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ROBUSTNESS CHECKS AND SENSITIVITY ANALYSIS OF ECONOMETRIC MODELS USING SIMULATED SYNTHETIC DATA: REPLICATING THE STATISTICAL PROPERTIES OF NIGERIAN MACROECONOMIC VARIABLES

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ABSTRACT

This study applies a Monte Carlo simulation approach to assess the robustness of macroeconomic growth models in Nigeria from 1990 to 2023 by generating 33 synthetic datasets that mimic the statistical behavior and interrelationships of key variables such as GDP, exchange rate, inflation, interest rate, industrialization, carbon emissions, and trade openness. Using parameters derived from time-series models like VAR, the simulated data preserve essential features including mean, variance, autocorrelation, and long-run relationships, enabling detailed sensitivity analysis. The findings show that log transformation generally stabilizes variance, although GDP, exchange rate, and inflation still exhibit skewness and heavy-tailed distributions, indicating persistent economic volatility. Stationarity tests confirm that all transformed variables are stable at levels. Long-run estimation using FMOLS reveals that inflation and interest rates positively influence GDP, while industrialization negatively affects growth, suggesting structural inefficiencies; other variables show limited long-run impact. Short-run dynamics from ARDL models highlight cyclical GDP adjustments, negative effects of exchange rate depreciation, and unstable trade openness impacts, alongside complex lag interactions among variables. Cointegration tests confirm stable long-run relationships, and VECM results indicate that, over time, industrialization, trade openness, carbon emissions, and inflation promote growth, whereas exchange rate depreciation and high interest rates hinder it, with short-run fluctuations largely driven by monetary and price adjustments.

Keywords: Monte Carlo Simulation, Economic Growth, ARDL, VECM, FMOLS, Cointegration

INTRODUCTION

Empirical econometric analysis, particularly in macroeconomics, is fundamentally constrained by the limitations of observed historical data. Researchers often work with a single, non-replicable realization of a time series process, typically characterized by a small sample size, as seen in the Nigerian context with annual data from 1990-2023. This limitation poses a significant challenge for rigorous model validation. Traditional in-sample goodness-of-fit measures, such as R-squared, can be misleading, as a model may appear robust while failing to capture the true data-generating process, a concern highlighted by the parameter instability detected in the Hansen test for the FMOLS model in the provided analysis. Consequently, the reliability of policy recommendations derived from such models remains uncertain without thorough testing of their performance under different, yet plausible, data conditions. The seminal work of Leamer (1983) in his book "Let's Take the Con Off Econometrics" famously critiqued this fragility, advocating for sensitivity analysis to determine if conclusions are robust to alternative assumptions and model specifications.

To address these inherent limitations, the use of simulated synthetic data has emerged as a powerful methodological tool in the econometrician's toolkit. The core premise, rooted in the principles of Monte Carlo simulation, is to generate multiple artificial datasets that faithfully replicate the key statistical properties such as mean, variance, autocorrelation, and cointegration structures of the original observed data **(**Hendry, 1984; Doornik & O'Brien, 2020**).** This synthetic data generation process allows researchers to perform extensive robustness checks and sensitivity analysis that are impossible with a single historical dataset. For instance, one can test how an Autoregressive Distributed Lag (ARDL) or Vector Error Correction Model (VECM) performs across thousands of simulated scenarios, assessing the stability of coefficients, the power of tests, and the model's resilience to misspecification. This approach moves beyond a single narrative of the past and explores the model's behavior across a "multiverse" of possible data realities.

In the context of Nigeria's volatile macroeconomic, characterized by oil dependence, exchange rate fluctuations, and inflationary pressures as vividly described in the descriptive statistics and time series plots the need for such robust validation is paramount. Previous studies on the Nigerian economy, such as those by Adenomon et al. (2020) and Adeoye et al. (2021), have applied various econometric techniques but often lack a comprehensive simulation-based validation framework. The preliminary analysis provided, which reveals issues like multicollinearity in the ARDL model and divergent results between FMOLS and VECM estimations, underscores the potential fragility of the findings. Therefore, this study is motivated by the critical need to bridge this methodological gap. By generating synthetic data that mirrors the complex dynamics of Nigerian macroeconomic variables, this research aims to rigorously stress-test existing econometric models, thereby providing a more definitive assessment of their reliability and strengthening the foundation for evidence-based economic policy in Nigeria.

This comprehensive body of empirical literature collectively demonstrates a paradigm shift in econometric methodology, where simulation and synthetic data generation have become indispensable tools for robust empirical analysis, moving beyond reliance on single historical datasets to establish more reliable and verifiable economic conclusions. The foundational work of Plagborg-Møller & Wolf (2021) set a crucial precedent by employing extensive Monte Carlo simulations to resolve a long-standing methodological debate between Local Projections (LP) and Vector Autoregressions (VARs), proving through synthetic datasets generated from known Data-Generating Processes (DGPs) that these competing estimators are theoretically equivalent, with any empirical differences arising purely from finite-sample biases and efficiency trade-offs rather than fundamental methodological superiority, thereby establishing that rigorous simulation-based analysis is essential for making informed choices between econometric techniques in practical applications. Building on this methodological imperative, Chan (2022) further demonstrated the critical role of synthetic data validation by developing a novel statistical test for comparing point and density forecasts, where Monte Carlo studies with artificially generated datasets mimicking the complex persistence and volatility of macroeconomic series provided the necessary "ground truth" to verify that his proposed test possessed excellent size and power properties, particularly for challenging nested model comparisons, thus establishing that any new econometric methodology requires simulation-based validation before it can be trusted with real-world data. This theme of using synthetic environments to test methodological boundaries was expanded by Goulet Coulombe et al. (2022), who employed factor-augmented synthetic data to systematically identify the specific conditions under which Machine Learning (ML) models outperform traditional time series approaches, creating controlled laboratories with artificial datasets embodying specific features like strong nonlinearities and complex interaction effects to demonstrate that ML methods only provide significant advantages when the underlying data-generating process is highly complex and nonlinear, while traditional models remain competitive in simpler environments, thus providing crucial nuance to the ML debate that could only be revealed through controlled simulation experiments. Similarly, Carriero et al. (2022) embraced this validation paradigm by subjecting their new Proxy VAR methodology for identifying structural economic shocks to rigorous Monte Carlo simulation exercises, generating synthetic datasets with known structural shocks and impulse responses to demonstrate that their data-rich approach yielded more precise and reliable estimates compared to standard methods, thereby establishing simulation-based validation as a necessary prerequisite for deploying complex econometric techniques to real economic data. Beyond methodological validation, the literature shows how synthetic data creation enables causal inference in otherwise intractable settings, as exemplified by Kremer et al. (2022), who applied the Synthetic Control Method to algorithmically construct a "synthetic Berlin" as a weighted combination of other German regions to serve as a counterfactual for assessing the economic impact of the 2016 terror attack, demonstrating that creating tailored synthetic datasets provides a powerful strategy for policy evaluation when randomized experiments are impossible. This causal inference framework was further strengthened by Huber et al. (2020), who addressed a critical weakness in Synthetic Control Methods by developing a bootstrap-based t-test validated through placebo simulation approaches that repeatedly applied the SCM to control units to generate empirical distributions of placebo effects, thereby providing reliable statistical inference for causal claims that previously lacked solid foundation. The utility of simulation extends to handling extraordinary economic circumstances, as demonstrated by Lenza & Primiceri (2022), who used Monte Carlo simulations to diagnose and solve the problem of extreme outliers like the COVID-19 shock in VAR models by generating synthetic data from stable VAR processes and artificially contaminating it with outliers to show how standard estimation produces severe bias while their proposed methods successfully recover true parameters, establishing simulation as essential for developing robust solutions to unprecedented economic events. For complex model specification, Goulet Coulombe (2023) illustrated how Bayesian estimation of sophisticated dynamic latent factor models linking the macroeconomy and yield curve requires prior validation through Monte Carlo studies with synthetic data to verify that estimation algorithms can accurately recover known parameters and latent states before trusting results with real data. Beyond model specification and estimation, Lewis et al. (2020) demonstrated how simulation-based sensitivity analysis provides crucial robustness checks by creating frameworks that simulate the impact of hypothetical unmeasured confounders across plausible parameter ranges, showing that many "significant" findings become fragile when confronted with modest confounding, thus establishing that any empirical conclusion requires quantification of its robustness to potential omitted variables. Finally, the study by Adenomon & Ojo (2020) on Nigerian stock market volatility during COVID-19 represents the endpoint of conventional analysis that motivates the entire simulation paradigm presenting findings from a single historical realization of data thereby highlighting the critical next step of assessing whether such conclusions remain robust across synthetic realizations of Nigerian financial data that preserve core statistical properties, thus completing the methodological circle where simulation and synthetic data generation emerge as the essential tools for advancing from fragile single-dataset conclusions to robust, verifiable economic knowledge. Together, these studies establish a comprehensive framework where simulation methodologies serve multiple critical functions: resolving theoretical equivalences, validating new statistical tests, identifying boundary conditions for methodological application, enabling causal inference, strengthening statistical foundations for policy evaluation, diagnosing model vulnerabilities, verifying complex estimation algorithms, conducting sensitivity analysis, and ultimately providing the necessary infrastructure for transitioning from potentially fragile empirical findings to genuinely robust economic conclusions.

MATERIALS AND METHODS

The methodology for this research will employ a rigorous Monte Carlo simulation framework to generate synthetic datasets that meticulously replicate the statistical properties of key Nigerian macroeconomic variables including GDP, exchange rates, inflation, interest rates, industrialization, carbon emissions, and trade openness over the period 1990-2023. The process will begin by using the original historical data to estimate the core characteristics of the data-generating process, such as means, variances, autocorrelation structures, co-integrating relationships, and volatility clustering, potentially employing Vector Autoregressive (VAR) or state-space models as the foundation for simulation. Following this, thousands of alternative, yet statistically congruent, synthetic time series will be generated, preserving the estimated multivariate dependencies and integration properties of the original data. These synthetic datasets will then serve as the laboratory for robust sensitivity analysis, where benchmark econometric models previously applied in the Nigerian context such as Fully Modified OLS (FMOLS), Autoregressive Distributed Lag (ARDL), and Vector Error Correction (VECM) models will be re-estimated repeatedly. The robustness of these models will be quantitatively assessed by examining the stability and distribution of their key coefficients, the power and size of their diagnostic tests (like parameter stability and serial correlation tests), and their resilience to specified perturbations, thereby providing a comprehensive, simulation-based evaluation of their reliability for policy analysis beyond what is possible with a single historical data realization.

Table 1: Descriptive Statistics of Simulated Gross Domestic Product (GDP), Exchange Rate (EXR), Carbon dioxide (C02), Industrialization (IND), Inflation Rate (INFR), Interest Rate (INTR) and Trade Openness (TO) in Nigeria (1990 – 2023)

	GDP	EXR	CO2	IND	INFR	INTR	TO
Mean	1559.597	168.4878	0.698042	27.53765	16.12721	3.472449	34.47263
Median	1618.399	156.2038	0.700721	27.34114	13.57918	1.861462	31.51624
Maximum	2928.469	471.7513	0.941155	38.09503	66.01299	24.98347	54.35698
Minimum	29.60885	-60.35395	0.394398	18.09312	-7.974403	-10.07882	22.47444
Std. Dev.	734.0380	117.5057	0.116958	4.359429	17.80786	9.494748	8.565102
Skewness	0.001703	0.354532	-0.064750	0.210797	0.598213	0.716083	0.747666
Kurtosis	2.380983	3.127065	3.087071	3.178588	3.198030	2.881527	2.689783
Jarque-Bera	0.542857	0.735131	0.034498	0.296984	2.083421	2.925609	3.304021
Probability	0.762290	0.692418	0.982899	0.862007	0.352851	0.231586	0.191664
Sum	53026.30	5728.586	23.73344	936.2800	548.3250	118.0633	1172.070
Sum Sq. Dev.	17780789	455650.4	0.451409	627.1526	10464.95	2974.958	2420.912
Observations	34	34	34	34	34	34	34

Sources: Computed Using EViews

The descriptive statistics for Nigeria's simulated macroeconomic variables from 1990 to 2023 reveal distinct patterns in central tendency, dispersion, and distribution. GDP has a mean of 1,559.60 and a median of 1,618.40, indicating a roughly symmetric distribution, though it exhibits high volatility (standard deviation = 734.04) with extreme highs and lows, reflecting Nigeria's economic fluctuations. Exchange rates show moderate right skewness (mean > median) and substantial variability (SD = 117.51), consistent with periods of sharp depreciation. Inflation and interest rates also exhibit right-skewed distributions with high volatility, highlighting the economy's unstable monetary environment. In contrast, carbon emissions, industrialization, and trade openness display near-symmetry and relatively lower variability, suggesting gradual structural changes over time. Kurtosis values close to 3 for most variables indicate distributions that are near normal, and the Jarque-Bera tests confirm no significant deviations from normality.

Table 2: Descriptive Statistics of Natural Log-transform Simulated Gross Domestic Product (GDP), Exchange Rate (EXR), Carbon dioxide (C02), Industrialization (IND), Inflation Rate (INFR), Interest Rate (INTR) and Trade Openness (TO) in Nigeria (1990 – 2023)

	LNGDP	LNEXR	LNCO2	LNIND	LNINFR	LNINTR	LNTO
Mean	7.152156	4.756672	-0.373879	3.303185	2.799202	1.646221	3.511879
Median	7.389147	5.051140	-0.355657	3.308392	2.799202	1.646221	3.450472
Maximum	7.982235	6.156452	-0.060648	3.640084	4.189852	3.218214	3.995573
Minimum	3.388073	-0.566844	-0.930394	2.895532	-0.766088	-1.000574	3.112379
Std. Dev.	0.848462	1.290901	0.175593	0.160889	0.934903	0.949006	0.238694
Skewness	-2.810329	-2.485243	-0.701100	-0.302491	-2.239588	-0.803810	0.339272
Kurtosis	12.71346	10.04424	4.301582	3.296750	9.132164	4.069195	2.316426
Jarque-Bera	178.4195	105.2966	5.185395	0.643256	81.69413	5.280798	1.314237
Probability	0.000000	0.000000	0.074818	0.724968	0.000000	0.071333	0.518343
Sum	243.1733	161.7269	-12.71189	112.3083	95.17286	55.97152	119.4039
Sum Sq. Dev.	23.75628	54.99204	1.017483	0.854215	28.84347	29.72020	1.880176
Observations	34	34	34	34	34	34	34

Sources: Computed Using EViews

The descriptive statistics for the log-transformed Nigerian macroeconomic variables from 1990 to 2023 show that the log transformation effectively reduces scale differences and stabilizes variance across the series. Most variables, such as $LNCO_2$, LNIND, and LNTO, exhibit near-symmetry with mean and median values closely aligned, suggesting successful normalization. However, LNGDP, LNEXR, and LNINFR remain strongly left-skewed, reflecting persistent asymmetry in GDP, exchange rate, and inflation even after transformation. Volatility is notably reduced for variables like industrialization and trade openness (SDs < 0.25), whereas LNEXR, LNGDP, and monetary indicators (LNINTR, LNINFR) still show higher standard deviations, indicating relatively greater short-term fluctuations. High kurtosis values for LNGDP, LNEXR, and LNINFR signal heavy-tailed distributions, suggesting occasional extreme events, while Jarque-Bera tests confirm non-normality for these variables, but normality cannot be rejected for $LNCO_2$, LNIND, and LNTO.

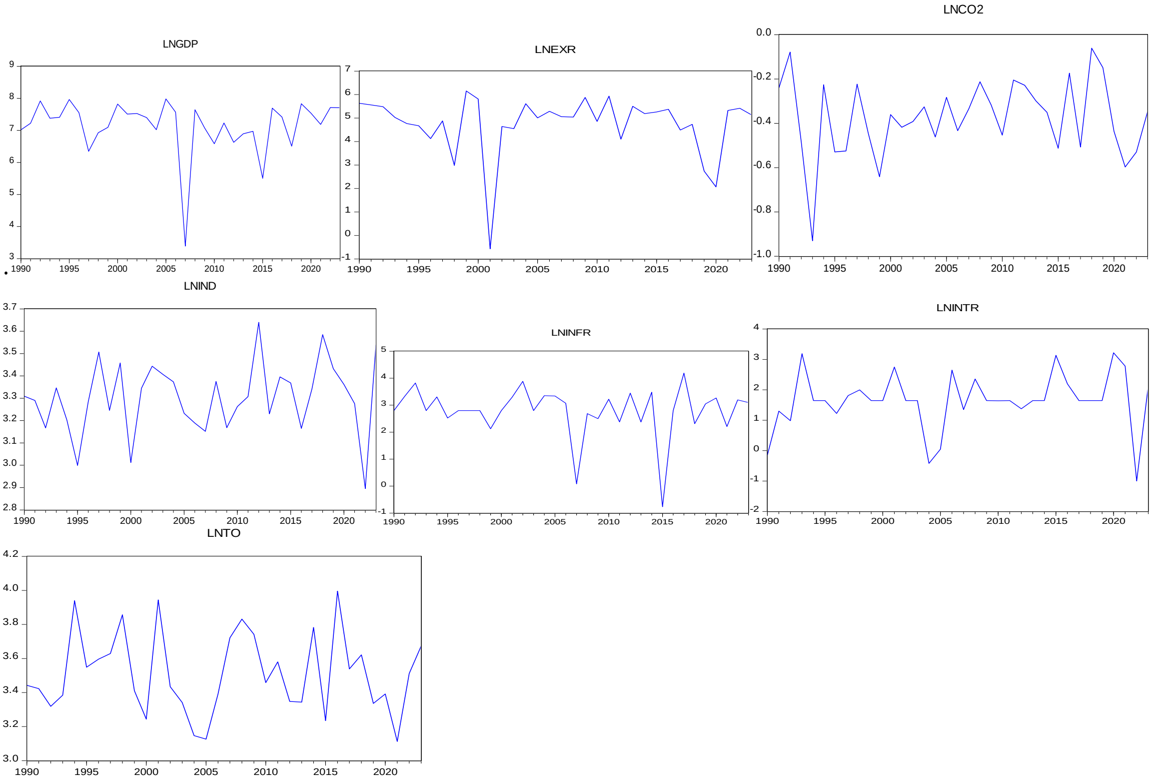

Figure 1: Graph of Natural Log of the Variables

Figure 1, which plots the natural logarithms of Nigeria's macroeconomic variables, illustrates both the long-term trends and short-term fluctuations in the economy from 1990 to 2023. LNGDP shows a generally upward trajectory, reflecting overall economic growth, but with noticeable cyclical dips corresponding to economic crises and external shocks. LNEXR exhibits high volatility, capturing periods of sharp currency depreciation and exchange rate instability, while LNINFR and LNINTR display pronounced fluctuations, indicating an unstable monetary environment and reactive policy adjustments. $LNCO_2$, LNIND, and LNTO demonstrate smoother, more gradual patterns, reflecting slower structural changes in emissions, industrialization, and trade openness. The log transformation highlights relative proportional changes, revealing that short-term shocks in GDP, exchange rates, inflation, and interest rates are amplified compared with structural variables, emphasizing the economy's vulnerability to external and domestic disturbances alongside the steady evolution of its industrial and trade sectors.

Table 3: Unit Root Test

Variables	Differences order	Critical values 1%	5%	10%	t-statistics	P-value	Remark
LNGDP	At level	-3.646342	-2.954021	-2.615817	-6.144767	0.0000	Stationary
LN exchange Rate	At level	-3.646342	-2.954021	-2.615817	-5.578422	0.0001	Stationary
LN Industrial	At level	-3.646342	-2.954021	-2.615817	-6.029986	0.0000	Stationary
LN Inflation Rate	At level	-3.646342	-2.954021	-2.615817	-6.317903	0.0000	Stationary
LN Trade Openness	At level	-3.646342	-2.954021	-2.615817	-5.255364	0.0001	Stationary
LN Carbon dioxide	At level	-3.646342	-2.954021	-2.615817	-5.700809	0.0000	Stationary
LN Interest Rate	At Level	-3.646342	-2.954021	-2.615817	-6.010637	0.0000	Stationary

Sources: Computed Using EViews

Table 3 presents the results of the Augmented Dickey-Fuller unit root tests for the log-transformed Nigerian macroeconomic variables. All variables LNGDP, LN exchange rate, LN industrialization, LN inflation rate, LN trade openness, LN carbon dioxide, and LN interest rate are stationary at level, as indicated by t-statistics that are more negative than the 1%, 5%, and 10% critical values and by p-values effectively equal to zero. This confirms that each series has constant statistical properties over time, with no stochastic trends, making them suitable for reliable time-series econometric modeling and ensuring that regression results are not spurious.

Table 4: Fully Modified Least Squares (FMOLS)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LNEXR	0.002786	0.069360	0.040164	0.9683
LNCO2	-0.049691	0.510384	-0.097360	0.9232
LNIND	-1.307783	0.551233	-2.372467	0.0254
LNINFR	0.761013	0.091979	8.273785	0.0000
LNINTR	0.252575	0.108183	2.334699	0.0276
LNTO	-0.546634	0.372210	-1.468617	0.1539
C	10.80805	2.608541	4.143331	0.0003
R-squared	0.609586	Mean dependent var		7.156247
Adjusted R-squared	0.519490	S.D. dependent var		0.861276
S.E. of regression	0.597026	Sum squared resid		9.267451
Long-run variance	0.199525			
Sources: Computed Using EViews				

From table 4.44, the FMOLS model is: $LNGDP_t = 10.808 + 0.003LNEXR_t - 0.050LNCO_{2t} - 1.308LNIND_t + 0.761LNINFR_t + 0.253LNINTR_t - 0.547LNTO_t + \varepsilon_t$

Overall Model Fit

Table 4 presents the results of the Fully Modified Ordinary Least Squares (FMOLS) estimation, which examines the long-run relationship between Nigeria's GDP and key macroeconomic variables. The model explains approximately 61% of the variation in GDP ($R^2 = 0.61$; adjusted $R^2 = 0.52$), indicating a reasonably good fit for long-run analysis. Industrialization (LNIND) has a significant negative impact on GDP (coefficient = -1.308, p = 0.025), suggesting structural inefficiencies in the sector may hinder growth. Inflation (LNINFR) shows a strong positive effect (coefficient = 0.761, $p < 0.001$), implying that rising prices are associated with higher GDP, possibly reflecting demand-driven expansion. Interest rates (LNINTR) also have a small but significant positive effect (coefficient = 0.253, p = 0.028), indicating a nuanced relationship between monetary policy and growth. Exchange rates (LNEXR), carbon emissions (LNCO2), and trade openness (LNTO) are statistically insignificant, suggesting their long-run impact on GDP is weak in this model. The highly significant intercept (C = 10.808, p = 0.0003) indicates a substantial baseline level of GDP when all explanatory variables are zero.

Autoregressive Distributed Lag (ARDL) Modelling

Table 5: Short-run Analysis

Variable	Coefficient	Std. Error	t-Statistic	Prob.*
LNGDP(-1)	-0.346049	0.120517	-2.871374	0.0349
LNGDP(-2)	-0.098070	0.086058	-1.139584	0.3061
LNGDP(-3)	0.249939	0.075913	3.292433	0.0217
LNEXR	-0.195247	0.052239	-3.737566	0.0135
LNEXR(-1)	-0.181009	0.061217	-2.956829	0.0316
LNEXR(-2)	0.238515	0.049340	4.834094	0.0047
LNEXR(-3)	0.144710	0.052328	2.765448	0.0396
LNCO2	-1.893927	0.515957	-3.670708	0.0144
LNCO2(-1)	-1.628613	0.460559	-3.536169	0.0166
LNCO2(-2)	-1.258454	0.540879	-2.326681	0.0675
LNCO2(-3)	2.187202	0.590611	3.703290	0.0140
LNIND	-1.202126	0.462277	-2.600448	0.0482
LNIND(-1)	-0.336495	0.450543	-0.746865	0.4888
LNIND(-2)	1.134106	0.372274	3.046432	0.0285
LNIND(-3)	-0.835948	0.662609	-1.261601	0.2627
LNINFR	0.512943	0.073134	7.013737	0.0009
LNINFR(-1)	-0.097104	0.085638	-1.133893	0.3083
LNINTR	0.057323	0.096190	0.595934	0.5772
LNINTR(-1)	-0.196355	0.091809	-2.138722	0.0855
LNINTR(-2)	0.065034	0.088314	0.736398	0.4946
LNINTR(-3)	0.579602	0.097488	5.945347	0.0019
LNTO	-1.779632	0.429209	-4.146305	0.0089
LNTO(-1)	0.069587	0.451894	0.153990	0.8836
LNTO(-2)	1.131216	0.447802	2.526149	0.0528
LNTO(-3)	-1.246716	0.343675	-3.627597	0.0151
C	15.99105	6.684308	2.392326	0.0622
R-squared	0.992387	Mean dependent var		7.129450
Adjusted R-squared	0.954322	S.D. dependent var		0.878013
S.E. of regression	0.187652	Akaike info criterion		-0.655585
Sum squared resid	0.176066	Schwarz criterion		0.547114

Log likelihood	36.16156	Hannan-Quinn criter.	-0.263535
F-statistic	26.07100	Durbin-Watson stat	1.851020
Prob(F-statistic)	0.000911		
Sources: Computed Using EViews			

Sources: Computed Using EViews

The estimated autoregressive distributed lag (ARDL) model for LNGDP is specified as follows:

$LNGDP_t = 15.991 - 0.346LNGDP_{t-1} - 0.098LNGDP_{t-2} + 0.250LNGDP_{t-3} - 0.195LNEXR_t - 0.181LNEXR_{t-1} + 0.239LNEXR_{t-2} + 0.145LNEXR_{t-3} - 1.894LNCO_{2t} - 1.629LNCO_{2t-1} - 1.258LNCO_{2t-2} + 2.187LNCO_{2t-3} - 1.202LNIND_t - 0.336LNIND_{t-1} + 1.134LNIND_{t-2} - 0.836LNIND_{t-3} + 0.513LNINFR_t - 0.097LNINFR_{t-1} + 0.057LNINTR_t - 0.196LNINTR_{t-1} + 0.065LNINTR_{t-2} + 0.580LNINTR_{t-3} - 1.780LNTO_t + 0.070LNTO_{t-1} + 1.131LNTO_{t-2} - 1.247LNTO_{t-3} + \varepsilon_t$

Table 5 presents the short-run ARDL estimates of Nigeria's economic growth dynamics, capturing both immediate and lagged effects of key macroeconomic variables. Past GDP exhibits a mixed influence, with a negative effect at lag 1 and a positive effect at lag 3, reflecting cyclical adjustment patterns. Exchange rate changes (LNEXR) show initial contractionary effects at current and first lags, followed by positive effects at later lags, suggesting short-term depreciation harms growth but may boost exports over time. Carbon emissions ($LNCO_2$) have significant negative short-run impacts in the first three periods, reversing to a positive effect at lag 3, highlighting complex environmental-growth interactions. Industrialization (LNIND) shows alternating negative and positive lagged effects, indicating delayed benefits of industrial activity. Inflation (LNINFR) has a strong immediate positive effect, reflecting demand-pull expansion, while interest rates (LNINTR) impact growth primarily at lag 3, consistent with delayed monetary policy transmission. Trade openness (LNTO) exhibits volatile short-run effects, with negative immediate impacts followed by alternating lagged effects. The model fits the data exceptionally well ($R^2 = 0.992$; Adjusted $R^2 = 0.954$), with no evidence of serious autocorrelation (Durbin-Watson ≈ 1.85), indicating that short-run GDP variations are largely explained by the combined dynamics of these variables.

Table 6: ARDL Long Run Form and Bounds Test

Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob.**
C	15.99105	6.684308	2.392326	0.0622
LNGDP(-1)*	-1.194180	0.237648	-5.024997	0.0040
LNEXR(-1)	0.006969	0.100308	0.069474	0.9473
LNCO2(-1)	-2.593792	1.149614	-2.256228	0.0737
LNIND(-1)	-1.240463	1.288647	-0.962609	0.3800
LNINFR(-1)	0.415839	0.124760	3.333113	0.0207
LNINTR(-1)	0.505605	0.244958	2.064049	0.0939
LNTO(-1)	-1.825546	0.816157	-2.236759	0.0755
D(LNGDP(-1))	-0.151869	0.146389	-1.037438	0.3471
D(LNGDP(-2))	-0.249939	0.075913	-3.292433	0.0217
D(LNEXR)	-0.195247	0.052239	-3.737566	0.0135
D(LNEXR(-1))	-0.383225	0.072960	-5.252505	0.0033
D(LNEXR(-2))	-0.144710	0.052328	-2.765448	0.0396
D(LNCO2)	-1.893927	0.515957	-3.670708	0.0144
D(LNCO2(-1))	-0.928748	0.816911	-1.136903	0.3071
D(LNCO2(-2))	-2.187202	0.590611	-3.703290	0.0140
D(LNIND)	-1.202126	0.462277	-2.600448	0.0482
D(LNIND(-1))	-0.298157	0.762848	-0.390848	0.7120
D(LNIND(-2))	0.835948	0.662609	1.261601	0.2627
D(LNINFR)	0.512943	0.073134	7.013737	0.0009
D(LNINTR)	0.057323	0.096190	0.595934	0.5772
D(LNINTR(-1))	-0.644636	0.134933	-4.777465	0.0050
D(LNINTR(-2))	-0.579602	0.097488	-5.945347	0.0019
D(LNTO)	-1.779632	0.429209	-4.146305	0.0089
D(LNTO(-1))	0.115500	0.386470	0.298859	0.7771
D(LNTO(-2))	1.246716	0.343675	3.627597	0.0151
*** p-value incompatible with t-Bounds distribution.**				
Levels Equation				
Case 2: Restricted Constant and No Trend				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
LNEXR	0.005836	0.084196	0.069310	0.9474
LNCO2	-2.172028	0.709262	-3.062378	0.0280

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LNIND	-1.038757	1.076564	-0.964882	0.3789
LNINFR	0.348222	0.114439	3.042870	0.0287
LNINTR	0.423391	0.212006	1.997066	0.1023
LNTO	-1.528702	0.612164	-2.497210	0.0547
C	13.39082	4.971307	2.693621	0.0431
EC = LNGDP - (0.0058*LNEXR -2.1720*LNCO2 -1.0388*LNIND + 0.3482 *LNINFR + 0.4234*LNINTR -1.5287*LNTO + 13.3908)				

Sources: Computed Using EViews

Long-run Model Interpretation

The long-run equilibrium equation is derived from the "Levels Equation" coefficients. The general form is: $LNGDP = \beta_0 + \beta_1 LNEXR + \beta_2 LNCO2 + \beta_3 LNIND + \beta_4 LNINFR + \beta_5 LNINTR + \beta_6 LNTO$

Plugging in the estimated coefficients from the table, the specific long-run model is: $LNGDP = 13.391 + 0.006 LNEXR - 2.172 LNCO2 - 1.039 LNIND + 0.348 LNINFR + 0.423 LNINTR - 1.529 LNTO$

Table 6 presents the ARDL long-run estimates and bounds test results, highlighting both the equilibrium relationships and short-run dynamics of Nigeria's economic growth. The highly significant and negative error correction term (LNGDP(-1) = -1.194, p = 0.004) confirms a stable long-run relationship and indicates a rapid adjustment mechanism, with deviations from equilibrium corrected forcefully within a year. In the long run, inflation (LNINFR) exerts a positive and significant effect on GDP, while carbon emissions ($LNCO_2$) and trade openness (LNTO) have negative long-run impacts, suggesting that higher emissions or over-reliance on trade may constrain sustainable growth. Industrialization (LNIND) and interest rates (LNINTR) show less robust significance, indicating that their long-run effects are weaker or context-dependent, whereas exchange rate (LNEXR) appears insignificant in the long run. The levels equation further confirms these relationships, providing a cointegrating vector that defines GDP's long-run equilibrium as a function of the macroeconomic determinants.

Table 7: ARDL Error Correction Regression

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(LNGDP(-1))	-0.151869	0.064881	-2.340746	0.0663
D(LNGDP(-2))	-0.249939	0.035750	-6.991342	0.0009
D(LNEXR)	-0.195247	0.024980	-7.815974	0.0005
D(LNEXR(-1))	-0.383225	0.028842	-13.28719	0.0000
D(LNEXR(-2))	-0.144710	0.021145	-6.843551	0.0010
D(LNCO2)	-1.893927	0.235783	-8.032504	0.0005
D(LNCO2(-1))	-0.928748	0.327528	-2.835633	0.0364
D(LNCO2(-2))	-2.187202	0.272364	-8.030425	0.0005
D(LNIND)	-1.202126	0.154840	-7.763678	0.0006
D(LNIND(-1))	-0.298157	0.207358	-1.437890	0.2100
D(LNIND(-2))	0.835948	0.184833	4.522732	0.0063
D(LNINFR)	0.512943	0.020351	25.20446	0.0000
D(LNINTR)	0.057323	0.032975	1.738357	0.1426
D(LNINTR(-1))	-0.644636	0.041535	-15.52038	0.0000
D(LNINTR(-2))	-0.579602	0.048870	-11.86004	0.0001
D(LNTO)	-1.779632	0.138226	-12.87477	0.0001
D(LNTO(-1))	0.115500	0.158556	0.728451	0.4990
D(LNTO(-2))	1.246716	0.158031	7.889038	0.0005
CointEq(-1)*	-1.194180	0.122670	-9.734891	0.0002
R-squared	0.996586	Mean dependent var		-0.006836
Adjusted R-squared	0.991464	S.D. dependent var		1.311046
S.E. of regression	0.121129	Akaike info criterion		-1.107198
Sum squared resid	0.176066	Schwarz criterion		-0.228302
Log likelihood	36.16156	Hannan-Quinn criter.		-0.820700
Durbin-Watson stat	1.851020			

Sources: Computed Using EViews

The short-run error correction model (ECM) derived from the ARDL estimation is specified as:

$\Delta LNGDP_t = -1.194 \cdot ECT_{t-1} - 0.152 \cdot \Delta LNGDP_{t-1} - 0.250 \cdot \Delta LNGDP_{t-2} - 0.195 \cdot \Delta LNEXR_t - 0.383 \cdot \Delta LNEXR_{t-1} - 0.145 \cdot \Delta LNEXR_{t-2} - 1.894 \cdot \Delta LNCO_{2t} - 0.929 \cdot \Delta LNCO_{2t-1} - 2.187 \cdot \Delta LNCO_{2t-2} - 1.202 \cdot \Delta LNIND_t - 0.298 \cdot \Delta LNIND_{t-1} + 0.836 \cdot \Delta LNIND_{t-2} + 0.513 \cdot \Delta LNINFR_t + 0.057 \cdot \Delta LNINTR_t - 0.645 \cdot \Delta LNINTR_{t-1} - 0.580 \cdot \Delta LNINTR_{t-2} - 1.780 \cdot \Delta LNTO_t + 0.116 \cdot \Delta LNTO_{t-1} + 1.247 \cdot \Delta LNTO_{t-2} + \varepsilon_t$

where ECT_{t-1} is the error correction term lagged one period. Table 7 presents the ARDL error correction model (ECM) results, capturing Nigeria's short-run economic growth dynamics and adjustment toward long-run equilibrium. The highly significant and negative error correction term (CointEq(-1) = -1.194, p = 0.0002) confirms a strong and rapid self-correcting mechanism, indicating that deviations from long-run GDP equilibrium are corrected by over 119% within one year. Short-run GDP growth exhibits negative lagged effects, reflecting cyclical adjustments. Exchange rate changes (LNEXR) have consistently significant negative short-run effects, suggesting that currency depreciation immediately constrains growth. Carbon emissions ($LNCO_2$) and industrialization (LNIND) also negatively impact GDP in the short run, though industrialization shows a delayed positive effect at lag 2. Inflation (LNINFR) has a strong immediate positive effect, reflecting demand-driven expansion, while interest rates (LNINTR) show delayed negative effects at lags 1 and 2, consistent with monetary policy transmission. Trade openness (LNTO) exhibits negative immediate effects but positive impacts at lag 2, indicating volatile short-term responses to trade changes.

Vector Error Correction Modelling of Simulated Data

Table 8: Cointegration Test

Unrestricted Cointegration Rank Test (Trace)				
Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.846721	185.4160	125.6154	0.0000
At most 1 *	0.742068	125.4002	95.75366	0.0001
At most 2 *	0.542694	82.03828	69.81889	0.0039
At most 3 *	0.439350	57.00139	47.85613	0.0055
At most 4 *	0.382304	38.48431	29.79707	0.0039
At most 5 *	0.323186	23.06801	15.49471	0.0030
At most 6 *	0.281448	10.57654	3.841466	0.0011

Sources: Computed Using EViews

Table 8 reports the Johansen trace test for cointegration among the variables, assessing the number of long-run equilibrium relationships (cointegrating equations, CEs) in the system. The results show that the Trace statistics for all hypothesized ranks from none up to six exceed their respective 5% critical values, and all associated p-values are highly significant (⩽ 0.0039). This indicates strong evidence that multiple cointegrating relationships exist among the variables, confirming that the Nigerian economy exhibits several interdependent long-run equilibria. In practical terms, this implies that the macroeconomic variables are not drifting independently over time but are linked through stable long-term relationships, necessitating careful modeling to capture these interconnected dynamics.

Table 9: Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized		**Max-Eigen**	**0.05**	
No. of CE(s)	Eigenvalue	Statistic	Critical Value	Prob.**
None *	0.846721	60.01580	46.23142	0.0010
At most 1 *	0.742068	43.36193	40.07757	0.0206
At most 2	0.542694	25.03689	33.87687	0.3825
At most 3	0.439350	18.51708	27.58434	0.4527
At most 4	0.382304	15.41630	21.13162	0.2608
At most 5	0.323186	12.49147	14.26460	0.0935
At most 6 *	0.281448	10.57654	3.841466	0.0011

Sources: Computed Using EViews

Table 9 presents the Johansen Maximum Eigenvalue test for cointegration, which evaluates the null hypothesis that there are at most *r* cointegrating relationships against the alternative of *r+1*. The results show that the Max-Eigen statistics are significant at the 5% level for "None" (r = 0) and "At most 1" (r ≤ 1), with p-values of 0.0010 and 0.0206, respectively, indicating rejection of the null hypothesis in these cases. However, for higher ranks (r ≥ 2), the test statistics are smaller than their critical values, and p-values are not significant, suggesting that there are only two significant cointegrating equations in the system. This implies that among the macroeconomic variables studied, there exist two stable long-run relationships, highlighting the interdependence of these variables in shaping Nigeria's economic dynamics.

Table 10: VECM Long-run Model

Standard Errors in ( ) & t-statistics in [ ]	
Cointegrating Eq:	**CointEq1**
LNGDP(-1)	1.000000
LNEXR(-1)	-0.406814
	(0.18112)
	[-2.24609]
LNCO2(-1)	5.338891

Standard Errors in ( ) & t-statistics in [ ]	
Cointegrating Eq:	**CointEq1**
	(1.73782)
	[3.07218]
LNIND(-1)	14.52571
	(1.72424)
	[8.42443]
LNINFR(-1)	1.184583
	(0.29861)
	[3.96697]
LNINTR(-1)	-3.149646
	(0.34005)
	[-9.26235]
LNTO(-1)	10.75642
	(0.99276)
	[10.8349]
C	-86.73282

Sources: Computed Using EViews

The cointegrating equation is written as CointEq1 = 0. Therefore, the normalized long-run mathematical model is derived by setting the equation to zero and solving for LNGDP:

$$LNGDP(-1) - 0.406814LNEXR(-1) + 5.338891LNCO2(-1) + 14.52571LNIND(-1) + 1.184583LNINFR(-1) - 3.149646LNINTR(-1) + 10.75642LNTO(-1) - 86.73282 = 0$$

Solving for LNGDP as the subject of the formula, we get the long-run equilibrium equation:

$$LNGDP = 86.73282 + 0.407LNEXR - 5.339LNCO2 - 14.526LNIND - 1.185LNINFR + 3.150LNINTR - 10.756LNTO$$

Table 10 presents the long-run cointegrating relationship from the Vector Error Correction Model (VECM), showing how macroeconomic variables jointly determine Nigeria's GDP over time. The coefficients reveal that industrialization (LNIND, 14.53), trade openness (LNTO, 10.76), carbon emissions ($LNCO_2$, 5.34), and inflation (LNINFR, 1.18) have positive and statistically significant long-run effects on GDP, indicating that growth is strongly driven by industrial activity, integration into global trade, energy-intensive production, and moderate inflationary demand. Conversely, the exchange rate (LNEXR, -0.41) and interest rates (LNINTR, -3.15) have significant negative impacts, suggesting that currency depreciation and higher borrowing costs constrain long-term economic growth. All t-statistics are substantial, confirming the robustness of these relationships.

Table 11: VECM Short-run Model

Standard errors in ( ) & t-statistics in [ ]							
Error Correction:	**D(LNGDP)**	**D(LNEXR)**	**D(LNCO2)**	**D(LNIND)**	**D(LNINFR)**	**D(LNINTR)**	**D(LNTO)**
CointEq1	-0.062588	0.161271	-0.046299	-0.035186	-0.417432	0.318732	-0.071348
	(0.10176)	(0.18560)	(0.01687)	(0.02263)	(0.10522)	(0.13792)	(0.02412)
	[-0.61505]	[0.86890]	[-2.74458]	[-1.55486]	[-3.96705]	[2.31102]	[-2.95851]
D(LNGDP(-1))	-0.300444	0.049943	-0.025197	-0.006988	0.655158	-0.375610	-0.096294
	(0.27911)	(0.50908)	(0.04627)	(0.06207)	(0.28861)	(0.37829)	(0.06615)
	[-1.07643]	[0.09810]	[-0.54456]	[-0.11258]	[2.27001]	[-0.99292]	[-1.45576]
D(LNGDP(-2))	-0.391215	-0.403642	0.045380	-0.029968	0.173202	0.167514	-0.018501
	(0.25260)	(0.46073)	(0.04188)	(0.05617)	(0.26120)	(0.34236)	(0.05986)
	[-1.54876]	[-0.87610]	[1.08370]	[-0.53349]	[0.66310]	[0.48930]	[-0.30905]
D(LNEXR(-1))	-0.400846	-0.395516	-0.026971	-0.020112	-0.414207	-0.233786	-0.031151
	(0.15905)	(0.29009)	(0.02637)	(0.03537)	(0.16446)	(0.21556)	(0.03769)
	[-2.52031]	[-1.36342]	[-1.02295]	[-0.56862]	[-2.51856]	[-1.08455]	[-0.82644]
D(LNEXR(-2))	-0.123006	-0.197375	-0.026200	-0.025820	-0.325389	-0.055444	-0.002114
	(0.14181)	(0.25864)	(0.02351)	(0.03154)	(0.14663)	(0.19219)	(0.03361)
	[-0.86743]	[-0.76311]	[-1.11453]	[-0.81876]	[-2.21905]	[-0.28848]	[-0.06292]
D(LNCO2(-1))	-0.289265	-4.430759	-0.427629	-0.044511	1.199520	-1.341768	-0.059525
	(1.24145)	(2.26433)	(0.20580)	(0.27608)	(1.28372)	(1.68258)	(0.29421)
	[-0.23301]	[-1.95676]	[-2.07784]	[-0.16123]	[0.93441]	[-0.79745]	[-0.20232]
D(LNCO2(-2))	-1.476819	-0.775865	-0.430028	0.348648	1.773013	-3.244593	-0.279921
	(1.31758)	(2.40319)	(0.21842)	(0.29301)	(1.36244)	(1.78576)	(0.31226)
	[-1.12086]	[-0.32285]	[-1.96877]	[1.18989]	[1.30135]	[-1.81693]	[-0.89645]
D(LNIND(-1))	1.018464	0.261742	0.730291	-0.081721	5.957072	-6.229617	0.184830
	(1.87670)	(3.42298)	(0.31111)	(0.41735)	(1.94060)	(2.54355)	(0.44476)

Standard errors in ( ) & t-statistics in [ ]							
Error Correction:	**D(LNGDP)**	**D(LNEXR)**	**D(LNCO2)**	**D(LNIND)**	**D(LNINFR)**	**D(LNINTR)**	**D(LNTO)**
	[0.54269]	[0.07647]	[2.34734]	[-0.19581]	[3.06970]	[-2.44918]	[0.41557]
D(LNIND(-2))	1.565560	-1.378105	0.363785	0.129052	4.552704	-1.368344	0.767105
	(1.64120)	(2.99345)	(0.27207)	(0.36498)	(1.69708)	(2.22437)	(0.38895)
	[0.95391]	[-0.46037]	[1.33708]	[0.35359]	[2.68266]	[-0.61516]	[1.97224]
D(LNINFR(-1))	-0.275471	-0.316892	0.022175	0.059212	-0.553623	-0.225426	0.105979
	(0.26360)	(0.48079)	(0.04370)	(0.05862)	(0.27258)	(0.35727)	(0.06247)
	[-1.04503]	[-0.65910]	[0.50744]	[1.01009]	[-2.03107]	[-0.63097]	[1.69645]
D(LNINFR(-2))	-0.060270	0.178583	-0.025729	0.087152	-0.120591	-0.494136	-0.021880
	(0.25476)	(0.46466)	(0.04223)	(0.05665)	(0.26343)	(0.34528)	(0.06037)
	[-0.23658]	[0.38433]	[-0.60923]	[1.53834]	[-0.45777]	[-1.43112]	[-0.36241]
D(LNINTR(-1))	-0.663561	0.016733	-0.138293	-0.156995	-1.355604	0.116694	-0.053020
	(0.30268)	(0.55207)	(0.05018)	(0.06731)	(0.31299)	(0.41023)	(0.07173)
	[-2.19230]	[0.03031]	[-2.75610]	[-2.33239]	[-4.33121]	[0.28446]	[-0.73914]
D(LNINTR(-2))	0.009639	0.219783	-0.186351	-0.083415	-0.311435	-0.410686	-0.174249
	(0.30269)	(0.55209)	(0.05018)	(0.06731)	(0.31300)	(0.41024)	(0.07173)
	[0.03185]	[0.39809]	[-3.71372]	[-1.23921]	[-0.99501]	[-1.00108]	[-2.42907]
D(LNTO(-1))	-0.706589	1.273133	0.332465	0.464517	1.327463	-1.390297	-0.058682
	(1.15319)	(2.10335)	(0.19117)	(0.25645)	(1.19246)	(1.56296)	(0.27330)
	[-0.61272]	[0.60529]	[1.73908]	[1.81133]	[1.11322]	[-0.88953]	[-0.21472]
D(LNTO(-2))	0.457275	0.414272	0.241221	-0.179816	-1.701406	1.688329	0.125021
	(0.92707)	(1.69091)	(0.15369)	(0.20616)	(0.95863)	(1.25648)	(0.21971)
	[0.49325]	[0.24500]	[1.56957]	[-0.87220]	[-1.77482]	[1.34369]	[0.56903]
C	-0.055178	-0.102068	0.011257	0.009274	-0.018909	-0.038271	0.026034
	(0.17317)	(0.31585)	(0.02871)	(0.03851)	(0.17907)	(0.23470)	(0.04104)
	[-0.31864]	[-0.32315]	[0.39211]	[0.24083]	[-0.10560]	[-0.16306]	[0.63435]
R-squared	0.742810	0.593771	0.803399	0.638662	0.779734	0.557815	0.788557
Adj. R-squared	0.485619	0.187542	0.606797	0.277325	0.559467	0.115630	0.577114
Sum sq. resids	13.26209	44.11971	0.364470	0.655868	14.18064	24.36149	0.744869
S.E. equation	0.940287	1.715026	0.155878	0.209104	0.972305	1.274402	0.222841
F-statistic	2.888169	1.461664	4.086434	1.767495	3.539957	1.261498	3.729411
Log likelihood	-30.82639	-49.45735	24.88402	15.77753	-31.86440	-40.25185	13.80518
Akaike AIC	3.021057	4.223055	-0.573163	0.014353	3.088026	3.629152	0.141601
Schwarz SC	3.761180	4.963177	0.166960	0.754475	3.828148	4.369274	0.881724
Mean dependent	-0.006836	-0.011413	0.004446	0.011990	-0.023363	0.037063	0.011446
S.D. dependent	1.311046	1.902699	0.248586	0.245975	1.464918	1.355156	0.342675
Sources: Computed Using EViews		5.26E-07					
Determinant resid covariance (dof adj.)							
Determinant resid covariance		3.27E-09					
Log likelihood		-5.040725					
Akaike information criterion		8.002627					
Schwarz criterion		13.50729					
Number of coefficients		119					

The short-run error correction model for the equation is specified as follows: $\Delta LNGDP_t = -0.0626 \cdot ECT_{t-1} - 0.3004 \cdot \Delta LNGDP_{t-1} - 0.3912 \cdot \Delta LNGDP_{t-2} - 0.4008 \cdot \Delta LNEXR_{t-1} - 0.1230 \cdot \Delta LNEXR_{t-2} - 0.2893 \cdot \Delta LNCO_{2t-1} - 1.4768 \cdot \Delta LNCO_{2t-2} + 1.0185 \cdot \Delta LNIND_{t-1} + 1.5656 \cdot \Delta LNIND_{t-2} - 0.2755 \cdot \Delta LNINFR_{t-1} - 0.0603 \cdot \Delta LNINFR_{t-2} - 0.6636 \cdot \Delta LNINTR_{t-1} + 0.0096 \cdot \Delta LNINTR_{t-2} - 0.7066 \cdot \Delta LNTO_{t-1} + 0.4573 \cdot \Delta LNTO_{t-2} - 0.0552 + \varepsilon_t$ where ECT_{t-1} is the error correction term lagged one period.

Table 11 presents the short-run dynamics of Nigeria's macroeconomic variables using the Vector Error Correction Model (VECM), capturing both immediate and lagged responses as the system adjusts toward long-run equilibrium. The error correction term (CointEq1) for GDP is negative but small and statistically insignificant (-0.0626, t = -0.615), indicating that short-run deviations from the long-run path are largely driven by other contemporaneous factors rather than rapid self-correction. Inflation (LNINFR, -0.4174, t = -3.967) and interest rates (LNINTR, 0.3187, t = 2.311) emerge as key short-run stabilizers, reflecting that monetary and price adjustments play critical roles in restoring equilibrium. Exchange rate changes (LNEXR) exert immediate contractionary effects on GDP and other variables, while past lags of GDP, industrialization, carbon emissions, and trade openness exhibit complex and sometimes opposing effects, highlighting cyclical patterns and delayed impacts in the economy.

RESULTS AND DISCUSSION

The results collectively paint a picture of an economy characterized by pronounced volatility alongside gradual structural transformation. The descriptive statistics and graphical analysis show that Nigeria's GDP, exchange rate, inflation, and interest rate are highly unstable, reflecting sensitivity to domestic policy shifts and external shocks, while carbon emissions, industrialization, and trade openness evolve more smoothly over time. Log transformation improves comparability and variance stability, yet persistent skewness and heavy tails in key macroeconomic variables especially GDP, exchange rate, and inflation—underscore the presence of extreme events and structural breaks. Unit root tests confirm stationarity at levels for all log-transformed series, validating the econometric framework and ensuring that subsequent long-run and short-run estimations are reliable and not driven by spurious trends.

Long-run estimations using FMOLS, ARDL bounds testing, and Johansen cointegration techniques reveal strong and multifaceted equilibrium relationships among the variables. Evidence from the bounds test and Johansen procedures confirms the existence of stable long-run linkages, though the number and nature of cointegrating relationships vary by method, reflecting the complexity of Nigeria's macroeconomic structure. Inflation consistently emerges as a significant long-run driver of GDP, suggesting that moderate inflation may be growth-enhancing in a demand-driven economy. However, the role of industrialization and trade openness is mixed across models: while FMOLS indicates inefficiencies that dampen growth, the VECM long-run results show strong positive contributions, implying that when structural and external linkages are well-coordinated, these sectors can substantially boost output. Exchange rate depreciation and high interest rates generally constrain long-run growth, highlighting the costs of macroeconomic instability and tight financial conditions.

In the short run, ARDL and VECM results emphasize rapid adjustment dynamics and pronounced cyclical behavior. The significant and negative error correction terms in the ARDL-based models indicate fast convergence toward long-run equilibrium, meaning that shocks are corrected relatively quickly, albeit sometimes overshooting. Short-run growth is particularly sensitive to exchange rate movements, inflation, and monetary policy, with depreciation and higher interest rates exerting contractionary effects, while inflation often stimulates output through demand-pull channels. Environmental and industrial variables display delayed and sometimes reversing impacts, reflecting trade-offs between growth, sustainability, and structural adjustment. Overall, the findings suggest that Nigeria's growth process is shaped by strong long-run interdependence among macroeconomic variables, but short-run outcomes remain highly vulnerable to policy shocks, financial conditions, and external disturbances—underscoring the need for coordinated, stability-oriented macroeconomic management alongside long-term structural reforms.

CONCLUSION

The empirical results indicate that Nigeria's economic growth is driven by complex and interdependent macroeconomic dynamics characterized by both long-run equilibrium relationships and pronounced short-run volatility. While inflation appears to support growth in both the long and short run—suggesting a demand-driven economy—exchange rate instability and high interest rates consistently constrain output, especially in the long term. Industrialization, trade openness, and carbon emissions show mixed effects across models, implying that their growth impacts depend heavily on structural efficiency, policy coordination, and sustainability considerations. The existence of multiple cointegrating relationships and strong adjustment mechanisms confirms that Nigeria's macroeconomic variables move together over time, but the economy remains highly vulnerable to policy shocks and external disturbances.

RECOMMENDATIONS

i. **Strengthen Macroeconomic Stability:** Prioritize exchange rate management and inflation control to reduce volatility that undermines investment and long-term growth.
ii. **Adopt Growth-Supportive Monetary Policy:** Balance interest rate policy to support productive investment while maintaining price stability and financial sector confidence.
iii. **Improve the Quality of Industrialization:** Focus on efficiency, value addition, and technology adoption in the industrial sector to ensure that industrial growth translates into real GDP gains.
iv. **Promote Sustainable Growth Strategies:** Integrate environmental considerations into growth policies by encouraging cleaner production and energy efficiency to mitigate the long-run negative effects of carbon emissions.
v. **Optimize Trade Openness:** Diversify exports and strengthen domestic productive capacity to ensure that trade integration supports growth rather than exposing the economy to external shocks.
vi. **Enhance Policy Coordination:** Align fiscal, monetary, industrial, and environmental policies to reinforce long-run equilibrium and minimize destabilizing short-run adjustments.

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