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APPLICATIONS OF INVERSE WEIBULL RAYLEIGH DISTRIBUTION TO FAILURE RATES AND VINYL CHLORIDE DATA SETS

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ABSTRACT

In this work, a new three parameter distribution called the Inverse Weibull Rayleigh distribution is proposed. Some of its statistical properties were presented. The PDF plot of Inverse Weibull Rayleigh distribution showed that it is good for modeling positively skewed and symmetrical datasets. The plot of the hazard function showed that the proposed distribution can fit datasets with bathtub shape. Method of maximum likelihood estimation was employed to estimate the parameters of the distribution, the estimators of the parameters of Inverse Weibull Rayleigh distribution is asymptotically unbiased and asymptotically efficient from the result of the simulation carried out. Applying the new distribution to a positively skewed Vinyl Chloride data set shows that the distribution performs better than Rayleigh, Generalized Rayleigh, Weibull Rayleigh, Inverse Weibull, Inverse Weibull Weibull, Inverse Weibull Inverse Exponential and Inverse Weibull Pareto distribution in fitting the data as it has the smallest AIC value. Also, applying the new distribution to a negatively skewed bathtub shape failure rates data shows that the distribution is a competitive model after Weibull Rayleigh and Inverse Weibull Weibull distributions in fitting the data because it has the third least AIC value.

Keywords: Inverse Weibull-G family, Rayleigh distribution, Inverse Weibull Rayleigh distribution, Maximum likelihood estimation, Applications.

INTRODUCTION

Many researchers and statisticians nowadays are interested in generating families of distribution thereby creating a new distribution that will fit a data better. The flexibility of a distribution can be enhanced by substituting it into a generalized family, by so doing, extra shape parameter or parameters, and scale parameter orparameters are added to the new distribution. Rayleigh distribution is an important distribution that is applicable to many known fields. According to Merovci and Elbatal (2015), the Rayleigh distribution has a wide range of applications including
life testing experiments, applied statistics, reliability analysis and clinical studies. Therefore, the need to add more flexibility to Rayleigh distribution in order to encourage more applicability. Tahir and Cordeiro (2016) stated that some well-known classical distributions such as Weibull, Rayleigh, Exponential and Gamma are limited in their characteristics and are unable to show wide flexibility. Rayleigh distribution has only one parameter, as a result, it suffers from lack of flexibility in order to model failure rates and vinyl chloride data sets that are positively skewed, symmetrical or assume bathtub shape, upside down bathtub shape or any other different shapes properly.

It is desirable to analyze data the way they were collected in their original form using the best probability distribution that best describe it instead of changing the original form of the data by any method of transformation. As a result of this, many researchers have worked on some generating family. Some popular known generating family are: The Beta-G by Eugene *et al.,* (2002),Cordeiro and de Castro (2011) introduced Kumaraswammy generalized family, the Log-Gamma G family by Amini *et al.,* (2014), Odd Frechet-G by Haq and Elgarhy (2018), Elbatal *et al.,* (2018) proposed a new Alpha power transformation family of distributions, Hassan and Nassr (2018) introduced Inverse Weibull-G family among others. Hassan and Nassr (2018) added that Inverse Weibull distribution is an important distribution which can be used to analyze lifetime data.

In this study, we intend to use the Inverse Weibull-G family to extend the Rayleigh distribution thereby increasing its flexibility in order to model data on failure rates and Vinyl Chloride.

MATERIALS AND METHOD

The Rayleigh distribution and Inverse Weibull G family

The cumulative distribution function (CDF) and the probability density function (PDF) of the Rayleigh distribution are given by

$$G_R(x,\sigma) = 1 - e^{-\frac{x^2}{2\sigma^2}} \tag{1}$$

$$g_R(x,\sigma) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} \tag{2}$$

where x > 0 and $\sigma > 0$ and σ is the scale parameter.

Hassan and Nassr (2018) introduced the Inverse Weibull generator of distributions. The cumulative density function and the probability distribution function are given by equations (3) and (4) respectively.

$$F(x) = e^{-\theta^{\beta}\left(\frac{G(x)}{1-G(x)}\right)^{-\beta}} \tag{3}$$

$$f(x) = \beta\theta^{\beta} \frac{g(x)G(x)^{-\beta-1}}{[1-G(x)]^{-\beta+1}} e^{-\theta^{\beta}\left(\frac{G(x)}{1-G(x)}\right)^{-\beta}} \tag{4}$$
$$x \,\epsilon\, \square, \theta, \beta > 0$$

Where θ and β are the scale and the shape parameters respectively, and G(.) is the CDF of any baseline distribution.

The Inverse Weibull Rayleigh (IWR) distribution

By inserting (1) in (3), then the CDF of IWR is

$$F_{IWR}(x) = e^{-\theta^{\beta}\left[\frac{\left(1-e^{-\frac{x^2}{2\sigma^2}}\right)}{1-\left(1-e^{-\frac{x^2}{2\sigma^2}}\right)}\right]^{-\beta}} , \sigma, \theta, \beta > 0 \tag{5}$$

The corresponding PDF to (5) is

$$f_{IWR}(x) = \frac{x\beta\theta^{\beta}}{\sigma^2}\left(1 - e^{-\frac{x^2}{2\sigma^2}}\right)^{-\beta-1}\left(e^{-\frac{x^2\beta}{2\sigma^2}}\right) e^{-\theta^{\beta}\left(e^{\frac{x^2}{2\sigma^2}}-1\right)^{-\beta}} \tag{6}$$

The survival function and the hazard function are respectively given by

$$S(x) = 1 - e^{-\theta^{\beta}\left[\frac{\left(1-e^{-\frac{x^2}{2\sigma^2}}\right)}{\left(1-\left(1-e^{-\frac{x^2}{2\sigma^2}}\right)\right)}\right]^{-\beta}} \tag{7}$$

$$H(x) = \frac{\frac{x\beta\theta^{\beta}}{\sigma^2}\left(1-e^{-\frac{x^2}{2\sigma^2}}\right)^{-\beta-1}\left(e^{-\frac{x^2\beta}{2\sigma^2}}\right)e^{-\theta^{\beta}\left(e^{\frac{x^2}{2\sigma^2}}-1\right)^{-\beta}}}{1-e^{-\theta^{\beta}\left[\frac{\left(1-e^{-\frac{x^2}{2\sigma^2}}\right)}{\left(1-\left(1-e^{-\frac{x^2}{2\sigma^2}}\right)\right)}\right]^{-\beta}}} \tag{8}$$

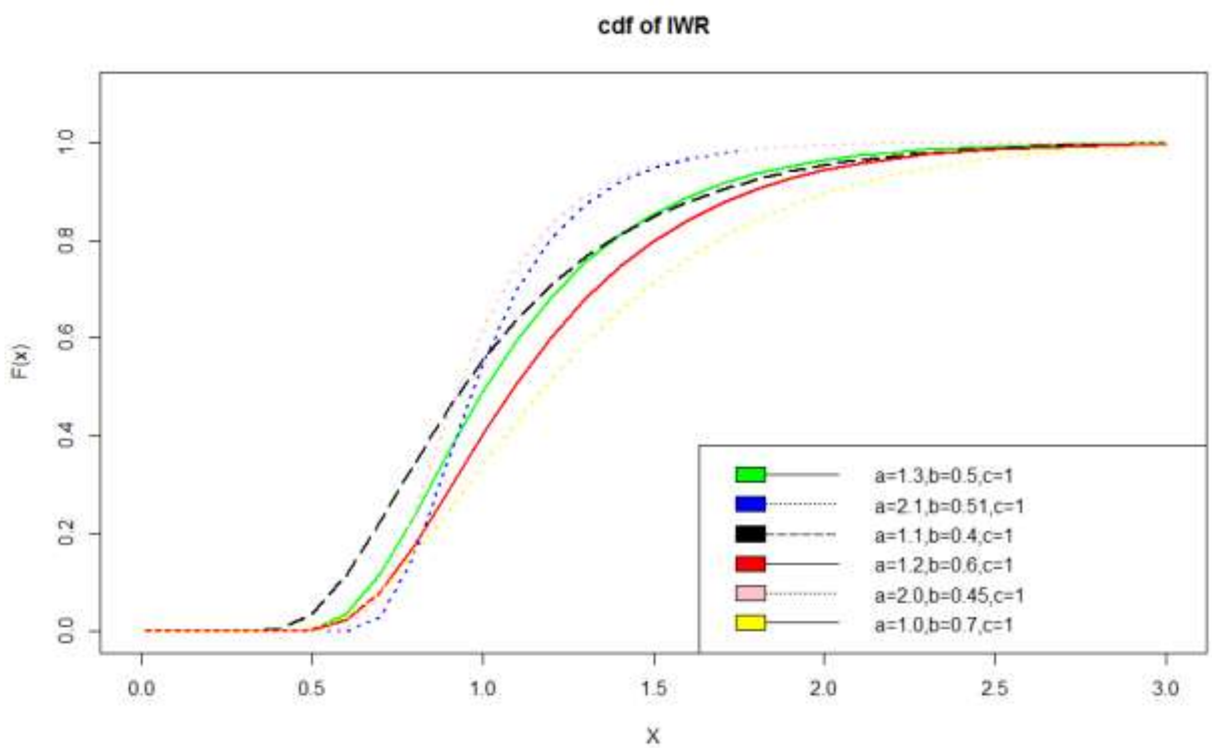

Figure 1: CDF Plot of Inverse Weibull Rayleigh distribution for different values of parameters

The CDF indicates that the distribution has an increasing upward trend that approaches the value one as required for any CDF plot.

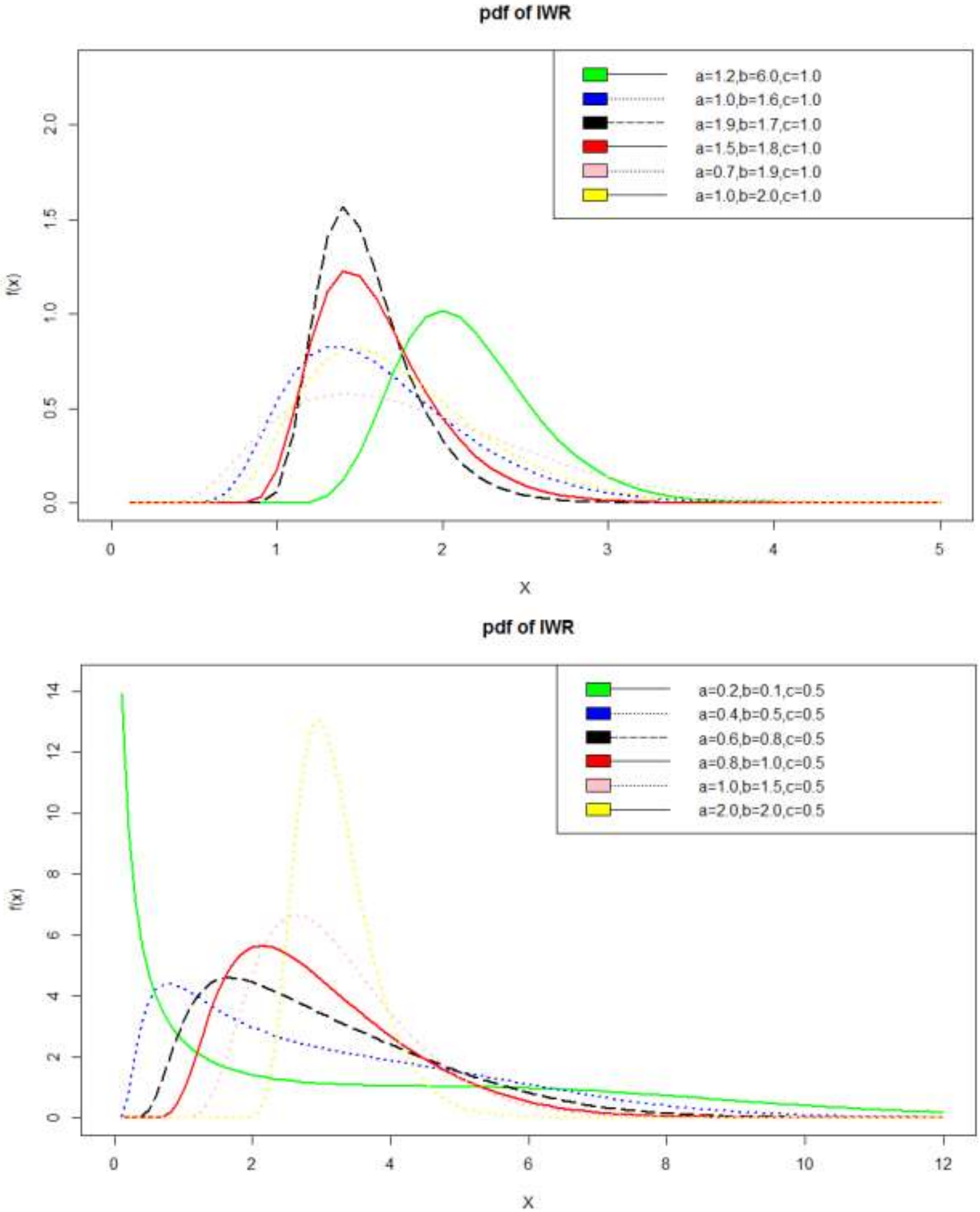

Figure 2: PDF Plot of Inverse Weibull Rayleigh distribution for different values of parameters

The PDF plots above show that the proposed distribution is right skewed, symmetrical and unimodal in nature.

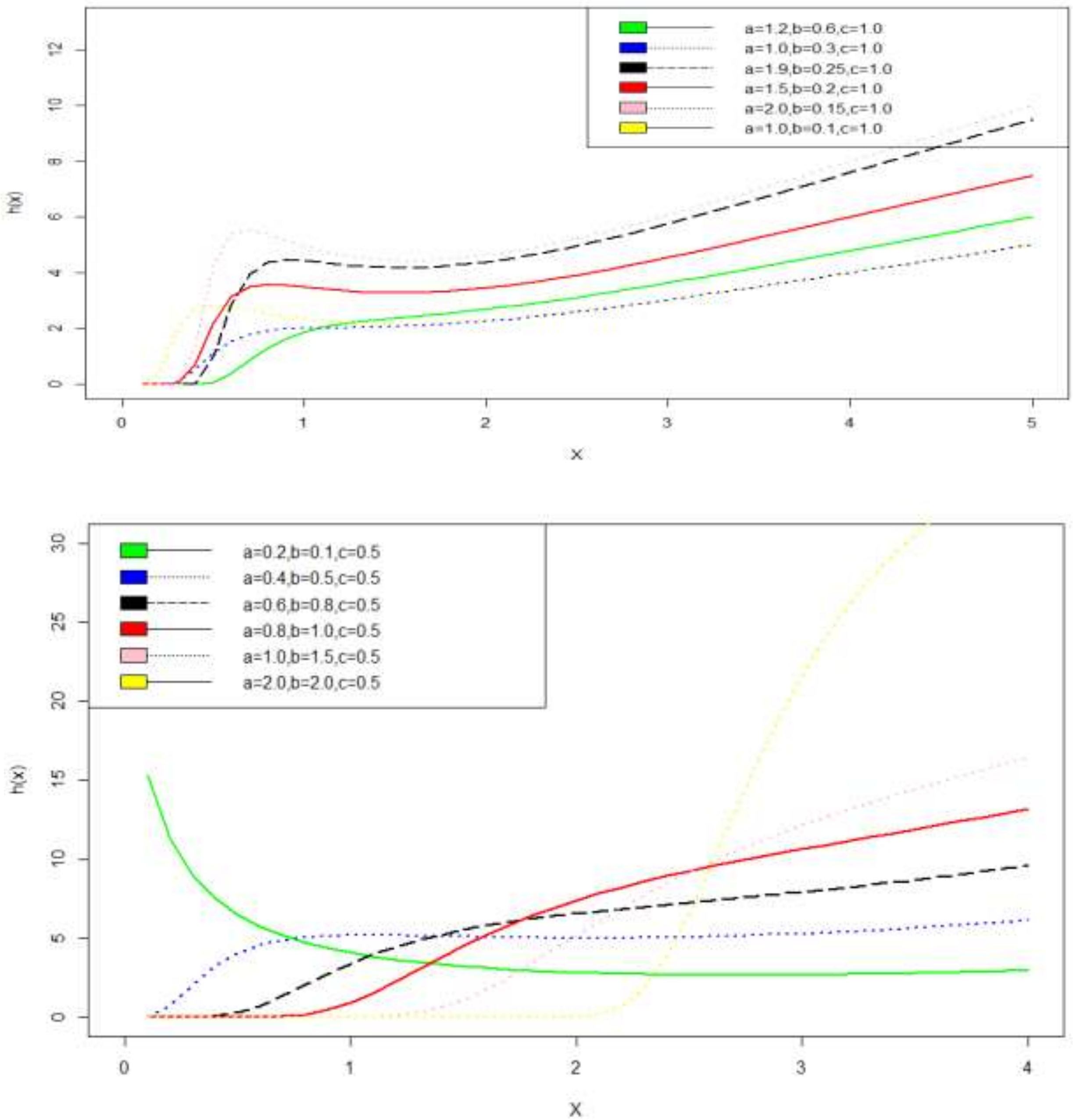

Figure 3: Plots of Hazard function of Inverse Weibull Rayleigh distribution for different values of parameters
It is evident that the hazard function can assume a bathtub shape and modified upside down bathtub shape for different values of parameters, and it increases as x increases.

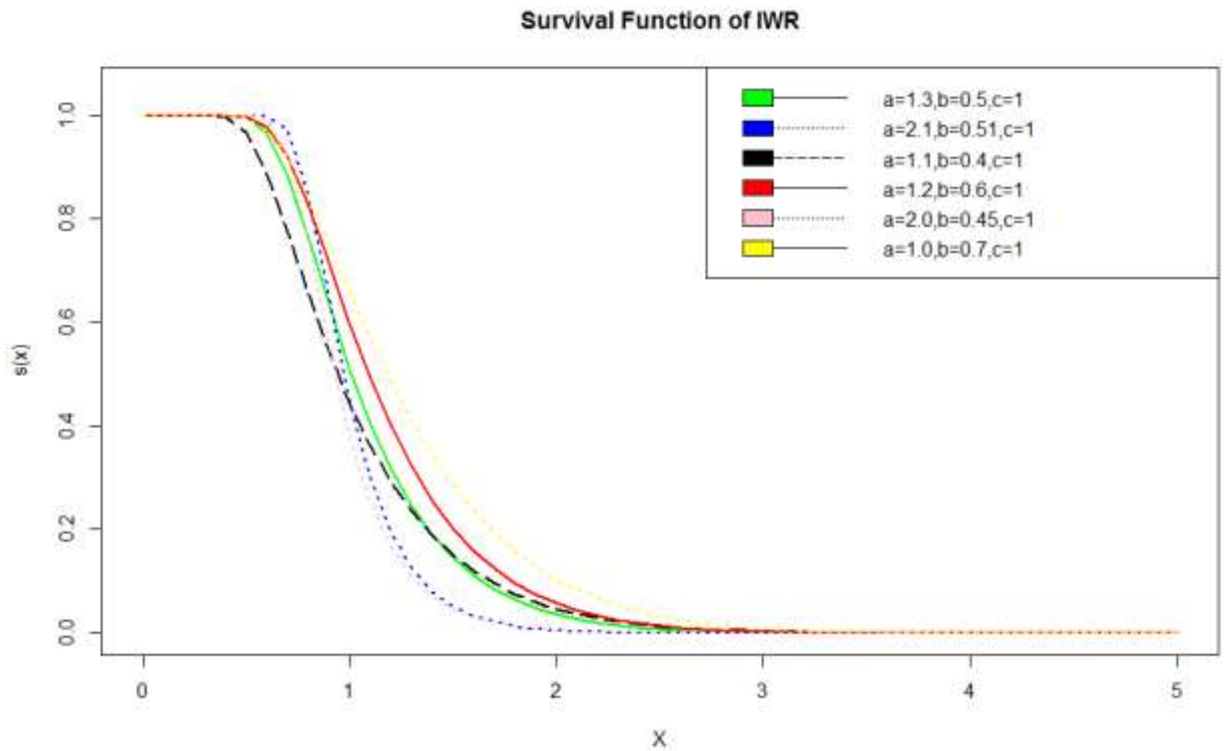

Figure 4: Plot of survival function of Inverse Weibull Rayleigh distribution for different values of parameters
It is observed from the survival function graph that the survival function equals to one at the origin and it decreases as x increases. It tends to zero as x gets larger.

Statistical Properties of the Inverse Weibull Rayleigh distribution
In this section, we derived the statistical properties of the distribution, including the quantile function, moments, moment generating function, probability weighted moments and Renyi entropy. The distribution of the order statistics is as well defined.
Quantile function
The quantile function of Inverse Weibull Rayleigh distribution, say$x$ = Q(q) can be obtained by inverting the cumulative density function ofthe Inverse Weibull Rayleigh distribution. The quantile function is obtainedusing the relation:$F_{IWR}(x;\theta,\beta,\sigma) = q$ and then making x the subject.

$$e^{-\theta^{\beta}\left(e^{\frac{x^2}{2\sigma^2}}-1\right)^{-\beta}} = q \tag{9}$$

$$x = Q(q) = \left[2\sigma^2\left(log\left(1+\theta(-logq)^{-\frac{1}{\beta}}\right)\right)\right]^{\frac{1}{2}} \tag{10}$$

The median of x is simply obtained by substituting q = 0.5
Skewness and Kurtosis based on quantile
Skewness measures the degree of asymmetry while kurtosis measures the degree of tail heaviness. The Bowley's (1962) measure of skewness and Moor's (1988) measure of kurtosis are respectively given by

$$B = \frac{Q\left(\frac{3}{4}\right)-2Q\left(\frac{1}{2}\right)+Q\left(\frac{1}{4}\right)}{Q\left(\frac{3}{4}\right)-Q\left(\frac{1}{4}\right)} \tag{11}$$

$$M = \frac{Q\left(\frac{7}{8}\right)-Q\left(\frac{5}{8}\right)+Q\left(\frac{3}{8}\right)-Q\left(\frac{1}{8}\right)}{Q\left(\frac{6}{8}\right)-Q\left(\frac{2}{8}\right)} \tag{12}$$

Where Q(.) denotes the quantile function
Useful expansion

$$f_{IWR}(x) = \frac{x\beta\theta^{\beta}}{\sigma^2}\left(1-e^{-\frac{x^2}{2\sigma^2}}\right)^{-\beta-1}\left(e^{-\frac{x^2\beta}{2\sigma^2}}\right)e^{-\theta^{\beta}\left(e^{\frac{x^2}{2\sigma^2}}-1\right)^{-\beta}}$$

$$e^{-\theta^{\beta}}\left(e^{\frac{x^2}{2\sigma^2}}-1\right)^{-\beta}=\sum_{i=0}^{\infty}\frac{(-1)^i}{i!}\theta^{i\beta}\left[\frac{1-e^{-\frac{x^2}{2\sigma^2}}}{e^{-\frac{x^2}{2\sigma^2}}}\right]^{-i\beta}=\sum_{i=0}^{\infty}\frac{(-1)^i}{i!}\theta^{i\beta}\left(1-e^{-\frac{x^2}{2\sigma^2}}\right)^{i\beta}\left(e^{-\frac{x^2}{2\sigma^2}}\right)^{i\beta}$$

$$=\sum_{i=0}^{\infty}\frac{x\beta\theta^{\beta(i+1)}(-1)^i}{i!\sigma^2}\left(1-e^{-\frac{x^2}{2\sigma^2}}\right)^{-(i\beta+\beta+1)}\left(e^{-\frac{x^2}{2\sigma^2}}\right)^{i\beta+\beta}$$

Using binomial theorem for negative powers

$$\left(1-e^{-\frac{x^2}{2\sigma^2}}\right)^{-(i\beta+\beta+1)}=\sum_{i=0}^{\infty}\binom{\beta+i\beta+j}{j}\left(e^{-\frac{x^2}{2\sigma^2}}\right)^{j}$$

$$f_{IWR(x;\sigma,\theta,\beta)}=\sum_{i-0}^{\infty}\sum_{j-0}^{\infty}\frac{x\beta\theta^{\beta(i+1)}(-1)^i}{i!\sigma^2}\binom{\beta+i\beta+j}{j}\left(e^{-\frac{x^2}{2\sigma^2}}\right)^{k},k=i\beta+\beta+j$$

Moment generating function

The moment generating function of an Inverse Weibull Rayleigh Random variable x can be obtained as:

$$M_x(t)=E(e^{tx})=\int_{-\infty}^{\infty}e^{tx}f_{IWR}(x)dx \tag{13}$$

$$\int_0^{\infty}e^{tx}\left(\frac{x\beta\theta^{\beta}}{\sigma^2}\left(1-e^{-\frac{x^2}{2\sigma^2}}\right)^{-\beta-1}\left(e^{-\frac{x^2\beta}{2\sigma^2}}\right)e^{-\theta^{\beta}\left(e^{\frac{x^2}{2\sigma^2}}-1\right)^{-\beta}}\right)dx \tag{14}$$

$$M_x(t)=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{n=0}^{\infty}\frac{\beta\theta^{\beta(i+1)}(-1)^i t^n\sigma^n 2^{\frac{n}{2}}}{i!n!k^{\frac{n+2}{2}}}\binom{\beta+i\beta+j}{j}\left(\frac{n}{2}\right)! \tag{15}$$

Where $k=i\beta+\beta+j$

Moments

The r[th] moment about the origin of an Inverse Weibull Rayleigh random variable x can be calculated from:

$$\mu_r'=\int_0^{\infty}x^r f_{IWR}(x)dx \tag{16}$$

$$=\int_0^{\infty}x^r\left(\frac{x\beta\theta^{\beta}}{\sigma^2}\left(1-e^{-\frac{x^2}{2\sigma^2}}\right)^{-\beta-1}\left(e^{-\frac{x^2\beta}{2\sigma^2}}\right)e^{-\theta^{\beta}\left(e^{\frac{x^2}{2\sigma^2}}-1\right)^{-\beta}}\right)dx \tag{17}$$

$$\mu_r'=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\frac{\beta\theta^{\beta(i+1)}(-1)^i\sigma^r 2^{\frac{r}{2}}}{i!k^{\frac{r+2}{2}}}\binom{\beta+i\beta+j}{j}\left(\frac{r}{2}\right)! \tag{18}$$

Where $k=i\beta+\beta+j$

Probability weighted moments

For a random variable X, the PWMs denoted by $\tau_{r,s}$ can be calculatedthrough the following relation.

$$\tau_{r,s}=E(X^r(F_{IWR}(x))^s)=\int_{-\infty}^{\infty}x^r f_{IWR}(x)((F_{IWR}(x))^s dx \tag{19}$$

$$\tau_{r,s}=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\frac{\beta\theta^{\beta(i+1)}(-1)^i(s+1)^i\sigma^r 2^{\frac{r}{2}}}{i!k^{\frac{r+2}{2}}}\binom{\beta+i\beta+j}{j}\left(\frac{r}{2}\right)! \tag{20}$$

Order statistics

Suppose, X_1, X_2, X_3 …., X_s is a random sample of size s from an Inverse Weibull Rayleigh (IWR) distribution, and let $X_{1:S}, X_{2:S}, X_{3:S}$ …., $X_{s:s}$ be thecorresponding order statistics, then the probability density function of thei[th] order statistics can be expressed as:

$$f_{i,s}(x) = \frac{s!}{(i-1)!(s-)!} f_{IWR}(x)[F_{IWR}(\times)]^{i-1}[1 - F_{IWR}(\times)]^{s-i} \quad (21)$$

$$f_{i,s}(x) = \frac{s!}{(i-1)!\,(s-)!} \frac{x\beta\theta^{\beta}}{\sigma^2}\left(1 - e^{-\frac{x^2}{2\sigma^2}}\right)^{-\beta-1} e^{-\frac{x^2\beta}{2\sigma^2}}$$

$$e^{-\theta^{\beta}\left(e^{\frac{x^2}{2\sigma^2}}-1\right)^{-\beta}}\left[e^{-\theta^{\beta}\left(e^{\frac{x^2}{2\sigma^2}}-1\right)^{-\beta}}\right]^{i-1}\left[1 - e^{-\theta^{\beta}\left(e^{\frac{x^2}{2\sigma^2}}-1\right)^{-\beta}}\right]^{s-1} \quad (22)$$

The expression for PDF of the smallest order statistics $X_{1:s}$and largestorder statistics $X_{s:s}$ of IWR distribution are gotten by letting i = 1 andi = s respectively in equation (15) and are given below:

$$f_{1:s}(x) = \frac{sx\beta\theta^{\beta}}{\sigma^2}\left(1 - e^{-\frac{x^2}{2\sigma^2}}\right)^{-\beta-1} e^{-\frac{x^2\beta}{2\sigma^2}}\left[e^{-\theta^{\beta}\left(e^{\frac{x^2}{2\sigma^2}}-1\right)^{-\beta}}\right]\left[1 - e^{-\theta^{\beta}\left(e^{\frac{x^2}{2\sigma^2}}-1\right)^{-\beta}}\right]^{s-1} \quad (23)$$

$$f_{s:s}(x) = \frac{sx\beta\theta^{\beta}}{\sigma^2}\left(1 - e^{-\frac{x^2}{2\sigma^2}}\right)^{-\beta-1} e^{-\frac{x^2\beta}{2\sigma^2}}\left[e^{-\theta^{\beta}\left(e^{\frac{x^2}{2\sigma^2}}-1\right)^{-\beta}}\right]\left[e^{-\theta^{\beta}\left(e^{\frac{x^2}{2\sigma^2}}-1\right)^{-\beta}}\right]^{s-1} \quad (24)$$

Renyi Entropy

The Renyi entropy of Inverse Weibull Rayleigh distribution is calculated by:

$$l_{\delta}(x) = \frac{1}{1-\delta} log \int_{-\infty}^{\infty}((f_{IWR}(x))^{\delta}dx, \;\; \delta > 0 \; and \; \delta \neq 1 \quad (25)$$

$$\frac{1}{1-\delta} log\left[\left(\frac{\beta\theta^{\beta}}{\sigma^2}\right)^{\delta} \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \frac{(-1)^i 2^{\frac{\delta-1}{2}} \sigma^{\delta+1}\delta^i\theta^i\beta}{i!k^{\frac{\delta+1}{2}}} \binom{i\beta + \delta\beta + \delta + j - 1}{j}\left(\frac{\delta-1}{2}\right)!\right] \quad (26)$$

Maximum likelihood Estimators

The maximum likelihood method of estimation will be employed in estimating the parameters of the model. Suppose x_1, x_2, x_3,….. x_n is a random sample from Inverse Weibull Rayleigh distribution. The likelihood function is given by:

$$L(x_1, x_2, x_3 \; …., x_n ; \theta, \beta, \sigma) = \prod_{i=1}^{n} f_{IWR}(x_i; \theta, \beta, \sigma)$$

$$\prod_{i=1}^{n}\left(\frac{x\beta\theta^{\beta}}{\sigma^2}\left(1 - e^{-\frac{x^2}{2\sigma^2}}\right)^{-\beta-1}\left(e^{-\frac{x^2\beta}{2\sigma^2}}\right) e^{-\theta^{\beta}\left(e^{\frac{x^2}{2\sigma^2}}-1\right)^{-\beta}}\right) \quad (27)$$

$$logL(.) = \sum_{i=1}^{n} logx_i + nlog\beta + n\beta(log\theta) - 2nlog\sigma - (\beta + 1)\sum_{i=1}^{n} log\left(1 - e^{-\frac{x_i^2}{2\sigma^2}}\right) - \frac{\beta}{2\sigma^2}\sum_{i=1}^{n} x_i{}^2 - \theta^{\beta}\sum_{i=1}^{n}\left(e^{\frac{x_i^2}{2\sigma^2}} - 1\right)^{-\beta} \quad (28)$$

$$\frac{\partial logL(.)}{\partial\theta} = \frac{n\beta}{\theta} - \beta\theta^{\beta-1}\sum_{i=1}^{n}\left(e^{\frac{x_i^2}{2\sigma^2}} - 1\right)^{-\beta} \quad (29)$$

$$\frac{\partial logL(.)}{\partial\beta} = \frac{n}{\beta} + nlog\theta - 2\sum_{i-1}^{n} log\left(1 - e^{-\frac{x_i^2}{2\sigma^2}}\right) - \frac{\sum_{i=1}^{n} x_i^2}{2\sigma^2} + \theta^{\beta}\sum_{i=1}^{n}\left(e^{\frac{x_i^2}{2\sigma^2}} - 1\right)^{-\beta} log\left(e^{\frac{x_i^2}{2\sigma^2}} - 1\right) + \theta^{\beta} log\theta\sum_{i=1}^{n}\left(e^{\frac{x_i^2}{2\sigma^2}} - 1\right)^{-\beta} \quad (30)$$

$$\frac{\partial InL(.)}{\partial\sigma} = \frac{-2n}{\sigma} - (\beta + 1)\sum_{i-1}^{n} e^{-\frac{x_i^2}{2\sigma^2}}\left(\frac{x_i^2}{\sigma^3}\right)\left(1 - e^{-\frac{x^2}{2\sigma^2}}\right)^{-1} + \frac{\beta\sum_{i=1}^{n} x_i^2}{\sigma^3} - \beta\theta^{\beta}\sum_{i=1}^{n}\left(e^{\frac{x_i^2}{2\sigma^2}} - 1\right)^{-\beta-1} e^{\frac{x_i^2}{2\sigma^2}}\left(\frac{x_i^2}{2\sigma^3}\right) \quad (31)$$

Solving the equations of $\frac{\partial logL(.)}{\partial\theta} = 0, \frac{\partial InL(.)}{\partial\beta} = 0, \frac{\partial logL(.)}{\partial\sigma} = 0$ for each parameter numerically with some statistical software will provide the maximum likelihood estimates of the parametersθ,β and σ.

RESULTS AND DISCUSSION

Simulation

In this subsection, a Monte Carlo Simulation is employed to check the performance of the maximum likelihood method in estimating the parameters for the IWR distribution using R-package and random sample of size 30,100, 300, 500, 1000.

The result of all simulation are obtained from 1000 replications and two different combinations of parametersθ,β,σ are chosen.

Table 1: Means, Bias and RMSEs for the parameter estimates when $\theta = 0.1$, $\beta = 0.4$, $\sigma = 0.2$,

n	Parameters	Means	Bias	RMSE
30	θ	0.1521	0.0521	0.1376
	β	0.4143	0.0143	0.0773
	σ	0.1976	-0.0024	0.0490
100	θ	0.1135	0.0135	0.0567
	β	0.4063	0.0063	0.0389
	σ	0.2011	0.0011	0.0274
300	θ	0.1045	0.0045	0.0287
	β	0.4011	0.0011	0.0221
	σ	0.2003	0.0003	0.0162
500	θ	0.1022	0.0022	0.0209
	β	0.4009	0.0009	0.0179
	σ	0.2005	0.0005	0.0124
1000	θ	0.1010	0.0010	0.0150
	β	0.4004	0.0004	0.0121
	σ	0.2001	0.0001	0.0089

Table 2: Means, Bias and RMSEs for the parameter estimate when $\theta = 2$, $\beta = 4$, $\sigma = 0.5$

n	Parameters	Means	Bias	RMSE
30	θ	2.0904	0.0904	0.2535
	β	4.1615	0.1615	0.6245
	σ	0.4949	-0.0051	0.0186
100	θ	2.0490	0.0490	0.1825
	β	4.0473	0.0473	0.3198
	□	0.4973	-0.0027	0.0134
300	□	2.0374	0.0374	0.1369
	□	3.9941	-0.0059	0.1878
	□	0.4977	-0.0023	0.0099
500	□	2.0322	0.0322	0.1180
	□	3.9920	-0.0080	0.1493
	□	0.4979	-0.0021	0.0086
1000	□	2.0374	0.0374	0.1034
	□	3.9852	-0.0148	0.1046
	□	0.4974	-0.0026	0.0074

The results in table (1) and (2) shows the average bias for some parameters to be negative after varying the values of the parameters, this indicates that the estimators are under estimated. As sample size increases, bias approach zero, this implies that the estimators of the parameters of IWR distribution are asymptotically unbiased.

As the sample size increases, the root mean square error decreases indicating that the estimators are asymptotically efficient. Increasing or decreasing the values of the parameters of IWR does not affect the value of the root mean square.
Therefore, the estimators of the parameters of IWR distribution being unbiased and asymptotically efficient from the results in the tables above show that the maximum likelihood estimates are precise.

Applications
In this subsection, we evaluated the fitness of the IWR distribution using two real data sets with other known distributions including Rayleigh, Generalized Rayleigh, Weibull Rayleigh, Inverse Weibull, Inverse Weibull Weibull, Inverse Weibull Inverse Exponential and Inverse Weibull Pareto distributions
Data set 1: This represent 34 observations of the Vinyl Chloride dataobtained from clean up gradient ground-water monitoring wells in mg/L.The data are obtained from Bhaumik*et al*., (2009).

Table 3: Summary of dataset 1

Parameter	N	Min	Q1	Median	Q3	Mean	Variance	Skewness	Kurtosis	Max
Value	34	0.100	0.500	1.150	2.475	1.879	3.813	1.604	5.004	8.00

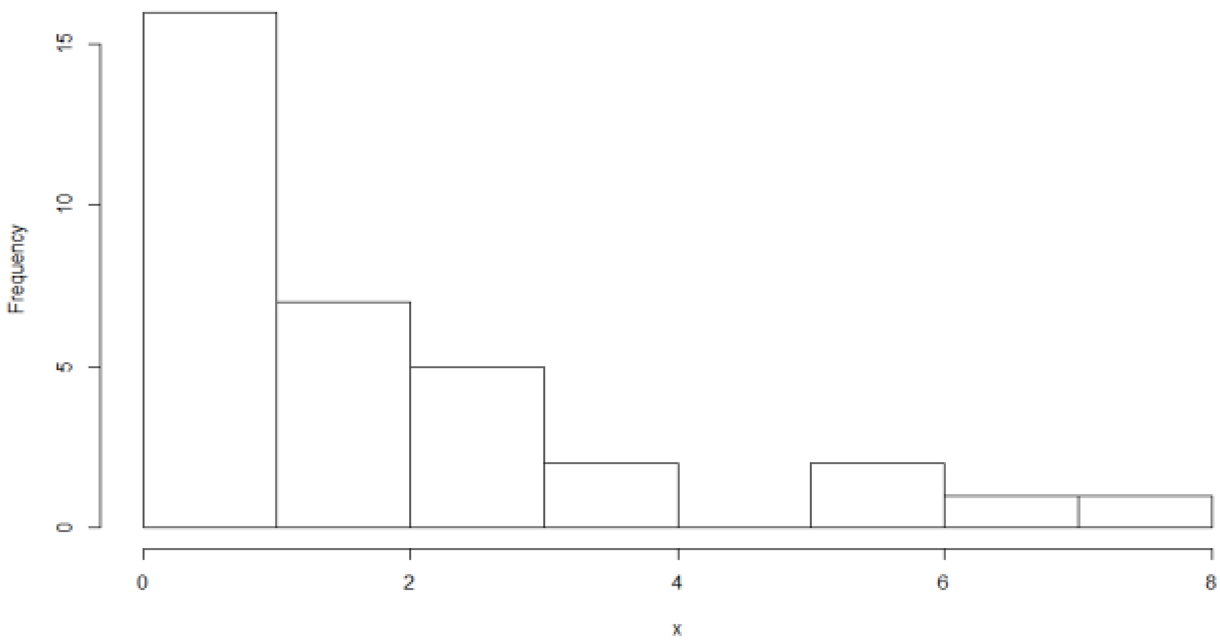

Figure 5: Histogram plot of dataset 1

Table 4: MLE's of the parameter(s) of some existing and the proposed distributions for Dataset 1

Distributions	MLE's of the parameter(s)
R(θ)	$\hat{\square} = 0.27654$
GR(α,λ)	$\hat{\square} = 0.36616$, $\hat{\square} = 0.25484$
WR(α, β,θ)	$\hat{\square} = 4.02715$, $\widehat{\square} = 0.4630$, $\hat{\square} = 0.02591$
IW(α,β)	$\hat{\square} = 0.8846$, $\hat{\square} = 0.6518$
IWW(α,β,θ, λ)	$\hat{\square} = 1.0928$, $\widehat{\square} = 0.5947$, $\hat{\square} = 0.5605$, $\hat{\square} = 0.6725$
IWIE(α, β,θ)	$\hat{\square} = 0.06584$, $\widehat{\square} = 0.79957$, $\hat{\theta} = 7.88321$
IWR(α, β,σ)	$\hat{\square} = 0.07679$, $\widehat{\square} = 0.37034$, $\hat{\square} = 1.85520$
IWP(α,β,θ, λ)	$\hat{\square} = 0.79830$, $\widehat{\square} = 0.78908$, $\hat{\square} = 03.79293$, $\widehat{\square} = 0.06104$

Table 5: Model performance comparison for dataset 1

Distributions	LL	AIC	BIC
R(θ)	-74.591665	151.5833	152.7097
GR(α,λ)	-80.79828	165.5966	168.6493
WR(α,□,θ)	-55.5311	117.0622	121.6413
IW(α,β)	-58.62734	121.2547	124.3074
IWW(α,β,θ, λ)	-54.31978	116.6396	122.745
IWIE(α, β,θ)	-59.59541	125.1908	129.7699
IWR(α, β,σ)	-54.40176	114.8035	119.3826
IWP(α,β,θ, λ)	-68.27471	144.5494	150.6549

Data set 2: The data represent the times of failure and running times fora sample of devices from aeld-tracking study of a larger system. The datawas studied by Merovci and Elbatal(2015). 30 units were installed innormal service condition. Two causes of failure were observed for eachunit that failed.

Table 6: Summary of dataset 2

Parameter	N	Min	Q1	Median	Q3	Mean	Variance	Skewness	Kurtosis	Max
Value	30	0.0200	0.6875	1.9650	2.9820	1.7700	1.3223	-0.2840	1.4537	3.0000

Table 7: MLE's of the parameter(s) of some existing and the proposed distributions for Dataset 2

Distributions	MLE's of the parameter(s)
R(θ)	$\hat{\theta} = 0.45326$
GR(α,λ)	$\hat{\alpha} = 0.10221, \hat{\lambda} = 0.05524$
WR(α, β,θ)	$\hat{\alpha} = 4.02715, \hat{\beta} = 0.4660, \hat{\theta} = 0.02591$
IW(α,β)	$\hat{\alpha} = 0.62761, \hat{\beta} = 0.67471$
IWW(α,β,θ, λ)	$\hat{\alpha} = 6.0950436, \hat{\beta} = 0.557911, \hat{\theta} = 0.6048508, \hat{\lambda} = 0.5930192$
IWIE(α, β,θ)	$\hat{\alpha} = 0.03577, \hat{\beta} = 0.54230, \hat{\theta} = 7.54341$
IWR(θ, β,σ)	$\hat{\theta} = 2.76802, \hat{\beta} = 0.18223, \hat{\sigma} = 0.64846$
IWP(α,β,θ, λ)	$\hat{\alpha} = 0.308729, \hat{\beta} = 1.143860, \hat{\theta} = 2.460591, \hat{\lambda} = 0.005866$

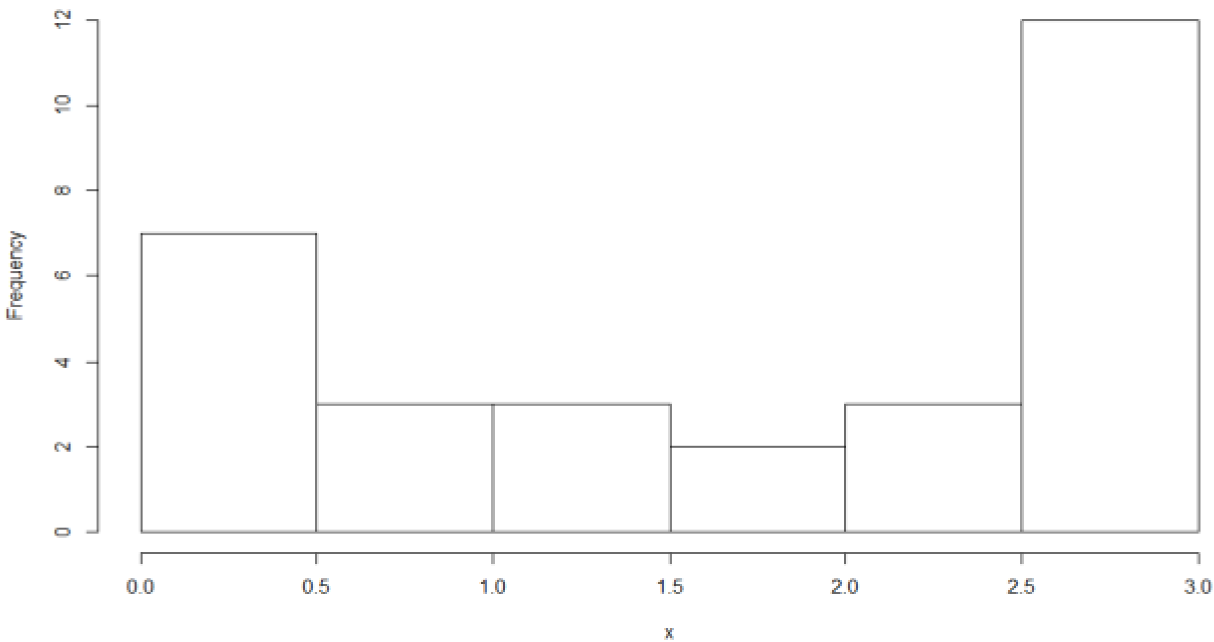

Figure 6: Histogram plot of Dataset 2

Table 8: Model performance comparison for dataset 2

Distributions	LL	AIC	BIC
R(θ)	-50.88295	103.7659	105.1671
GR(α,λ)	-64.5769	133.1538	135.9562
WR(α, β,θ)	-35.41096	76.82192	81.02551
IW(α,β)	-60.29261	124.5852	127.3876
IWW(α,β,θ, λ)	-32.29217	72.58435	78.77349
IWIE(α, β,θ)	-64.38682	134.7760	138.9772
IWR(α, β,σ)	-39.56269	85.12539	89.32897
IWP(α,β,θ, λ)	-73.03204	154.0641	159.6689

DISCUSSION

The table of summary and the histogram plot of data set one as shown in Table (3) and Figure (5) respectively shows that the data set is positively skewed, and also the PDF of Inverse Weibull Rayleigh distribution in Figure (2) shows that the distribution is skewed to the right. As a result of this, the distribution can be applied on the data set.

Application of eight competing distributions to data set one as shown in Table (5) shows that Inverse Weibull Rayleigh distribution has the least value of AIC and BIC. The result

indicates that Inverse Weibull Rayleigh distribution could be used as the best model compared to the other seven distributions used for fitting the dataset. Hence, Inverse Weibull Rayleigh distribution is a competitive distribution for analyzing data sets that are skewed to the right.
The table of summary and the histogram plot of data set two as shown in Table (6)and Figure (6)respectively shows that the data set is negatively skewed and has abathtub shape.
Application of eight competing distributions to data set two as shown in Table (8) shows that Inverse Weibull Rayleigh distribution performed third to Inverse Weibull Weibull distribution and Weibull Rayleigh distribution respectively, reason being that the data is negatively skewed. Hence, Inverse Weibull Rayleigh distribution is also a competing distribution for analyzing bathtub shape data.

CONCLUSION

We propose a three parameter distribution called the Inverse Weibull Rayleigh distribution.
Some mathematical properties of the new distribution like moments, moment generating function, survival function, hazard function, skewness and kurtosis have been studied. In addition, the expression for the Probability weighted moments and Rènyi entropy was also derived. The parameters of the new distribution was estimated using maximum likelihood method of estimation, the performance of these estimators was examined by simulation study, the result indicates that the maximum likelihood estimates are precise. We fit the model to two real data sets to show its usefulness. The new model provides a better fit compared to Rayleigh distribution, Generalized Rayleigh distribution, Inverse Weibull distribution, Inverse Weibull Weibull distribution, Weibull Rayleigh distribution, Inverse Weibull Inverse Exponential distribution and Inverse Weibull Pareto distribution in modeling a positively skewed Vinyl Chloride data set.
The new model is also a competing model in modeling a bathtub shape failure rates with negative skewness behind Weibull Rayleigh distribution and Inverse Weibull Weibull distribution among the list of the competing models.

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