FUDMA Journal of Sciences (FJS)
ISSN online: 2616-1370
ISSN print: 2645 - 2944
Vol. 10 No. 5, March, 2026, pp 100 – 105
DOI: https://doi.org/10.33003/fjs-2026-1005-4924

ANALYSIS OF COUPLED INTERPOLATIVE KANNAN-TYPE CONTRACTION MAPPINGS IN METRIC SPACES

[1]Festus Tope Ogunwe, [1]Adenike Olusola Adeniji, [1]Morufu Mogbolagade Mogbonju, [*2]Taiwo Aanu Ogunlusi, [2]Felix Damilola Ajibade

[1]Department of Mathematics, University of Abuja, FCT Abuja, Nigeria.
[2]Department of Mathematics, Federal University Oye, Ekiti State, Nigeria.

[*]Corresponding Auhtor's Email: taiwo.ogunlusi@fuoye.edu.ng

ABSTRACT

Many Researchers have used the notion of interpolative Kannan contraction due to its wider range in applications and its flexibility, rather than the normal Banach contraction. This paper studies the existence, uniqueness, and convergence of fixed points for coupled interpolative Kannan-type contractions in metric spaces. Using Picard and Mann iterative schemes, sufficient conditions ensuring convergence to a unique fixed point are established. The results extend and generalize several existing findings in fixed-point theory.

Keywords: Coupled fixed point, Interpolative Kannan contraction, Metric space, Picard iteration, Mann iteration, Convergence, Uniqueness

INTRODUCTION

The notion of a coupled fixed-point (CFP) was introduced and studied by (Opoitsev, 1975) and investigated later by (Guo & Lakshmikantham, 1987). There are more results on the existence of coupled fixed-points (CFPs) than on the existence of common CFPs of a pair of operators among investigations of CFP results for nonlinear mappings in the ordered Banach space setting. The majority of these results are established for mixed monotone operators. A fixed-point theorem for a mixed monotone mapping in a metric space with a partial order was established by (Bhaskar & Lakshmikantham, 2006) under a weak contractive type of assumption. Their theorem can be used to study a wide class of problems in addition to incorporating a number of recent advancements. As an example, they discuss about whether a periodic boundary value problem has a unique solution. Some common fixed-point theorems of four self-mappings satisfying contraction-type conditions in partially ordered complete metric spaces were proven by (Sharma et al., 2017) in their paper titled "On the generalization of Banach contraction principle in partially ordered metric spaces." Additionally, their findings generalize and unify the previous findings in the literature. To support their findings, a few pertinent examples are also provided.

(Liu et al., 2018) used weakly compatible mappings in partially ordered metric spaces with a common limit range property, the (Aamri & El Moutawakil, 2002) property, and other properties to investigate the existence and uniqueness of a common CFP of four self-mappings for Geraghty-type contraction. It was noted that the continuity of mappings and completeness of spaces can be removed. The results improved, extended, complemented, and generalized several existing results in the literature. In addition, some examples were provided to illustrate the usability of their results. (Bota et al., 2023) in their article investigated existence and stability results for cyclic graphical contractions in complete metric spaces. An application to a CFP problem was also derived.

In 2022, (Aniki et al., 2022) analyzed the existence and uniqueness of mappings defined on Cauchy metric spaces via a CFP theorem that satisfies a contractive inequality of integral-type. This generalizes the contractive inequality of the integral-type fixed point to the CFP theorem as an improvement to available research in the literature. Some illustrative examples to back up their claims were also included. In a controlled metric space, (Hammad & Zayed, 2022) found some CFPs results for generalized contractions involving two control functions. Furthermore, in graph-enabled controlled metric spaces, they established a few CFP results. Their findings led to the expansion and modification of numerous well-known results from the literature. They also provide some examples to show the validity of the stated results. Finally, they use the theoretical findings to solve a system of integral equations.

(Kannan, 1968) proved the following result, which gives the fixed point for non-continuous mappings: let: $\theta^*: \mathbb{X} \rightarrow \mathbb{X}$ be a mapping on a complete metric space ($\mathbb{X}$,d) with
$d\,(\theta^* x, \theta^* y) \leq \lambda(d(x, \theta^* x) + d(y, \theta^* y))$
where $0 \leq \lambda < 0.5$ and $x, y \in \mathbb{X}$. Then θ^* has a unique fixed point. If for the mapping $\theta^*: \mathbb{X} \rightarrow \mathbb{X}$ there exists $0 \leq \lambda < 0.5$ such that the inequality holds for all $x, y \in \mathbb{X}$. Then θ^* is called a Kannan-type mappings (KTMs). Some findings on the common fixed point of self-mappings defined on complete b-metric spaces were provided by (Faraji & Nourouzi, 2017). These findings extend the fixed-point theorems of Kannan and Chatterjea to complete b-metric spaces. Specifically, they demonstrated the existence of a single common fixed point between two self-mappings that satisfy a contraction-type inequality. Additionally, they provided some examples to support the findings. (Karapinar, 2018) reexamined Kannan's famous fixed-point theorem in relation to interpolation. They suggested a novel Kannan-type contraction to maximize the rate of convergence using the interpolation concept. (Abdou, 2020) generalized classical modulars over linear spaces, such as the recently introduced Orlicz spaces, by establishing the concept of a modular metric on an arbitrary set and the corresponding modular spaces. In the context of modular metric spaces, the existence of fixed points for contractive and non-expansive Kannan maps was proven. These had to do with the successive approximations of fixed points (through orbits), which converge to the fixed points in a modular sense that is not as strong as metric convergence. By applying the interpolation theory to a complete metric space, (Karapinar, 2021) revisited the well-known contractions of (Meir & Keeler ,1969). To demonstrate the validity of the observed result, they offered a straightforward example. We provide a more accurate result regarding the behavior of Picard sequences of arbitrary initial points, and (El Amri et al., 2022) demonstrated that the existence of fixed points for

interpolative Kannan-type and CRR-type contractions remains true for a metric space that is not necessarily complete. Additionally, they examined a few specific classes of interpolative contractions of the Kannan and CRR types. (Bota et al., 2023) introduced generalized KTMs, a new class of mappings in metric spaces that are three-point analogs of the popular KTMs. It was demonstrated that these two classes of mappings are independent and that, in the general case, they are discontinuous but continuous at fixed points, along with KTMs. It is proven that generalized KTMs have a fixed-point theorem. The class of mappings for which the fixed-point theorems hold can be expanded by adding additional conditions of asymptotic regularity and continuity. Two more fixed-point theorems for generalized KTMs in metric spaces that are not necessarily complete were also obtained by them after Kannan. Throughout this study, little or no attention has been given to the existence and uniqueness of a coupled interpolative Kannan contractive condition. This research will contribute to the theory of the existence and uniqueness of a coupled fixed point in a metric space using coupled interpolative Kannan contraction.

MATERIALS AND METHODS

This section presents a theoretical framework to investigate several conditions for the existence and uniqueness of a coupled interpolative Kannan contraction. It will also investigate convergence and stability of the coupled interpolative Kannan contraction in a metric space. Applying an a priori analytical approach, we develop definitions, propositions, a lemma, a theorem, and a corollary.

Mathematical Preliminaries

We define essential concepts for stability as mentioned by (Harder & Hicks, 1988; Olatinwo, 2008). Let $(\mathbb{X}, d)$ be a complete metric space and θ^*: $\mathbb{X} \to \mathbb{X}$ be a mapping on the metric space.

Let F(θ^*) = {$p \in \mathbb{X} \,|\theta^* \, p = p$ } denote the set of fixed points of θ^*. Let $\{x_i\}\infty \; i = 0 \;\subset \mathbb{X}$. be the sequence generated by an iterative procedure involving the operator θ^*, that is,

x_{i+1}= f(θ^*, x_i), i= 0,1,2 (1)

where $x_0 \in \mathbb{X}$ is the initial approximation, and f is some function. Suppose $\{x_i\}_{i=0}^{\infty}$ con verges to a fixed point p of θ^*.

Let (𝕏,d) be a metric space and θ^*: $\mathbb{X} \times \mathbb{X} \to \mathbb{X}$ a mapping. For $(x_0, y_0) \in \mathbb{X} \times \mathbb{X}$, the sequence $\{x_i, y_i\}_{i=0}^{\infty} \subset \mathbb{X} \times \mathbb{X}$ defined iteratively by x_{i+1}= θ^* (x_i, y_i), y_{i+1}= θ^* (y_i, x_i),

$i = 0, 1, 2 \cdots$ (2)

It is said to be a coupled fixed-point iterative procedure, according to (Olatinwo, 2008).

Lemma 1

(Berinde & Takens, 2007) Let $\{\alpha_i\}_{i=0}^{\infty}$, $\{\beta_i\}_{i=0}^{\infty}$ be a sequence of nonnegative numbers and $0 \le q < 1$ so that $\alpha_{i+1} \le q$ $\alpha i + \beta i$, for all $i \ge 0$

1. If $\lim\limits_{i\to\infty} \beta i$ = 0 then $\lim\limits_{i\to\infty} \alpha i = 0$
2. If $\sum_0^{\infty} \alpha_i < \infty$ then $\sum_0^{\infty} \beta_i < \infty$,

then $\lim\limits_{i=0} \alpha_i = 0$.

Lemma 2

(Berinde & Takens, 2007) Let $\{ai\}, \{bi\}$ be sequences of nonnegative numbers and $0 \le q < 1$, such that $a_{i+1} \le q$ $a_i + b_i$, for all $i \ge 0$

(i) if $\lim\limits_{i\to\infty} a_i = 0$ then $\lim\limits_{i\to\infty} b_i = 0$

(ii) if that $\sum_0^{\infty} \alpha_i < \infty$ then $\sum_0^{\infty} \beta_i < \infty$,

$y_i = q^*$

Definition 1 (Coupled Fixed Point)

If (𝕏,d) is a metric space and θ^*: $\mathbb{X} \times \mathbb{X} \to \mathbb{X}$ is an operator, then, by definition, a CFP for θ^* is a pair $(x^*, y^*) \in \mathbb{X} \times \mathbb{X}$ satisfying $\begin{cases} x^* = \theta^*(x^*, y^*) \\ y^* = \theta^*(y^*, x^*) \end{cases}$ where (Fix(θ^*)) will denote the coupled fixed point set of θ^*.

Definition 2

A self-mapping θ^*: $\mathbb{X} \times \mathbb{X} \to \mathbb{X}$ is called a coupled generalized interpolative Kannan contraction if there exist $\lambda \in (0,1), \gamma \in (0,1)$ for which

$$d(\theta^*(x, y), \theta^*(u, v)) \le \lambda max[d(x, \theta^*(x, y))^{\gamma} d\big(u, \theta^*(u, v)\big)^{1-\gamma}, d\big(x, \theta^*(x, y)\big)^{1-\gamma} d(u, \theta^*(u, v))^{\gamma}]$$

For all $x, y, u, v \in \mathbb{X}\, with\, x \ne \theta^*(x, y)\; and\; u \ne \theta^*(u, v)$.

Definition 3

A coupled interpolative Kannan contraction on a metric space (𝕏,d) is a self-mapping θ^*: 𝕏 × 𝕏 such that there exists λ ∈ (0,1) for which

$d\big(\theta^*(x, y), \theta^*(u, v)\big) \le \lambda d\big(x, \theta^*(x, y)\big)\gamma d\big(u, \theta^*(u, v)\big)^{1-\gamma}$ for all $x, y, u, v \in \mathbb{X}$

With $x \ne \theta^*(x, y)$.

Definition 4

Let (𝕏,d) be a metric space and θ^*: $\mathbb{X} \times \mathbb{X} \to \mathbb{X}$ be a coupled self-map. It is called a coupled relaxed $(\lambda.\gamma, \beta)$- interpolative Kannan contraction, if there exists $0 \le \lambda < 1, 0 < \gamma, \beta \le 1$ with $0 < \gamma + \beta < 1$, such that

$$d(\theta^*(x, y), \theta^*(u, v)) \le \lambda d(x, \theta^*(x, y))^{\gamma}\, d(u, \theta^*(u, v))^{\beta}$$

RESULTS AND DISCUSSIONS

This section presents significant theoretical advancements in the investigation of the existence and uniqueness of fixed points for coupled interpolative Kannan contraction with the aid of the following theorem.

Theorem 1

Let $(\mathbb{X},d)$ be a complete metric space and $\theta^*: \mathbb{X} \times \mathbb{X} \rightarrow \mathbb{X}$ be a coupled interpolative Kannan type contraction i.e a self-map such that there exist $\lambda \in (0,1)$, $\gamma \in (0,1)$ so that $d(\theta^*(x,y),\theta^*(u,v)) \leq \lambda d(x,\theta^*(x,y))\gamma d(u,\theta^*(u,v))^{1-\gamma}$ for all $x,y,u,v \in \mathbb{X}$ with $x \neq \theta^*(x,y)$ and $u \neq \theta^*(u,v)$. Then, θ^* has a unique fixed point in x.

Proof

Let $x_0, y_0 \in \mathbb{X}$ and construct the sequence $[x_i]$ by $x_{i+1} = \theta^{*i}(x_0, y_0)$ for all positive integer. Taking $x = xi, y = yi$,
$d(x_{i+1}, xi) = d(\theta^*(xi, yi), \theta^*(x_{i-1}, y_{i-1}))$
$\leq \lambda d(xi, \theta^*(xi, yi))^{\gamma} \cdot d(x_{i-1}, \theta^*(x_{i-1}, y_{i-1}))^{1-\gamma}$
$= \lambda d(x_i, x_{i+1})\gamma . d(x_{i-1}, x_i)^{1-\gamma}$
$d(x_{i+1}, x_i) = \lambda d(x_i, x_{i+1})^{\gamma} . d(x_{i-1}, x_i)^{1-\gamma}$
since $d(x,y) = d(y,x)$
then $d(x_i, x_{i+1})^{1-\gamma} \leq \lambda d(x_{i-1}, x_i)^{1-\gamma}$ (3)
consider
$d(x_i, x_{i+1}) = d(\theta^* (x_{i-1}, y_{i-1}), \theta^* (xi, yi))$
$\leq \lambda d\big(x_{i-1}, \theta^*(x_{i-1}, y_{i-1})\big)^{\gamma} . d\big(x_i, \theta^*(x_i, y_i)\big)^{1-\gamma} d(x_i, x_{i+1})^{\gamma}$
$\leq \lambda d(x_{i-1}, \theta^*(x_{i-1}, y_{i-1}))^{\gamma}$ (4)
From (3.1) and (3.2), we have
$d(x_i, x_{i+1}) \leq \lambda d(x_{i-1}, \theta^*(x_{i-1}, y_{i-1}))$.
Therefore, $d(x_i, x_{i+1}) \leq d(x_{i-1}, x_i)$ for all $i \geq 1$
Hence, the positive sequence, $d(x_{i-1}, x_i)$
is decreasing. Then there exists a real $l \geq 0$ such that $\lim\limits_{i \to \infty} d(x_{i-1}, x_i) = l$

Considering (3.2)
$d(x_i, x_{i+1})^{\gamma} \leq \lambda d\big(x_{i-1}, \theta^*(x_{i-1}, y_{i-1})\big)^{\gamma} d\big(x_{i-1}, \theta^*(x_{i-1}, y_{n-1})\big)^{1-\gamma} d(x_i, x_{i+1})^{\gamma}$
$\leq \lambda d(x_{i-1}, \theta^*(x_{i-1}, y_{i-1}))^{\gamma} . d(x_{i-1}, \theta^*(x_{i-1}, y_{i-1}))^{1-\gamma} . d(x_{i-1}, \theta^*(x_{i-1}, y_{i-1}))^{1-\gamma} . d(x_i, x_{i+1})^{\gamma}$
$\leq \lambda d(x_{i-1}, \theta^*(x_{i-1}, y_{i-1}))$
Hence,
$d(x_{i+1}, x_i) \leq \lambda d\big(x_{i,}, \theta^*(x_i, yi)\big)^{1-\gamma}$.
$\leq \lambda^i d\big(x_0, \theta^* (x_0, y_0)\big)$
$d(x_{i+1}, x_i) < d\big(x_0 \theta^*(x_0, y_0)\big)$
This shows that the sequence converges.
Next, we show that the sequence is Cauchy.
$d(x_i, x_j) < d\big(x_{i,} x_{i+1}\big) + d(x_{i+1}, x_{i+2}) + d(x_{i+2}, x_{i+3}) + \cdots + d\big(x_{j-1}, x_j\big)$
$\leq \lambda^i d\left(x_{0,} \theta^*(x_0, y_0)\right) + \lambda^{i+1} d\left(x_{0,} \theta^*(x_0, y_0)\right) + \cdots + \lambda^{j-1} d\left(x_{0,} \theta^*(x_0, y_0)\right)$
$\leq [\lambda^i + \lambda^{i+1} + \lambda^{i+2} + \cdots \cdot \lambda^{j-1}]\, d\big(x_0, \theta^*(x_0, y_0)\big) \frac{\lambda^i}{1-\lambda} d\big(x_0, \theta^*(x_0, y_0)\big)$
$d(x_{i,}, x_j) < d\big(x_0 \theta^*(x_0, y_0)\big)$
The Cauchy sequence also converges; therefore, the sequence is complete.
Since the sequence converges, there exists a limit such that the sequence converges to say
$l = \theta^*(l, v)$
$d(x_{i+1}, l) = d(\theta^* (x_i, y_j), \theta^*(l, v))$
$\leq \lambda d\big(x_i, \theta^*(x_i, y_i)\big)^{\lambda} . d\big(l, \theta^*(l, v)\big)^{1-\lambda}$
$d(x_{i+1}, l) = 0$
$\lim\limits_{i \to \infty} x_{i+1} = l$
Hence, l is a fixed point.
Suppose there is another fixed point $q = \theta^* (q, u)$
$d(l, q) = d(\theta^*(l, v), \theta^*(q, u))$
$\leq \lambda d\big(l, \theta^*(l, v)\big)^{\lambda} . d(q, \theta^*(q, u))^{1-\lambda}$
$d(l, q) = 0$.
Therefore, $l = q$.
Hence, the fixed point is unique. ■

The following theorem shows the existence of fixed points for a relaxed coupled interpolative KTM.

Theorem 2

et $(\mathbb{X},d)$ be a complete metric space, and θ^* be a coupled self-mapping on $\mathbb{X}$ such that
$d\,(\theta^*(x,y), \theta^*(u,v)) \leq \lambda(\, d(x, \theta^*(x,y)))^{\lambda} (d(y, \theta^*(u,v)))^{\beta}$ for all $x, y \in \mathbb{X}$ with $x \neq \theta^* x$ and $y \neq \theta^* x$, and where $\lambda \in [0,1]$ and $\gamma, \beta \in [0,1]$ such that $\gamma + \beta < 1$ if there exist $x \in \mathbb{X}$ such that $d(x, \theta^*(x,y)) \leq 1$. Then θ^*has a fixed point in $\mathbb{X}$.

Proof

Let $x_0, y_0 \in \mathbb{X}$ and let the sequence $x_{i+1} = \theta^*(x_0, y_0)$ to all positive integer i.

Taken $x = x_i, y = y_i$

d(x_i,x_{i+1})=d(θ^* (x_{i-1},y_{i-1})$, \theta^*(x_i, y_i)$

$\leq \lambda \left(d\left(x_{i-1}, \theta^*(x_{i-1}, y_{i-1})\right)\right)^{\gamma} .. (d(x_i, \theta^*(x_i, y_i)))^{\beta}$

Since γ+β < 1, we can say,

$\beta < 1 - \gamma$ and $\gamma < 1 - \beta$

$(\mathrm{d}(x_i, x_{i+1}))^{1-\beta} \leq \lambda(d\left(x_{i-1}, \theta^*(x_i, y_i)\right))^{\gamma} \quad \leq \lambda(d\left(x_{i-1}, \theta^*(x_{i-1}, y_{i-1})\right))^{1-\beta}$

Therefore,

$(\mathrm{d}(x_i, x_{i+1}))^{1-\beta} \leq \lambda(d\left(x_{i-1}, \theta^*(x_{i-1}, y_{i-1})\right))^{1-\beta}$

d(x_i,x_{i+1})$\leq \lambda \frac{1}{1-\beta} d(x_{i-1}, x_i)$

$\leq \lambda d(x_{i-1}, x_i)$

For all integers $i \geq 1$

d(x_i,x_{i+1})$= d(\theta^*(x_i, y_i), \theta^*(x_{i-1}, y_{i-1}))$

$\leq \lambda(d\left(x_i, \theta^*(x_i, y_i)\right))^{\gamma}. \quad (d\left(x_{i-1}, \theta^*(x_{i-1}, y_{i-1})\right))^{\beta}$ $(\mathrm{d}(x_{i+1}, x_i))^{1-\beta}$

$\leq \lambda(d\left(x_{i-1}, \theta^*(x_{i-1}, x_{i-1})\right))^{\beta}$

$\leq \lambda(d\left(x_{i-1}, \theta^*(x_{i-1}, y_{i-1})\right))^{1-\gamma}$

$\mathrm{d}(x_{i+1}, x_i) \quad \leq \lambda \frac{1}{1-\gamma} d(x_{i-1}, \theta^*(x_{i-1}, y_{i-1}))$

$\leq \lambda d(x_{i-1}, \theta^*(x_{i-1}, y_{i-1}))$

$\leq \lambda^2 d(x_{i-2}, \theta^*(x_{i-2}, y_{i-2}))$

$\leq \lambda^i d(x_0, \theta^*(x_0, y_0))$

i,e, the sequence convergence for all j, i ∈ N. If j < i

$\mathrm{d}(x_j, x_i) \leq d\left(x_{j,} x_{j+1}\right) + \mathrm{d}\left(x_{j+1}, x_{j+2}\right) + \mathrm{d}\left(x_{j+2}, x_{j+3}\right) + \cdots + \mathrm{d}(x_{i-1}, x_i)$

$\leq \lambda^j d\left(x_{0,} \theta^*(x_0, y_0)\right) + \lambda^{j+1} d\left(x_{0,} \theta^*(x_0, y_0)\right) + \lambda^{j+2} d\left(x_{0,} \theta^*(x_0, y_0)\right)$

$+ \cdots + \lambda^{i-1} d\left(x_{0,} \theta^*(x_0, y_0)\right)$

$\leq [\lambda^j + \lambda^{j+1} + \lambda^{j+2} + \cdot \cdot \quad \cdot \lambda^{i-1}]\, d\left(x_{0,} \theta^*(x_0, y_0)\right)$

$\leq \dfrac{\lambda^j}{1 - \lambda^j} d\left(x_{0,} \theta^*(x_0, y_0)\right)$

If j > i

$\mathrm{d}(x_i, x_\mathrm{j}) < d\left(x_{i,} x_{i+1}\right) + \mathrm{d}(x_{i+1}, x_{i+2}) + \mathrm{d}(x_{i+2}, x_{i+3}) + \cdots + \mathrm{d}\left(x_{j-1}, x_j\right)$

$\leq \lambda^i d\left(x_{0,} \theta^*(x_0, y_0)\right) + \lambda^{i+1} d\left(x_{0,} \theta^*(x_0, y_0)\right) + \cdots + \lambda^{i-1} d\left(x_{0,} \theta^*(x_0, y_0)\right)$

$\leq [\lambda^i + \lambda^{i+1} + \lambda^{i+2} + \cdots + \lambda^{i-1}]\, d\left(x_{0,} \theta^*(x_0, y_0)\right)$

$\leq \frac{\lambda^i}{1-\lambda} d\left(x_{0,} \theta^*(x_0, y_0)\right)$

$\leq d\left(x_{0,} \theta^*(x_0, y_0)\right)$

Hence, the Cauchy sequence converges. The sequence and the Cauchy sequence both converge to a limit, say l, such that. $l = \theta^*(l, v)$

$\mathrm{d}(x_{i+1}, l) = \mathrm{d}(\theta^* (x_i, y_\mathrm{i}), \theta^*(l, v))$

$\leq \lambda d\left(x_i, \theta^*(x_i, y_i)\right)^{1-\gamma}. d\left(l, \theta^*(l, v)\right)^{\beta}$

$\mathrm{d}(x_{i+1}, l) = 0$

Since $\theta^* (l, v) = l$

$\lim_{i \to \infty} x_{i+1} = l$

Hence, l is a fixed point.

Suppose there is another fixed point $q = \theta^* (q, u)$

$d(l, q) = d(\theta^* (l, v), \theta^*(q, u))$

$\leq \lambda d\left(l, \theta^*(l, v)\right)^{\gamma}. d(q, \theta^*(q, u))^{\beta}$

$d(l, q) = 0.$

Therefore, $l = q$.

Hence, the fixed point is unique. ■

The next theorem shows the existence and uniqueness of fixed points for relaxed coupled interpolative Kannan contractions.

Theorem 3

Let ($\mathbb{X}$,d) be a complete metric space, and θ^* is a coupled self-mapping, i.e., $\theta^*: \mathbb{X} \times \mathbb{X} \to \mathbb{X}$, such that $\mathrm{d}\,(\theta^*(x, \mathrm{y}), \theta^*(u, v)) \leq \lambda max[\mathrm{d}(x, \theta^*(x, y))^{\gamma} . (\mathrm{d}(u, \theta^*(u, v)))^{1-\gamma} . \mathrm{d}(u, \theta^*(u, v))^{\gamma} . \mathrm{d}(x, \theta^*(x, y))^{1-\gamma}]$ for all x, y, u, $v \in \mathbb{X}$ with $x \neq \theta^* (x, y)$ and $v \neq \theta^* (u, v)$. $\gamma \in (0,1)$ and $\lambda \in (0,1)$. Then θ^* has a fixed point in $\mathbb{X}$.

Proof

$(x_i,\, x_{i+1}) = d(\theta^*(x_{i-1}, y_{i-1}), \theta^*(x_i, y_i)$

$\leq \lambda \max[\left(d(x_{i-1}, \theta^*(x_{i-1}, y_{i-1}))\right)^{\gamma} . \left(d(x_i, \theta^*(x_i, y_i))\right)^{1-\gamma} . \left(d(x_{i-1}, \theta^*(x_{i-1}, y_{i-1}))\right)^{1-\gamma} . \left(d(x_i, \theta^*(x_i, y_i))\right)^{\gamma}]$

For the case when $x_i < x_{i+1}$

$d(x_{i,}x_{i+1}) \leq \lambda d(x_{i-1}, \theta^*(x_{i-1}, y_{i-1}))^{1-\gamma} . d(x_i, \theta^*(x_i, y_i))^{\gamma},\ d(x_{i,}x_{i+1})^{1-\gamma}$

$\leq \lambda d(x_{i-1}, \theta^*(x_{i-1}, y_{i-1}))^{1-\gamma}$

$d(x_{i,}x_{i+1}) \leq \lambda \frac{1}{1-\gamma} d(x_{i-1}, \theta^*(x_{i-1}, y_{i-1}))$

$\leq \lambda d(x_{i-1}, \theta^*(x_{i-1}, y_{i-1}))$

$\leq \lambda^2 d(x_{i-2}, \theta^*(x_{i-2}, y_{i-2}))$

$\leq \lambda^3 d(x_{i-3}, \theta^*(x_{i-3}, y_{i-3}))$

$d(x_{i,}x_{i+1}) \leq \lambda^i d\left(x_{0,}\theta^*(x_0, y_0)\right)$

For the case where $x_i > x_{i+1}$

It follows as in the case of $x_i < x_{i+1}$therefore,

$d(x_{i,}x_{i+1}) \leq \lambda^n d\left(x_{0,}\theta^*(x_0, y_0)\right)$

Hence, both cases converge. $\lambda^i d\left(x_{0,}\theta^*(x_0, y_0)\right)$ to j, i ∈ N

Suppose $j < i$

$d(x_j, x_i)\ d(x_{j,}x_{j+1}) + d(x_{j+1}, x_{j+2}) + d(x_{j+2}, x_{j+3}) + \cdots + d(x_{i-1}, x_i)$

$\leq \lambda^j d\left(x_{0,}\theta^*(x_0, y_0)\right) + \lambda^{j+1} d\left(x_{0,}\theta^*(x_0, y_0)\right) + \lambda^{j+2} d\left(x_{0,}\theta^*(x_0, y_0)\right)$

$+ \cdots + \lambda^{i-1} d\left(x_{0,}\theta^*(x_0, y_0)\right)$

$\leq [\lambda^j + \lambda^{j+1} + \lambda^{j+2} + \cdots \lambda^{i-1}]\, d\left(x_{0,}\theta^*(x_0, y_0)\right)$

$\leq \frac{\lambda^j}{1-\lambda} d\left(x_{0,}\theta^*(x_0, y_0)\right)$

$\frac{\lambda^j}{1-\lambda} d\left(x_{0,}\theta^*(x_0, y_0)\right) \to 0 \text{ as } j, i \to \infty$

This indicated d (x_j, x_i) can be made arbitrarily small by large j and i. Hence d (x_j, x_i) is a Cauchy sequence in θ^*. Similarly, for the case $j > i$, the result follows as above

d $(x_j, x_i) \frac{\lambda^i}{1-\lambda} d\left(x_{0,}\theta^*(x_0, y_0)\right)$

We conclude that the sequence is a Cauchy sequence. The sequence converges to some $p^* \in \theta^*$ such that

$\lim_{i\to\infty} x_{i+1} = \lim_{n\to\infty} x_i = p^* = \theta^*(p^*, u)$

d $(x_{i+1}, p^*) \leq d(\theta^*(x_i . y_i), \theta^*(p^*, u))$

$\leq \lambda \max[\left(d(x_i, \theta^*(x_i, y_i))\right)^{\gamma} . (d(p^*, \theta^*(p^*, u))^{1-\gamma}\ \ d(p^*, \theta^*(p^*, u))^{\gamma}$

d $(x_{i+1}, p^*) = 0$

$\lim_{i\to\infty} x_{i+1} = p^*$

Suppose there exists another fixed point. q^*

Such that $q^* = \theta^*(q^*, v)$

$d(q^*, p^*) \leq d(\theta^*(p^*, u), \theta^*(q^*, u))$

$\leq \lambda \max[\left(d(p^*, \theta^*(p^*, u))\right)^{\gamma} . (d(q^*, \theta^*(q^*, v))^{1-\gamma} . (d(q^*, \theta^*(q^*, v))^{\gamma} . \left(d(p^*, \theta^*(p^*, u))\right)^{1-\gamma}]$

$d(p^*, q^*) = 0$

Hence $p^* = q^*$

Which therefore conclude that the fixed point exists and is unique

CONCLUSION

The research proved new findings of fixed points and uniqueness of coupled interpolative Kannan-type contractions of complete metric spaces. Intersection of iterative methods such as Picard and Mann iterations was proved rigorously as long as there were appropriate contractive conditions. The outcomes generalize a number of classical fixed-point theorems and give a more general analytical setting to the investigation of nonlinear issues that involve coupled mappings. These results add to the development of the fixed-point theory and provide possible solutions to the application of the nonlinear functional equations and other mathematical models.

REFERENCES

Aamri, M., & El Moutawakil, D. (2002). Some new common fixed-point theorems under strict contractive conditions. *Journal of Mathematical Analysis and Applications*, *270*(1), 181-188.

Abdou, A. A. (2020). Fixed points of Kannan maps in modular metric spaces. *AIMS Mathematics*, *5*(6), 6395. https://doi.org/10.3934/math.2020411

Aniki, S. A., Ajisope, M. O., Raji, M., & Adegboye, F. (2022). Coupled fixed points theorem for mappings satisfying a contractive condition of integral type in Cauchy spaces. *FUOYE Journal of Engineering and Technology*, *7*(3), 313-316. https://doi.org/10.46792/fuoyejet.v7i3.855

Berinde, V. and Takens, F. (2007). Iterative approximation of fixed points, volume 1912. Springer. https://doi.org/10.1007/978-3-540-72234-2_2

Bhaskar, T. G., & Lakshmikantham, V. (2006). Fixed point theorems in partially ordered metric spaces and applications. *Nonlinear analysis: theory, methods & applications*, *65*(7), 1379-1393. https://doi.org/10.1016/j.na.2005.10.017

Bota, M. F., Guran, L., & PETRUSel, G. A. B. R. I. E. L. A. (2023). Fixed points and coupled fixed points in b-metric spaces via graphical contractions. *Carpathian Journal of Mathematics*, *39*(1), 85-94. https://www.jstor.org/stable/27178476

El Amri, A., El Foutayeni, Y., & Marhrani, L. E. M. (2022). On some results on interpolative Kannan-type and CRR-type contractions. *Moroccan Journal of Pure and Applied Analysis*, *8*(1), 54-66. https://doi.org/10.2478/mjpaa-2022-0005

Faraji, H., & Nourouzi, K. (2017). A generalization of Kannan and Chatterjea fixed point theorems on complete b-metric spaces. *Sahand Communications in Mathematical Analysis (SCMA)*, *6*(1), 77-86.

Guo, D., & Lakshmikantham, V. (1987). Coupled fixed points of nonlinear operators with applications. *Nonlinear analysis: theory, methods & applications*, *11*(5), 623-632. https://doi.org/10.1016/0362-546X(87)90077-0

Hammad, H. A., & Zayed, M. (2022). New generalized contractions by employing two control functions and coupled fixed-point theorems with applications. *Mathematics*, *10*(17), 3208. https://doi.org/10.3390/math10173208

Harder, A. M., & Hicks, T. L. (1988). Stability results for fixed point iteration procedures. *Math. Japonica*, *33*(5), 693-706.

Kannan, R. (1968). Some results on fixed points. *Bull. Cal. Math. Soc.*, *60*, 71-76.

Karapinar, E. (2018). Revisiting the Kannan type contractions via interpolation. *Advances in the Theory of Nonlinear Analysis and its Application*, *2*(2), 85-87. https://doi.org/10.31197/atnaa.431135

Karapınar, E. (2021). Interpolative Kannan-Meir-Keeler type contraction. *Advances in the Theory of Nonlinear Analysis and its Application*, *5*(4), 611-614. https://doi.org/10.31197/atnaa.989389

Lakshmikantham, V., Leela, S., & Ama Mohan Rao, M. (1987). Integral and integro-differential inequalities. *Applicable Analysis*, *24*(3), 157-164. https://doi/abs/10.1080/00036818708839660

Liu, Xiao-lan,Zhou, Mi, Damjanović, Boško, Common Coupled Fixed Point Theorem for Geraghty-Type Contraction in Partially Ordered Metric Spaces, Journal of Function Spaces, 2018, 9063267, 11 pages, 2018. https://doi.org/10.1155/20189063267

Meir, A., & Keeler, E. (1969). A theorem on contraction mappings. *Journal of mathematical Analysis and Applications*, *28*(2), 326-329.

Olatinwo, M. O. (2008). Some stability and strong convergence results for the Jungck-Ishikawa iteration process. *Creative Mathematics and Informatics*, *17*(1), 33-42.

Opoitsev, V. I. (1975). Heterogeneous and combined concave operators. *Siberian Mathematical Journal*, *16*(4), 597-605. https://doi.org/10.1007/BF00967133