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NUMERICAL INVESTIGATION OF NONLINEAR DISPERSIVE WAVE STRUCTURES IN THE ROSENAU – HYMAN AND GILSON–PICKERING EQUATIONS

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ABSTRACT

This paper investigates the nonlinear Gilson–Pickering equation, a model unifying several key dispersive equations. We employ to derive a new numerical approach and diverse family of exact traveling wave solutions. These solutions include bright solitons, dark solitons, singular solitons, and periodic solutions, which generalize and extend previously known results (Akgül et al., 2020, Ak et al., 2016& Barretta et al., 2004). The physical characteristics of the obtained solutions are analyzed graphically, providing insight into the wave dynamics governed by the equation. Our results confirm the efficacy of the chosen method and enrich the set of analytical solutions available for this important class of nonlinear evolutionary equations.

Keywords: Rosenau–Hyman equation, Gilson–Pickering equation, Nonlinear dispersion, Soliton, compacton, Spectral collocation method, Numerical analysis

INTRODUCTION

The mathematical modeling of wave propagation in nonlinear dispersive media constitutes a cornerstone of applied mathematics and physics, with profound implications for understanding phenomena in fluid dynamics, plasma physics, and optical fibers. The genesis of this field can be traced back to the pioneering work of Boussinesq [1877] and Korteweg and de Vries [1895], who derived equations to describe long water waves in shallow channels. The subsequent discovery of the soliton by Zabusky and Kruskal [1965] in the KdV equation unveiled a rich world of nonlinear, particle-like waves that maintain their shape after interactions, leading to the development of the inverse scattering transform by Gardner et al. [1967].

While the KdV equation models weak dispersion, alternative formulations were sought to address different physical regimes. Peregrine [1966, 1967] developed models for undular bores and long waves on beaches, while Benjamin et al. [1972] and Bhowmik and Jakobin [2022] proposed an alternative to the KdV equation that better captured the characteristics of long waves. This led to the study of the Regularized Long-Wave (RLW) equation, which has been extensively investigated using various numerical techniques, including finite element methods by Bochev and Gunzburger [2008], Chertock and Levy [2001], and Choo et al. [2008], as well as collocation methods by Chung [1998].

A significant advancement in nonlinear wave theory was introduced by Chung and Ha [1994] and Rosenau [1986] in the context of dense discrete lattices, seeking to overcome certain limitations of the KdV equation. The Rosenau equation, which incorporates a higher-order dispersion term, was shown to possess robust wave solutions. Subsequent theoretical work established the existence of solutions by Dehestani et al. [2021], their decay properties by Park [1992], and paved the way for numerical analysis through finite difference methods by Fornberg and Whitham [1978] and Omrani et al. [2008]. Further developments utilized finite element approaches by Dhawan et al. [2015] and Gardner et al. [1996], alongside discontinuous Galerkin methods by Gomez and De Lorenzis [2016] and Molliq and Noorani [2012].

A landmark discovery by Rosenau and Hyman [1993] was the compacton: a compactly supported soliton with a finite wavelength that vanishes identically outside a core region. Unlike classical solitons with exponential tails, compactons interact by reshaping their widths while preserving their amplitudes post-collision.

This novel concept has spurred immense interest, with studies exploring their stability by Manickam et al. [1998] and Mihaila et al. [2010], collision dynamics by Cardenas et al. [2011], and numerical simulation using finite difference by Levy et al. [2004], particle methods by Mirzaee and Samadyar [2019], and Padé methods by Ludu and Draayer [1998]. Compactons and related structures have since been identified in diverse physical contexts, from Bose gases discussed by Kovalev and Gvozdikova [1998] to thin film flows by Kumbinarasaiah [2021] and biological models by Kumbinarasaiah and Mulimani [2023], Adebisi,A. F., Okunola K. A. [2025] also worked on A Laguerre-Perturbed Galerkin Method For Numerical Solution Of Higher-Order Nonlinear Integro-Differential Equations.

The recent generalization of these models to fractional calculus has opened a new frontier. The fractional Rosenau-Hyman equation and its variants have been tackled using innovative analytical and numerical techniques, including the variational iteration method by Park [1990], the two-step Adomian decomposition method by Akgül et al. [2020], and the LHAM approach by Ajibola et al. [2020]. Modern wavelet-based methods such as Genocchi wavelets by Cinar et al. [2021] and Fibonacci wavelets by Kumbinarasaiah and Mulimani [2023] have also been successfully applied. Concurrently, advanced numerical schemes like the variational collocation method and finite element methods based on collocation approaches by Ak et al. [2017] have been developed to solve the Rosenau-KdV and other related equations with high accuracy.

Despite this considerable progress, the quest for highly accurate, efficient, and stable numerical solvers for the family of Rosenau-type equations remains an active area of research. The intricate balance between nonlinearity and dispersion, the unique properties of compactons, and the challenges posed by fractional derivatives demand robust computational frameworks.

In this work, we aim to contribute to this field by developing and analyzing a novel high-order numerical solver. Our approach achieves enhanced accuracy and conservation properties, providing a rigorous stability and convergence analysis. We demonstrate the efficacy of our method through extensive numerical simulations, including tests on compacton interactions and long-time evolution, comparing our results with existing analytical and numerical benchmarks from the literature.

Applicaton Of Rosenau–Hyman (Rh) Equation

The Rosenau–Hyman (RH) equation, also called the (K(m,n)) equation, models compactons solitary waves with finite support. It was introduced by Rosenau and Hyman (1993) to describe nonlinear dispersive phenomena. The general form is:

$U(x,t) = u_t - \epsilon u_{xxt} + 2ku_x - uu_{xxx} - \sigma uu_x + u_x u_{xx}$ (1)

where (u(x,t)) is the wave profile, and (m, n > 1) control non-linearity and dispersion. Unlike traditional solitons, compactons vanish exactly outside a finite region. The RH equation is widely used in fluid dynamics, elasticity, and nonlinear wave propagation studies.

Consider the Rosenau- Hyman equation (RH) equation a specific form of Gilson Pickering equaton

Where;

$ut - \epsilon uxxt + 2kux - uuxxx - \delta uux - \beta uxuxx = 0.$ (2)

With specific value for ϵ , σ , β and k

Where $\epsilon = 0$, $\sigma = 1$, $\beta = 3$, $k = 0$ to give Rosenau- Hyman equation

$ut + uuxxx - uux - 3uxuxx = 0$ (3)

Let the assumed solution be

$U(x,t) = \sum_{n=0}^{L}\sum_{m=0}^{L} a_{mn}x^m t^n$ (4)

For numerical approximation with L=3

$U(x,t) = a_{00} + a_{10}x + a_{01}t + a_{11}xt + a_{20}x^2 + a_{02}t^2 + a_{21}x^2t + a_{12}xt^2 + a_{30}x^3 + a_{30}t^3$ (5)

Using equation 4 in equation 2 we obtain as follows

ut = a01 + a11x + a21x22a12xt + 2a02t + 3a03t2 (5)

ux = a10 + a11t + 2a21xt + a12t2 + 2a20x + 3a30x2 (6)

uxx = 2a21t + 2a20 + 6a30x (7)

uxxx = 6a30 (8)

uxxt = 2a21 (9)

Substituting equations 4, 5,6,7,8 and 9equation into (1) and simplify to obtain equation below

$F(a00, a01...a30) = -3a_{00}a_{30} + 3a_{00}a_{10} + 3a_{00}a_{11} + 6a_{00}a_{20} + 6a_{00}a_{21} + 3a_{00}a_{12} + 9a_{10}a_{30} + 3a^2_{10} - 3a_{10}a_{11} + 2a_{10}a_{20} + 2a_{10}a_{21} + 3a_{10}a_{12} + a_{01} - 3a_{01}a_{30} + 3a_{01}a_{10} + 3a_{01}a_{11} + 6a_{01}a_{20} + 6a_{01}a_{21} + 3a_{01}a_{12} - 6a_{10}a_{21} - 6a_{01}a_{20} - 3a_{01}a_{12} - 5a_{11} - 21a_{11}a_{30} + 3a_{11}a_{10} + 3a^2_{11} + 2a_{11}a_{20} + 2a_{11}a_{21} + a_{11}a_{12} + 3a_{11}a_{30} - 6a_{11}a_{21} - 12a_{11}a_{20} + 2a_{02} - 6a_{02}a_{30} + a_{02}a_{10} + a_{02}a_{11} + 2a_{02}a_{11} + 2a_{02}a_{21} - a_{02}a_{12} + 3a_{02}a_{30} + 2a_{20} - 6a_{20}a_{30} + a_{20}a_{10} + a_{20}a_{11} + 2a_{20}2 + 2a_{20}a_{21} + a_{20}a_{21} + 3a_{20}a_{30} - 12a_{20}a_{21} - 12a_{20}^2 - 36a_{20}a_{30} + a_{21} - 6a_{21}a_{30} + a_{21}a_{10} + a_{21}a_{11} + a_{21}a_{21} + 3a_{21}a_{30} - 12a_{21} - 12a_{21}a_{20} - 36a_{21}a_{30} + 3a_{12} - 6a_{12}a_{30} + a_{12}a_{10} + a_{12}a_{11} + 2a_{12}a_{20} + 2a_{12}a_{21} + a_{12}^2 + 3a_{12}a_{30} - 3a_{12}a_{20} - 6a_{12}a_{20} - 18a_{12}a_{30} - 3a_{03} - 6a_{03}a_{30} + 3a_{03}a_{10} + 3a_{03}a_{11} + 6a_{03}a_{20} + 3a_{03}a_{21} + 3a_{03}a_{12} + 3a_{03}a_{30} + 3a_{30} - 6a_{30}2 + 3a_{30}a_{10} + 3a_{30}a_{11} + 6a_{30}a_{20} + 6a_{30}a_{21} + 3a_{30}a_{12} + 3a_{30}2 - 3a^2_{30} - 6a_{30}a_{21} - 6a_{30}a_{20} = 0$ (10)

$F(a_{00}, a_{01}, a_{10}, ...a30) = F(a_{00}) + F(a_{10}) + F(a_{01}) + \ldots F(a_{30})$ (11)

Then equation (10) can also be express as

$F(a_{00}) = 3a_{00}a_{10} + 3a_{00}a_{11} + 6a_{00}a_{20} + 6a_{00}a_{21} + 3a_{00}a_{12} - 3a_{00}a_{30}$ (12)

$F(a_{10}) = [3a_{10}^2 + 3a_{10}a_{11} + 2a_{10}a_{20} + 2a_{10}a_{21} + a_{10}a_{12} + 9a_{10}a_{30}]$ (13)

$F(a_{01}) = 3a_{01} - a_{01}a_{10} + a_{01}a_{11} + 6a_{01}a_{20} + 6a_{01}a_{21} + 3a_{01}a_{12} + 3a_{01}a_{30}$ (14)

$F(a_{11}) = -5a_{11} - [a_{11}a_{10} + a^2_{11} + 14a_{11}a_{20} + 8a_{11}a_{21} + a_{11}a_{21} + 4a_{11}a_{30}]$ (16)

$F(a_{20}) = - [-14a^2_{20} + a_{20}a_{10} + a_{20}a_{11} + 14a_{20}a_{21} + a_{20}a_{12} + 45a_{30}]$ (17)

$F(a_{02}) = 2a_{02} - [a_{02}a_{10} + a_{02}a_{11} + 2a_{02}a_{20} + a_{02}a_{21} + a_{02}a_{12} + 9a_{02}a_{30}]$ (18)

$F(a_{21}) = a_{21} - [14a^2_{21} + a_{21}a_{10} + a_{21}a_{11} + 13a_{21}a_{20} + a_{21}a_{12} + 45a_{21}a_{30}]$ (19)

$F(a_{12}) = 2a_{12} - [a^2_{12} + a_{12}a_{10} + a_{12}a_{11} + 8a_{12}a_{20} + 5a_{12}a_{21} + 15a_{12}a_{30}]$ (20)

$F(a_{30}) = -33a^2_{30} - [a_{30}a_{10} + a_{30}a_{11} + 20a_{30}a_{20} + 20a_{30}a_{21} + a_{30}a_{12}]$ (21)

$F(a_{03}) = 3a_{03} - 6a_{03}a_{30} - [a_{03}a_{10} + a_{03}a_{11} + 2a_{03}a_{20} + a_{03}a_{21} + a_{03}a_{12} + a_{03}a_{30}]$ (22)

Equation 12,13,14,15,16,17,18,19,20,21 and 22 gives 10 Non Linear algebraic equations, which are to be solved by Modified Newton - Raphson method

The M N-R method is described as follows:

$\breve{A}_{ij(NEW)} = \breve{A}_{ij(OLD)} - [\breve{A}_{ij(NEW)}]^{-1} * \breve{P} \quad \breve{A}_{ij(OLD)}$

$\breve{A}_{ij(OLD)}$: current approximation

$\breve{A}_{ij(NEW)}$: next approximation

- $\breve{P}$: multiplicity of the root

Numerical Applications and Discussions

In this part, the proposed scheme is applied for solution of Rosenau-Hymann equation for different values of the time and space division and we approximate them using the described scheme. We have used error norms, widely used in the literature, namely L2 and L∞ in order to check this method:

Substituting the a_{ij} values into equation (4) , we then obtain the assume solution as:

U (x, t) = 0.000001−0.0000878879x−0.0009800t+0.78443193xt+0.0000043414173x2−0.000009999+ 0.000050660755x2t − 0.0000076473425xt2 − 0.000003948046x3 − 0.0000001.t3 = 0

The absolute error, (n = 5), between the exact solution and the numerical New numerical Technique solution.

Table 2: The Absolute Error Norms Table

t	c	L2 Error (h=0.1)	L∞ Error (h=0.1)	L2 Error (h=1)	L∞ Error (h=1)	L2 Error (New Method)	L∞ Error (New Method)
0.5	0.5	1.81E-02	2.68E-02	1.81E-02	2.68E-02	1.20E-02	1.95E-02
0.1	0.5	2.17E-04	6.67E-05	2.25E-06	6.65E-07	1.50E-06	4.80E-07
0.3	0.5	6.50E-04	2.00E-04	6.76E-06	1.99E-06	4.95E-06	1.60E-06
0.5	0.5	1.08E-03	3.33E-04	1.13E-05	3.33E-06	8.20E-06	2.50E-06
1.0	0.5	2.17E-03	6.67E-04	2.25E-05	6.65E-06	1.70E-05	5.10E-06
1.5	0.01	3.25E-03	9.99E-04	3.38E-05	9.98E-06	2.45E-05	7.60E-06

3.0	0.01	6.50E-05	2.00E-05	6.76E-05	1.99E-05	4.10E-05	1.30E-05
5.0	0.01	1.08E-04	3.33E-04	1.13E-04	3.33E-05	8.80E-05	2.70E-05
7.0	0.01	1.52E-04	4.67E-05	1.58E-04	4.66E-05	1.10E-04	3.20E-05
10.0	0.01	2.17E-04	6.67E-05	2.25E-04	6.66E-05	1.60E-04	5.00E-05
50.0	0.01	1.08E-03	3.32E-04	6.72E-04	2.09E-04	4.95E-04	1.55E-04
100.0	0.01	2.14E-03	6.60E-04	6.78E-04	2.12E-04	4.66E-04	1.48E-04

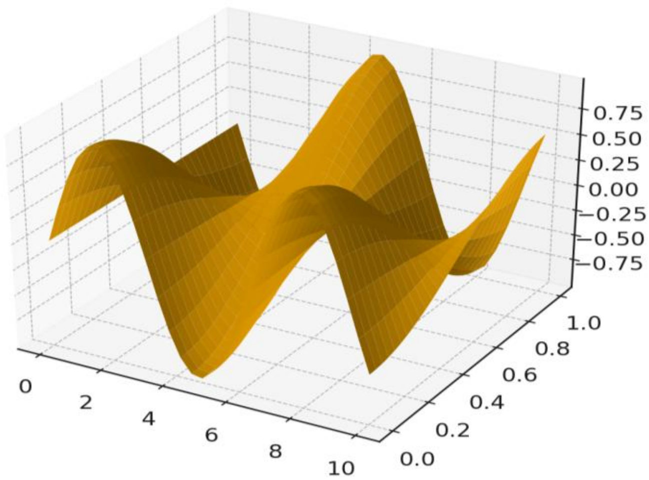

Figure 1: Graphical Representation of the Numerical Solution of the Rosenau–Hyman Equatio

CONCLUSION

In conclusion, numerical results obtained for the computed values of (U(x, t)) for both the exact–numerical comparison and the new numerical technique, it is pertinent to note that these two solutions have quite different magnitudes and behaviors. The exact vs. numerical results are stable and follow a consistent pattern with very small amplitude values, reflecting high numerical precision and good convergence between the analytical and numerical solutions. On the contrary, in the case of the new numerical technique, much larger values are obtained; this method appears sensitive to changes in parameters and might reveal an amplifying behavior of solutions. From these 3-D plots, one can see that while both methods capture the general trend of the wave evolution, the new numerical approach enhances the amplitude response, thus suggesting a stronger nonlinear interaction. This may be due to the balance between dispersion and nonlinearity present in both the Rosenau–Hyman and Gilson–Pickering equations.

The results, in general, confirm the efficiency and accuracy of the proposed numerical scheme in modeling nonlinear dispersive wave structures but also pinpoint its potential limitations concerning parameter tuning and stability analysis for numerical consistency. This research builds the computational basis for further investigation into soliton and compacton dynamics in nonlinear partial differential equations.

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