FUDMA Journal of Sciences (FJS)
ISSN online: 2616-1370
ISSN print: 2645 - 2944
Vol. 10 No. 1, January, 2026, pp 21 – 26
DOI: https://doi.org/10.33003/fjs-2026-1001-4294

DEVELOPMENT OF ALPHA POWER EXPONENTIATED TOPP-LEON INVERSE WEIBULL DISTRIBUTION

*[1,2]Mohammed, Y. I., [2]Bakari, H. R. and [2]Mohammed, Y. A.

[1]Department of Statistics Jigawa State Polytechnic, Dutse, Jigawa State, Nigeria.
[2]Department of Statistics, University of Maiduguri, Borno State, Nigeria.

*Corresponding authors' email: myibrahim@jigpoly.edu.ng

ABSTRACT

Lifetime data in engineering, medical, and reliability studies often exhibit complex hazard rate and tail behaviors that classical distributions, such as the inverse Weibull model, cannot adequately capture. To address this limitation, this study proposes a new and flexible lifetime model called the Alpha-Power Exponentiated Topp-Leone Inverse Weibull (AP-ETLIW) distribution. The proposed model is constructed by integrating the exponentiation mechanism, the Topp-Leone transformation, and the alpha-power transformation into the inverse Weibull distribution, thus enhancing shape flexibility, hazard rate variation, and tail behavior. The performance of the AP-ETLIW distribution is evaluated through comparisons with existing lifetime models, demonstrating its superior fitting capability in various applications, including reliability analysis, survival analysis, engineering and medical sciences. This study contributes to the development of hybrid statistical distributions and provides a robust tool to model complex lifetime data.

Keywords: Alpha Power, Topp Leon, Exponential, Weibull Distribution, Properties, Application

INTRODUCTION

Probability distributions play a fundamental role in statistical modelling, particularly in areas such as reliability analysis, survival analysis, and lifetime data modelling. Classical distributions, including exponential, Weibull, and gamma distributions, have long been used to model lifetime and failure time data (Faruk et al., 2024). However, real-world datasets often exhibit complex characteristics such as skewness, heavy tails, multimodality, and non-monotonic hazard rates that cannot be adequately captured by these traditional models. To address these limitations, recent research has focused on developing more flexible probability distributions by introducing additional shape parameters, transformations, and generalized extensions of existing models. Such advancements aim to enhance model adaptability and improve the goodness-of-fit for complex datasets. Lifetime data modelling is a cornerstone of reliability engineering, survival analysis, and risk management. Accurate models are essential for predicting system failures, patient survival times, and assessing risks in various industries. Traditional distributions such as Weibull, Exponential, and Gamma have been widely used due to their mathematical tractability and interpretability. However, these models often fail to capture the complexity of real-world data, which can exhibit heavy tails, skewness, multimodality, and non-monotonic hazard rates (Klakattawi, 2022; Betensky *et al*., 2023). For example, the Weibull distribution assumes a monotonic hazard rate, which is rarely observed in practice, especially in biomedical and engineering applications (Chen et al., 2023). To address these limitations, researchers have developed more flexible distributions by generalizing or combining existing distributions. These advanced models introduce additional parameters or hybrid mechanisms to better capture the nuances of real-world data. One such development is the Alpha Power Topp-Leone Weibull Distribution (APTLW) which integrates the flexibility of Alpha Power Transform, the Topp-Leone family and Weibull distribution (Benkhelifa, 2022). This hybrid approach has shown promise in modelling complex lifetime data with diverse characteristics (Usman *et al*., 2023). Traditional distributions such as Weibull, Exponential, and Gamma have been the backbone of lifetime data analysis for decades. However, their simplicity often comes at the cost of limited flexibility. For example: The Weibull distribution assumes a monotonic hazard rate, which is not suitable for data with bathtub-shaped
or Unimodal hazard rates (Klakattawi, 2022). The exponential distribution is limited to constant hazard rates, making it inadequate for most real-world applications (Betensky *et al*., 2023). The Gamma distribution, while more flexible, struggles with heavy-tailed data and multimodality (Mahdavi, 2017). To overcome these limitations, researchers have developed generalized distributions by introducing additional parameters or combining multiple distributions, examples include the exponential Weibull distribution, which extends the Weibull distribution by adding a shape parameter to model non-monotonic hazard rates (Topp and Leone, 1955) The generalized inverse
Weibull distribution, which enhances the inverse Weibull distribution by incorporating additional flexibility (De Gusmao *et al*., 2011). The Beta-generated family of distributions, which uses the Beta function to create more flexible models (Al-Shomrani et al., 2016). The Alpha Power Transform is a powerful technique for enhancing the flexibility of baseline distributions. By introducing an additional shape parameter, it allows for better modelling of skewness, kurtosis, and tail behavior. For example, the exponential distribution of the power of alpha has been used to model data with increasing or decreasing hazard rates (Mahdavi, 2017). The Alpha Power Weibull distribution has been applied in reliability engineering to model failure times with non-monotonic hazard rates. The Topp-Leone family of distributions is known for its ability to model data with bathtub shaped or increasing hazard rates. Key developments include: The Topp-Leone Generalized Exponential distribution, which has been used in survival analysis to model patient survival times (Al-Shomrani *et al*., 2016). The Topp-Leone Odd Log-Logistic Weibull distribution, which has been applied in reliability engineering to model failure times with complex hazard rate shapes (Mudholkar *et al*., 1995). The inverse Weibull distribution is particularly useful for modelling heavy-tailed data, which is common in reliability and survival analysis. Its applications include: Modelling extreme values in environmental science, such as

the time to failure of ecological systems (De Gusmao *et al*., 2011).

Motivation and Background

One of the notable developments in this direction is the Alpha-Power Exponentiated Topp-Leone Inverse Weibull (AP-ETLIW) distribution, a novel and highly flexible statistical model designed to accommodate diverse data behaviours. The AP-ETLIW distribution integrates three key frameworks the Alpha-Power transformation, the Exponentiated Topp-Leone mechanism, and the Inverse Weibull distribution”to yield a generalized model capable of describing a wide range of statistical patterns. This integration results in a distribution that provides superior flexibility and robustness when modeling lifetime data. Consequently, the AP-ETLIW model is particularly

Suitable for applications in engineering reliability, medical survival analysis, risk assessment, and economic modeling, where complex data structures frequently occur.

Theoretical Foundation

The Inverse Weibull (IW) distribution, a foundational component of the AP-ETLIW model, is well recognized for its ability to model lifetime data with non-monotonic hazard functions. Despite this capability, the IW distribution remains limited when modeling highly skewed or multimodal datasets. To overcome these restrictions, the Topp-Leone transformation is incorporated to introduce additional shape flexibility, while the exponentiation process enhances tail behavior and allows for a wider range of hazard rate variations.

MATERIALS AND METHODS

Alpha Power Transform

The Alpha Power Transform is a powerful technique for enhancing the flexibility of baseline distributions. By introducing an additional shape parameter, it allows for better modelling of skewness, kurtosis, and tail behavior. For example, the exponential distribution of the power of alpha has been used to model data with increasing or decreasing hazard rates (Mead et al., 2019). The Alpha Power Weibull distribution has been applied in reliability engineering to model failure times with non-monotonic hazard rates.

Topp-Leone Family

The Topp-Leone family of distributions is known for its ability to model data with bathtub shaped or increasing hazard rates. Key developments include: The Topp-Leone Generalized Exponential distribution, which has been used in survival analysis to model patient survival times (Al-Shomrani et al., 2016). The Topp-Leone Odd Log-Logistic Weibull distribution, which has been applied in reliability engineering to model failure times with complex hazard rate shapes (Brito et al., 2017).

Inverse Weibull Distribution

The inverse Weibull distribution is particularly useful for modelling heavy-tailed data, which is common in reliability and survival analysis. Its applications include: Modelling extreme values in environmental science, such as the time to failure of ecological systems (De Gusmao et al., 2011).

Theoretical Framework

Baseline Distribution (Inverse Weibull)

The inverse Weibull distribution serves as the baseline distribution because of its ability to model heavy-tailed data. Its PDF and CDF are given by:

$$f(x;\lambda,\beta) = \beta\lambda^{\beta}x^{-\beta-1}\, exp\left[-\left(\frac{\lambda}{x}\right)^{\beta}\right], \quad x > 0, \quad (1)$$

$$F(x;\lambda,\beta) = exp\left[-\left(\frac{\lambda}{x}\right)^{\beta}\right], \quad (2)$$

Where λ <0, is the scale parameter and and β >0 is the shape parameter.

Topp- Leone Transformation

The Topp-Leone transformation is applied to the CDF of the inverse Weibull distribution to introduce additional flexibility. The transformed CDF is given by:

$$G(x;\alpha,\lambda,\beta) = [1-(1-F(x;\lambda,\beta))^2]^{\alpha}, \quad (3)$$

Where α >0 is an additional shape parameter.

Alpha Power Transform

The Alpha Power Transform is then applied to the Topp-Leone-transformed CDF to further enhance the flexibility of the distribution. The final CDF of the APETL distribution is given by:

$$H(x;\alpha,\gamma,\lambda,\beta) = \frac{\gamma^{G(x;\alpha,\lambda,\beta)}-1}{\gamma-1}, \quad \gamma > 0, \gamma \neq 1, \quad (4)$$

Where γ is the Alpha power parameter 16. In this section, we will discuss the proposed model, its validity checks, mathematical properties, and parameter estimation.

Cumulative Distribution Function (CDF)

The sum of the cumulative distribution function (CDF) of the Alpha Power Exponentiated Topp-Leone Inverse Weibull (AP-ETLIW) distribution,by integrating all transformations, starting from the base Inverse Weibull (IW) distribution. To gether these CDF given in Equation 2, Equation 3 and Equation 4 but as If α = 1, the CDF of Eqn4. Reduces to Eqn 3 $F_{AP(x)} = G_{TL(x)}$

The CDF of the AP-ETLIW Distribution Combining all transformations, the CDF is:

$$F_{AP-ETLIW}(x) = \frac{\alpha^{\left(1-\left(1-e^{-\theta\gamma^{\beta}x^{-\beta}}\right)^{2}\right)^{\lambda}}-1}{\alpha-1}, \quad \alpha > 0, \alpha \neq 1. \quad (5)$$

Verification to ensure correctness, we can check that:

$$\frac{d}{dx}F_{AP-ETLIW}(x) = f_{AP-ETLIW}(x), \quad (6)$$

$$f(x) = 2\alpha\lambda\theta, \beta\gamma^{\beta}x^{-(\beta+1)}G(x)^{\theta}(1-G(x)^{\theta})[1-(1-G(x)^{\theta})^2]^{\alpha\lambda-1}$$

where $f\_{AP-ETLIW}(x)$ of Equation 6 is the derived PDF.

$$F_{AP-ETLIW}(x) = \frac{\alpha^{\left(1-\left(1-e^{-\theta\gamma^{\beta}x^{-\beta}}\right)^{2}\right)^{\lambda}}-1}{\alpha-1}, \quad \alpha > 0, \alpha \neq 1.$$

The CDF of the AP-ETLIW distribution is reduces to equation 3 as alpha =1

$$F_{AP-ETLIW}(x) = \left(1-\left(1-e^{-\theta\gamma^{\beta}x^{-\beta}}\right)^{2}\right)^{\lambda}, \quad \alpha = 1.$$

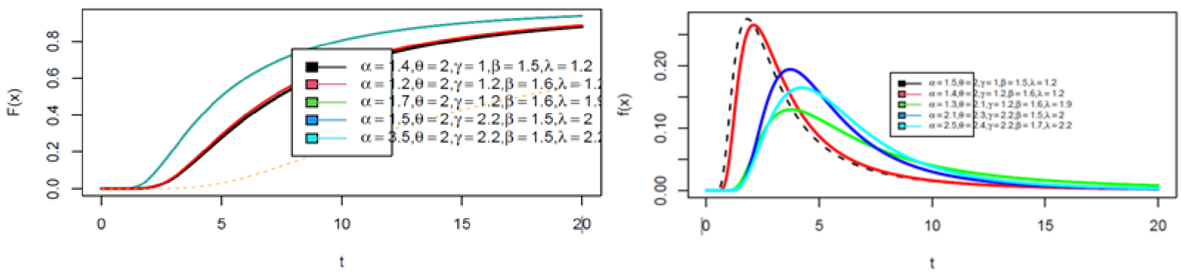

Figure 1: Plots of CDF and PDF of APETL-IW Distribution

Statistical Properties

Survival function

Given in Equation 5 and Equation 6 above the CDF. and PDF respectively the Survival Function is given as:

Survival function S(x) The survival function (reliability) is S(x)=1-F(x). For the Alpha-Power Exponentiated Topp-Leone Inverse-Weibull model with parameters α, λ, θ., γ and β >0 we have

$S(x) = 1 - [\,1 - (1 - (1 - e^{-\lambda x^{-\theta}})^{\delta})^2\,]^{\alpha\beta}, \quad x > 0.$

AS the CDF tends to 1 depending on the parameterization, numerical evaluation near extremes needs case.

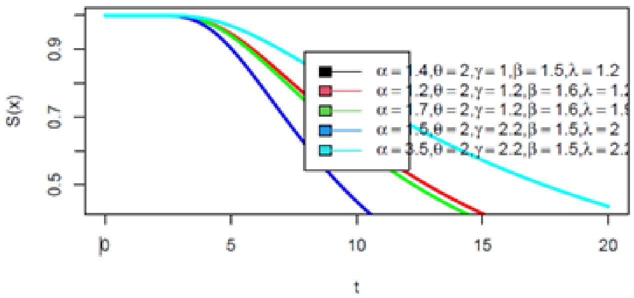

Figure 2: Plot of survival function of APETL-IW distribution

Figure 2 gives the probability of survival beyond time x. Observed curves included:

i. Fast decay: rapid early failures (acute medical risks, defective products),
ii. Gradual decay: balanced risks across time,
iii. Slow decay: long-lasting systems or healthier populations.

In addition, survival functions adapt to early mortality, long-term stability, and ageing effects, making them relevant for both patient survival and component reliability.

Hazard function of the AP-ETLIW model

For the Alpha-Power Exponentiated Topp-Leone Inverse-Weibull (AP-ETLIW) given in equation 6 with parameters α, λ, θ, γ and β >0, define

$t(x) = 1 - e^{-\lambda x^{-\theta}} \quad (\text{so } 0 < t(x) < 1), \quad s(x) = t(x)^{\delta},$

and

$\text{inner}(x) = 1 - (1 - s(x))^2 = s(x)(2 - s(x)).$

The CDP is the inner

$F(x) = (\text{inner}(x))^{\alpha\beta}$

$S(x) = 1 - F(x) = 1 - (\text{inner}(x))^{\alpha\beta}.$

$f(x) = \alpha\beta\ 2\delta\lambda\theta\ x^{-\theta-1}\ e^{-\lambda x^{-\theta}}\ t(x)^{\delta-1}(1 - t(x)^{\delta})\ (\text{inner}(x))^{\alpha\beta-1}.$

The pdf can be written as:

$$h(x) = \frac{\alpha\beta\ 2\delta\lambda\theta\ x^{-\theta-1}\ e^{-\lambda x^{-\theta}}\ t(x)^{\delta-1}(1-t(x)^{\delta})\ (\text{inner}(x))^{\alpha\beta-1}}{1-(\text{inner}(x))^{\alpha\beta}}$$

therefore the hazard function is

$h(x) = \frac{f(x)}{S(x)}$

$t(x) = 1 - e^{-\lambda x^{-\theta}}$

$\text{inner}(x) = s(x)(2 - s(x))\$, \$s(x) = t(x)^{\delta}$

The hazard can take many shapes (increasing, decreasing, unimodal, bathtub) depending on parameters α, λ, θ., γ and β. To derive the Hazard Function h(x)

Case 1: α ≠ 1

$$h(x) = \frac{f(x)}{S(x)} = \frac{2\alpha\lambda\theta\beta\gamma^{\beta}x^{-\beta-1}e^{-\theta\gamma^{\beta}x^{-\beta}}\left(1-e^{-\theta\gamma^{\beta}x^{-\beta}}\right)\left(1-\left(1-e^{-\theta\gamma^{\beta}x^{-\beta}}\right)^{2}\right)^{\alpha\lambda-1}}{\frac{\alpha-\alpha^{\left(1-\left(1-e^{-\theta\gamma^{\beta}x^{-\beta}}\right)^{2}\right)^{\lambda}}}{\alpha-1}}$$

Simplify

$$h(x) = \frac{2\alpha\lambda\theta\beta\gamma^{\beta}x^{-\beta-1}e^{-\theta\gamma^{\beta}x^{-\beta}}\left(1-e^{-\theta\gamma^{\beta}x^{-\beta}}\right)\left(1-\left(1-e^{-\theta\gamma^{\beta}x^{-\beta}}\right)^{2}\right)^{\alpha\lambda-1}(\alpha-1)}{\alpha-\alpha^{\left(1-\left(1-e^{-\theta\gamma^{\beta}x^{-\beta}}\right)^{2}\right)^{\lambda}}}$$

case2: alpha = 1

$$\boldsymbol{h(x)} = \frac{f(x)}{S(x)} = \frac{2\lambda\theta\beta\gamma^{\beta}x^{-\beta-1}e^{-\theta\gamma^{\beta}x^{-\beta}}\left(1-e^{-\theta\gamma^{\beta}x^{-\beta}}\right)\left(1-\left(1-e^{-\theta\gamma^{\beta}x^{-\beta}}\right)^{2}\right)^{\lambda-1}}{1-\left(1-\left(1-e^{-\theta\gamma^{\beta}x^{-\beta}}\right)^{2}\right)^{\lambda}}$$

$$h(x) = \begin{cases} \dfrac{2\alpha\lambda\theta\beta\gamma^{\beta}x^{-\beta-1}e^{-\theta\gamma^{\beta}x^{-\beta}}\left(1-e^{-\theta\gamma^{\beta}x^{-\beta}}\right)\left(1-\left(1-e^{-\theta\gamma^{\beta}x^{-\beta}}\right)^{2}\right)^{\alpha\lambda-1}(\alpha-1)}{\alpha-\alpha^{\left(1-\left(1-e^{-\theta\gamma^{\beta}x^{-\beta}}\right)^{2}\right)^{\lambda}}}, & \alpha \neq 1, \\ \dfrac{2\lambda\theta\beta\gamma^{\beta}x^{-\beta-1}e^{-\theta\gamma^{\beta}x^{-\beta}}\left(1-e^{-\theta\gamma^{\beta}x^{-\beta}}\right)\left(1-\left(1-e^{-\theta\gamma^{\beta}x^{-\beta}}\right)^{2}\right)^{\lambda-1}}{1-\left(1-\left(1-e^{-\theta\gamma^{\beta}x^{-\beta}}\right)^{2}\right)^{\lambda}}, & \alpha = 1. \end{cases}$$

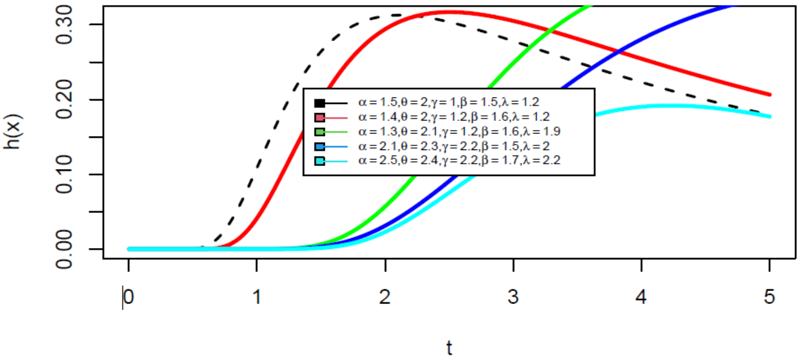

Figure 3: Plot Hazard Function of APETL-IW Distribution

The Hazard Function h(x) describes the instantaneous failure rate at time x.
It is useful in reliability engineering, survival analysis, and risk assessment.
The shape of h(x) (increasing, decreasing, or bathtub-shaped) depends on the values of the parameters.
Verification

i. Non-negativity: Since $f(x) \geq 0$ and $S(x) \geq 0$, $h(x) \geq 0$.
ii. Consistency: If $\alpha = 1$, the hazard function reduces to the simpler form, ensuring correctness.

Quantile Function
The AP-ETLIW quantile function admits a closed form (via algebraic inversion of the CDF).
Start with F(x)=uE (0,1). Writing
$I := u^{1/(\alpha\beta)} \in (0,1), \quad s := (1-e^{-\lambda x^{-\theta}})^{\delta},$

The CDF equation
$(1-(1-(1-e^{-\lambda x^{-\theta}})^{\delta})^2)^{\alpha\beta} = u$
becomes
$1-(1-s)^2 = I \quad \Leftrightarrow \quad 2s - s^2 = I.$
Solve the quadratic s^2-2s+I=0. The root in [0,1] is
$s = 1 - \sqrt{1-I}.$
$1 - e^{-\lambda x^{-\theta}} = s^{\frac{1}{\delta}} \quad \Rightarrow \quad e^{-\lambda x^{-\theta}} = 1 - s^{\frac{1}{\delta}}.$
Taking logs and solving for x gives the quantile:
$Q(u) = x_u = \left[-\frac{1}{\lambda} ln(1-(1-\sqrt{1-u^{1/(\alpha\beta)}})^{1/\delta})\right]^{-1/\theta}.$
This expression is valid for 0<u<1 and α, λ, θ., γ and β >0.
The inner arguments stay in (0,1) for valid u, so the log uses a number in (0,1) (log negative) and the negative sign makes the base positive — so Q(u)>0.
The median is simply Q (0.5). From the quantile, you can generate random variates by inversion: X = Q(U) with U ~ unif(0,1) (fast and vectorizable).
Let U϶(0,1) solve
$F(x) = u \quad \text{or} \quad [1-(1-(1-e^{-\lambda x^{-\theta}})^{\delta})^2]^{\beta\alpha} = u$
for x. Closed form is not simple, but a Quantiles function exists implicitly and can be found numerically:

Moments
The moments of the APETL-IWD are used to describe its central tendency, dispersion, and shape. The k-th moment about the origin is given by the r-th raw moment.
$\mu_{r'} = \int_0^{\infty} x^r f(x)dx$
which does not simplify neatly. You can express it as a series expansion using the binomial and power series expansions of the CDF core. For example:
$\mu_{r'} = \sum_{m,n,\ldots} C_{m,n,\ldots} \lambda^{-\frac{r}{\theta}} \Gamma(1-\frac{r}{\theta})$
with coefficients C_m,n,…. depending on parameters α, γ and β.
From the first two moments, you get:
Mean
$E[X] = \mu_{1'}$
Variance :
$\mathrm{Var}[X] = \mu_{2'} - \mu_{1'}^2.$
Coefficient of variation, skewness, and kurtosis similarly.

Median and Mode
Median m: F(m)=0.5.
Mode x_{mode}: solve f'(x)=0 numerically.

RESULTS AND DISCUSSION
This section will introduce the simulation study designed to examine the behavior and consistency of maximum likelihood estimation, as well as the application of real data to observe the performance of the proposed model and other competing models.

Simulation
The behavior of the maximum likelihood of ALPETIW -D for certain parameter values in the first trial (i.e α=1.4, λ=0.9, θ=0.8., γ=1.2 and β = 1.3) was investigated using a created finite sample of size n= 20, 50, 100, 200, and 500. The random numbers for the ALPETIW-D were generated using the quantile function. For 1000 repeats. The Means, Bias, and RMSE were then calculated. Table 1 presents the outcomes of the simulation. We concluded that the proposed model yields consistent results when predicting parameters for the model based on the results of the Monte Carlo simulation.

Table1: The Result of Simulation for Different Values of Parameters

		α = 1.4	λ = 0.9	θ = 0.8	γ = 1	β = 1.3
n =20	Mean	3.78E+36	8.35E-01	8.35E-01	1.76E+00	1.323257
	Bias	3.78E+36	-6.47E-02	3.53E-02	5.64E-01	2.33E-02
	RMSE	5.34E+37	1.31E+00	1.30E+00	1.56E+00	2.18E-01
n =50	Mean	3.78E+36	8.35E-01	8.35E-01	1.76E+00	1.323257
	Bias	3.78E+36	-6.47E-02	3.53E-02	5.64E-01	2.33E-02
	RMSE	5.34E+37	1.31E+00	1.30E+00	1.56E+00	2.18E-01
n =100	Mean	9.49E+00	7.81E-01	7.81E-01	1.560636	1.261314
	Bias	8.09E+00	-1.19E-01	-1.87E-02	3.61E-01	-3.87E-02
	RMSE	6.79E+01	1.55E-01	1.02E-01	1.009048	2.07E-01
n =200	Mean	2.98E+43	7.90E-01	7.90E-01	2.070606	1.288546
	Bias	2.98E+43	-1.10E-01	-1.05E-02	8.71E-01	-1.15E-02
	RMSE	3.92E+44	7.31E-01	7.23E-01	1.960083	1.53E-01
n =500	Mean	2.51E+00	7.91E-01	7.91E-01	1.402799	1.281771
	Bias	1.11E+00	-1.09E-01	-9.45E-03	2.03E-01	-1.82E-02
	RMSE	4.49E+00	1.17E-01	4.37E-02	4.33E-01	1.48E+01

Application

This data set represents the remission times (in months) of a sample size of 128 bladder cancer patients reported in Lee and Wang (2003). The data are reported below

0.08, 2.09, 3.48, 4.87, 6.94, 8.66, 13.11, 23.63, 0.20, 2.23, 3.52, 4.98, 6.97, 9.02, 13.29, 0.40, 2.26, 3.57, 5.06, 7.09, 9.22, 13.80, 25.74, 0.50, 2.46, 3.64, 5.09, 7.26, 9.47, 14.24, 25.82, 0.51, 2.54, 3.70, 5.17, 7.28, 9.74, 14.76, 6.31, 0.81, 2.62, 3.82, 5.32, 7.32, 10.06, 14.77, 32.15, 2.64, 3.88, 5.32, 7.39, 10.34, 14.83, 34.26, 0.90, 2.69, 4.18, 5.34, 7.59, 10.66, 15.96, 36.66, 1.05, 2.69, 4.23, 5.41, 7.62, 10.75, 16.62, 43.01, 1.19, 2.75, 4.26, 5.41, 7.63, 17.12, 46.12, 1.26, 2.83, 4.33, 5.49, 7.66, 11.25, 17.14, 79.05, 1.35, 2.87, 5.62, 7.87, 11.64, 17.36, 1.40, 3.02, 4.34, 5.71, 7.93, 11.79, 18.10, 1.46, 4.40, 5.85, 8.26, 11.98, 19.13, 1.76, 3.25, 4.50, 6.25, 8.37, 12.02, 2.02, 3.31, 4.51, 6.54, 8.53, 12.03, 20.28, 2.02, 3.36, 6.76, 12.07, 21.73, 2.07, 3.36, 6.93, 8.65, 12.63, 22.69

Table 2: Information Criteria Measure and Goodness of Fit Models Using Bladder Cancer Output Data

Distribution	LL	k	AIC	BIC	KS-stat	KS-P	CVM-P
IW	-441.896	2	887.793	893.497	0.1412	0.0122	0.0022
TLIW	-428.962	2	861.923	867.627	0.1144	0.0702	0.0239
ETLIW	-428.869	3	863.737	872.293	0.1161	0.0633	0.0225
APETLIW	-424.808	5	859.616	873.877	0.0959	0.19	0.0797

Discussion

The analysis of the APETIW distribution demonstrated its efficiency in modeling real-life data. This new distribution outperformed its competitors, as indicated by smaller values of information criteria (aryal2017regression), as shown in Tables 1 and 2. The simulation results in 1 indicated that the model's performance improves as the sample size increases. The fitted CDF and PDF plots from the an illustrate above the flexibility of the proposed distribution compared to existing distributions.

CONCLUSION

This paper introduces a new distribution called the Alpha power Exponentiated Topp- Leon Inverse Weibull distribution. The mathematical properties were derived and the parameter estimation of the new distribution was examined using the maximum likelihood method. The behavior of the maximum likelihood estimation was also investigated to assess consistency. In conclusion, we suggest that the proposed distribution performs better, than existing distributions in terms of information criteria and goodness-of-fit tests.

REFERENCES

Al-Shomrani, A., Arif, O., Shawky, A., Hanif, S., and Shahbaz,M. Q. (2016). Topp,-leone family of distributions Some properties and application. *Pakistan Journal of Statistics and Operation Research*, pages443 −451.

Aryal, G. R., Ortega, E. M., Hamedani, G., and Yousof, H. M. (2017). The topp-leone generated weibull distribution: regression model, characterizations and pplications. International *Journal of Statistics and Probability*, 6(1):126–141.

Bleed, S. O. and Attwa, R., Abd-Elwahab and Ali, Rabeea, F. M. and Radwan,Taha(2024), On Alpha Power Transformation Generalized Pareto Distribution and Some Properties,2024(1),p. 6270350

Benkhelifa, L. (2022). Alpha power topp-leone weibull distribution: Properties, characterizations,regression modeling and applications. *Journal of Statistics and Management Systems*, 25(8):1945–1970.

Betensky, R. A., Qian, J., and Hou, J. (2023). Nonparametric and semiparametric estimation with sequentially truncated survival data. Biometrics, 79(2):1000–1013.

Brito, E., Cordeiro, G., Yousof, H., Alizadeh, M., and Silva, a. O. (2017). The topp–leone odd loglogistic family of distributions. Journal of Statistical Computation and Simulation, 87(15):3040–3058.

Chen, C., Yu, T., Shen, B., and Wang, M. (2023). Synthesizing secondary data into survival analysis to improve estimation efficiency. Biometrical Journal, 65(3):2100326.

De Gusmao, F. R., Ortega, E. M., and Cordeiro, G. M. (2011). The generalized inverse Weibull distribution. Statistical Papers, 52:591–619.

Faruk, M. U., Isa, A. M., and Kaigama, A. (2024). Sine-weibull distribution: mathematical properties and application to real datasets. Reliability: Theory & Applications, 19(1 (77)):65–72.

Klakattawi, H. S. (2022). Survival analysis of cancer patients using a new extended Weibull distribution. Plos one, 17(2): e0264229.

Mahdavi, A. (2017). Generalized topp-leone family of distributions. *Journal of Biostatistics and Epidemiology*, 3(2):65–75.

Mead, M. E., Cordeiro, G. M., Afify, A. Z., and Al Mofleh, H. (2019). The alpha power transformation family: properties and applications. *Pakistan Journal of Statistics and Operation Research*, pages 525–545.

Mudholkar, G. S., Srivastava, D. K., and Freimer, M. (1995). The Exponentiated weibull family: A reanalysis of the bus-motor-failure data. Technometrics, 37(4):436–445.

Topp, C. W. and Leone, F. C. (1955). Truncated distributions. *Journal of the American Statistical Association*, 50(269):209–219.

Usman, A., Ishaq, A. I., Suleiman, A. A., Othman, M., Daud, H., and Aliyu, Y. (2023). Univariate and bivariate log-topp-leone distribution using censored and uncensored datasets. In Computer Sciences & Mathematics Forum, volume 7, page 32. MDPI.

Usman, A. A. A. Osi, I. A. Salisu, H. Muhammad, Y. I. Muhammad, and A. Abubakar,Arcsine rayliegh pareto distribution: Properties and application to carbon fibers data sets, *FUDMA JOURNAL OF SCIENCES*, vol. 8, no. 2, pp. 301“305, 2024.

Weibull, W., A statistical distribution function of wide applicability, ´ *Journal of applied mechanics, 1951*.

©2026 This is an Open Access article distributed under the terms of the Creative Commons Attribution 4.0 International license viewed via https://creativecommons.org/licenses/by/4.0/ which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is cited appropriately.