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Symmetric Garch Modeling of Some Cryptocurrency Returns: Implications For Coin Investment Decision-Making

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ABSTRACT

The rapid growth of cryptocurrency markets has intensified interest in understanding the volatility dynamics of digital assets due to their implications for investment decision-making, risk management, and financial forecasting. Although numerous studies have examined cryptocurrency volatility using GARCH-type models, limited attention has been given to the comparative analysis of volatility mean reversion and half-life measures across multiple cryptocurrencies and their implications for investment horizons. This study addresses this gap by investigating the volatility behaviour, persistence, mean reversion, and volatility half-lives of eight major cryptocurrencies, namely Bitcoin (BTC), Dogecoin (DOGE), Ethereum (ETH), Litecoin (LTC), Tether (USDT), Stellar (XLM), Monero (XMR), and Ripple (XRP) using daily closing price data from January 2014 to August 2024. Descriptive statistics, unit root tests, ARCH effect tests, and symmetric GARCH (1,1) models were employed to estimate volatility persistence and compute volatility half-life measures. The results revealed that all cryptocurrency return series exhibited non-normal and leptokurtic distributions, significant ARCH effects, and strong volatility clustering. The estimated GARCH models indicated high volatility persistence across all cryptocurrencies, with persistence coefficients ranging from approximately 0.8706 for USDT to 0.99995 for BTC. The computed volatility half-lives varied substantially, ranging from 5 days for USDT and 12 days for ETH to 1,708 days for XRP and 13,864 days for BTC, indicating marked differences in the speed of adjustment to market shocks. Cryptocurrencies with shorter volatility half-lives, such as USDT and ETH, were found to be more suitable for short-term trading strategies, whereas BTC and XRP exhibited characteristics consistent with longer-term investment horizons. These findings have important practical implications for investors, portfolio managers, and financial institutions by providing empirical evidence for aligning asset selection with investment horizons, improving portfolio diversification strategies, and enhancing volatility forecasting and risk management practices. The study's principal contribution lies in its comprehensive comparative ranking of major cryptocurrencies based on volatility persistence, mean reversion, and half-life measures, thereby offering a novel framework for evaluating cryptocurrency investment suitability from both short-term and long-term perspectives.

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INTRODUCTION

Cryptocurrencies have attracted considerable attention from investors, researchers, and policymakers due to their decentralized nature, high returns, and extreme price volatility. Unlike conventional financial assets, cryptocurrencies operate in highly speculative markets where prices are influenced by investor sentiment, technological innovations, regulatory announcements, and macroeconomic developments. Consequently, understanding the volatility dynamics of cryptocurrency returns has become an important area of research for portfolio management, risk assessment, asset pricing, and financial forecasting (Katsiampa, 2019; Kim *et al*., 2021).

A fundamental characteristic of financial market volatility is mean reversion, which refers to the tendency of volatility to return to its long-run equilibrium level following market shocks. Closely related to this concept is volatility half-life, which measures the time required for a volatility shock to decay by one-half. These measures provide valuable information regarding the persistence of market uncertainty and the speed at which financial assets recover from shocks. For investors and portfolio managers, volatility half-life serves as a useful indicator for determining appropriate investment horizons, timing market entry and exit decisions, and designing risk management strategies (Belliler & Yildirim, 2021a; Cortese *et al*., 2023).

Previous studies have extensively examined cryptocurrency volatility using GARCH-type models and related econometric frameworks. For example, Naimy and Hayek (2018), Katsiampa (2019), Kim *et al*. (2021), and Wang (2021) investigated volatility dynamics and forecasting performance across major cryptocurrencies, while Koutmos (2018), Corbet *et al*. (2018), and Etienne *et al*. (2022) focused on volatility spillovers and interdependence among digital assets. Other studies, such as John *et al*. (2019), Ahmed *et al*. (2018), David *et al*. (2021), and Huang *et al*. (2021), examined volatility persistence and mean-reverting behaviour using GARCH-family models and stochastic volatility approaches. Although these studies have significantly improved understanding of cryptocurrency market behaviour, their primary focus has largely been on volatility forecasting, spillovers, persistence, and return dynamics rather than on comparative evaluation of volatility half-lives across multiple cryptocurrencies for investment decision-making.

Despite the growing body of literature on cryptocurrency volatility, empirical evidence on the comparative mean reversion rates and volatility half-lives of major cryptocurrencies remains limited. Existing studies often focus on a single cryptocurrency, a small subset of digital assets, or volatility forecasting performance without explicitly linking volatility persistence and half-life measures to practical investment horizons. Furthermore, few studies have provided a comprehensive ranking of cryptocurrencies based on volatility half-life as a basis for distinguishing between short-term and long-term investment opportunities. This gap limits investors' ability to understand how quickly different cryptocurrencies recover from volatility shocks and how such behaviour may influence portfolio allocation decisions.

This study contributes to the cryptocurrency volatility literature by providing a comprehensive comparative analysis of volatility persistence, mean reversion, and half-life behaviour for eight major cryptocurrencies, namely Bitcoin (BTC), Dogecoin (DOGE), Ethereum (ETH), Litecoin (LTC), Tether (USDT), Stellar (XLM), Monero (XMR), and Ripple (XRP). Unlike previous studies that primarily emphasized volatility forecasting or spillover effects, this research employs symmetric GARCH (1,1) models and volatility half-life measures to rank cryptocurrencies according to their speed of mean reversion and to evaluate their suitability for short-term and long-term investment strategies. By linking volatility half-life directly to investment decision-making, the study provides practical insights for traders, portfolio managers, financial analysts, and policymakers.

The main objective of this study is to model the volatility behaviour of selected cryptocurrencies and examine the implications for investment decision-making. Specifically, the study seeks to (i) examine the statistical and distributional characteristics of cryptocurrency returns (ii) determine the stationarity and heteroskedastic properties of the selected cryptocurrency return series (iii) estimate symmetric GARCH (1,1) models for the selected cryptocurrencies and evaluate volatility persistence (iv) compute volatility mean reversion rates and half-lives for the cryptocurrencies under study (v) rank the cryptocurrencies based on volatility half-life and assess their suitability for short-term and long-term investment strategies.

Based on these objectives, the study tests the proposition that cryptocurrency returns exhibit significant volatility clustering, stationary mean-reverting behaviour, and differing volatility half-lives that can be utilized to inform investment horizon decisions. The findings are expected to contribute to the growing literature on cryptocurrency volatility while providing empirical evidence to support portfolio management, risk assessment, and financial forecasting in digital asset markets.

Empirical research on cryptocurrencies has extensively employed GARCH-type models and other volatility frameworks to analyze price dynamics, return behaviour, and spillover effects. Early studies by Salisu et al. (2018) highlighted the importance of pre-testing residuals to select the appropriate GARCH model for Bitcoin volatility, emphasizing the role of conditional heteroskedasticity in capturing market behaviour. Naimy and Hayek (2018) compared GARCH, EGARCH, and EWMA models for Bitcoin/USD, finding that EGARCH provided superior out-of-sample forecasting accuracy. Similarly, Jinan and Apostolos (2019) applied GARCH-in-mean models to leading cryptocurrencies and confirmed significant shock and volatility spillovers both within the cryptocurrency market and from cryptocurrencies to major financial markets.

Subsequent studies have examined volatility spillovers, persistence, and mean-reversion dynamics. Koutmos (2018) and Corbet et al. (2018) documented return and volatility spillovers among Bitcoin, Ethereum, Ripple, and other major cryptocurrencies, identifying Bitcoin as the primary transmitter of shocks. John et al. (2019) measured half-life volatility for Bitcoin, Litecoin, and Ripple using GARCH family models, finding short half-lives indicative of strong mean reversion. Pinar et al. (2020) and Han (2019) incorporated long-memory and jump components into GARCH and FIGARCH models to capture persistent and abrupt volatility changes in cryptocurrencies, demonstrating the effectiveness of fractional GARCH extensions for modeling return dynamics.

Recent research has refined volatility modeling with advanced econometric and stochastic methods. Kim et al. (2021) showed that Bayesian stochastic volatility models outperformed GARCH models for nine leading cryptocurrencies, particularly for longer forecasting horizons. Cortese et al. (2023) applied a sparse regime-switching model to identify bull, neutral, and bear regimes in cryptocurrency returns, emphasizing the role of first-moment returns, trend, and market attention. Ahmed et al. (2023) modeled Bitcoin, Dash, Monero, and Stellar using comparative GARCH approaches and SEM, detecting return-volatility spillovers and identifying Monero as a key shock transmitter. Karim et al. (2023) examined asymmetric return-volatility relationships in Bitcoin and Ethereum using NARDL and quantile regression, highlighting the behavioural drivers of volatility asymmetry.

Empirical evidence further underscores the integration and interconnectedness of cryptocurrency markets. Etienne et al. (2022) found return-volatility spillovers among Bitcoin, Ethereum, Ripple, and Litecoin, with limited correlation to US equity markets but sensitivity to US bond market shocks. Joseph et al. (2024) applied BEKK and DCC GARCH models to African markets, confirming growing spillover effects from cryptocurrencies to select national financial markets. Lee and Wang (2024) and Seabe et al. (2024) emphasized the predictive role of intraday variances, retail investor behaviour, and factor-investing strategies in modeling cryptocurrency returns. Collectively, these studies demonstrate the importance of advanced volatility models including GARCH variants, stochastic volatility models, and regime-switching frameworks for capturing the complex dynamics, persistence, and spillovers in cryptocurrency markets.

MATERIALS AND METHODS

Source of Data

This study employs secondary time series data on daily closing prices of eight (8) cryptocurrencies collected from the coinmarketcap.com database. The data spanned from 2nd January, 2014 to 30th August, 2024. The selected time period is due to the availability of the data. The cryptocurrencies return series (r_t) was calculated as log of first difference of daily closing price as shown in Equation (1):

$$r_t = \ln\left[\frac{R_t}{R_{t-1}}\right] \times 100 \quad (1)$$

where r_t is the daily cryptocurrency return at time t, R_t is the closing price at time t, and R_{t-1} is the corresponding price in the period at time $t-1$.

Methods of Data Analysis

The following data analysis techniques were employed in this study.

Preliminary Test Statistics

This section focuses on the preliminary tests such as descriptive statistics and normality measures, unit root test and heteroskedasticity test for ARCH effect.

Descriptive Statistics and Normality Measures

The sample standard deviation ($\hat{\sigma}$) of cryptocurrency returns over a given period of time is computed as given in Equation (2):

$$\hat{\sigma} = \sqrt{\frac{1}{n-1}\sum_{t=1}^{n}(r_t - \bar{r})^2} \qquad (2)$$

where $\bar{r}$ is the sample mean return defined by Equation (3):

$$\bar{r} = \frac{1}{n}\sum_{i=1}^{n} r_i \qquad (3)$$

Jarque and Bera (1980, 1987) proposed a normality test which is a goodness-of fit test of whether sample data have the skewness and kurtosis matching a normal distribution. The test is usually used to test the null hypothesis that the series is normally distributed. Given a return series $\{r_t\}$ the test statistic JB is defined as given by Equation (4):

$$JB = \frac{T}{6}\left(g_1^2 + \frac{1}{4}(g_2 - 3)^2\right) \qquad (4)$$

where T is the number of observations, g_1 is the sample skewness which is estimated by Equation (5):

$$g_1 = \frac{\mu_3}{\mu_2^{3/2}} = T^{1/2}\sum_{t=1}^{T}(r_t - \bar{r})^3 / (\sum_{t=1}^{T}(r_t - \bar{r})^2)^{3/2} \qquad (5)$$

and g_2 is the sample kurtosis which is estimated by Equation (6):

$$g_2 = \frac{\mu_4}{\mu_2^2} = T\sum_{t=1}^{T}(r_t - \bar{r})^4 / (\sum_{t=1}^{T}(r_t - \bar{r})^2)^2 \qquad (6)$$

The JB normality test checks the following pair of hypotheses:

H_0: $\hat{\mu}_3 = 0$ and $\hat{\mu}_4 = 0$ (i.e. r_t is from a normal distribution) against the alternative

H_1: $\hat{\mu}_3 \neq 0$ and $\hat{\mu}_4 \neq 0$ (i.e. r_t is not from a normal distribution)

Augmented Dickey-Fuller (ADF) Unit Root Test

To check the unit root and stationarity properties of the series, Augmented Dickey-Fuller unit root test is employed. The ADF test addresses autocorrelation by including lagged differences of the dependent variable. The ADF test modifies the basic Dickey-Fuller equation by adding lagged differences of the variable r_t to capture higher-order autoregressive processes (Dickey and Fuller, 1979). The general form of the ADF test is expressed in Equation (7) as:

$$\Delta r_t = \alpha + \beta t + \delta r_{t-1} + \sum_{i=1}^{p}\phi_i \Delta r_{t-i} + \varepsilon_t \qquad (7)$$

where r_t is the value of the return series at time t, α is the intercept (constant), β is the coefficient of the deterministic time trend, δ is the coefficient of the lagged level of the series, ϕ_i are the coefficients of the lagged differences Δr_{t-i}, p is the number of lagged terms (chosen to account for autocorrelation in the error terms), ε_t is the white noise error term.

The ADF test is specified in two different forms in this study including an intercept only, and an intercept with a linear time trend. The ADF test with intercept only is specified in Equation (8):

$$\Delta r_t = \alpha + \delta r_{t-1} + \sum_{i=1}^{p}\phi_i \Delta r_{t-i} + \varepsilon_t \qquad (8)$$

where α is the intercept. This version is used when the series may have a non-zero mean but no deterministic trend. The ADF test with an intercept and a linear time trend is expressed in Equation (9) as:

$$\Delta r_t = \alpha + \beta t + \delta r_{t-1} + \sum_{i=1}^{p}\phi_i \Delta r_{t-i} + \varepsilon_t \qquad (9)$$

where α is the intercept, βt is the time trend term. The test checks the following pairs of hypothesis:

H_0: $\delta = 0$ (the series has a unit root)

H_1: $\delta < 0$ (the series is stationary).

The test statistic is the t-statistic for the estimated coefficient δ as represented in Equation (10) (Dickey and Fuller, 1979):

$$\mathrm{l}t_{\hat{\delta}} = \frac{\hat{\delta}}{SE(\hat{\delta})} \qquad (10)$$

The ADF test checks whether the coefficient δ is significantly different from zero by looking at the t-statistic for δ. If the t-statistic is more negative than the critical value, we reject the null hypothesis and conclude that the series is stationary (Kuhe, 2024).

Heteroskedasticity Test

To test for heteroskedasticity or ARCH effects in the residuals of cyptocurrency returns, we apply the Lagrange Multiplier (LM) test due to Engle (1982). The procedure of performing the Engle's LM test is to first obtain the residuals e_t from an ordinary least squares regression of the conditional mean equation which could be an AR, MA or ARMA model that best fit the data. For an ARMA (1,1) model, the conditional mean equation is specified in Equation (11) as:

$$r_t = \phi_1 r_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1} \qquad (11)$$

where r_t is the return series, ϕ_1 and θ_1 are the coefficients of the AR and MA terms while ε_t is the random error term. Having obtained the residuals e_t, we then regress the squared residuals on a constant and q lags such as in the following Equation (12):

$$e_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 e_{t-2}^2 + \alpha_3 e_{t-3}^2 + \cdots + \alpha_q e_{t-q}^2 + v_t \qquad (12)$$

The null hypothesis of no ARCH effect up to lag q is then formulated as follows:

H_0: $\alpha_1 = \alpha_2 = \alpha_3 = \cdots = \alpha_q$ versus the alternative H_1: $\alpha_i > 0$ for at least one $i = 1,2,3,\ldots,q$.

There are two test statistics for the joint significance of the q-lagged squared residuals. The F-statistic and the number of observations times R-squared (nR^2) from the regression. The F-statistic is estimated in Equation (13) as:

$$F = \frac{SSR_0 - SSR_1/q}{SSR_1(n-2q-1)} \qquad (13)$$

where $SSR_1 = \sum_{t=q+1}^{T} e_t^2$, $SSR_0 = \sum_{t=q+1}^{T}(r_t^2 - \bar{r})^2$ and $\bar{r} = \frac{1}{\mathrm{n}}\sum_{\mathrm{t}=1}^{\mathrm{T}} \mathrm{r}_\mathrm{t}^2$ (14)

$\hat{e}_t$ is the residual obtained from least squares linear regression, $\bar{r}$ is the sample mean of r_t^2. The nR^2 is evaluated against $\chi^2(q)$ distribution with q degrees of freedom under H_0. The decision is to reject the null hypothesis of no ARCH effect in the residuals of returns if the p-values of the F-statistic and nR^2 statistic are less than $\alpha = 0.05$.

Model Specification

The following conditional heteroskedastic time series models are specified for this study.

Autoregressive Conditional Heteroskedasticity (ARCH) Model

The Autoregressive Conditional Heteroskedasticity model of order q, ARCH (q) proposed by Engle (1982) variable is given in Equations (14, 15 and 16) by:

$$r_t = \mu_t + \varepsilon_t \qquad (14)$$

$$\varepsilon_t = \sigma_t e_t; \quad e_t \sim N(0,1) \qquad (15)$$

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \cdots + \alpha_q \varepsilon_{t-q}^2 = \omega + \sum_{i=1}^{q}\alpha_i\, \varepsilon_{t-i}^2 \qquad (16)$$

where ε_t is the innovation/shock at day t and it follows heteroskedastic error process, σ_t^2 is the volatility at day t

(conditional variance), ε_{t-i}^2 is the squared innovation at day $t-i$, ω is a constant term. A sufficient condition for the conditional variance to be positive is that parameters of the model should satisfy the following constraints: $\omega > 0$, $\alpha_i \geq 0$ for $i > 0$. The ARCH (1) process is defined by the following Equation (17):

$$\sigma_t^2 = \omega + \alpha_1\varepsilon_{t-1}^2 \quad (17)$$

where $\omega > 0$ and $\alpha_1 \geq 0$. As the persistence (as measured by α_1) increases towards unity, the process explodes.

Generalized Autoregressive Conditional Heteroskedasticity (GARCH) Model

Bollerslev (1986) extended the ARCH model of Engle (1982) to Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model. A GARCH (p, q) process is specified in Equations (18, 19 and 20) as:

$$r_t = \mu_t + \varepsilon_t \quad (18)$$

$$\varepsilon_t = \sigma_t e_t; \quad e_t \sim N(0,1) \quad (19)$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{q}\alpha_i\varepsilon_{t-i}^2 + \sum_{j=1}^{p}\beta_j\sigma_{t-j}^2 \quad (20)$$

where ε_t is the innovation/shock at day t and it follows heteroskedastic error process, σ_t^2 is the volatility at day t (conditional variance), ε_{t-i}^2 is squared innovation at day $t-i$, ω is a constant term, μ_t can be any adapted model for the conditional mean; p is the order of the autoregressive GARCH term; q is the order of the moving average ARCH term. The GARCH (1,1) model is capable of capturing all volatilities in any return series and is defined in Equation (21) as:

$$\sigma_t{}^2 = \omega + \alpha_1\varepsilon^2{}_{t-1} + \beta_1\sigma^2{}_{t-1} \quad (21)$$

The requirements for stationarity in GARCH (1, 1) model are that $\alpha_1 + \beta_1 < 1,\ \alpha_1 \geq 0,\ \beta_1 \geq 0$ and $\omega > 0$.

Modeling Mean Reversion using GARCH (1,1) Model

Although excessive volatility may be experienced in the financial markets from time to time, it will eventually settle down to a long run level. Given that the long level of variance ε_t for stationary GARCH (1,1) model is as shown in Equation (22):

$$\bar{\sigma}^2 = \frac{\omega}{(1-\sigma_1-\beta_1)} \quad (22)$$

In this case, volatility is always pulled toward this long run level by rewriting the ARMA representation as given by Equations (23) and (24).

$$\varepsilon_t^2 = \omega + (\alpha_1+\beta_1)\varepsilon_{t-1}^2 + \mu_t - \beta_1\mu_{t-1} \quad (23)$$

as follows

$$\left(\varepsilon_t^2 - \frac{\omega}{1-\sigma_1-\beta_1}\right) = (\sigma_1+\beta_1)\left(\varepsilon_{t-1}^2 - \frac{\omega}{1-\sigma_1-\beta_1}\right) + \mu_t - \beta_1\mu_{t-1} \quad (24)$$

If the above equation is iterated k times, it can be shown in Equation (25) that

$$\left(\varepsilon_{t+k}^2 - \frac{\omega}{1-\sigma_1-\beta_1}\right) = (\alpha_1+\beta_1)^k\left(\varepsilon_t^2 - \frac{\omega}{1-\sigma_1-\beta_1}\right) + \eta_{t+k} \quad (25)$$

where η_t is the moving average process since $(\alpha_1+\beta_1) < 1$ for stationary GARCH (1,1) model, $(\alpha_1+\beta_1)^k \to 0$ as $k \to \infty$. Although at time t there may be a large deviation between ε_t^2 and the long-run variance $\varepsilon_{t+1}^2 = \omega/(1-\sigma_1-\beta_1)$ will approach zero on average as k gets large. That is, the volatility mean reverts to its long-run level $\omega/(1-\sigma_1-\beta_1)$. In contrast, if $\omega/(1-\sigma_1-\beta_1)$ >1 and the GARCH model is non-stationary, the volatility will eventually explode to infinity as $k \to \infty$. Similar arguments can be easily constructed for GARCH (p,q) model (Kuhe & Audu, 2016).

Half-Life of Volatility Stock

The half-life of volatility represents the time taken by the volatility shock to cover half the distance backwards its mean volatility after a deviation from it. For a stationary GARCH (1,1) model $\sigma_t^2 = \omega + \alpha_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2$. The mean reverting form of the basic GARCH (1,1) model is given by Equation (26).

$$(\varepsilon_t^2 - \bar{\sigma}^2) = (\alpha_1+\beta_1)(\varepsilon_{t-1}^2 - \bar{\sigma}^2) + \mu_t - \beta_1\sigma_{t-1}^2 \quad (26)$$

where $\bar{\sigma}^2 = \omega/(1-\alpha_1-\beta_1)$ is unconditional long-run level of volatility and $\mu_t = (\varepsilon_t^2 - \bar{\sigma}^2)$ the mean reverting rate $(\alpha_1+\beta_1)$ implied by most fitted models is usually very close to 1. The magnitude of $(\alpha_1+\beta_1)$ controls the speed of mean reversion. The half-life of a volatility stock is given by the formula as shown in Equation (27):

$$L_{Half} = 1 - \left\{\frac{\ln(2)}{\ln(\alpha_1+\beta_1)}\right\} \quad (27)$$

The half-life measures the average time it takes for $|\varepsilon_t^2 - \bar{\sigma}^2|$ to decrease by one half. The closer $(\alpha_1+\beta_1)$ is to one, the longer the half-life of a volatility stock. If $(\alpha_1+\beta_1) > 1$, the GARCH (1,1) model is non-stationary and the volatility explodes to infinity (Kuhe & Audu, 2016).

To test for mean reverting properties of the return series and assuming a GARCH (1,1) model, we check the following pair of hypothesis.

$H_0 = (\alpha_1+\beta_1) > 1$ (i.e., the return series is non-stationary) against the alternative

$H_1 = (\alpha_1+\beta_1) < 1$ (i.e., the return series is mean reverting).

Model Order Selection using Information Criteria

GARCH model order and error distribution selection involves selecting a model order that minimizes one or more information criteria evaluated over a range of model orders. In this work, we employed Schwarz information Criterion (SIC) due to (Schwarz, 1978). The criterion is given in Equation (28) as:

$$SIC(P) = -2\ln(L) + Pln(T) \quad (28)$$

where P is the number of free parameters to be estimated in the model, T is the number of observations and L is the maximum likelihood function for the estimated model defined by Equations (29) and (30):

$$L = \prod_{t=0}^{n}\left(\frac{1}{2\pi\sigma_t^2}\right)^{1/2} exp\left[-\sum_{t=1}^{n}\frac{(r_t-\mu)^2}{2\sigma_t^2}\right] \quad (29)$$

$$\ln(L) = In\left[\prod_{t=1}^{n}\left(\frac{1}{2\pi\sigma_t^2}\right)^{1/2}\right] - \frac{1}{2}\sum_{t=1}^{n}\frac{(r_t-\mu)^2}{\sigma_t^2} \quad (30)$$

Thus given a set of estimated GARCH models for a given set of data, the preferred model is the one with the minimum information criterion and highest log likelihood value.

Error Distribution for Modeling Volatility

To estimate the time-varying volatility in the cryptocurrency returns and account for the excess kurtosis and fat tails that are present in the residuals of the return series, we model the error term in the GARCH models with normal (Gaussian) distribution, Student's-t distribution, and Generalized Error Distribution (GED). These distributions are appropriate to capture the excess kurtosis and the skewness in the residuals return series (Greene, 2010).

i. *Normal (Gaussian) Distribution (ND):* The normal distribution is given by Equation (31):

$$f(z) = \frac{1}{\sqrt{2\pi}}e^{\frac{-z^2}{2}}, -\infty < z < \infty \quad (31)$$

The normal distribution to the log likelihood for observation t is given in Equation (32) as:

$$l_t = \frac{-\frac{1}{2}\log(2\pi) - \frac{1}{2}\log\sigma_t^2 - \frac{1}{2}\left(y_t - X_t'\theta\right)^2}{\sigma_t^2} \quad (32)$$

ii. *Student's-t Distribution (STD)*: The student's-t distribution is given in Equation (33) as:

$$f(z) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{v\pi}\Gamma\left(\frac{v}{2}\right)}\left(1 + \frac{z^2}{v}\right)^{-\left(\frac{v+1}{2}\right)}, -\infty < z < \infty \quad (33)$$

For student's $t-$ distribution, the log-likelihood contributions are of the form given in Equation (34):

$$l_t = \frac{1}{2}\log\left[\frac{\pi(v-2)\Gamma(v/2)^2}{\Gamma((v+1)/2)}\right] - \frac{1}{2}\log\sigma_t^2 - \frac{(v+1)}{2}\log\left[1+\frac{(y_t-X_t'\theta)^2}{\sigma_t^2(v-2)}\right] \quad (34)$$

where $\Gamma(.)$ is the gamma function. This distribution is always fat-tailed and produces a better fit than the normal distribution for most asset return series. The degree of freedom $v > 2$ controls the tail behaviour. The distribution is only well defined if $v > 2$ since the variance of a student's-t with $v \leq 2$ is infinite, that is, the t $-$distribution approaches the normal distribution as $v \rightarrow \infty$.

iii. *The Generalized Error Distribution (GED):* The Generalized Error Distribution (GED) is given in Equation (35) as:

$$f(z;\, \mu,\sigma,v) = \frac{\sigma^{-1} v e^{\left(-0.5\left|\left(\frac{z-\mu}{\sigma}\right)/\lambda\right|^{v}\right)}}{\lambda 2^{(1+(1/v))}\Gamma(\frac{1}{v})},\ 1 < z < \infty \quad (35)$$

where $v > 0$ is the degree of freedom or tail-thickness parameter and

$$\lambda = \sqrt{2^{(-2/v)}\,\Gamma\left(\frac{1}{v}\right)/\Gamma\left(\frac{3}{v}\right)} \quad (36)$$

For GED, the log-likelihood contributions are of the form given in Equation (36):

$$l_t = -\frac{1}{2}\log\left[\frac{\Gamma(1/r)^3}{\Gamma(3/r)(r/2)^2}\right] - \frac{1}{2}\log\sigma_t^2 - \left[\frac{\Gamma(3/r)(y_t-X_t'\theta)^2}{\sigma_t^2\Gamma(1/r)}\right]^{r/2} \quad (37)$$

If $v = 2$ the GED yields a normal distribution, if $v < 2$ the density function has thicker or fat-tails than the normal density function, whereas for $v > 2$ is has thinner tails. In order for this distribution to be used for estimating GARCH parameters, it is necessary that $v \geq 1$ since the variance is infinite when $v < 1$.

RESULTS AND DISCUSSION

Descriptive Statistics and Normality Measures of Cryptocurrency Returns

Summary statistics and normality measures including mean, range, standard deviation, skewness, kurtosis, and the Jarque-Bera test reported in Table 1 were computed to examine the descriptive and distributional characteristics of cryptocurrency returns.

Table 1: Summary Statistics and Normality Measures of Cryptocurrency Returns

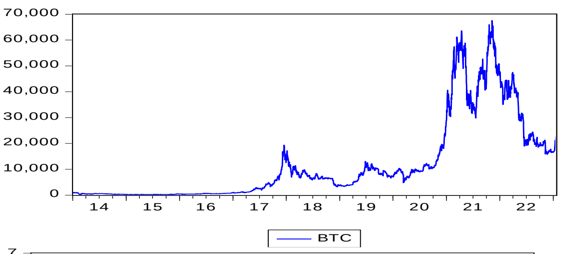
	Mean	Range	SD	Skewness	Kurtosis	JB	P-value
BTC	0.1009	232.30	5.2837	5.6594	230.5759	7158317	0.0000
DOGE	0.1611	829.77	15.7289	0.4349	481.3009	19645839	0.0000
ETH	0.1964	84.823	5.5559	-0.5846	11.2251	7218.182	0.0000
LTC	0.1346	109.38	5.9967	0.5355	16.4191	17699.13	0.0000
USDT	0.0005	10.276	0.4455	0.0453	37.3860	103953.5	0.0000
XLM	0.1775	116.88	7.3523	1.6431	19.8950	26686.30	0.0000
XMR	0.2183	249.33	11.4588	-0.0039	45.2555	216939	0.0000
XRP	0.1119	202.45	8.8230	0.8072	32.8519	108887.4	0.0000

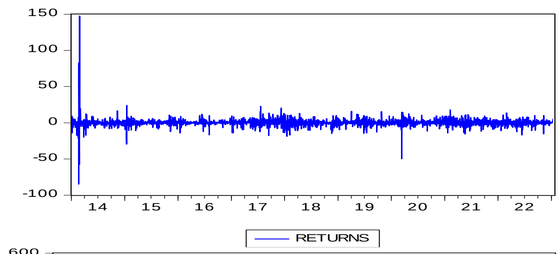
The descriptive statistics reported in Table 1 indicate that all cryptocurrency returns have positive mean values, reflecting overall gains during the study period, while their relatively high standard deviations and wide ranges suggest substantial volatility and dispersion in returns. In terms of distributional properties, ETH and XMR exhibit negative skewness, whereas the others show positive skewness, and all series display excess kurtosis, indicating heavy tails.

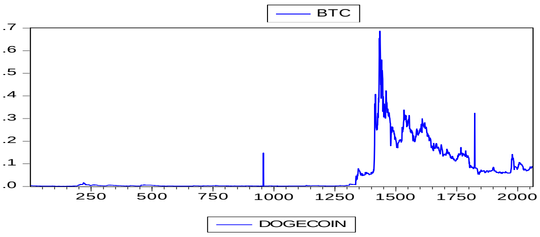
Furthermore, the Jarque-Bera test confirms that all return series are non-Gaussian, with significant p-values rejecting normality, implying the presence of fat-tailed distributions and frequent extreme events. These findings align with existing literature and reinforce the view that cryptocurrency returns are highly volatile and do not follow normal distribution assumptions.

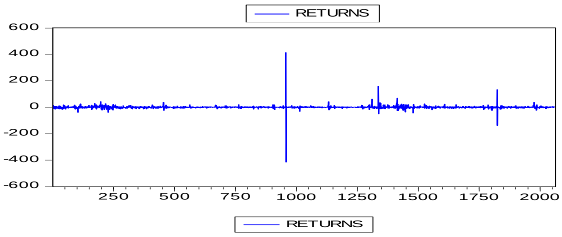
Time Plots of Daily Cryptocurrency Prices and Returns

The original series (daily cryptocurrency prices) as well as the transformed series (daily cryptocurrency log returns) were plotted against time and the graphical properties of the series were observed. The plots were presented in Figures 1 and 2.









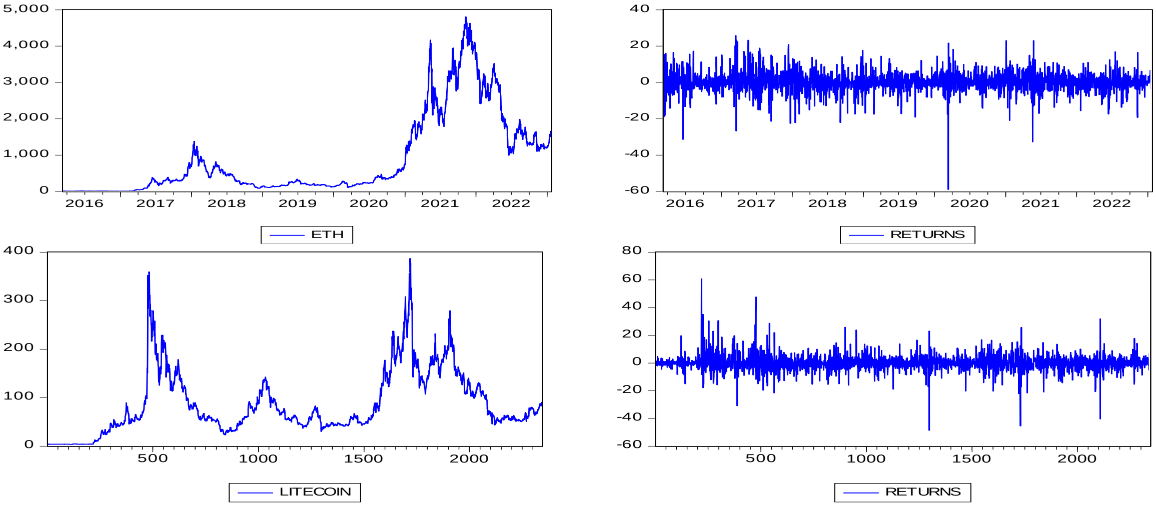


Figure 1: Time Plots of BTC, DODE, ETH and LTC Prices and Returns

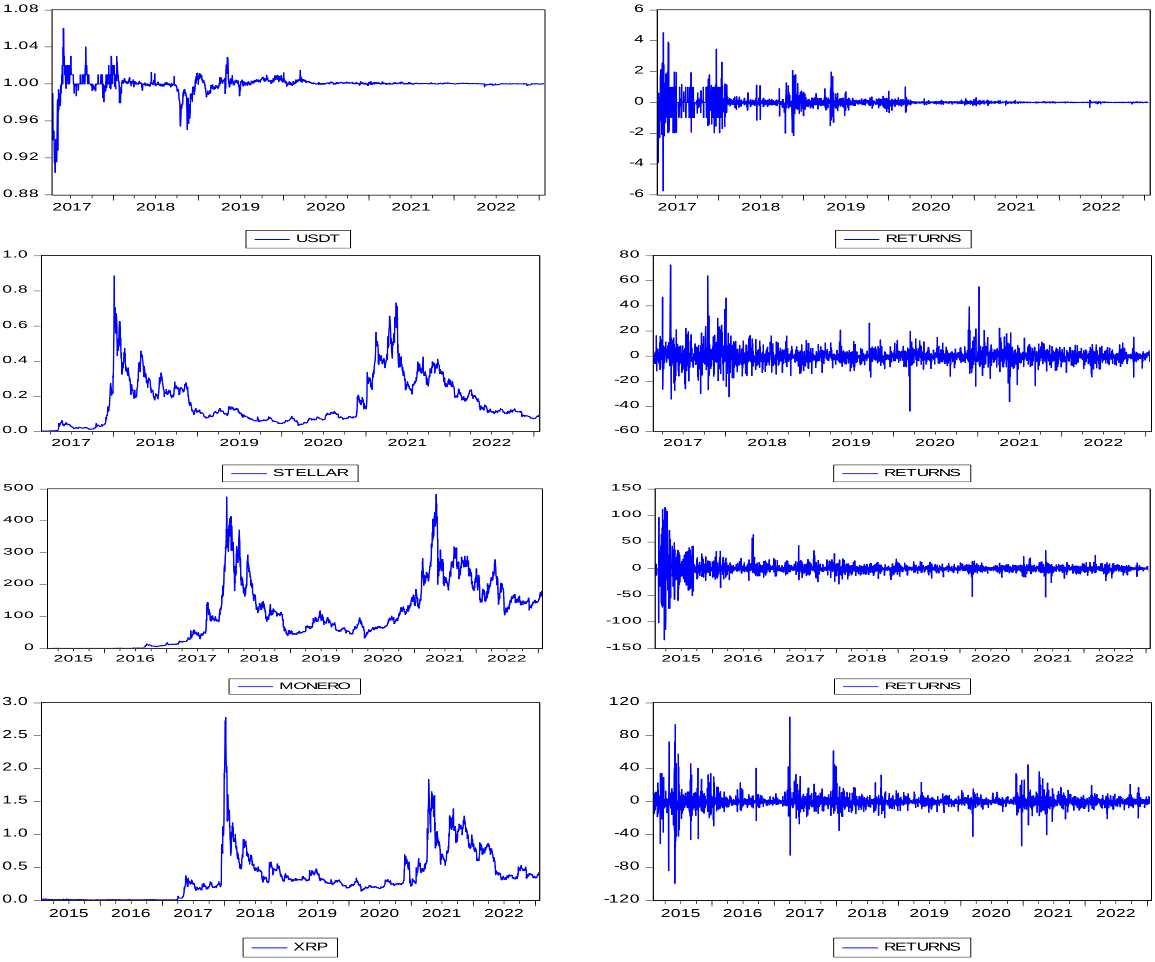


Figure 2: Time Plots of USDT, XLM, XMR and XRP Prices and Returns

Time plot analysis shows that cryptocurrency prices are non-stationary with time-varying means and variances (Figures 1 and 2 (left panels)), while their log returns are stationary with constant statistical properties and exhibit volatility clustering particularly in ETH, XRP, and XLM consistent with established characteristics of financial time series (Figures 1 and 2 (right panels)).

Unit Root and Heteroskedasticity Test Results

The study employed the Augmented Dickey-Fuller (ADF) unit root test reported in Tables 3 and 4 to examine unit roots in cryptocurrency prices and returns, and used Engle's LM test to detect ARCH effects and heteroskedasticity in the return series as shown in Table 5.

Table 2: ADF Unit Root Test Results of Cryptocurrency Prices

Variable	Option	ADF Test Statistic	P-value	5% Critical Value
BCT Prices	Intercept only	-1.2923	0.5876	-2.8622
	Intercept & trend	-2.0752	0.5589	-3.4112
DOGE Prices	Intercept only	-2.3155	0.1684	-2.8627
	Intercept & trend	-2.7437	0.2189	-3.4120
ETH Prices	Intercept only	-1.6308	0.4662	-2.8625
	Intercept & trend	-2.1915	0.4936	-3.4116
LTC Prices	Intercept only	-1.0167	0.3335	-2.8626
	Intercept & trend	-3.0843	0.1102	-3.4117
USDT Prices	Intercept only	-1.0057	0.1461	-2.8627
	Intercept & trend	-2.9972	0.1578	-3.4119
XLM Prices	Intercept only	-1.9126	0.1440	-2.8627
	Intercept & trend	-2.8616	0.1754	-3.4119
XRM	Intercept only	-2.5439	0.1052	-2.8623
	Intercept & trend	-2.3632	0.1727	-3.4114
XRP Prices	Intercept only	-1.6806	0.4500	-2.8625
	Intercept & trend	-1.5933	0.1768	-3.4117

Table 3: ADF Unit Root Test Results of Cryptocurrency Returns

Variable	Option	ADF Test Statistic	P-value	5% Critical Value
BCT Returns	Intercept only	-21.4321	0.0000	-2.8622
	Intercept & trend	-21.9608	0.0000	-3.4112
DOGE Returns	Intercept only	-44.7265	0.0000	-2.8627
	Intercept & trend	-44.7199	0.0000	-3.4120
ETH Returns	Intercept only	-52.3998	0.0000	-2.8625
	Intercept & trend	-52.4101	0.0000	-3.4116
LTC Returns	Intercept only	-51.2745	0.0001	-2.8626
	Intercept & trend	-51.9620	0.0000	-3.4117
USDT Returns	Intercept only	-31.5512	0.0000	-2.8627
	Intercept & trend	-31.5591	0.0000	-3.4119
XLM Returns	Intercept only	-44.3243	0.0000	-2.8627
	Intercept & trend	-44.4020	0.0000	-3.4119
XRM Returns	Intercept only	-13.9701	0.0000	-2.8623
	Intercept & trend	-14.0695	0.0000	-3.4114
XRP Returns	Intercept only	-39.7098	0.0000	-2.8625
	Intercept & trend	-39.7058	0.1768	-3.4117

The ADF test results reported in Tables 3 and 4 reveal that cryptocurrency prices are non-stationary at levels (Table 3) but become stationary after first differencing (Table 4), indicating that while price series contain unit roots, their log returns are stable over time with constant statistical properties.

Table 4: Heteroskedasticity Test Results for ARCH Effects

Variable	F-statistic	P-value	nR^2	P-value
BTC	8.99866	0.0027	8.97966	0.0027
DOGE	192.3716	0.0000	176.0906	0.0000
ETH	60.1727	0.0000	58.8087	0.0000
LTC	57.0347	0.0000	55.7253	0.0000
USDT	241.7833	0.0000	217.0895	0.0000
XLM	428.9750	0.0000	358.1735	0.0000
XMR	385.9910	0.0000	341.0494	0.0000
XRP	237.2957	0.0000	219.6114	0.0000

From the result of the Engle’s LM test for ARCH effects reported in Table 5, the p-values of the F-statistics and nR^2 are all statistically significant at the 1% marginal significance levels ($p < 0.01$). This means that the cryptocurrency log returns of all the eight cryptocurrencies under review exhibit heteroskedasticities (time-varying conditional variances) and can only be modeled using ARCH or GARCH variants.

Model Order and Error Distribution Selection

To select the optimal error distributions that best fit the cryptocurrency returns, three error distributions including Normal distribution (ND), Student’s-t distribution (STD) and Generalized Error distribution (GED) were employed as reported in Table 6.

Table 5: Error Distribution Selection for Fitting Volatility Model using SIC

Variable	ND		STD		GED	
	LogL	**SIC**	**LogL**	**SIC**	**LogL**	**SIC**
BTC	-9034.57	5.4704	-8555.39	5.1832	-8556.53	5.1839
DOGE	-7423.52	7.2186	-6962.83	6.7753	-6322.92	6.1543
ETH	-7687.61	6.1381	-7497.27	5.9895	-7491.32	5.9848
LTC	-7347.39	6.2823	-6993.09	5.9833	-7028.65	6.0137
USDT	1382.80	-1.2962	1766.07	-1.6559	1741.46	-1.6325
XLM	-6957.53	6.4504	-6774.98	6.2851	-6795.65	6.3042
XMR	-9849.60	6.7665	-9643.64	6.5943	-9367.52	6.4386
XRP	-9618.59	6.5899	-9027.59	6.1885	-9061.47	6.2116

The results of model order and error distribution selection reported in Table 6 show that non-Gaussian error distributions best fit cryptocurrency returns, with Student's-t selected for BTC, LTC, USDT, XLM, and XRP, and GED for DOGE, ETH, and XMR, confirming that digital asset returns are heavy-tailed and consistent with existing empirical literature like Katsiampa (2019), Yukun and Aleh (2018), Naimy and Hayek (2018), Woebbeking (2021), Kim *et al*. (2021), Takaishi (2021) and others who also modeled cryptocurrency returns using non-Gaussian distributions.

Estimation of Symmetric GARCH Models

This study employs lower symmetric GARCH (1,1) models to investigate the symmetric properties of cryptocurrency returns for all the eight cryptocurrency log returns under review, results are presented in Table 7 while post-estimation heteroskedasticity for ARCH effects is reported in Table 8.

Table 6: Parameter Estimates of Symmetric GARCH (1,1) Volatility Models

	Variable	Coefficient	Std. Error	z-Statistic	P-value	$\boldsymbol{\alpha_1 + \beta_1}$
BTC	μ	0.131262	0.038451	3.413780	0.0006	**0.99995**
	ω	0.335123	0.098057	3.417634	0.0006	
	α_1	0.284188	0.062216	4.567751	0.0000	
	β_1	0.715762	0.012135	58.98327	0.0000	
	v	2.543991	0.153913	16.52877	0.0000	
DOGE	μ	-0.078138	0.042830	-1.824347	0.0681	**0.997735**
	ω	7.877644	1.086565	7.250045	0.0000	
	α_1	0.637754	0.098113	6.500199	0.0000	
	β_1	0.359981	0.037645	9.562489	0.0000	
	v	0.716114	0.014070	50.89550	0.0000	
ETH	μ	0.078158	0.069933	1.117614	0.2637	**0.939401**
	ω	2.049555	0.472302	4.339502	0.0000	
	α_1	0.146252	0.022974	6.366076	0.0000	
	β_1	0.793149	0.029143	27.21594	0.0000	
	v	1.022355	0.032274	31.67720	0.0000	
LTC	μ	0.004745	0.074569	0.063632	0.9493	**0.994511**
	ω	0.566762	0.170492	3.324276	0.0009	
	α_1	0.120104	0.021731	5.526964	0.0000	
	β_1	0.874407	0.112937	7.742432	0.0000	
	v	3.069761	0.222128	13.81981	0.0000	
USDT	μ	9.33E-05	0.000614	0.151994	0.8792	**0.849627**
	ω	1.65E-05	6.04E-06	2.735983	0.0062	
	α_1	0.206822	0.027574	7.500583	0.0000	
	β_1	0.642805	0.112959	5.690605	0.0000	
	v	3.433195	0.243291	14.11147	0.0000	
XLM	μ	-0.092455	0.094697	-0.976326	0.3289	**0.994162**
	ω	1.601320	0.383377	4.176884	0.0000	
	α_1	0.163270	0.026309	6.205889	0.0000	
	β_1	0.830892	0.020075	41.38985	0.0000	
	v	3.513746	0.312007	11.26175	0.0000	
XMR	μ	0.024661	0.062823	0.392548	0.6947	**0.988127**
	ω	1.431899	0.314426	4.554007	0.0000	
	α_1	0.142386	0.019630	7.253510	0.0000	
	β_1	0.845741	0.016505	51.24092	0.0000	
	v	0.896497	0.018600	48.19893	0.0000	
XRP	μ	-0.156807	0.066823	-2.346582	0.0189	**0.999594**
	ω	4.261064	0.948757	4.491209	0.0000	
	α_1	0.331726	0.083552	3.970294	0.0000	
	β_1	0.667868	0.022369	29.85631	0.0000	
	v	2.568712	0.148402	17.30914	0.0000	

Table 7 reported the parameter estimates of the symmetric GARCH (1,1) models for all the eight cryptocurrency log returns. From the results presented in Table 7, all the coefficients of the models in the conditional variance equations for all the eight cryptocurrency log returns are statistically significant and satisfied the non-negativity constraints of the models. The positive and significant coefficients of the ARCH terms (α_1) and GARCH terms (β_1) clearly show that the digital coin market news about past volatilities have explanatory powers on current volatilities across the cryptocurrency log returns under review. The models showed evidence of volatility clustering, leptokurtosis (fat-tails) and volatility shocks are quite persistent in the digital coin market. The sums of ARCH and GARCH terms are less than unity (i.e., $\alpha_1 + \beta_1 < 1$) in all the eight symmetric GARCH models. If sum of ARCH and GARCH terms are less than one, it implies that the conditional variance of the cryptocurrency log return is stationary, stable, predictable, mean reverting and the conditional volatility is quite persistent. The empirical works of Ahmed *et al*. (2018), Alrabadi (2018), John *et al*. (2019), Gbenro and Moussa (2019), David *et al.* (2021), Huang *et al*. (2021) among others also found mean reversion in some cryptocurrency prices. However, when the sum of ARCH and GARCH terms is greater than one (i.e., $\alpha_1 + \beta_1 > 1$), it implies that the conditional variance of the cryptocurrency log return is non-stationary, unstable, unpredictable, does not mean revert and the conditional volatility is over-persistence and explosive.

The volatility shock persistence is less in USDT, ETH and XMR cryptocurrency log returns indicating faster reactions of volatility to digital coin market changes, whereas the volatility shock persistence is quite high in XRP and BTC cryptocurrency log returns indicating delayed reactions of volatility to digital coin market changes.

The high persistence coefficients observed for BTC (0.99995) and XRP (0.99959) imply that volatility shocks remain in the market for prolonged periods. Consequently, periods of heightened uncertainty may persist for several years, increasing long-term investment risk and requiring larger risk premiums from investors. Conversely, USDT and ETH exhibit relatively low persistence and shorter half-lives, indicating faster absorption of market shocks. Such assets may be more attractive to active traders seeking rapid volatility correction and shorter holding periods

When GARCH models are estimated with student's-t error distribution, the t-distribution degree of freedom, the shape parameter, (v) is greater than 2 for the distributions to be fat-tailed. Also, when GARCH models are estimated using generalized error distribution (GED), the shape parameter, (v) is less than 2 for the distributions to be fat-tailed. From the results of parameter estimates for symmetric GARCH models reported in Table 7, the shape parameter ($v > 2$) for all the GARCH models estimated using STD and ($v < 2$) for all the GARCH models estimated with GED indicating that the cryptocurrency log return series under review are heavy or fat-tailed (leptokurtic).

Table 7: Post Estimation of Heteroskedasticity Test Results for ARCH Effects

Variable	F-statistic	P-value	nR^2	P-value
BTC	6.47E-05	0.9936	6.47E-05	0.9936
DOGE	0.001059	0.9740	0.001060	0.9740
ETH	0.107404	0.7431	0.107484	0.7430
LTC	0.021519	0.8834	0.021537	0.8833
USDT	0.130400	0.7181	0.130515	0.7197
XLM	0.063857	0.5439	0.059980	0.5439
XMR	0.031831	0.8584	0.031853	0.8584
XRP	0.039513	0.8425	0.039539	0.8424

The heteroskedasticity test result for ARCH effects reported in Table 8 indicate that the symmetric GARCH (1,1) models successfully captured all ARCH effects in the cryptocurrency return series, as evidenced by insignificant p-values, confirming that the models are adequate, valid, and reliable for describing market volatility.

Results of Volatility Half-Life

The study computed volatility mean reversion rates and half-lives for the eight cryptocurrencies and used these measures to rank volatility and inform investment decisions (see Tables 9 and 10).

Table 8: Volatility Mean Reversion and Half-Life of Cryptocurrency Returns

Variable	$\alpha_1 + \beta_1$	$\ln(2)$	$\ln(\alpha_1 + \beta_1)$	$1 - \dfrac{\ln(2)}{\ln(\alpha_1 + \beta_1)}$
BTC	0.99995	0.693147	-5.00013E-05	13863.6
DOGE	0.997735	0.693147	-0.002267569	306.6785
ETH	0.939401	0.693147	-0.062512841	12.08808
LTC	0.994511	0.693147	-0.00550412	126.9324
USDT	0.849627	0.693147	-0.162957849	5.253537
XLM	0.994162	0.693147	-0.005855108	119.3833
XMR	0.988127	0.693147	-0.011944047	59.03286
XRP	0.999594	0.693147	-0.000406082	1707.912

The results of volatility mean reversion and half-life reported in Table 9 reveals that the sums of ARCH and GARCH terms are less than unity ($\alpha_1 + \beta_1 < 1$) for the eight cryptocurrency log returns. When the sum of ARCH and GARCH terms is less than unity, it indicates that the cryptocurrency prices can mean revert to their historical price values after some certain periods of time. When the sum of ARCH and GARCH terms is greater than unity, it indicates that the cryptocurrency prices can never mean revert to their historical price values after some certain periods of time. This means that the price will explode to infinity.

It is also observed that the higher the values of the sums of ARCH coefficients (α_i) and GARCH coefficients (β_i), the higher the volatility, and hence the slower the rate of mean reverting process. It is clearly evident from Table 11 that Bitcoin (BTC) and Ripple coin (XRP) cryptocurrencies have the slowest mean reversion with the highest volatilities as compared to other cryptocurrencies under review, whereas USDT, ETH and XMR have the fastest mean reversion, with the lowest volatility as compared to other digital currencies under investigation. This result agrees with the empirical findings of Corbet *et al*. (2018), Ahmed *et al*. (2018), Alrabadi (2018), John *et al*. (2019), Gbenro and Moussa (2019), Agyarko *et al*. (2019), Aalborg *et al*. (2019), Jinan and Apostolos (2019), Huthaifa *et al*. (2020), Jimoh and Oluwasgun (2020), David *et al*. (2021), Huang *et al*. (2021), Hamed *et al*. (2021) who also found some digital currencies having faster mean reversion than others in independent studies.

Bitcoin (BTC) and Ripple coin (XRP) have higher sum of ARCH and GARCH coefficients ($\alpha_1 + \beta_1 = 0.99995$ and $\alpha_1 + \beta_1 = 0.999594$) respectively, and the volatilities of Bitcoin and Ripple coin cryptocurrencies take about 13,864 and 1,708 days respectively to mean revert to half of their historical average values, these are the longest time periods when compared to other cryptocurrencies under study. In contrast, USDT and ETH have the lowest sum of ARCH and GARCH coefficients ($\alpha_1 + \beta_1$) and it takes only 5 and 12 days respectively for USDT and ETH to mean revert to half of their historical averages. These represent the shortest possible time periods as compared to other cryptocurrencies under review.

Table 9: Half-Life Volatility Ranking and Choice of Investment

	Half-life (in days)	Rank	Investment Decision Open (days)	Close (days)	Choice of investment
BTC	13864	14	0	27728	Long-term
DOGE	307	12	0	614	Short-term
ETH	12	4	0	24	Short-term
LTC	127	11	0	254	Short-term
USDT	5	1	0	10	Short-term
XLM	119	10	0	238	Short-term
XMR	59	8	0	118	Short-term
XRP	1708	13	0	3416	Long-term

Table 10 depicts the results of volatility half-life and ranking of the mean reverting process among the cryptocurrency log returns. Since USDT displayed that the log returns can mean revert to half of its mean value within 5 days, it suggests that investors of the USDT cryptocurrency must open a position at day 0 and must close after the 10th day. The ETH showed that its log returns can mean revert to half of it mean values within 10 days, indicating that investors of ETH cryptocurrencies must open a position at day 0 and must close after the 20th day. Hence, for short-term trading and investment, Tether (USDT), Ethereum (ETH) etc with short volatility half-lives could be the better choice of market.

On the other hand, Bitcoin (BTC) cryptocurrency has the slowest mean reversion process; it takes about 13,864 days to revert to half of its mean value. Therefore, the investors of Bitcoin cryptocurrency should open at 0 days and must close after 27728th day (approximately 110 years). Also, the Ripple coin (XRP) cryptocurrency has the second slowest mean reversion process; it takes about 1708 days to revert to half of its mean value. Therefore, the investors of Ripple coin cryptocurrency should open at 0 days and must close after 3416th day (approximately 14 years). Thus, Bitcoin and Ripple coin cryptocurrencies provide maximum leverages for investors to operate as compared to Tether (USDT) and Ethereum (ETH) etc, which provide the smallest time period for investors operate freely. Hence, for long-term trading and investment, Bitcoin (BTC) and Ripple coin (XRP) with high volatility half-lives could be the better choice of market.

Limitations of the Estimated Models

Although the symmetric GARCH (1,1) model effectively captured volatility clustering and persistence, it assumes that positive and negative shocks have identical effects on volatility. Cryptocurrency markets are often characterized by leverage effects and asymmetric reactions to information, which cannot be captured by symmetric specifications. Furthermore, the analysis focused on eight cryptocurrencies and did not account for spillover effects among digital assets. Structural breaks arising from regulatory changes, exchange collapses, and macroeconomic shocks were also not explicitly modeled. This shows awareness of model constraints.

CONCLUSION

This study investigated the volatility dynamics of eight major cryptocurrencies (BTC, DOGE, ETH, LTC, USDT, XLM, XMR, and XRP) using symmetric GARCH (1,1) models with the specific objectives of: (i) examining the statistical and volatility characteristics of cryptocurrency returns, (ii) estimating volatility persistence and mean reversion behaviour, (iii) determining the volatility half-life of each cryptocurrency, and (iv) identifying their suitability for short-term and long-term investment strategies. Using daily closing prices from January 2014 to August 2024, the study found that all cryptocurrency return series were non-Gaussian, leptokurtic, stationary, and characterized by significant ARCH effects. The estimated GARCH models revealed strong volatility clustering, high but stationary volatility persistence, and predictable conditional variances that reverted to their long-run means.

A major contribution of this study to the cryptocurrency volatility literature is the comparative estimation and ranking of volatility half-lives across eight leading cryptocurrencies over an extended ten-year period. By linking volatility mean reversion rates and half-life measures to investment decision-making, the study provides an empirical framework for distinguishing cryptocurrencies that are more appropriate for short-term trading from those better suited for long-term investment horizons. The findings showed that USDT, ETH, XMR, XLM, LTC, and DOGE exhibited relatively faster mean reversion and shorter volatility half-lives, whereas BTC and XRP demonstrated slower mean reversion and considerably longer volatility half-lives.

The results have important practical implications for portfolio management, risk assessment, and financial forecasting. Investors and portfolio managers can utilize volatility half-life information to align asset selection with investment horizons, optimize portfolio allocation, improve timing strategies, and enhance volatility forecasting. Similarly, financial institutions and market analysts can incorporate mean reversion estimates into risk management systems and predictive models to better anticipate future market behaviour and volatility shocks.

Despite these contributions, the study has some limitations. First, the analysis was restricted to symmetric GARCH (1,1) models, which do not account for potential asymmetric responses of volatility to positive and negative market shocks. Second, the study focused on only eight selected cryptocurrencies and may not fully represent the broader digital asset market. Third, the findings are based on data covering the period from January 2014 to August 2024 and may be influenced by market conditions specific to that sample period.

Future studies should extend this research by employing asymmetric volatility models such as EGARCH, TGARCH, APARCH, and GJR-GARCH to capture leverage effects and volatility asymmetries in cryptocurrency markets. Further research may also utilize multivariate GARCH frameworks, including DCC-GARCH and BEKK-GARCH models, to investigate volatility spillovers, dynamic correlations, and interdependence among cryptocurrencies and other financial assets. Such extensions would provide deeper insights into cryptocurrency market behaviour and improve forecasting accuracy for investment and policy purposes.

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