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DEVELOPING THE ODD BURR III–BURR III (OBIII–BIII) DISTRIBUTION: THEORETICAL FOUNDATIONS AND APPLICATIONS TO SURVIVAL DATA

*[1]Abdulaziz M. Ndatsu and [2]Boubakar Dodo

[1]Department of Statistics, Usmanu Danfodiyo University Sokoto, Nigeria
[2]Département d'Informatique, IUT/ Université Djibo Hamani de Tahoua, Niger

*Corresponding authors' email: da6thabdul@gmail.com

ABSTRACT

The normal, exponential, gamma, Dagum, and Burr distributions are commonly used to model uni-modal datasets. While the Burr III distribution is widely applied in reliability and income modeling, it is limited by its inherently decreasing hazard rate, inability to analyze complex survival datasets, bathtub shape hazard rates and asymmetric datasets, which makes it unsuitable for multimodal observations, hence the need to extend the classical Burr III distribution. In response, the study introduced the modified form of Burr III distribution called Odd Burr III-Burr III distribution with four parameters. The new distribution has the ability to capture various data behaviors, including heavy tails, skewness other shapes that the conventional Burr III distribution can't captured. The probability density function (PDF) and cumulative distribution function (CDF) of the propose model were derived and Log-likelihood function were also sorted out, the goodness of fit and information criteria were adapted to assess the performance of the proposed model using simulated data and real-life applications.

Keywords: Odd Burr III family, Burr III distribution, Cumulative distribution function, Probability density function, Survival data

INTRODUCTION

Probability distributions play a vital role in statistical theory and applications. They describe the likelihood of different outcomes in a random experiment and provide mathematical models for real-life phenomena. Continuous probability distributions, in particular, are essential in modeling various types of lifetime data, reliability systems, and environmental measurements (Manu, et al., 2023). In many scientific and engineering fields such as hydrology, climatology, and survival analysis, the modeling of extreme events or lifetime behavior is of great importance. This need has led to the development of several flexible families of probability distributions capable of capturing skewness, kurtosis, and tail behavior in data more accurately (Abdulhameed *et al*, 2025)

Burr (1942) introduced a comprehensive system of probability distributions designed for modeling cumulative frequency data. This system consists of twelve distinct cumulative distribution functions, each capable of capturing a wide range of distributional shapes. Its appeal lies in the combination of mathematical simplicity and flexibility, particularly in its extensive coverage of the skewness kurtosis space. Numerous classical distributions including the Weibull, exponential, logistic, generalized logistic, Gompertz, normal, extreme value, and uniform distributions emerge as special or limiting cases within the Burr system. Among the twelve forms, the Burr type XII (BXII) distribution and its inverse, the Burr type III (BIII), have attracted substantial interest across fields such as physics, actuarial science, reliability analysis, and applied statistics. Their versatility has supported diverse applications, Nadarajah and Kotz (2007) analyzed fracture toughness and fracture stress data using Burr-type models; Gove et al. (2008) applied the Burr III distribution in forestry-related studies; Mielke (1973) employed it to model precipitation amounts in meteorological research; and Shao et al. (2008) proposed and implemented an extended Burr III distribution for low-flow frequency analysis, with particular emphasis on its behavior in the lower tail. The Burr Type III distribution is one of the most versatile members of this family, it is a continuous distribution defined on the positive real line and is known for its heavy-tailed nature and flexible hazard rate. However, despite its flexibility, the Burr III model may still be limited in describing data sets with more complex skewness or kurtosis patterns. To overcome these limitations, statisticians have proposed several generalization techniques such as the exponentiated, Marshall–Olkin, and odd transformation methods. The odd transformation approach is particularly powerful because it introduces an additional shape parameter that increases the model's ability to capture different hazard-rate structures ranging from increasing and decreasing to bathtub-shaped patterns. This approach led to the formulation of the Odd Burr III distribution, which modifies the cumulative distribution function of Burr III through an odd transformation of the baseline model.

Building on this idea, in this study we propose the Odd Burr III–Burr III distribution, sometimes denoted as the Odd Burr III-G family where the baseline G is again Burr III. This model combines the strengths of both the Burr III baseline and the odd transformation, producing a new family with enhanced flexibility. The additional parameters allow it to adapt to various shapes of empirical data, making it suitable for modeling lifetime and reliability data where other models fail to provide adequate fits.

Although the Burr III distribution has been extensively used in modeling lifetime and reliability data, its flexibility is still limited when dealing with data characterized by complex hazard rate patterns or heavy tails. Many real-world data sets such as those arising from biological survival times, mechanical failure times, or economic losses require models that can adapt to multiple shapes of hazard functions and varying degrees of skewness.

Existing generalizations of the Burr III distribution, such as the exponentiated or Marshall–Olkin extensions, provide some improvement but may not be sufficiently versatile for all data structures. The Odd Burr III–Burr III distribution offers a promising alternative by introducing additional parameters that enhance the model's ability to fit a broader range of data types. However, the mathematical properties of

this distribution, including its reliability functions, moments, and parameter estimation procedures, are not yet fully explored or documented. The main aim of this study is to introduce the Odd Burr III Burr III Distribution as an extension of the classical Burr III distribution, to improve its flexibility and applicability in modeling lifetime data.
However, similar works existed in the literature such as: Agboola & Adebiyi (2023), Haq, M. *et al.* (2019) introduced the generalized Odd Burr III (GOBIII) family, which incorporates additional shape parameters and thus yields a more flexible class capable of capturing more extreme tail behavior and varied skewness than the original single-parameter form. Their work also develops key distributional properties, quantile functions, and estimation methods, and shows that the GOBIII models provide superior fits to multiple datasets when compared with simpler alternatives, Jamal *et al* (2017) introduced the odd transformer applied to Burr-III, derived its PDF, CDF and exploring fundamental statistical properties of the model, Oluyede, *et al* (2021) introduced the Weibull Odd Burr III-G Family (2021) through T-X generator, Maokafi *et al* (2022) proposed the Topp-Leone Odd Burr III-G family, which models data on bounded supports and captures a variety of complex hazard rates shapes including bathtub and increasing/decreasing shapes, Oluyed & Chipepa (2025) introduced Power-series Odd Burr III (OBIII-GPS) using flexible mixture mixture distribution families. The authors concluded that the approach marks a modern trend of constructing flexible mixture and power-series generalizations to capture complex distributional features such as over-dispersion and heterogeneity. Similarly, On the Generalized Log Burr III Distribution was introduced by Bhatti et al, (2019). The authors addresses log-scale modelling through log-Burr III variants, providing transformation and parameter estimation techniques that enhanced Burr III applications in various fields , Noori, et al, (2023) presented the Modified Burr III (MBIII) distribution. The authors derived key mathematical properties, such as the moments, hazard rate function, and survival function and carried out parameter estimation using MLE. Other including: Behairy *et al* (2016) showed that using exponentiation in models with complex hazard shape and quantile function leads to efficient simulation and application outcomes.

The Odd Burr III-Burr III (OBIII-BIII) Distribution

The probability generator adapted in this research is the Odd Burr III generalized family of distribution, the general form of the Odd Burr III-G distribution is constructed by applying a generator function G(.) to the CDF of the Odd Burr III distribution.

$$G(x,c,k) = \left(1+\frac{F(x)^{-c}}{1-F(x)}\right)^{-k} \quad c>0, k>0. \tag{1}$$

The corresponding probability density function (PDF) is:

$$g(x;c,k) = ckf(x)\frac{(1-F(x))^{c-1}}{(F(x))^{c+1}}\left(1+\left(\frac{F(x)}{1-F(x)}\right)^{-c}\right)^{-k-1} \quad c>0, k>0 \tag{2}$$

Similarly, the baseline distribution used in this research is the Burr III distribution. The cumulative distribution function (CDF) and probability density function (PDF) are defined as:

$$F(x,\alpha,\lambda) = (1+x^{-\alpha})^{-\lambda} \quad \alpha>0, \lambda>0, x>0. \tag{3}$$

The corresponding PDF is:

$$f(x;\alpha,\lambda) = \alpha\lambda x^{-c-1}(1+x^{-\alpha})^{-\lambda-1} \quad x>0, \alpha, \lambda>0. \tag{4}$$

The proposed distribution (Odd Burr III Burr III distribution) has its cumulative distribution function (CDF) and probability density function (PDF) as:

$$G(x) = \left(1+\left(\frac{(1+x^{-\alpha})^{-\lambda}}{1-(1+x^{-\alpha})^{-\lambda}}\right)^{-c}\right)^{-k} \quad c,k,\alpha,\lambda>0. \tag{5}$$

And

$$f(x) = \frac{ck\lambda\alpha x^{-\alpha-1}(1+x^{-\alpha})^{-\lambda-1}}{\left(1+(1+x^{-\alpha})^{-\lambda}\right)^2}\left(\frac{(1+x^{-\alpha})^{-\lambda}}{1-(1+x^{-\alpha})^{-\lambda}}\right)^{-c-1}\left(1+\left(\frac{(1+x^{-\alpha})^{-\lambda}}{1-(1+x^{-\alpha})^{-\lambda}}\right)^{-c}\right)^{-k-1} \quad c,k,\alpha,\lambda>0 \tag{6}$$

By substituting Eq. (3) in (1), yields (5). Similarly, we obtained Eq. (6) by inputting (3) and (4) in (2).

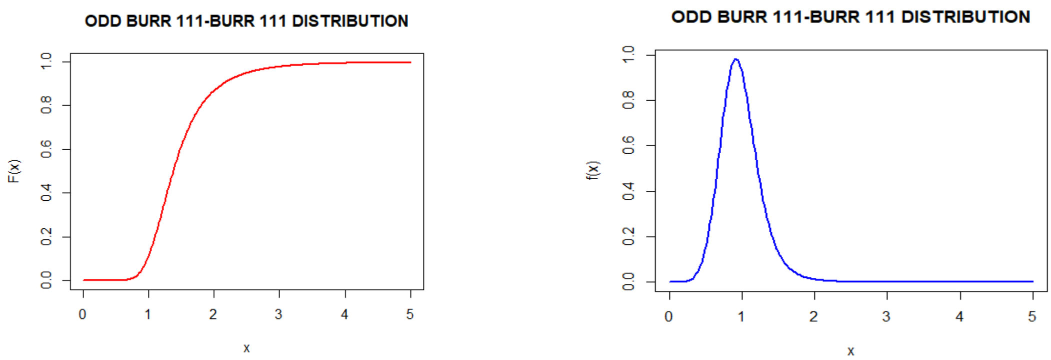

Figure 1: Plots showing the shapes of the CDF and PDF of the Odd Burr III-Burr III model

Theorem (Normalization of PDF)

To normalize the Odd Burr III Burr III distribution, we need to ensure that the total probability over all possible outcomes integrates to 1. That is:

$$\int_{-\infty}^{\infty} f(x)d(x) = 1. \tag{7}$$

Proof:

Substituting Eq. (6) in (7), yields;

$$\int_{-\infty}^{\infty} f(x)d(x) = \int_{-\infty}^{\infty} \frac{ck\lambda\alpha x^{-\alpha-1}(1+x^{-\alpha})^{-\lambda-1}}{(1+(1+x^{-\alpha})^{-\lambda})^{2}}\left(\frac{(1+x^{-\alpha})^{-\lambda}}{1-(1+x^{-\alpha})^{-\lambda}}\right)^{-c-1}\left(1+\left(\frac{(1+x^{-\alpha})^{-\lambda}}{1-(1+x^{-\alpha})^{-\lambda}}\right)^{-c}\right)^{-k-1} d(x). \quad (8)$$

Let$y = (1+x^{-\alpha})^{-\lambda} 0 < y < 1, \frac{d(y)}{d(x)} = -\lambda(1+x^{-\alpha})^{-\lambda-1} - \alpha x^{-\alpha-1}$

$d(x) = \frac{1}{\lambda(1+x^{-\alpha})^{-\lambda-1}\alpha x^{-\alpha-1}} = \frac{1}{\lambda\alpha x^{-\alpha-1}(1+x^{-\alpha})^{-\lambda-1}} d(y).$

Limits: $x \to 0, y \to 1, x \to \infty, y \to 0.$

Imputing the following terms in Eq. (8) yields Eq. (9);

$$\int_{-\infty}^{\infty} f(x)d(x) = \int_{0}^{1} \frac{ck\lambda\alpha x^{-\alpha-1}(y)^{-1}}{(1-y)^{2}}\left(\frac{y}{1-y}\right)^{-c-1}\left(1+\left(\frac{y}{1-y}\right)^{-c}\right)^{-k-1} \times \frac{1}{\lambda\alpha x^{-\alpha-1}(y)^{-1}} d(y). \quad (9)$$

Cancelling out common terms in the left and right hand side of Eq. (9) yield Eq. (10);

$$\int_{-\infty}^{\infty} f(x)d(x) = \int_{0}^{1} \frac{ck}{(1-y)^{2}}\left(\frac{y}{1-y}\right)^{-c-1}\left(1+\left(\frac{y}{1-y}\right)^{-c}\right)^{-k-1} d(y). \quad (10)$$

Let$m = \frac{y}{1-y}$, 0<m<1, $\frac{dm}{dy} = \frac{1}{(1-y)^{2}}$, $dy = (1-y)^{2}dm$, $y \to 0, m \to 1, y \to 1, m \to 0$.

By substituting these terms into Eq. (10), we generate Eq. (11):

$$\int_{-\infty}^{\infty} f(x)d(x) = \int_{0}^{1} \frac{ck}{(1-y)^{2}} m^{-c-1}(1+m^{-c})^{-k-1} \times (1-y)^{2}dm. \quad (11)$$

Cancelling out common terms in Eq. (11) would yield Eq. (12):

$$\int_{-\infty}^{\infty} f(x)d(x) = ck\int_{0}^{1} m^{-c-1}(1+m^{-c})^{-k-1}\, dm. \quad (12)$$

Let$p = (1+m)^{-c}$, 0<P<1, $\frac{dp}{dm} = -c(m)^{-c-1}$,$dm = \frac{1}{-c(m)^{-c-1}}dp.$

Limits: $x \to 0, p \to 1, x \to 1, p \to 0.$

By substituting these terms into Eq. (12) we generate Eq. (13):

$$\int_{-\infty}^{\infty} f(x)d(x) = ck\int_{0}^{1} m^{-c-1}p^{-k-1} \times \frac{1}{-c(m)^{-c-1}}dp. \quad (13)$$

Cancelling out common terms in Eq. (13) would yield:

$$\int_{-\infty}^{\infty} f(x)d(x) = -k\int_{0}^{1} p^{-k-1}dp. \quad (14)$$

Therefore integrating Eq. (14) with the given limits yielded 1.

$$\int_{-\infty}^{\infty} f(x)d(x) = -k\left(\frac{p^{-k-1+1}}{-k-1+1}\right)_{0}^{1} = p^{-k}/_{0}^{1} = (1)^{-k} - (0)^{-k} = 1.$$

Hence; $\int_{-\infty}^{\infty} f(x)d(x) = 1.$

Reliability Properties

The Odd Burr III Burr III distribution offers a robust framework for reliability analysis with properties like survival function, hazard function and cumulative hazard function thereby providing valuable insights into the systems performance.

Survival function S(x)

$$S(x) = 1 - F(x) \quad (15)$$

By substituting Eq. (5) into Eq. (15), we generate Eq. (16).

$$S(x) = 1 - \left(1+\left(\frac{(1+x^{-\alpha})^{-\lambda}}{1-(1+x^{-\alpha})^{-\lambda}}\right)^{-c}\right)^{-k} \quad (16)$$

Hazard function h(x);

$$h(x) = \frac{f(x)}{S(x)} \quad (17)$$

By substituting Eq. (6) into Eq. (17) ,we generate Eq. (18).

$$h(x) = \frac{\frac{ck\lambda\alpha x^{-\alpha-1}(1+x^{-\alpha})^{-\lambda-1}}{(1+(1+x^{-\alpha})^{-\lambda})^{2}}\left(\frac{(1+x^{-\alpha})^{-\lambda}}{1-(1+x^{-\alpha})^{-\lambda}}\right)^{-c-1}\left(1+\left(\frac{(1+x^{-\alpha})^{-\lambda}}{1-(1+x^{-\alpha})^{-\lambda}}\right)^{-c}\right)^{-k-1}}{1-\left(1+\left(\frac{(1+x^{-\alpha})^{-\lambda}}{1-(1+x^{-\alpha})^{-\lambda}}\right)^{-c}\right)^{-k}} \quad (18)$$

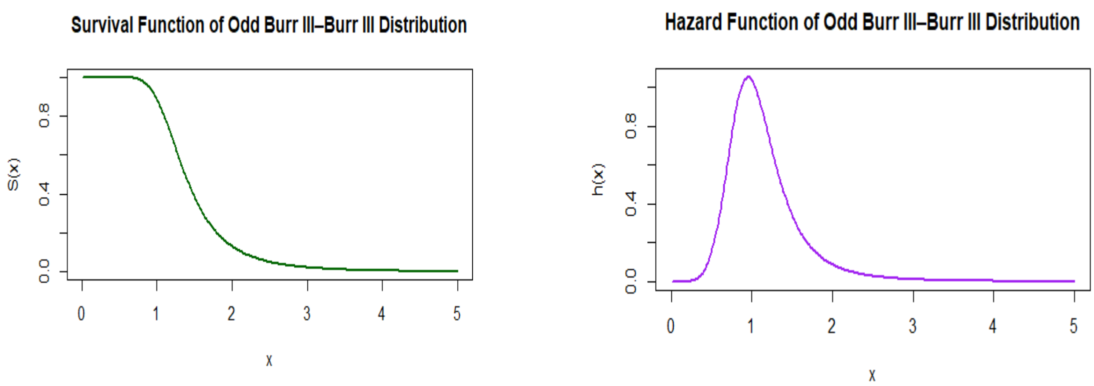

Figure 2: Plots showing the shapes of the Survival and Hazard of the Odd Burr III-Burr III model

Cumulative Hazard Function (CHF):

The cumulative hazard function for the Odd Burr III Burr III distribution is the negative logarithm of the survival function;

$$h(x) = -ln\left(S(x)\right) \tag{19}$$

Substituting the survival function in Eq. (16), yield:

$$h(x) = -ln\left(1 - \left(1 + \left(\frac{(1+x^{-\alpha})^{-\lambda}}{1-(1+x^{-\alpha})^{-\lambda}}\right)^{-c}\right)^{-k}\right) \tag{20}$$

Quartile Function

The quantile function, is the inverse of the cumulative distribution function (CDF) and is often used to find specific percentiles of a distribution such as the first, second (median) and third quartiles. It is usually denoted as$Q(x)$. For Odd Burr III Burr III distribution is defined as follows:

$$Q(u) = \left(\left(1 + \left(U^{\frac{1}{-k}} - 1\right)^{\frac{1}{c}}\right)^{\frac{1}{\lambda}} - 1\right)^{-\frac{1}{\alpha}} \tag{21}$$

Proof:

Let X be a random variable following the Odd Burr III Burr III distribution with CDF given by

$$U = \left(1 + \left(\frac{(1+x^{-\alpha})^{-\lambda}}{1-(1+x^{-\alpha})^{-\lambda}}\right)^{-c}\right)^{-k} \quad c, k, \alpha, \lambda > 0$$

By raising both sides to the power of $\frac{1}{-k}$,

$$U^{\frac{1}{-k}} = 1 + \left(\frac{(1+x^{-\alpha})^{-\lambda}}{1-(1+x^{-\alpha})^{-\lambda}}\right)^{-c}, U^{\frac{1}{-k}} - 1 = \left(\frac{(1+x^{-\alpha})^{-\lambda}}{1-(1+x^{-\alpha})^{-\lambda}}\right)^{-c}$$

Raise both sides to the power of $-\frac{1}{c}$, $\left(U^{\frac{1}{-k}} - 1\right)^{-\frac{1}{c}} = \frac{(1+x^{-\alpha})^{-\lambda}}{1-(1+x^{-\alpha})^{-\lambda}}$

Let $Z = (1 + x^{-\alpha})^{-\lambda}$, $\left(U^{\frac{1}{-k}} - 1\right)^{-\frac{1}{c}} = \frac{Z}{1-Z}$

Make Z the subject formula

$$Z = \frac{\left(U^{\frac{1}{-k}}-1\right)^{-\frac{1}{c}}}{1+\left(U^{\frac{1}{-k}}-1\right)^{-\frac{1}{c}}}, (1 + x^{-\alpha})^{-\lambda} = \frac{\left(U^{\frac{1}{-k}}-1\right)^{-\frac{1}{c}}}{1+\left(U^{\frac{1}{-k}}-1\right)^{-\frac{1}{c}}}$$

Raise both sides to the power of $-\frac{1}{\lambda}$,

$$1 + x^{-\alpha} = \left(\frac{1+\left(U^{\frac{1}{-k}}-1\right)^{-\frac{1}{c}}}{\left(U^{\frac{1}{-k}}-1\right)^{-\frac{1}{c}}}\right)^{\frac{1}{\lambda}}, \frac{1+\left(U^{\frac{1}{-k}}-1\right)^{-\frac{1}{c}}}{\left(U^{\frac{1}{-k}}-1\right)^{-\frac{1}{c}}} = 1 + \left(U^{\frac{1}{-k}} - 1\right)^{\frac{1}{c}}$$

Raise both sides to the power of $-\frac{1}{\alpha}$

$$x = \left(\left(1 + \left(U^{\frac{1}{-k}} - 1\right)^{\frac{1}{c}}\right)^{\frac{1}{\lambda}} - 1\right)^{-\frac{1}{\alpha}}$$

Hence the expression of X in terms of U is:

$$(u) = \left(\left(1 + \left(U^{\frac{1}{-k}} - 1\right)^{\frac{1}{c}}\right)^{\frac{1}{\lambda}} - 1\right)^{-\frac{1}{\alpha}}$$

Where u ~ uniform distribution (0,1).

Log Likelihood Function

The likelihood function for the Odd Burr III-Burr III Distribution is expressed as follows:

$$L(c, k, \lambda, \alpha) = \prod_{i=1}^{n} \frac{ck\lambda\alpha x^{-\alpha-1}(1+x^{-\alpha})^{-\lambda-1}}{\left(1+(1+x^{-\alpha})^{-\lambda}\right)^2} \left(\frac{(1+x^{-\alpha})^{-\lambda}}{1-(1+x^{-\alpha})^{-\lambda}}\right)^{-c-1} \left(1 + \left(\frac{(1+x^{-\alpha})^{-\lambda}}{1-(1+x^{-\alpha})^{-\lambda}}\right)^{-c}\right)^{-k-1} \tag{22}$$

And taking the log of both sides of Eq. (22), yields:

$$InL(c, k, \lambda, \alpha) = \sum_{i=1}^{n} \begin{bmatrix} InC + InK + In\lambda + In\alpha + (-\alpha - 1)Inx_i + (-\lambda - 1)In(1 + x_i^{-\alpha}) - 2In\left(1 + (1 + x^{-\alpha})^{-\lambda}\right) + \\ (c - 1)In\left(\frac{(1+x^{-\alpha})^{-\lambda}}{1-(1+x^{-\alpha})^{-\lambda}}\right) + (-k - 1)In\left(1 + \left(\frac{(1+x^{-\alpha})^{-\lambda}}{1-(1+x^{-\alpha})^{-\lambda}}\right)^{-c}\right)^{-k-1} \end{bmatrix} \tag{23}$$

$$InL(c, k, \lambda, \alpha) = n(InC + InK + In\lambda + In\alpha) - (\alpha - 1)\sum_{i=1}^{n} Inx_i - (\lambda - 1)\sum_{i=1}^{n} In(1 + x_i^{-\alpha}) - 2\sum_{i=1}^{n} In\left(1 + (1 + x^{-\alpha})^{-\lambda}\right)$$

$$+$$

$$(c - 1)\sum_{i=1}^{n} In\left(\frac{(1+x^{-\alpha})^{-\lambda}}{1-(1+x^{-\alpha})^{-\lambda}}\right) - (k - 1)\sum_{i=1}^{n} In\left(1 + \left(\frac{(1+x^{-\alpha})^{-\lambda}}{1-(1+x^{-\alpha})^{-\lambda}}\right)^{-c}\right)^{-k-1} \tag{24}$$

The Competing Models

In this section, we would be listing out other competing models, their CDFs and PDFs.

Burr III distribution, introduced by Irving W. Burr (1942), the author defined the CDF and PDF as follows:

$$F(x) = (1 + x^{-c})^{-k} \quad (25)$$

And

$$f(x) = ckx^{-c-1}(1 + x^{c})^{-k-1}, x > 0, c, k > 0 \quad (26)$$

Burr XII distribution was introduced by Irving W. Burr (1942). The CDF and PDF was given as:

$$F(x) = 1 - (1 + x^{c})^{-k} \quad (27)$$

$$f(x) = ckx^{c-1}(1 + x^{c})^{-(k+1)}, x > 0, c > 0, k > 0 \quad (28)$$

Burr X distribution, introduced by Irving W. Burr (1942). The CDF and PDF was defined as:

$$F(x) = 1 - e^{-\left(\frac{k}{x}\right)^{c}}, \quad x > 0 \quad (29)$$

$$f(x) = ck^{c}x^{-c-1}e^{-\left(\frac{k}{x}\right)^{c}}, \quad x > 0, c, k > 0 \quad (30)$$

MO-BIII distribution: The Marshall-Okin Burr III distribution, introduced by Bhatti *et al* (2019), the authors defined the CDF and PDF as follows;

$$F(x) = \frac{(1+x^{-c})^{-k}}{1-\lambda+\lambda(1+x^{-c})^{-k}} \quad (31)$$

$$f(x) = \frac{(1-\lambda)ckx^{-c-1}(1+x^{-c})^{-k-1}}{(1-\lambda+\lambda(1+x^{-c})^{-k})^{2}} x > 0, c, k, \lambda > 0 \quad (32)$$

Model Comparison and Selection Criteria

In this section, the flexibility of the proposed model would be initiated by comparing its performance to other selected models using information criteria in R software. The information criteria to be used are Akaike Information Criteria (AIC), the Bayesian Information Criteria (BIC), the Consistent Akaike Information Criteria (CAIC) and Hannan-Quinn Information Criteria (HQIC). The formulations for the information criteria are given below:

i. Akaike Information Criterion (AIC): AIC = -2ln(L) + 2k.
ii. Bayesian Information Criterion (BIC): BIC = -2 ln(L) + k ln (n).
iii. Consistent Akaike Information Criterion (CAIC): CAIC = -2 ln(L) + k(ln(n) +1).
iv. Hannan-Quinn Information Criterion (HQIC): HQIC = -2 ln (L) + 2k ln(ln(n)).

Where n is for sample size, and k is the number of parameters to be estimated.

The Survival Datasets

Data 1: The data set considered in this study consists of 100 observations of breaking stress of carbon fibers given by [22] as presented below; the data was termed to be normally distributed and it was originally studied by Khalif *et al.*

0.920, 0.9280, 0.9997, 0.9971, 1.0610, 1.117,1.1620, 1.183, 1.187, 1.1920, 1.196, 1.2130,1.215, 1.2199, 1.220, 1.2240, 1.225, 1.2280, 1.237, 1.240, 1.244, 1.259, 1.2610, 1.263, 1.276, 1.310, 1.3210, 1.3290, 1.3310, 1.337, 1.351, 1.359, 1.388, 1.4080, 1.449, 1.4497, 1.450, 1.459, 1.471, 1.475, 1.477, 1.480, 1.489, 1.501, 1.507, 1.515, 1.530, 1.5304, 1.533, 1.544, 1.5443,1.552, 1.556, 1.5620, 1.566, 1.585, 1.586, 1.599, 1.602, 1.6140, 1.6160, 1.617, 1.6280, 1.6840, 1.71100,1.7180, 1.733,1.7380, 1.7380, 1.7430,1.7590, 1.777, 1.7940, 1.799, 1.806, 1.814, 1.8160, 1.8280, 1.830, 1.884, 1.892, 1.944, 1.972, 1.9840, 1.987, 2.02, 2.0304, 2.0290, 2.0350, 2.0370, 2.0430, 2.0460, 2.0590, 2.111, 2.165, 2.686, 2.778, 2.972, 3.504, 3.863, 5.3060.

Data 2: this dataset represents 101 observations that show the failure times (in hours) of Kevlar 49/epoxy strands subjected to constant sustained pressure at a 90 percent stress level. The data was originally given by (R. Barlow *et al* 1984) and has been analyzed in several studies. The observations are as follows:

0.01, 0.02, 0.02, 0.02, 0.03, 0.03, 0.04, 0.05, 0.06, 0.07, 0.07,0.08, 0.09, 0.10, 0.10, 0.11, 0.11,0.12, 0.13, 0.18, 0.19, 0.20, 0.23, 0.24, 0.29, 0.34, 0.35, 0.36, 0.38, 0.40, 0.42, 0.43, 0.52, 0.54, 0.56, 0.60, 0.63,0.65, 0.67, 0.68, 0.72, 0.72, 0.72, 0.73, 0.79, 0.79, 0.80, 0.80,0.85, 0.90, 0.92, 0.95, 0.99, 1.00, 1.01, 1.02, 1.03, 1.05, 1.10, 1.10, 1.15, 1.18, 1.20, 1.29,1.31, 1,33, 1.34, 1.40, 1.43, 1.45, 1.50, 1.51, 1.53, 1.54, 1.55, 1.58, 1.60, 1.63, 1.64, 1.80, 1.80, 1.81, 2.02, 2.14, 2.17, 2.33, 3.03, 3.34, 4.20, 4.69, 7.89.

For the analysis of data, R-language possessing packages was used, The function provides some key comparison criteria. Loglikelihood, AIC, BIC, CAIC and HQIC were considered, The criteria for good fitted model is that the values of these comparison measures should be smaller as compared to others. The analysis of this work would be presented using table.

RESULTS AND DISCUSSION

Results

In this section, the results of the data analysis and the performance of models based on the Odd Burr III–Burr III distribution are presented. All analyses were carried out using the R statistical software in accordance with the methodology outlined in Section 2.

Table 1: Simulation Results for MLE Parameter Performance for Odd Burr III–Burr III and Burr III Models

	n	Parameter	True-value	Estimates	Bias	Variance	MSE
Odd Burr III–Burr III Model	100	C	2.5	2.5404	0.404	0.0017	0.0033
		K	2.5	0.3000	-2.2000	0.0005	4.8400
		λ	1.1	3.0000	1.9000	0.0004	3.6100
		α	1.8	1.1575	-0.6425	0.0141	0.4269
	200	C	2.5	2.5358	0.0358	0.0008	0.0021
		K	2.5	0.2900	-2.1000	0.0004	4.8300
		λ	1.1	2.9000	1.8000	0.0003	3.6000
		α	1.8	1.1573	-0.6410	0.0079	0.4185
	500	C	2.5	2.5355	0.0355	0.0003	0.0015
		K	2.5	0.2600	-2.0000	0.0001	4.8000
		λ	1.1	2.7000	1.6000	0.0002	3.5800
		α	1.8	1.1570	-0.6407	0.0036	0.4155
	1000	C	2.5	2.5350	0.0350	0.0001	0.0010
		K	2.5	0.2200	-1.6000	0.0000	4.7500

		λ	1.1	2.3000	1.1000	0.0001	3.5300
		α	1.8	1.1566	-0.6402	0.0018	0.4150
Burr III Model	100	δ	2.5	2.5267	0.0267	0.0371	0.0378
		γ	2.5	2.5558	0.0558	0.0631	0.0660
	200	δ	2.5	2.5139	0.0139	0.0198	0.0199
		γ	2.5	2.5055	0.0055	0.0283	0.0283
	500	δ	2.5	2.5023	0.0023	0.0070	0.0070
		γ	2.5	2.5038	0.0038	0.0136	0.0136
	1000	δ	2.5	2.5017	0.0017	0.0036	0.0036
		γ	2.5	2.5083	0.0083	0.0058	0.0058

The simulation results above shows clear differences in the estimation efficiency between the two competing distributions. Across all sample sizes, the variance of the parameter estimates under Odd Burr III-Burr III distribution is consistently lower, implying greater stability and less fluctuation across repeated samples. As the sample size increases, the estimators of the Odd Burr III-Burr III distribution converge more rapidly to the true parameter values than those of Burr III distribution demonstrating superior asymptotic efficiency. Although both distributions improve with increasing sample size (as expected from MLE theory), Odd Burr III-Burr III distribution maintains its advantage at every level of n. This suggests that the Odd Burr III-Burr III distribution is better suited for modeling data similar to the one used in this study, and it may offer practical benefits in real-world applications involving reliability and lifetime modeling.

Table 2: Goodness of fit test for Odd Burr III-Burr III Distribution and Competing Models for data-1

Models	LLE	AIC	BIC	CAIC	HQIC	AD	CR-M	K-S
OBIII-BIII	991.27	-1974.54	-1964.241	-1974.105	-1970.376	40.869	8.9269	0.5659
BIII	51.1297	106.2596	111.409	106.3872	108.3417	0.5359	0.0705	0.0694
BXII	-50.3893	107.6786	115.4027	107.9366	110.8018	0.4759	0.0662	0.0675
BX	-74.0433	152.8066	157.956	152.9832	154.8888	8.9515	1.7554	0.2307
MO-BIII	-50.2832	108.5664	118.8652	109.1011	112.7307	0.4308	0.0613	0.0613

Table 3: Goodness of fit test for Odd Burr III-Burr III Distribution and Competing Models for data-2

Models	LLE	AIC	BIC	CAIC	HQIC	AD	CR-M	K-S
OBIII-BIII	11641.63	-23275.25	-23265.17	-23274.79	-23271.18	12.26603	2.4406	0.2763
BIII	-103.911	211.8222	216.8657	211.957	213.8578	1.7296	0.3403	0.1261
BXII	-104.1619	214.3237	221.8891	214.5964	217.3772	1.3189	0.2263	0.0907
BX	-105.3125	214.625	219.6686	214.7598	216.6606	1.8356	0.3352	0.1129
MO-BIII	-101.2099	210.4199	220.507	210.8796	214.4911	0.6225	0.0896	0.0687

Table 2 & 3, above provides details of the model comparison between the proposed model and several existing models for both data1 and data 2, it is evident that the proposed model has the lowest values for AIC, BIC, CAIC and HQIC and also the highest value for the loglikelihood estimation both in data1 and in data2. This indicates that the proposed model (the Odd Burr III-Burr III Distribution, OBIII-BIII) offers the best fit amongst the compared models. The values of the Cramer-von Mises, Anderson-Darling, and Kolmogorov-Smirnov tests are shown in the table, with the p-value for the K-S test also included.

Table 4: Parameter Estimates for Odd Burr III-Burr III Distribution and Competing Models for data-1.

Models	C	K	α	λ
OBIII-BIII	1.9665	0.0790	10.0000	0.2225
BIII	6.1437	4.9219	_	_
BXII	0.4489	10.0152	_	_
BX	0.1078	19.6956	_	_
MO-BIII	0.0256	2.8687	4.3221	_

Table 5: Parameter Estimates for Odd Burr III-Burr III Distribution and Competing Models for Data-2.

Models	C	K	α	λ
OBIII-BIII	0.0267	0.3136	9.4839	7.5768
BIII	0.5709	1.7081	_	_
BXII	1.0272	24.8426	_	_
BX	1.147085	1.5637	_	_
MO-BIII	1.4325	2.4216	0.2879	_

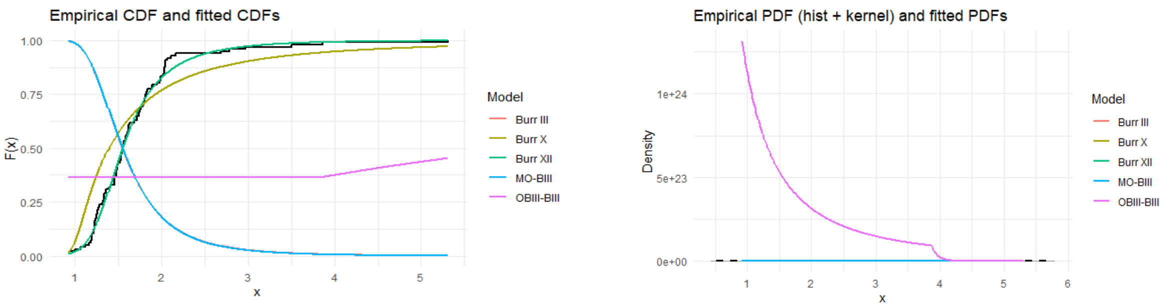

Figure 3: Empirical PDF and CDF Plots for Data 1

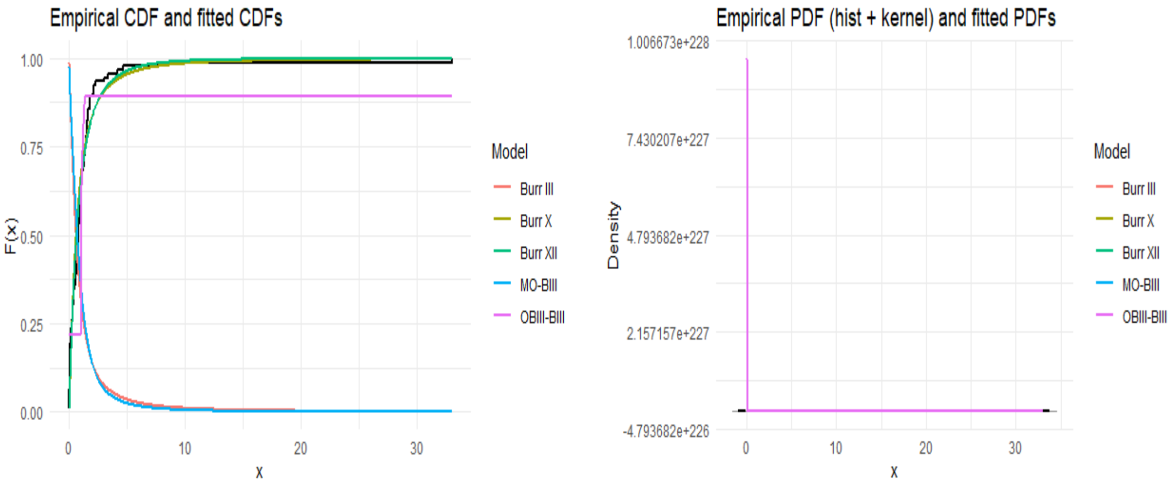

Figure 4: Empirical PDF and CDF Plots for Data 2

CONCLUSION

This study proposes a modified form of Burr III distribution, termed the Odd Burr III–Burr III (OBIII–BIII) distribution, to enhance its flexibility and applicability to real-world datasets. The model retains core properties of the Burr III while introducing parameter adjustments for greater flexibility. The Odd Burr III–Burr III distribution's probability density function, cumulative distribution function, log-likelihood expression, and reliability properties are formally derived. In a comparative analysis against competing models such as the Burr III, Burr XII, Burr X, and Marshall–Olkin-Burr III models, the proposed Odd BIII–BIII distribution consistently achieved the lowest goodness-of-fit and simulation values, demonstrating superior precision and adaptability. These findings highlight its potential for survival analysis, reliability, and risk modeling. Future work should refine its theoretical foundation, improve estimation methods, and test it across more diverse, complex datasets to further establish its applicability in interdisciplinary statistics.

REFERENCES

Adeleke, O., Femi, A., & Yusuf, T. (2024). The Odd Burr III–Lindley distribution: Properties and applications. Journal of Reliability and Statistical Studies, 12(3), 44–59.

Agboola, S., & Adebiyi, O. O. (2023). APPLICATION OF POWER NUMERICAL METHOD FOR THE STATIONARY DISTRIBUTION OF MARKOV CHAIN. *FUDMA JOURNAL OF SCIENCES*, *7*(2), 19-24.

Barlow, R. E., Toland, R. H., & Freeman, T. L. (1984). Statistical analyses of Kevlar 49/epoxy strand data subjected to constant sustained pressure. Lockheed Palo Alto Research Laboratory Report, California.

Bhatti, F. A. (2019). On the Generalized Log Burr III distribution. Journal of Statistical Computation and Simulation, 89(5), 802–818.

Burr, I. W. (1942). *Cumulative frequency functions.* Annals of Mathematical Statistics, 13(2), 215–232.

Gove, J. H., Ducey, M. J., Leak, W. B., & Zhang, L. (2008). Rotated sigmoid structures in managed uneven-aged northern hardwood stands: A look at the Burr Type III distribution. Forestry, 81(2), 161–168.

Gove, J. H., Ducey, M. J., Leak, W. B., & Zhang, L. (2008). Rotated sigmoid growth curves and the expression of stem-profile models in height–diameter form. Forest Science, 54(5), 521–531.

Haq, M. A. U. (2019). The Generalized Odd Burr III family of distributions. Journal of Probability and Statistics, 2019, 1–15.

Haq, M. A. ul, Afify, A. Z., Mofleh, H. A., Usman, R. M., Alqawba, M., & Sarg, A. M. (2021). *The Extended Marshall-Olkin Burr III Distribution: Properties and Applications. Pakistan Journal of Statistics and Operation Research, 17*(1), 1–14.

Hassan, R., Malik, M., & Tariq, H. (2024). The Power Transformed Odd Burr III distribution. Communications in Statistics: Theory and Methods, 53(7), 1831–1845.

Khalaf, A. A., Isa, A., Bashiru, S., & Kaigama, A. (2024). Topp-Leone Exponentiated Burr XII Distribution: Theory and Application to Real-Life Data Sets. Iraqi Statisticians Journal, 63-72.

Khan, Z., & Jamal, M. (2022). The Exponentiated Odd Burr III distribution: Statistical properties and applications. International Journal of Applied Statistics, 11(2), 99–112.

Manu, F., Usman, U., & Audu, A. (2023). New Life-Time Continuous Probability Distribution With Flexible Failure Rate Function. *FUDMA JOURNAL OF SCIENCES*, *7*(1), 323-329.

Mielke, P. W. (1973). Another family of distributions for describing and analyzing precipitation data. Journal of Applied Meteorology, 12(2), 275–280.

Mielke, P. W. (1973). Statistical theory and applications of the kappa distribution for precipitation amounts.

Moakofi, L. (2022). The Topp–Leone Odd Burr III-G family of distributions. Statistics and Probability Letters, 183, 109304.

Mohammed, A., Ibrahim, T., & Sadiq, M. (2023). The Generalized Odd Burr III–Inverse Exponential distribution. Reliability Engineering and System Safety, 235, 109054.

Nadarajah, S., & Kotz, S. (2007). Modeling fracture toughness and fracture stress data. Journal of Applied Statistics, 34(5), 507–524.

Nadarajah, S., & Kotz, S. (2007). On the alternative to the Weibull function. Engineering Fracture Mechanics, 74(3), 451–456.

Nasir, M., Jamal, S., Silvia, R., & Tahir, M. (2018). The Odd Burr-G Poisson family of distributions. Journal of Statistics Applications & Probability, 7(3), 493–504.

Nkomo, W., Oluyede, B., & Chipepa, F. (2024). The New Odd Burr III-G Power-Series class of distributions. International Journal of Mathematical Modelling and Numerical Optimisation, 14(2), 120–136.

Noori, A., Abdullah, M., & Ahmed, S. (2023). The Modified Burr III distribution: Derivation and applications. Trikit Journal of Pure Science, 6(1), 27–41.

Olabatuyi, F., Asiribo, O., & Dawodu, A. (2018). The Generalized Gamma Burr III distribution. Communications in Statistics: Simulation and Computation, 47(9), 2664–2678.

Oluwafemi, S. (2022). The Odd Burr III–Lomax distribution and its applications. International Journal of Statistical Analysis, 10(4), 210–224.

Oluyede, B., & Chipepa, F. (2025). The Power-Series Odd Burr III (OBIII-GPS) family of distributions. Journal of Statistical Modelling and Theory, 15(1), 1–19.

Peter, O. P., Musa, D., & Okoro, C. (2021). The Weibull Odd Burr III-G family of distributions. Journal of Applied Probability and Statistics, 16(2), 122–139.

Shao, Q., Chen, Y. D., & Zhang, L. (2008). An extension of three-parameter Burr III distribution for low-flow frequency analysis. Computational Statistics & Data Analysis, 52(3), 1304–1314.

Shao, Q., Wong, H., & Xia, J. (2008). Models for low-flow frequency analysis and an extension of the Burr type III distribution. Hydrological Processes, 23(4), 673–680.

Yousof, H., Ahmad, M., & Tariq, S. (2023). The Transmuted Odd Burr III distribution: Characterizations and applications. Statistical Papers, 64(3), 823–845.