



## FRACTIONAL-ORDER MATHEMATICAL MODELING FOR POPULATION PROJECTION AND MANAGEMENT: A CASE STUDY OF OGUN STATE, NIGERIA

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### ABSTRACT

Population projection is an essential tool for effective population management and sustainable development planning at the sub-national level in Nigeria. This study applies classical and fractional-order logistic growth models to project the population of Ogun State up to the year 2050 and to examine the implications of projected growth for policy and development planning. Population data obtained from the 2006 National Population Census and annual projections from the National Bureau of Statistics (NBS) and the World Bank for 2007–2019 were used for model calibration. Model parameters were initially estimated using a four-point initialization approach and subsequently refined through nonlinear least-squares optimization implemented in MATLAB R2016a. Numerical solutions of the fractional-order model were obtained using the Adams-Bashforth-Moulton predictor-corrector scheme. Model performance was evaluated using goodness-of-fit measures and information criteria. The findings showed that the classical logistic model provided the most parsimonious representation of the observed population dynamics, although the fractional-order and numerical models offered additional insight into long-term uncertainty and structural variation. The projections indicate that Ogun State's population is expected to rise steadily from approximately 3.75 million in 2006 to about 12.35 million by 2050 under the classical logistic model, while the fractional-order and Adams-Bashforth-Moulton models produced projected values of approximately 10.21 million and 10.75 million respectively by 2050. The results further suggest that Ogun State is likely to exceed the 10 million population threshold between 2040 and 2042. These findings provide a quantitative basis for threshold-based planning, scenario analysis, and evidence-driven policy decisions in housing, healthcare, education, transportation, and urban infrastructure development within Ogun State.

**Keywords** Fractional-Order, Population Projection, Ogun State, ABM Numerical Method, Logistic Growth

### INTRODUCTION

Population growth remains a major challenge to sustainable development in Nigeria, particularly at the sub-national level where planning decisions are implemented. Reliable population projections are required for effective planning in education, healthcare, housing, transportation, and employment generation.

Ogun State has experienced steady population growth due to natural increase, internal migration, and its strategic location adjacent to Lagos State. Population projections in Nigeria are commonly obtained using simple exponential or classical logistic growth models. While these approaches are easy to implement, they assume that current population growth depends only on present population size and ignore the possible influence of past demographic trends.

Fractional-order differential equations provide an alternative framework that allows historical population dynamics to influence current growth through a memory effect. With the increasing availability of numerical computing tools such as MATLAB, it is now feasible to implement and compare both classical and fractional population models in a transparent and reproducible manner. This study applies MATLAB-based numerical techniques to evaluate the suitability of fractional-order logistic models for population projection in Ogun State. Several studies have applied mathematical models to population growth in Nigeria, with classical logistic and exponential models being the most widely used. These models have been evaluated using error-based criteria such as RMSE and SSE. However, few studies have explored the use of fractional-order models or modern numerical solvers in population projection at the state level. Agog et al. (2020) applied logistic growth to national data, while Oladimeji et al.

(2022) assessed the suitability of exponential and logistic models using RMSE.

Abubakar and Usaini (2019) conducted an empirical study using the exponential (Malthusian) model. The study being a review, focused on the population projection of Niger state, Nigeria for a period of 20 years, using the data from the 2006 census figures. Similarly, Akaligwo et al. (2023) conducted an in-depth study using both the exponential and logistic growth models to predict the long-term population of Abuja (capital of Nigeria).

Also, Folorunsho et al. (2024) applied exponential growth model to assess the urban growth of Ibeju-Lekki Local Government Area (LGA), Lagos state, Nigeria, using census data from 1991, 2006 to project human population up to 2022. Fractional-order models have been successfully applied in biological and socio-economic systems to capture long-term dependence and delayed responses. However, fractional-order models, which consider the impact of historical population dynamics, are underutilized in the Nigerian context. By including a memory effect through fractional derivatives, these models better reflect the complex nature of population change influenced by migration, policy decisions, and environmental factors.

The present study contributes to population modeling literature by combining fractional-order modeling with MATLAB-based numerical implementation to assess population growth dynamics in Ogun State and also examining whether fractional – order dynamics provides additional insight beyond classical population growth models for ogun state. The study jointly analyzes classical, fractional analytical and fractional numerically for a quantitative long term projection of Ogun state population to the year 2050.

**MATERIALS AND METHODS**

**Data Sources**

2006 Census: Ogun State population = 3,751,140 (NPC, 2006)

Projections: Data from World Bank / NBS (2007-2019) were used for calibration.

**Model Formulation**

The classical logistic model is expressed as:

$$\frac{dP(t)}{dt} = rP(t) \left(1 - \frac{P(t)}{K}\right) \quad (1)$$

The fractional-order logistic model incorporates a memory effect and it is expressed as:

$${}^c D_t^\alpha P(t) = rP(t) \left(1 - \frac{P(t)}{K}\right), \quad 0 < \alpha \leq 1 \quad (2)$$

The Caputo derivative is defined as:

$${}^c D_t^\alpha P(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{P'(s)}{(t-s)^\alpha} ds \quad (3)$$

In the context of population dynamics, the fractional order parameter  $\alpha$  represents the degree to which current population growth depends on historical demographic states. Values of which imply the presence of memory effects, where past population levels, migration patterns, and socio-economic conditions exert a delayed influence on present growth rates. Such effects may arise from gradual urbanization processes, delayed fertility responses, policy inertia, or long-term migration adjustments.

**Table 1: Parameters for the Models**

Parameters	Description
$r$	intrinsic growth rate
$K$	carrying capacity
$P(t)$	population at time $t$
$t$	Time
${}^c D_t^\alpha$	Caputo fractional derivative of order $0 < \alpha \leq 1$
$\alpha$	Fractional order (memory parameter)

**Parameters Estimation**

The study employed the four- point logistic model based on four strategically selected pollution observations solely to generate initial estimates of the carrying capacity  $K$  and the logistic slope formula for  $r$  estimate using the data from World Bank / NBS (2007–2019).  $K$  and  $r$  are refined using nonlinear least square fitting.

Four point estimate formula:  $\frac{(K-P_1)(K-P_3)^3 P_2^3 P_4}{(K-P_2)^3 (K-P_4) P_1 P_3^3} = 1 \quad (4)$

Logistic slope formula:  $r_0 = \left(\frac{1}{h}\right) \log\left(\frac{P_2(K-P_1)}{P_1(K-P_2)}\right) \quad (5)$

Residuals =  $\min_{r,k} = \sum (P_{ob} - P_{cal})^2 \quad (6)$

Unequally spacing of the four selected observations was adopted to avoid degeneracy and numerical instability inherent in equally spaced four-point parameter estimation. Parameters are iteratively adjusted to minimize the sum of squared residuals until convergence is achieved.

Using equation (4) and (5), results give  $K_0 = 5.96M, r_0 = 0.093 \quad (7)$

Using equation (1) and (6), estimation of initial guess of  $K$  and  $r$  give:

$K_0 = 18.24M, r_0 = 0.04 \quad (8)$

Substituting equations (7) and (8) respectively into equation (3), we have table 2.

**Table 2: Computation of the Fractional Logistic Model Using the Initial Estimates**

Model Estimated Parameters	Fractional Logistic model result--
Classical logistic	$K(\sim 18M)---r(0.04\%)---\alpha \sim (1.0)-$
Four-point	$K(\sim 18M)---r(0.04\%)---\alpha \sim (1.0)-$

Equation model (3) has the same computational results from the two different initial parameters estimate models. The fractional order converged to  $\sim 1$ , which implies that addition

parameter  $\alpha$  in the fractional model does not provide additional explanatory power for the dataset

**Table 3: Model Selection Criteria**

Model	AIC	BIC	SSE
Classical Logistic	44.514	45.742	$2.16e^{-5}$
Fractional Logistic	42.514	44.355	$2.16e^{-5}$

The classical logistic model was favored by AIC/BIC. The models achieved the same goodness of fit, model selection reduces to parsimony. Consequently, no statistically significant memory effect was detected in the population dynamics over the observed period. Then, the classical logistic model is preferred to provide an adequate and reliable representation of the underlying growth dynamic.

require accounting for structural uncertainty and potential memory effects that are not captured by classical dynamics. Consequently, projections based solely on the classical logistic model may underrepresent long-term uncertainty, justifying the complementary use of fractional and numerically solved models.

**RESULTS AND DISCUSSION**

Thou, model selection criteria identify the classical logistic model as the most parsimonious representation of the observed population data. However, long-term projections

The Adams-Bashforth-Moulton predictor-corrector method was used with a step size of one year from 2006 to 2050 to allow accurate computation of fractional derivatives while capturing the gradual influence of past population levels on future growth.

**Table 4: Comparison of Models for Projection of Ogun State Population**

Year	Classical	Classic	Fractional	Fractional	ABM	ABM	LowCI/UpCI	LowCL/UpCI	LowCL/UpCI
2006	3.75	3.7382	3.7618	3.75	3.5906	3.9094	3.75	3.6141	3.8859
2007	3.869	3.8573	3.8808	3.8714	3.712	4.0308	3.878	3.7421	4.0139
2008	3.9913	3.9795	4.0031	3.9873	3.8279	4.1467	3.9945	3.8586	4.1303
2009	4.1168	4.1051	4.1286	4.1029	3.9435	4.2623	4.111	3.9751	4.2469
2010	4.2457	4.2339	4.2575	4.2191	4.0597	4.3784	4.2285	4.0926	4.3644
2011	4.378	4.3662	4.3897	4.3363	4.1769	4.4957	4.3475-	4.2116	4.4834
2012	4.5137	4.5019	4.5254	4.4548	4.2954	4.6142	4.4682	4.3323	4.6041
2013	4.6528	4.6411	4.6646	4.5748	4.4154	4.7342	4.5908	4.4549	4.7267
2014	4.7955	4.7838	4.8073	4.6963	4.5369	4.8557	4.7154	4.5795	4.8513
2015	4.9418	4.93	4.9536	4.8195	4.6601	4.9789	4.842	4.7061	4.9779
2016	5.0917	5.0799	5.1035	4.9444	4.785	5.1038	4.9709	4.835	5.1068
2017	5.2452	5.2334	5.257	5.071	4.9116	5.2304	5.1019	4.966	5.2378
2018	5.4024	5.3906	5.4142	5.1994	5.04	5.3588	5.2353	5.0994	5.3712
2019	5.5633	5.5515	5.5751	5.3296	5.1702	5.489	5.3709	5.235	5.5068
2020	5.7279	5.7161	5.7397	5.4616	5.3022	5.621	5.5089	5.373	5.6448
2021	5.8963	5.8845	5.9081	5.5954	5.436	5.7548	5.6493	5.5134	5.7852
2022	6.0685	6.0567	6.0802	5.7311	5.5717	5.8904	5.7921	5.6562	5.928
2023	6.2444	6.2326	6.2562	5.8686	5.7092	6.0279	5.9373	5.8014	6.0732
2024	6.4242	6.4124	6.4359	6.0079	5.8485	6.1673	6.0849	5.949	6.2208
2025	6.6077	6.596	6.6195	6.149	5.9897	6.3084	6.235	6.0991	6.3709
2026	6.7951	6.7834	6.8069	6.292	6.1326	6.4514	6.3876	6.2517	6.5235
2027	6.9863	6.9746	6.9981	6.4368	6.2774	6.5962	6.5426	6.4067	6.6785
2028	7.1813	7.1696	7.1931	6.5834	6.424	6.7428	6.7001	6.5642	6.836
2029	7.3801	7.3683	7.3919	6.7318	6.5724	6.8912	6.8601	6.7242	6.996
2030	7.5827	7.5709	7.5945	6.882	6.7226	7.0414	7.0225	6.8866	7.1584
2031	7.789	7.7772	7.8008	7.0339	6.8745	7.1933	7.1873	7.0514	7.3232
2032	7.9991	7.9873	8.0108	7.1875	7.0281	7.3469	7.3546	7.2187	7.4905
2033	8.2128	8.201	8.2246	7.3428	7.1834	7.5022	7.5244	7.3885	7.6603
2034	8.4302	8.4184	8.4419	7.4998	7.3404	7.6592	7.6965	7.5606	7.8324
2035	8.6512	8.6394	8.6629	7.6585	7.4991	7.8179	7.8711	7.7352	8.007
2036	8.8757	8.8639	8.8874	7.8187	7.6594	7.9781	8.048	7.9121	8.1839
2037	9.1037	9.0919	9.1155	7.9806	7.8212	8.14	8.2272	8.0913	8.3631
2038	9.3351	9.3234	9.3469	8.144	7.9846	8.3033	8.4088	8.2729	8.5447
2039	9.5699	9.5582	9.5817	8.3089	8.1495	8.4682	8.5927	8.4568	8.7286
2040	9.808	9.7962	9.8198	8.4752	8.3158	8.6346	8.7788	8.6429	8.9147
2041	10.049	10.037	10.061	8.643	8.4836	8.8024	8.9671	8.8312	9.103
2042	10.294	10.282	10.305	8.8122	8.6528	8.9716	9.1575	9.0216	9.2934
2043	10.541	10.529	10.553	8.9827	8.8233	9.1421	9.3501	9.2142	9.486
2044	10.791	10.779	10.803	9.1545	8.9952	9.3139	9.5448	9.4089	9.6807
2045	11.044	11.032	11.056	9.3276	9.1682	9.487	9.7415	9.6056	9.8774
2046	11.3	11.288	11.312	9.5019	9.3425	9.6613	9.9402	9.8043	10.076
2047	11.558	11.546	11.57	9.6773	9.5179	9.8367	10.141	10.005	10.277
2048	11.819	11.807	11.831	9.8538	9.6944	10.013	10.343	10.207	10.479
2049	12.082	12.07	12.093	10.031	9.872	10.191	10.547	10.412	10.683
2050	12.347	12.335	12.359	10.21	10.051	10.369	10.753	10.617	10.889

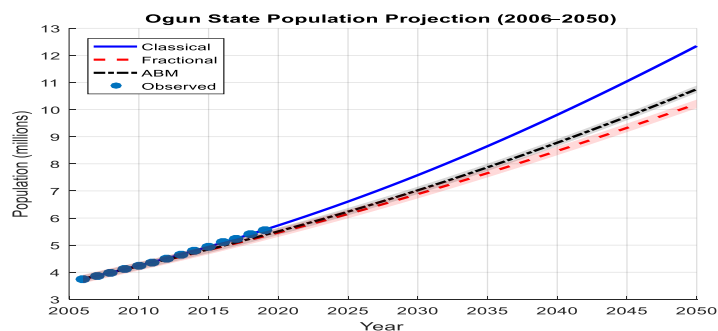


Figure 1: Comparison of Models for Data Used

The model selection criteria presented in Table 3 indicate that the classical logistic model provides the most parsimonious representation of the observed population data, as reflected by its lower AIC and BIC values compared with the fractional-order logistic model. Although both models produced similar goodness-of-fit statistics and identical SSE values, the classical model achieved this fit with fewer effective parameters, thereby satisfying the principle of parsimony. This finding suggests that the available population data for Ogun State do not provide sufficient statistical evidence to strongly justify the inclusion of memory-dependent fractional dynamics within the calibration period.

The estimated fractional-order parameter converged close to unity, implying that the influence of long-range historical dependence on population growth is relatively weak over the observed period. Consequently, population changes in Ogun State appear to depend more strongly on present demographic conditions than on delayed historical effects. However, this outcome should not be interpreted as evidence that memory effects are entirely absent in population systems. Rather, it reflects the limited temporal depth and aggregation level of the available demographic dataset.

The population projections presented in Table 4 and illustrated in Figure 1 demonstrate close agreement among the classical logistic, fractional-order analytical, and Adams–Bashforth–Moulton (ABM) numerical models during the early projection years. For example, in 2025 the projected population values were approximately 6.61 million for the classical logistic model, 6.15 million for the fractional-order model, and 6.24 million for the ABM numerical model. This close correspondence indicates that the estimated parameters and numerical procedures are stable and internally consistent. However, the divergence among the models becomes more noticeable over longer forecasting horizons. By the year 2050, the classical logistic model projects a population of approximately 12.35 million, while the fractional-order and ABM numerical models project approximately 10.21 million and 10.75 million respectively, as shown in Table 4. The wider confidence intervals associated with the fractional-order and ABM projections suggest increased uncertainty in long-term forecasts when memory effects and numerical approximations are incorporated. This observation is important for development planning because long-term policy decisions require not only point estimates but also realistic uncertainty bounds capable of supporting contingency planning.

Figure 1 further reveals that all projection curves maintain a steady upward trajectory throughout the forecast horizon, confirming the persistence of population growth in Ogun State. The graphical comparison also shows that the classical logistic model predicts a relatively faster long-term increase compared with the fractional-order formulations. This difference may reflect the moderating influence of fractional dynamics, which distribute growth effects across both present and past demographic states.

The projected growth trajectory has important implications for population management and infrastructure planning in Ogun State. Based on the projections in Table 4, the population is expected to exceed 10 million inhabitants between 2040 and 2042. This demographic threshold carries significant implications for housing demand, transportation systems, healthcare delivery, educational facilities, employment generation, and urban expansion. The findings therefore reinforce the need for proactive and long-term investments in public infrastructure and social services. The results also support the adoption of threshold-based planning strategies. For instance, projected population milestones such

as 8 million residents in the early 2030s and over 10 million residents by the early 2040s can be used as planning benchmarks for phased expansion of healthcare facilities, school enrollment capacity, housing schemes, and transportation infrastructure. Such an approach would enable policymakers to respond proactively to demographic pressures rather than relying on reactive interventions after infrastructure deficits have already emerged.

Furthermore, the use of multiple projection approaches in this study provides policymakers with a broader analytical framework for scenario planning. While the classical logistic model offers a parsimonious baseline forecast, the fractional-order and ABM numerical models provide complementary perspectives on uncertainty and possible long-term variation in demographic trends. This combination improves the robustness and policy relevance of the population forecasts. Population Management and Policy Implications for Ogun State

The population projections presented in this study have clear and actionable implications for the planning and management of human resources and infrastructure in Ogun State. As the state's population continues to grow, driven both by natural increase and in-migration, especially from neighboring Lagos, there is an urgent need for strategic policy responses to ensure that demographic growth supports sustainable development rather than exacerbates existing pressures on infrastructure and public services Adedotun (2025).

Ogun State has embarked on a series of strategic development initiatives aimed at economic transformation, urban renewal, and social welfare improvement. The Ogun State Economic Transformation Project (OGSTEP) is a flagship initiative designed to stimulate economic growth, expand livelihoods, and improve the quality of life for residents through targeted investment in infrastructure, agriculture, skills development, and the business environment (Ogun State Economic Transformation project, OGSTEP). Population projections thus serve as a crucial input into these development plans by quantifying future demand for housing, education, healthcare, and employment opportunities.

**Housing and Urban Development:** The state government has set ambitious targets to deliver affordable housing units, including a cumulative goal of 10,000 homes by 2027 and additional housing projects planned for expanding peri-urban corridors (Ayomide, 2026). Given the projected population growth toward and beyond 10 million by 2040-2050, these housing initiatives must be scaled up and spatially integrated with transportation and utilities planning to avoid informal settlement pressures and urban congestion.

**Infrastructure and Settlement Expansion:** Rapid population growth in border towns such as Mowe-Ibafo, Ado-Odo/Ota, and Ifo, stimulated by economic opportunities and proximity to Lagos has already strained existing infrastructure, including roads, drainage, water supply, and public services, Adedotun O.(2025). Long-term population forecasts can be incorporated into corridor development and land-use planning frameworks to prioritize infrastructure investment ahead of demographic pressures.

**Healthcare and Social Services:** A growing population will require expanded preventive and curative health services. Although Ogun State has previously launched reproductive health strategies, the magnitude of population growth demands updated maternal and child health planning, strengthened primary care networks, and family planning outreach to ensure demographic trends support healthy communities rather than overburdened systems.

**Threshold-Based Planning and Early Action**

Population projections allow for threshold-based policy triggers. For instance, if the state's population projection curve indicates crossing 8 million residents by the early 2030s and 10 million by the early 2040s, planners can use these thresholds to schedule phased infrastructure and service expansions. This proactive approach helps avoid reactive planning that often results in over-stretched services and unsustainable urban sprawl.

Forecasting models also support scenario planning, comparing baseline growth with higher-migration scenarios could help authorities prepare for sudden influxes of residents due to economic opportunities or urban spillover from Lagos. Scenario outputs from MATLAB simulations can be directly fed into state budget planning cycles, improving alignment between demographic expectations and fiscal allocations.

Further, these projections can support community engagement and accountability by making demographic trends transparent to citizens and stakeholders, which aligns with recent government emphasis on participatory budgeting and inclusive governance.

### CONCLUSION

This study applied both classical and fractional-order logistic models to population projection in Ogun State, Nigeria. While the classical logistic model was found to be the most parsimonious, the fractional-order model served as a useful complementary tool for assessing long-term uncertainty. The population projections up to 2050 provide valuable quantitative input for development planning and policy formulation at the state level. Future studies may extend this work by incorporating migration effects, age-structured populations, or longer historical datasets.

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