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EVALUATING THE VALIDITY OF CONSTRUCTING BALANCED INCOMPLETE BLOCK DESIGN USING GALOIS FIELD MULTIPLICATION

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ABSTRACT

The paper focused on the construction of Balanced Incomplete Block Designs (BIBDs) using Galois Fields with prime factors $p = 3,5$, based on multiplicative binary operations. For each prime, multiplication tables modulo p were created and used to construct designs from irreducible functions over $G(p)$. In $G(2)$, $G(3)$ and $G(5)$, the minimal functions were computed, and the corresponding elements of each field were generated and employed to construct Mutually Orthogonal Latin Squares (MOLS), and consequently, BIBDs. The resulting constructions were verified against the BIBD parameters (v, b, v, k, λ), and the findings revealed that the prime factors 3, and 5 do not satisfy the necessary conditions for BIBD existence. Therefore, BIBDs cannot be constructed using multiplicative binary operations with any of these prime factors.

Keywords: Galois field, Irreducible function, Multiplicative binary operation, Block Design, Prime factor

INTRODUCTION

Balanced Incomplete Block Designs (BIBDs) constitute a fundamental class of combinatorial designs that balance experimental comparisons while reducing the number of experimental units required for treatments that cannot all be observed together in a single block (Bose, 1939). A BIBD with parameters (v, b, r, k, λ) consists of v treatments arranged in b blocks of size k, each treatment occurring in r blocks, and every unordered pair of distinct treatments occurring together in exactly λ blocks; these parameters satisfy the standard relations $vr = bk$ and $\lambda(v-1) = r(k-1)$ (Cochran & Cox, 1957). The balanced concurrence property of BIBDs ensures that pairwise comparisons of treatments enjoy uniform precision, making BIBDs attractive in agricultural trials, industrial experiments, sensory studies, and survey sampling where full randomization or complete-block layouts are impractical (Federer, 1955; John & Williams, 1995). The algebraic and combinatorial theory underpinning BIBDs has matured over decades, linking design existence and construction to finite geometries, difference sets, group actions, and algebraic structures such as finite fields (Galois fields) and cyclotomic classes (Beth et al., 1999; Dinitz & Stinson, 2024; Colbourn & Dinitz, 2007; Street, 1987). Classical constructions — including those derived from symmetric designs, affine and projective planes, and difference sets provide families of BIBDs with rich structural properties and wide applicability (Hedayat et al., 1999; Shrikhande & Raghavarao, 1994; Bailey, 2008).

More recent work emphasizes algorithmic generation, classification up to isomorphism, and the exploitation of algebraic automorphisms to obtain large classes of nonisomorphic designs with prescribed parameters (Ionin & Shrikhande, 2006; Street, 2010; Kang & Jungnickel, 2021). Constructed balanced incomplete block design using Galois field (Janardan, 2018). Examined algebraic structures in BIBDs and exploration of optimal BIBDs through various constructions (Akra et al., 2023, 2024). Further, Akra et al., (2025) investigated isomorphisms and automorphisms of BIBDs, highlighting their structural symmetries. Constructed balanced incomplete block design (BIBD) using finite Euclidean and projective geometry approach (Akra et al., 2021, 2025).

From a statistical perspective, BIBDs yield desirable inferential properties: when the design is connected, treatment contrasts are estimable and the information matrix has a simple form determined entirely by the parameter (v, r, λ), enabling closed-form expressions for variances of elementary contrasts and facilitating comparisons of efficiency against completely randomized and randomized block designs (Pearce, 1984). Advances in computational linear algebra and simulation methods have also allowed practitioners to assess robustness to missing observations, heteroscedastic errors, and departures from model assumptions, thereby expanding the practical utility of BIBDs in modern applications (Kageyama & Kubota, 2016; Mukerjee & Das, 2017; Li & Wang, 2023).

Despite their theoretical strengths, several practical and theoretical challenges remain. Existence results for BIBDs are incomplete for many parameter sets, and constructions that rely on algebraic operations (e.g., multiplicative subgroups of finite fields) require careful verification of balance and intersection properties for each parameter regime (Araujo & Pardo, 2022; Isaac et al., 2025). Moreover, modern applications increasingly demand flexible designs that tolerate missing plots and adapt to complex nuisance structures; new procedure for constructing N-point D-optimal symmetric and asymmetric designs; bridging combinatorial existence with statistical robustness and algorithmic scalability continues to be an active area of research (Onyeka & Akra, 2024; Isaac et al., 2025). In this work we investigate the viability of a multiplication-based construction in Galois fields for producing BIBDs, and we assess the resulting designs' combinatorial properties and statistical performance.

MATERIALS AND METHODS

Galois Field Design

An algebraic structure satisfying all the axioms of the field but with F being a finite set of elements is known as a Galois field and it is denoted by $GF(q)$ or GF(p^n), where p is a prime number and n is a positive. These fields have well-defined operations of addition and multiplication with properties that are useful for constructing designs. The concept of a polynomial in ordinary algebra can be extended to any field. If $a_1, a_2 \ldots\ldots, b_0, b_1, b_2, \ldots .$ are elements of any field F, then the elements of the form;

$$\boldsymbol{f(x) = a_0 + a_1x_1 + a_2x^2 + \ldots .} \quad (1)$$

Constitute the sets of polynomials belonging to what may be called the commutative ring$F[x]$, under addition and multiplication defined in an ordinary way.$(a_0 + a_1x_1 + a_2x^2 + \ldots) + (b_0 + b_1x_1 + b_2x^2 + \ldots) =\rightleftarrows (a_0 + b_0) + (a_1 + b_1)x + \ldots$ and$(b_0 + b_1x_1 + b_2x^2 + \ldots) = a_0b_0 + (a_1b_0 + a_0b_1)x + \ldots + (a_0 + a_1x_1 + a_2x^2 + \ldots)$ (2)

This field exist for every finite number of elements which is the power of a prime. It is clear that every number of elements contained by a Galois field (a field with a finite number of elements) must be of the formp^n, where p is a prime integer and n any positive integer. Thus every element of $GF(p^m)$ can be expressed in the standard form;

$$f(x) = a_0 + a_1x_1 + a_2x^2 + \ldots . + a_{p-1}x^{n-1} \quad (3)$$

Where $a_0, a_1, a_2 \ldots$ are integers ranging from 0 to$p - 1$.

Definition: Irreducible Polynomial

In the field of rational polynomials$Q[x]$ (i.e. polynomials$f(x)$ with rational coefficients), $f(x)$ is said to be irreducible if there do not exist two none constant polynomials$g(x)$ and$h(x)$ in *x* with rational coefficients such that;

$$f(x) = g(x)h(x). \quad (4)$$

Definition: Primitive Root

In$GF(p^m)$, a nonzero element *is* said to be primitive if the order of$x$is$p - 1$.

The powers of a primitive element generate all the nonzero elements of$GF(p^m)$,$x$is the primitive root of$GF(p^m)$ if $x$satisfies the equation;

$$x^{p^m - 1} + (p - 1) = 0 \quad (5)$$

Definition: Minimum function

If the function$f(x)$ can be factorized with the help of$GF(p^m)$ then the function$f(x)$ is called the minimum function of$GF(p^m)$. The function$p(x)$ is said to be a minimum function for generating the elements of$GF(p^m)$, the non-zero elements may be represented either as polynomials degree at most$(m - 1)$ as we know the power of primitive root *x* such that$x^{p^m-1} + (p - 1) = 0$. To obtain the minimum function we divide$x^{p^m-1} + (p - 1)$ by the least common multiple of all factors lies$x^d + 1$, where d is a divisor of$p^m - 1$. The order of the equation will be$\psi_k(p^m - 1)$, where (ψ_k) denotes the number of positive integers less than *k* and relatively prime to it. In this equation, by replacing each coefficient by its least non-zero residue to modulus$p$, we get the cyclotomic polynomial of order$\psi_k(p^m - 1)$.

Definition: Latin square and Orthogonal and Mutual Orthogonal Latin Square

Latin square is an $n \times n$ array filled with *n* different symbols, each occurring exactly once in each row and exactly once in each column. Note that a Latin Square is an incomplete design, which means that it does not include observations for all possible combinations of *i*, *j* and *k*. Once we know the row and column of the design, then the treatment is specified.

When two Latin squares of same order are superimposed on one another, in the resultant array if every ordered pair of symbols occurs exactly once, then the two Latin squares are said to be orthogonal.

Balanced Incomplete Block Design

The construction of balanced incomplete block design depends on the total arrangement of the treatments into blocks. Balanced incomplete block design (BIBD) are satisfied by the following relations. The relations (i) – (iii) are some necessary but not sufficient conditions for the existence of BIBDs.

The parameters$v, b, \rightleftarrows k, rand\lambda$ of a BIBD on $X = \{x_i\}_{i=1}^t$satisfies the following conditions:

(i) $b \rightleftarrows k = vr$ (6)

(ii) $\lambda(v - 1) = r(k - 1)$ (7)

(iii) $b \geq v$ (8)

Construction of Galois Field

The element of Galois field $GF(p^m)$ is defined as;

$GF(p^m)$ =(0,1,2, $\ldots, p - 1) \cup (p, p + 1, p + 2, \ldots, p + p - 1) \cup (p^2, p^2 + 1, p^2 + 2, \ldots, p^2 + p - 1) \cup (p^m, p^{m-1} + 1, p^{m-1} + 2, \ldots, p^{m-1} + p - 1)$ (9)

The order of the field is given by $m \in Z^+$while $m \in Z^+$ is called the characteristics of the field.

The function p(^{x}) is said to be a minimum function for generating the elements of *GF* (*p* $^{m)}$. The non-zero elements may be represented either as polynomials degree at most (m–^{1}) as we know the power of primitive root *x* such that $x^{\text{pm-1}} + (p-1) = 0$. To obtain a cyclotomic equation we divide $_x{}^{pm-1}{}_+(p-1)$ by the least common multiple of all factors lies $_x{}^d$ +1, where *d* is a divisor of $_{\text{pm-1.}}$ If the cyclotomic equation is factorized, minimum function(s) is or are obtained which is also the factor(s) or the reducible and the irreducible polynomial of a lower degree

Let p(^{x}) be an irreducible factor of this polynomial, then p(^{x}) is a minimum function which is in general not unique. Construction of Galois field of p^melements fromp^{th}order field$GF(p)$. Thep^{th}elements of$GF(p)$ are 0, 1, … , $(p - 1)$ and a new symbol.

Construct BIBD using Galois field

Construct BIBD using Galois field of the form GF (q) involved the following steps:

(i) Chose a prime factor(s) and check whether the prime factors satisfy the axioms of a field.

(ii) Construct Galois field, by obtaining cyclotomic equation and primitive root,

(iii) Factorize the cyclotomic equation to obtain a minimum function which is the factor(s) or polynomial of a lower degree

(iv) Use the minimum function and the elements of the Galois field GF (p) to obtain the multiplicative binary operation

(v) Substitute the primitive roots to obtain a Latin square.

(vi) Superimpose the different Latin squares to obtain mutual orthogonal Latin square and then have a set of design that will form a BIBD.

(vii) After the construction of BIBD, use the parameters v, b, k, r and λ of a BIBD to check if it satisfies the following conditions, then draw a generalized conclusion to further ascertain the result obtained.

RESULTS AND DISCUSSION

Construction of BIBD GF (3) Using Multiplicative Operation

The elements of Galois field GF (3) are 0, 1 and$\alpha$with minimum function$2x + 1$

Then GF (3) is used to obtain the following Latin squares as follows:

$$L_1 = \begin{bmatrix} \times & 0 & 1 & \alpha \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & \alpha \\ \alpha & 0 & \alpha & \alpha^2 \end{bmatrix} \Rightarrow \begin{bmatrix} \times & 0 & 1 & \alpha \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 2 \\ 2 & 0 & 2 & 1 \end{bmatrix} \text{ at } \alpha = 2$$

implies $\begin{bmatrix} \times & 1 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ Latin square

Blocks for the first Latin square design is given as;

Column blocks Row blocks

$\overset{B1}{\begin{bmatrix} 1 \\ 2 \end{bmatrix}}, \overset{B2}{\begin{bmatrix} 2 \\ 1 \end{bmatrix}}$ $\quad B1:[12]\ B2:[21]$

Multiplying the vertical column of L_1 by α to get the 2nd Latin Square

$$L_2 = \begin{bmatrix} \times & 0 & 1 & \alpha \\ 0 & 0 & 0 & 0 \\ \alpha & 0 & \alpha & \alpha^2 \\ \alpha^2 & 0 & \alpha^2 & \alpha^3 \end{bmatrix} \Rightarrow \begin{bmatrix} \times & 0 & 1 & \alpha \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 2 & 1 \\ 2 & 0 & 1 & 2 \end{bmatrix}$$

at $\alpha = 2$implies $\begin{bmatrix} \times & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ Latin square

Blocks for the second Latin square design is given as;

Column blocks Row blocks

$\overset{B1}{\begin{bmatrix} 2 \\ 1 \end{bmatrix}}, \overset{B2}{\begin{bmatrix} 1 \\ 2 \end{bmatrix}}$ $\quad B1:[21]\ B2:[12]$

=Multiplying the vertical column of L_1 by α^2 to get the 3rd Latin Square

$$L_3 = \begin{bmatrix} \times & 0 & 1 & \alpha \\ 0 & 0 & 0 & 0 \\ \alpha^2 & 0 & \alpha^2 & \alpha^3 \\ \alpha^3 & 0 & \alpha^3 & \alpha^4 \end{bmatrix} \Rightarrow \begin{bmatrix} \times & 0 & 1 & \alpha \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 2 \\ 2 & 0 & 2 & 1 \end{bmatrix}$$

at $\alpha = 2$ implies $\begin{bmatrix} \times & 1 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ Latin square

Blocks for the third Latin square design is given as;

Column blocks Row blocks

$\overset{B1}{\begin{bmatrix} 1 \\ 2 \end{bmatrix}}, \overset{B2}{\begin{bmatrix} 2 \\ 1 \end{bmatrix}}$ $\quad B1:[12]$

$B2:[21]$

Multiplying the vertical column of L_1 by α^3 to get the 4th Latin Square

$$L_4 = \begin{bmatrix} \times & 0 & 1 & \alpha \\ 0 & 0 & 0 & 0 \\ \alpha^3 & 0 & \alpha^3 & \alpha^4 \\ \alpha^4 & 0 & \alpha^4 & \alpha^5 \end{bmatrix} \Rightarrow \begin{bmatrix} \times & 0 & 1 & \alpha \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 2 & 1 \\ 2 & 0 & 1 & 2 \end{bmatrix}$$

at $\alpha = 2$ implies $\begin{bmatrix} \times & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ Latin square

Blocks for the fourth Latin square design is given as;

Column blocks Row blocks

$\overset{B1}{\begin{bmatrix} 2 \\ 1 \end{bmatrix}}, \overset{B2}{\begin{bmatrix} 1 \\ 2 \end{bmatrix}}$ $\quad B1:[21]\ B2:[12]$

Superimpose the four Latin squares to obtain mutually orthogonal Latin square (MOLS) design, thus;

$$L_p = \begin{bmatrix} \times & 1 & 2 \\ 1 & 1212 & 2121 \\ 2 & 2121 & 1212 \end{bmatrix}$$

Neglecting the first row and column ofL_p the MOLS with the defining parameters: r = 8, v = 3 b = 4, k = 4, and λ = 12

Now using the parameters v, b, k, r and λ to check the three conditions of a BIBD. That is;

(1) bk $=$ vr $\Rightarrow (4 \times 4) \neq 3(8)$ (Not satisfy)

(2) $\lambda(v-1) = r(k-1) \Rightarrow 12(2) = 8(3)$ (Satisfy)

(3) $b \geq v \Rightarrow 2 < 3$ (Not satisfy)

Since the two parametric relations of BIBD are not satisfied, then the design is a not a BIBD under multiplicative binary operation.

Construction of BIBD GF (5) Using Multiplicative Operation

The elements of Galois field GF (5) are 0, 1, α, α^2 and α^3 with minimum function$2x+1$

Then GF (5) is used to obtain the following Latin squares as follows:

$$L_1 = \begin{bmatrix} \times & 0 & 1 & \alpha & \alpha^2 & \alpha^3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & \alpha & \alpha^2 & \alpha^3 \\ \alpha & 0 & \alpha & \alpha^2 & \alpha^3 & \alpha^4 \\ \alpha^2 & 0 & \alpha^2 & \alpha^3 & \alpha^4 & \alpha^5 \\ \alpha^3 & 0 & \alpha^3 & \alpha^4 & \alpha^5 & \alpha^6 \end{bmatrix} \Rightarrow \begin{pmatrix} \times & 1 & 2 & 4 & 3 \\ 1 & 1 & 2 & 4 & 3 \\ 2 & 2 & 4 & 3 & 1 \\ 4 & 4 & 3 & 1 & 2 \\ 3 & 3 & 1 & 2 & 4 \end{pmatrix}$$

at$\alpha =$ 2is a Latin square (first)

Blocks for the first Latin square design is shown as;

Column blocks (b) Row blocks

$\overset{B1}{\begin{bmatrix} 1 \\ 2 \\ 4 \\ 3 \end{bmatrix}}, \overset{B2}{\begin{bmatrix} 2 \\ 4 \\ 3 \\ 1 \end{bmatrix}}, \overset{B3}{\begin{bmatrix} 4 \\ 3 \\ 1 \\ 2 \end{bmatrix}}, \overset{B4}{\begin{bmatrix} 3 \\ 1 \\ 2 \\ 4 \end{bmatrix}}$ $B1:[1234]\ B2:[2431]\ B3:[4312]\ B4:[3124]$

Multiplying the vertical column of L_1 by α to get the 2nd Latin Square

$$L_2 = \begin{bmatrix} \times & 0 & 1 & \alpha & \alpha^2 & \alpha^3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & 0 & \alpha & \alpha^2 & \alpha^3 & \alpha^4 \\ \alpha^2 & 0 & \alpha^2 & \alpha^3 & \alpha^4 & \alpha^5 \\ \alpha^3 & 0 & \alpha^3 & \alpha^4 & \alpha^5 & \alpha^6 \\ \alpha^4 & 0 & \alpha^4 & \alpha^5 & \alpha^6 & \alpha^7 \end{bmatrix} \Rightarrow \begin{pmatrix} \times & 1 & 2 & 4 & 3 \\ 2 & 2 & 4 & 3 & 1 \\ 4 & 4 & 3 & 1 & 2 \\ 3 & 3 & 1 & 2 & 4 \\ 1 & 1 & 2 & 4 & 3 \end{pmatrix}$$

at $\alpha = 2$is a Latin square

The blocks for the second Latin square design is shown as;

Column blocks (b) Row blocks

$\overset{B1}{\begin{bmatrix} 2 \\ 4 \\ 3 \\ 1 \end{bmatrix}}, \overset{B2}{\begin{bmatrix} 4 \\ 3 \\ 1 \\ 2 \end{bmatrix}}, \overset{B3}{\begin{bmatrix} 3 \\ 1 \\ 2 \\ 4 \end{bmatrix}}, \overset{B4}{\begin{bmatrix} 1 \\ 2 \\ 4 \\ 3 \end{bmatrix}}$ $B1:[2431]\ B2:[4312]\ B3:[3124]\ B4:[1243]$

Multiplying the vertical column of L_1 by α^2 to get the 3rd Latin Square

$$L_3 = \begin{bmatrix} \times & 0 & 1 & \alpha & \alpha^2 & \alpha^3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha^2 & 0 & \alpha^2 & \alpha^3 & \alpha^4 & \alpha^5 \\ \alpha^3 & 0 & \alpha^3 & \alpha^4 & \alpha^5 & \alpha^6 \\ \alpha^4 & 0 & \alpha^4 & \alpha^5 & \alpha^6 & \alpha^7 \\ \alpha^5 & 0 & \alpha^5 & \alpha^6 & \alpha^7 & \alpha^8 \end{bmatrix} \Rightarrow \begin{pmatrix} \times & 1 & 2 & 4 & 3 \\ 4 & 4 & 3 & 1 & 2 \\ 3 & 3 & 1 & 2 & 4 \\ 1 & 1 & 2 & 4 & 3 \\ 2 & 2 & 4 & 3 & 1 \end{pmatrix}$$

at $\alpha =$ 2is a Latin square

Blocks for the third Latin square design is shown as;

Column blocks (b) Row blocks

$\overset{B1}{\begin{bmatrix} 4 \\ 3 \\ 1 \\ 2 \end{bmatrix}}, \overset{B2}{\begin{bmatrix} 3 \\ 1 \\ 2 \\ 4 \end{bmatrix}}, \overset{B3}{\begin{bmatrix} 1 \\ 2 \\ 4 \\ 3 \end{bmatrix}}, \overset{B4}{\begin{bmatrix} 2 \\ 4 \\ 3 \\ 1 \end{bmatrix}}$ $B1:[4312]\ B2:[3124]\ B3:[1243]\ B4:[2431]$

Multiplying the vertical column of L_1 by α^3 to get the 4th Latin Square.

$$L_4 = \begin{bmatrix} \times & 0 & 1 & \alpha & \alpha^2 & \alpha^3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha^3 & 0 & \alpha^3 & \alpha^4 & \alpha^5 & \alpha^6 \\ \alpha^4 & 0 & \alpha^4 & \alpha^5 & \alpha^6 & \alpha^7 \\ \alpha^5 & 0 & \alpha^5 & \alpha^6 & \alpha^7 & \alpha^8 \\ \alpha^6 & 0 & \alpha^6 & \alpha^7 & \alpha^8 & \alpha^9 \end{bmatrix} \Rightarrow \begin{pmatrix} \times & 1 & 2 & 4 & 3 \\ 3 & 3 & 1 & 2 & 4 \\ 1 & 1 & 2 & 4 & 3 \\ 2 & 2 & 4 & 3 & 1 \\ 4 & 4 & 3 & 1 & 2 \end{pmatrix}$$

at $\alpha =$ 2is a Latin square

The blocks for the fourth Latin square design is shown as;

Column blocks (b) Row blocks

$\overset{B1}{\begin{bmatrix} 3 \\ 1 \\ 2 \\ 4 \end{bmatrix}}, \overset{B2}{\begin{bmatrix} 1 \\ 2 \\ 4 \\ 3 \end{bmatrix}}, \overset{B3}{\begin{bmatrix} 2 \\ 4 \\ 3 \\ 1 \end{bmatrix}}, \overset{B4}{\begin{bmatrix} 4 \\ 3 \\ 1 \\ 2 \end{bmatrix}}$

$B1:[3124]\ B2:[1243]\ B3:[2431]\ B4:[4312]$

Superimpose the four Latin squares to obtain mutually orthogonal Latin square as;

$$L_p = \begin{pmatrix} \times & 1111 & 2222 & 4444 & 3333 \\ 1243 & 1243 & 2431 & 3124 & 4312 \\ 2431 & 2431 & 4312 & 1243 & 3124 \\ 4312 & 4312 & 3124 & 2431 & 1243 \\ 3124 & 3124 & 1243 & 4312 & 2431 \end{pmatrix}$$

Neglecting the first row and column of L_p gives the MOLS design with the defining parameters: v = 5, b = 16, : r = 16, k = 4, and λ = 12.
Now using the parameters v, b, k, r and λ to check the three conditions of a BIBD. That is;
(1) $\mathrm{bk} = \mathrm{vr} \Rightarrow (16 \times 4) \neq 5(16)$ (Not satisfy)
(2) $\lambda(v-1) = r(k-1) \Rightarrow 12(4) = 16(3)$ (Satisfy)
(3) $b \geq v \Rightarrow 16 > 5$ (Satisfy)
Since the one of the parametric relations of BIBD is not satisfied, then the design is a not a BIBD under multiplicative binary operation.

CONCLUSION

The results presented demonstrate the feasibility of constructing BIBDs using GF (p) with prime factors 3 and 5. On the contrary, construction of BIBDs under multiplicative binary operation does not give a precise result because some of the parametric relations of BIBD are not satisfied. Hence, balanced incomplete block design cannot be constructed under multiplicative binary operation using Galois field approach and recommended that additive binary operation should be considered to construct balanced incomplete block design.

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