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ON THE POLYGONAL AND CIRCULAR REPRESENTATIONS OF THE (123) AVOIDING CLASS OF AUNU PERMUTATION PATTERNS

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ABSTRACT

In this communication, another theoretic Property of the (123) - Avoiding class of AUNU permutation patterns is x-rayed. Our results are obtained by utilizing the Cayley tables of these patterns as used by Ibrahim and Abubakar, 2016 to study their Non-associativity/non-commutativity of these important Permutation avoiding Patterns. Their approach describes how these non-associative and non-commutative properties can be established by using the Cayley table on which a binary operation is defined to act on the 123-avoiding and 132-avoiding patterns of AUNU permutations using a pairing scheme. Here, we consider the Polygonal and Circular representations of these patterns as studied by Paulus Gerdes with the aim of exploring more of their pictorial applications in relation to interconnection Networks, etc. Our results generated larger matrices from permutations of points of the AUNU patterns of prime cardinality. Our goal is achieved by considering these Patterns for $n=5, 7, 11, ...$ *where n is prime*. The polygonal and circular Representation of these patterns are thus obtained which gave some interesting graphical structures such as the kite in Fig (g) etc.

Keywords: AUNU Permutation Patterns, Cayley Tables, Polygonal Representation, Circular Representation, Non-Associative/Non-Commutativity Property

INTRODUCTION

An overview of the AUNU permutations patterns, AUNU integer sequence, the (123)/ (132) - avoiding patterns and their applications was explored by the author(s) in [2]. This algebraic structures had found wide applications in almost all facets of applied mathematics such as Association/Succession schemes, Thin cyclic design, Latin squares, lattices, automate theory, Graph theory, Coding theory e.t.c (Ibrahim and Abubakar, 2016a), (Chun et al, 2016a), (Ibrahim and Abubakar, 2016b), (Chun et al, 2016b), (chun, 2025) and (Ibrahim, 2006). In this paper, we consider the Polygonal and Circular representations of the rows(as (123)-avoiding permutations words) of the Cayley tables of the patterns used by Ibrahim and Abubakar to study their Non-associativity/non-commutativity using Paulus Gerdes' approach (Paulus, 2007) with a view of exploring more of their pictorial applications in relation to interconnection Networks etc.

Some Basic Concepts

Polygonal Representation

In an attempt to answer the question that "Does there exist an easier or a more beautiful notation to represent the permutation that corresponds to the transformation of a positive alternating cycle matrix that has a row of the first matrix as one of its rows? Paulus Gerdes (Paulus, 2007), proposed that the Permutation denoted by (14325) may be represented by connecting the five points placed at the vertices of a regular Pentagon as illustrated in figure (a) below which is a polygonal representation of the permutation (14325).

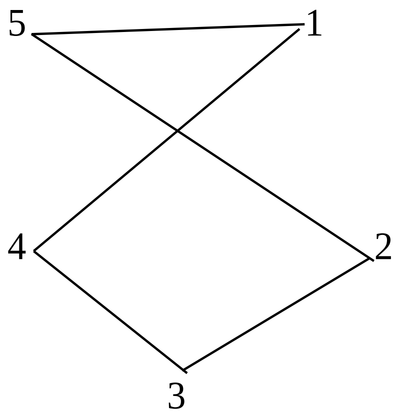

Figure 1: Polygonal Representation of (14325)

In the same way, two permutations (13542) and (12453) respectively, may be presented by the figure (b) below;

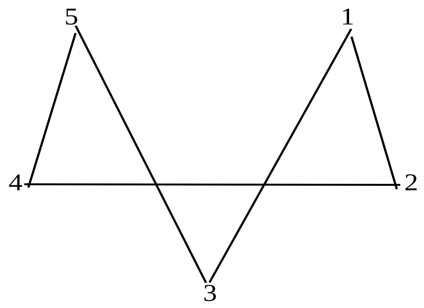

Figure 2: Polygonal Representation of the Permutations (13542) and (12453)

Note: For a permutation of length *n =7*, the polygonal representation is a symmetrical design presented in fig (c) below for the permutation (1473265) for instance;

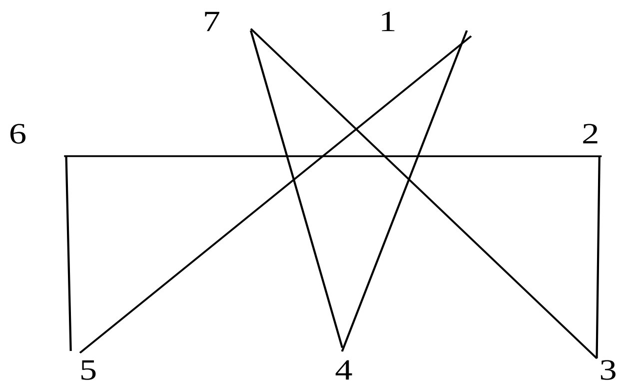

Figure 3: Polygonal Representation of the Permutation (1473265)

Circular Representation

If we consider what will the representation be of the permutation (1473265) that had let us the polygonal representation in Figure 3? Starting from point (1) at the left and advancing clockwise to point (4) at the right side, e.t.c , we obtain the beautiful representation illustrated in Figure 4 below. We are dealing with a regular heptagonal star!

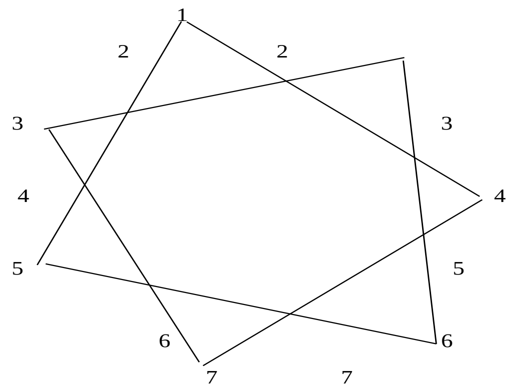

Figure 4: Circular Representation of the Permutation (1473265)

Behind the sequence of the numbers in the cycle notation of the permutation, (1473265)- which appears to lack a certain order, exist the perfect order of the regular heptagonal star. We may call this type of representation a circular representation to be able to distinguish it from the previous Polygonal representation.

Example 1 Find the circular representation of the permutations (14325) and (13542) whose polygonal representations are as shown in Figure 1 and Figure 2 respectively.

Solution: The permutation (14325) may be represented by the pentagonal star as shown in Figure 5 below;

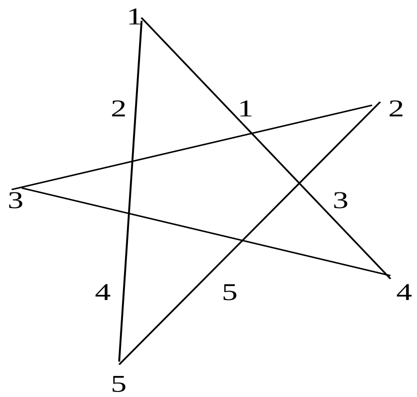

Figure 5: Circular Representation of the Permutation (14325)

Similarly, the permutation (13542), however may be represented by the regular pentagon as shown in the Figure 6 below

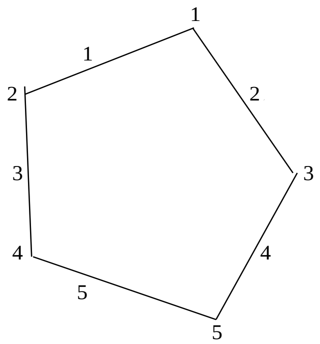

Figure 6: Circular Representation of the Permutation (13542)

Methodology

In our methods, we adopted the rows of the Cayley tables for *$5 \leq n \leq 17$, where n is Prime* showing generated points of Ω as permutations of (123)-avoiding patterns of AUNU schemes under the action of Θ (Ibrahim and Abubakar, 2016a). In (Ibrahim and Abubakar, 2016a), Ibrahim and Abubakar used the entries of the generated Cayley tables to demonstrate the non- associative/non-commutativity properties of these patterns. In our approach, we consider these (123)-avoiding permutations as rows of the Cayley tables and project their Polygonal and Circular representations. The Cayley tables with the representations are thus presented;

Table 1: Cayley Table for *n =5*

Θ	1	2	3	4	5
1	1	3	5	2	4
2	1	4	2	5	3
3	1	5	4	3	2

The three rows of the above table give the following permutations whose polygonal and circular representations are demonstrated: (13524), (14253) and (15432)

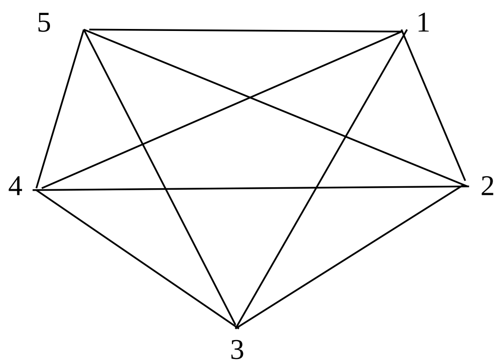

Figure 7: Polygonal Representation of (13524), (14253) and (15432)

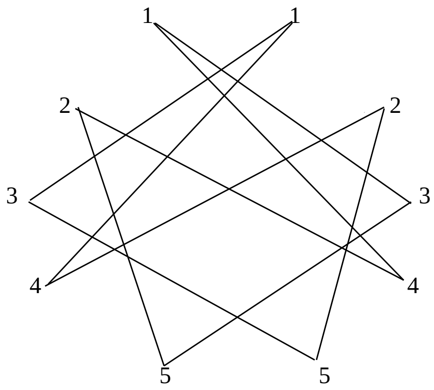

Figure 8: Circular Representation of (13524) and (14253)

Table 2: Cayley Table for *n =7*

Θ	1	2	3	4	5	6	7
1	1	3	5	7	2	4	6
2	1	4	7	3	6	2	5
3	1	5	2	6	3	7	4
4	1	6	4	2	7	5	3
5	1	7	6	5	4	3	2

The five rows of the above table give the following permutations whose polygonal and circular representations are demonstrated: (1357246), (1473625), (1526374), (1642753) and (1765432)

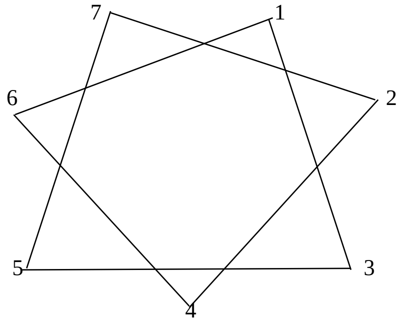

Figure 9: Polygonal Representation of (1357246) and (1642753)

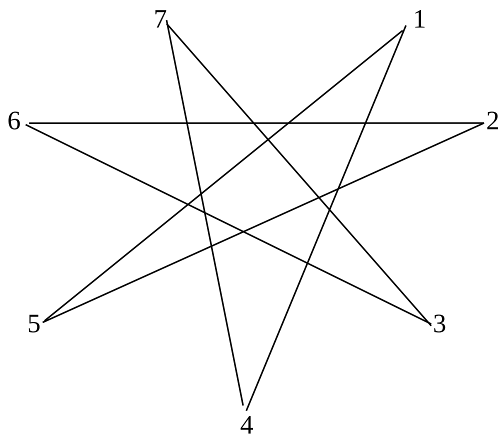

Figure 10: Polygonal Representation of (1473625) and (1526374)

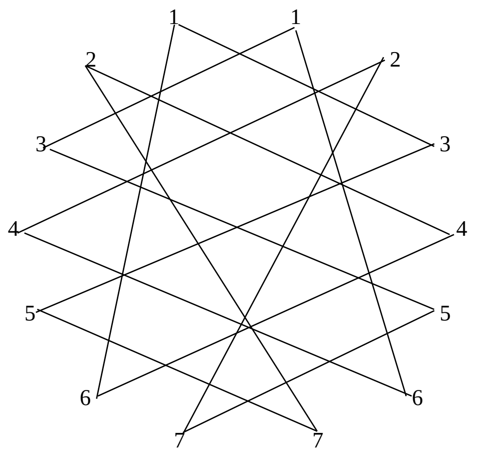

Figure 11: Double Circular Representation of the Permutations (1357246) and (1642753)

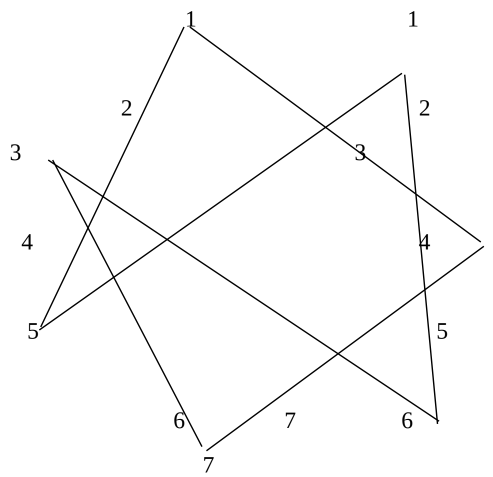

Figure 12: Circular Representation of the Permutations (1473625) and (1526374)

Remark

More interesting pictorial shapes can be explored in a similar manner using higher values of n *=11, 13 and 17* respectively.

RESULTS AND DISCUSSION

It can easily be seen that the polygonal and circular representation of the AUNU (123) - avoiding Permutation Patterns has resulted to very interesting symmetrical designs/figures/shapes which could fit in some special purposeful interconnections or network designs. In particular, the Polygonal Representation of (13524), (14253) and (15432) Fig (g) gives rise to a beautiful shape of a kite. Figure 8: Circular Representation of (13524) and (14253) represents a regular star nonagon. Figures i and j respectively represents regular heptagons though in different forms. Same applies to Figure 12: Circular Representation of the Permutations (1473625) and (1526374) showing the convergence of Polygonal and Circular representations of these numbers at some point. For the interpretation of tables 1 and 2, see (Ibrahim and Abubakar, 2016a).

CONCLUSION

This Paper had exposed yet another important application of the (123) - avoiding patterns of the AUNU permutations in respect to their circular and polygonal representations resulting to some interesting mathematical shapes and graphical structures.

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