomes, including temperature and concentration patterns and the influence of different factors on flow behavior, in our analysis, we aim to deepen our understanding of fluid flow in porous media under these conditions. The presence of a porous medium changes the flow dynamics, reducing resistance to the fluid motion and enhancing concentration and temperature functions, this is as a result of improved heat and mass facilitated by the porous structure which lead to intensified gradient near surface Parameter like Prandtl number (*Pr*) and the heat source parameter (*Nu)* influence momentum and thermal diffusivity in fluid flow, A lower *Pr* indicates higher thermal conductivity, increasing in velocity profile. Similarly, increasing *Nu* reduces temperature gradients, affecting temperature and velocity profiles. The Dufour number characterizes heat and mass transfer ratios with concentraton gradients. A higher Dufour number enhances velocity and temperature distribution, indicating a stronger heat-mass transfer coupling.

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