FUDMA Journal of Sciences (FJS)
ISSN online: 2616-1370
ISSN print: 2645 - 2944
Vol. 9 No. 11, November, 2025, pp 466 – 472
DOI: https://doi.org/10.33003/fjs-2025-0911-4133

TRANSMUTED NEW CLASS SINE – G (TNCS-G) FAMILY OF DISTRIBUTION: DISTRIBUTIONAL PROPERTIES AND APPLICATIONS

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ABSTRACT

This research introduces a novel family of probability distribution referred to as Transmuted New Class Sine-G (TNCS-G) family of distribution with Transmuted New Class Sine-Weibull (TNS-W) distribution as its sub model. The new TNCS-G family is an extension of the New Class Sine-G Family of Distribution developed by Sapkota (2023). Using the transmutation map, the probability density function (pdf) and the cumulative distribution function (cdf) of the proposed family of distribution and its sub model with weibull distribution as baseline distribution were derived. The Transmuted New Class Sine Weibull (TNCS-W) distribution is developed by combining the transmutation technique with the sine function and the Weibull distribution. Existing Weibull-based models often fail to adequately capture complex behaviours such as skewness, heavy tails, or non-monotonic hazard rates, motivating the need for more flexible models. The objective of this study is to propose and investigate the TNCS-W distribution as a more adaptable model for real-life data. Distributional properties including the probability density function (PDF), cumulative distribution function (CDF), hazard rate function, moments, and quantile function are derived. Parameters are estimated using Maximum Likelihood Estimation (MLE), and the T-SGW is compared with the New Weighted Weibull (NWWD) and standard Weibull distributions using real datasets. Results show that the TNCS-W consistently yields superior fits, as indicated by lower AIC and log-likelihood values (e.g., for rainfall data: AIC = 609.57 for TNCS-W versus 656.28 for NWWD). These findings demonstrate that the TNCS-W provides a more accurate and flexible alternative for applications in fields such as reliability analysis, survival studies, and environmental modelling.

Keywords: Distribution, Transmutation, Maximum likelihood estimation, Transmuted New Class Sine-G family of distribution, Weibull distribution, Generalisation

INTRODUCTION

For the past two decades, developing compound distributions has received widespread attention from both the mathematics and statistics research communities with the aim of developing flexible probabilistic models that fit unique stochastic phenomena. This phenomenon occurs in several sectors of human endeavor, such as engineering, survival analysis, marine sciences and many more. The unique nature of different behaviors of datasets from several random processes as a result of technological development has been the key motivational factor for developing new probability distributions. A variety of techniques has been developed in the literature for defining new compound distributions such as link function, odd function, generalisation, transmutation and many more. Several families for generating new distributions and their respective sub-families have been developed in the literature. The Weibull distribution (Weibull, 1951) has long served as a fundamental model in reliability and lifetime data analysis, appreciated for its flexibility in modelling various hazard rates. However, it often falls short when data exhibit complex, non-monotonic behaviour or heavy tails.

To address these limitations, several generalisations such as the generalised Weibull (Mudholkar et al., 1996) and exponentiated Weibull (Mudholkar & Srivastava, 1993) distributions were proposed. Meanwhile, Shaw and Buckley (2007) introduced the transmutation technique to further enhance distributional flexibility by adding a shape-controlling parameter.

Building on these ideas, Kumar et al (2015) and Souza (2015) developed the Sine-G family of distributions, using sine transformations to generate more adaptable models. Recent efforts combine the Sine-G approach with transmutation techniques to maximize flexibility. Despite the usefulness of existing Weibull generalisations, many still struggle to adequately capture datasets with strong skewness, multimodality, or heavy-tailed behaviour. This gap motivates the development of more versatile families that integrate both sine transformations and transmutation techniques.

The Transmuted New Class Sine-Weibull (TNCS-W) distribution emerges from this lineage, blending the Weibull baseline with sine transformation and a transmutation map. It offers a powerful model capable of capturing complex hazard shapes and diverse data behaviors, making it a strong candidate for applications in survival analysis, engineering, and biostatistics.

Recent advancements have explored the integration of transmutation techniques with the Sine-G family to enhance distributional flexibility. For instance, Kumar et al. (2015) have defined a new class of distribution using the sine trigonometric function and defined the sin-exponential model as its member, Souza (2015) introduced another trigonometric model based on the sine function, Isa et al. (2022) have developed a new two-parameter model called the sine Burr XII distribution, Chesneau and Jamal (2020) developed the Sine Kumaraswamy-G family; Mahmood et al. (2019) have developed the new sin-G family and analyzed the sin-inverse Weibull model in particular, Oramulu et al. (2024) introduced the Sine Generalized-G family, which combines the Sine-G and Generalized-G families to model complex data behaviors effectively., Sakthivel and Rajkumar (2024) also worked on the transmuted Sine-G family highlighting the effectiveness of combining transmutation maps with sine transformations

to achieve greater control over skewness and tail behavior in probability distributions, Muhammad et al. (2021) have defined the exponentiated sine-G family and analyzed the particular distribution as an exponentiated sine-Weibull distribution.
Using the New Class sine-G family of distributions, this paper proposes the Transmuted New Class Sine- (TNCS-G) family of distribution and the Transmuted New Class Sine-Weibull (TNCS-W) distribution as its sub-model to overcome these limitations and provide a more flexible model for real-life data across fields such as reliability, survival analysis, and environmental sciences.
A random variable X is said to have a transmuted distribution function as developed by Shaw and Buckley (2009) if its *pdf* and *cdf* are respectively given by:

$$f(x) = g(x)[1 + \lambda - 2\lambda G(x)] \quad (1)$$

And

$$F(x) = (1 + \lambda)G(x) - \lambda \quad (2)$$

Where $x > 0$, and $-1 \leq \lambda \leq 1$ is the transmuted parameter, *G(x)* is the *cdf* of any continuous distribution while *f(x)* and *g(x)* are the associated *pdf* of *F(x)* and *G(x),* respectively.

MATERIALS AND METHODS

The New Class Sine-G family of distribution

The cumulative distribution function (cdf) of New Class Sine-G is given as a random variable X is said to follow a Sine-G distribution with parameter(s) if its cumulative distribution function (cdf) is given as,

$$F(x;\eta) = sin\left\{\pi\frac{G(x;\eta)}{1+G(x;\eta)}\right\}; x \in \mathbb{R} \quad (3)$$

With corresponding probability density function (*pdf*) as

$$f(x;\eta) = \pi\, cos\left\{\pi\frac{G(x;\eta)}{1+G(x;\eta)}\right\}\left(\frac{g(x;\eta)}{(1+G(x;\eta))^2}\right); x \in \mathbb{R} \quad (4)$$

The Proposed Transmuted New Class Sine-G (TNCS-G) family of distribution

The *cdf* and *pdf* of the transmuted New Class sine-G family of distribution is stated as follows,

$$f(x) = \pi\, cos\left\{\pi\frac{G(x;\eta)}{1+G(x;\eta)}\right\}\left(\frac{g(x;\eta)}{(1+G(x;\eta))^2}\right)\left[1 + \lambda - 2\lambda\, sin\left\{\pi\frac{G(x;\eta)}{1+G(x;\eta)}\right\}\right]; x \in \mathbb{R} \quad (5)$$

$$F(x;\eta) = (1 + \lambda)sin\left\{\pi\frac{G(x;\eta)}{1+G(x;\eta)}\right\} - \lambda\left(sin\left\{\pi\frac{G(x;\eta)}{1+G(x;\eta)}\right\}\right)^2; x \in \mathbb{R} \quad (6)$$

The proposed TNCS-W Distribution as a sub model of the TNCS-G

To define the TNCS-W distribution, we first define the *pdf* and *cdf* of a two-parameter Weibull distribution with parameters $\alpha, \beta > 0$as:

$$f(x;\alpha,\beta) = \frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1} e^{-(x/\alpha)^\beta}; x \geq 0 \quad (7)$$

$$F(x;\alpha,\beta) = 1 - e^{-\left(\frac{x}{\alpha}\right)^\beta}; x \geq 0 \quad (8)$$

Where $\alpha > 0$is the scale parameter, and $\beta > 0$is the shape parameter.
A random variable X is said to have a new class sine-G family if its *pdf* and *cdf* are respectively given by:

$$f(x;\eta) = sin\left\{\pi\frac{G(x;\eta)}{1+G(x;\eta)}\right\}; x \in \mathbb{R} \quad (9)$$

And

$$F(x;\eta) = \pi\, cos\left\{\pi\frac{G(x;\eta)}{1+G(x;\eta)}\right\}\left(\frac{g(x;\eta)}{(1+G(x;\eta))^2}\right); x \in \mathbb{R} \quad (10)$$

Where $x > 0$, the notation "η" is used generally for the parameters of the baseline distribution. In our case, the parameters are αandβ;*G(x)* is the *cdf* of any continuous distribution while *f(x)* and *g(x)* are the associated *pdf* of *F(x)* and *G(x),* respectively.
Using equations (7) and (8) into equation (9) and (10) and simplifying, we obtain the *cdf* and *pdf* of the transmuted New Class Sine Weibull (TNCSW) distribution, respectively, as follows:

$$F(x;\alpha,\beta,\lambda) = (1 + \lambda)[1 - cos(\theta(x))] - \lambda[1 - cos(\theta(x))]^2 \quad (11)$$

$$f(x;\alpha,\beta,\lambda) = \pi\frac{\beta}{\alpha}\frac{(x/\alpha)^{\beta-1}e^{-(x/\alpha)^\beta}}{\left(2-e^{-(x/\alpha)^\beta}\right)^2}cos(\theta(x))[(1-\lambda) + 2\lambda\, cos(\theta(x))] \quad (12)$$

Where; $\theta(x) = \pi\frac{1-e^{-(x/\alpha)^\beta}}{2-e^{-(x/\alpha)^\beta}}$, $\alpha > 0$is a scale parameter and $\beta > 0$is the shape parameter. Illustratively, the *pdf* and *cdf* of the *TNCSW distribution* using some parameter values are displayed in Figures 1 and 2 as follows:

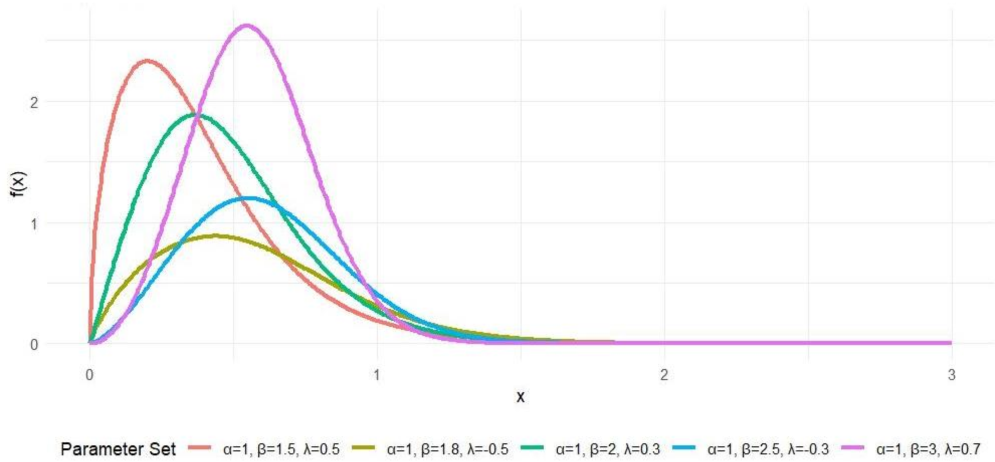

Figure 1: The graph of *pdf* of the *TNCSW* at some chosen parameter values of α and β

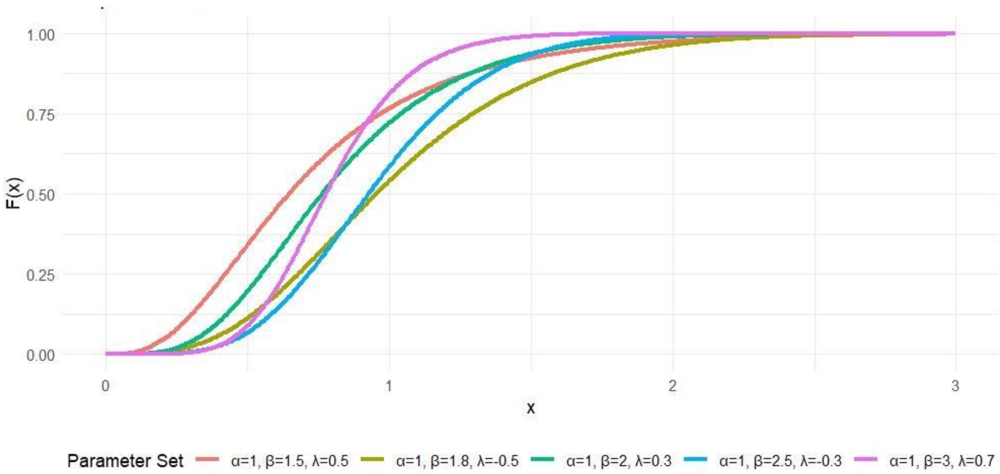

Figure 2: The graph of *cdf* of the *TNCSW* at some chosen parameter values of α and β

The PDF plot illustrates the effect of varying the shape parameter β and the transmutation parameter λ, while keeping α=1 constant. The curves show a wide range of skewness and peak behavior, the CDF shows that Larger β and positive λ values result in a steeper CDF slope, suggesting that values accumulate quickly near the mode. Overall, these plots highlight the TNCSW distribution's adaptability in capturing different data behaviors, from peaked and compact to broad and skewed forms, depending on the choice of parameters.

Properties

In this section, we define and discuss some properties of the *TNCSW* distribution.

Moments

Let *X* denote a continuous random variable, the n^{th} moment of *X* is given by;

$$\mu'_n = E(X^n) = \int_{-\infty}^{\infty} x^n f(x)dx = \int_0^{\infty} x^n f(x)dx \quad (13)$$

Taking *f(x)* as the *pdf* of the *TNCSW* as given in equation (12), the n^{th} moment of *X* is obtained using integration by substitution and is given as:

$$\mu'_n = \frac{\pi}{2}(1-\lambda)E\left[X^n \sin\left(\frac{\pi}{2}G(X)\right)\right] + \lambda\pi E\left[X^n \cos\left(\frac{\pi}{2}G(X)\right)\sin\left(\frac{\pi}{2}G(X)\right)\right] \quad (14)$$

Where: $G(x)$and$g(x)$are the CDF and PDF of the Weibull distribution, respectively.

The Mean

The mean of the TNCSW can be obtained from the nth moment of the distribution when $n = 1$as follows:

$$\mu'_1 = \frac{\pi}{2}(1-\lambda)E\left[X \sin\left(\frac{\pi}{2}G(X)\right)\right] + \lambda\pi E\left[X \cos\left(\frac{\pi}{2}G(X)\right)\sin\left(\frac{\pi}{2}G(X)\right)\right] \quad (15)$$

Also, the second moment of the TNCSW is obtained from the nth moment of the distribution when$n = 2$ as:

$$\mu'_2 = \frac{\pi}{2}(1-\lambda)E\left[X^2 \sin\left(\frac{\pi}{2}G(X)\right)\right] + \lambda\pi E\left[X^2 \cos\left(\frac{\pi}{2}G(X)\right)\sin\left(\frac{\pi}{2}G(X)\right)\right] \quad (16)$$

Generally, the expression cannot be evaluated analytically; thus, it is mostly computed numerically.

The Variance

The variance of *X* for *TNCSW* distribution is obtained from the central moment when $n = 2$, that is;

$$Var(X) = E(X^2) - \{E(X)\}^2 \quad (17)$$

$$Var(X) = \mu'_2 - \{\mu'_1\}^2 \quad (18)$$

Whereμ'_1andμ'_2 are the mean and second moment of the TNCS-W distribution, respectively.

The variance is usually evaluated numerically because of the complexity of the sine and cosine functions involving the Weibull CDF.

The variation, skewness, and kurtosis measures can also be calculated from the non-central moments using some well-known relationships.

Moment Generating Function

The moment-generating function (*mgf*) is an important shape characteristic of a distribution and is always a function that represents all the moments. In other words, the *mgf* produces all the moments of the random variable *X* by differentiation.

The *mgf* of a random variable *X* can be obtained by:

$$M_X(t) = E(e^{tx}) = \int_0^{\infty} e^{tx} f(x)dx \quad (19)$$

Thus, the *mgf* of the *TNCSW* distribution is given as:

$$M_X(t) = \frac{\pi}{2}\left((1-\lambda)\int_0^{\infty} e^{tx} g(x)\sin\left(\frac{\pi}{2}e^{-(x/\beta)^{\alpha}}\right)dx - \lambda\int_0^{\infty} e^{tx} g(x)\sin\left(\frac{3\pi}{2}e^{-(x/\beta)^{\alpha}}\right)dx\right) \quad (20)$$

Estimation of Parameters of TNCSW distribution using the Maximum Likelihood Method

Let $X_1 \ldots X_n$ be a sample of size 'n' of independently and identically distributed random variables from the *TNCSW* distribution with unknown parameters $\alpha$and β defined previously. The *pdf* of the *TNCSW* is given as:

$$f(x;\alpha,\beta,\lambda) = \pi\frac{\beta}{\alpha}\frac{(x/\alpha)^{\beta-1}e^{-(x/\alpha)^{\beta}}}{\left(2-e^{-(x/\alpha)^{\beta}}\right)^2}\cos(\theta(x))[(1-\lambda) + 2\lambda\cos(\theta(x))] \quad (21)$$

The likelihood function is given by:

$$L(X_1 \ldots X_n;\alpha,\beta,\lambda) = \left(\frac{\pi}{2}\right)^n \alpha^n \beta^{-n\alpha} \left(\prod_{i=1}^{n} x_i^{\alpha-1}\right) exp\left(-\sum_{i=1}^{n}\left(\frac{x_i}{\beta}\right)^{\alpha}\right) \times \prod_{i=1}^{n}\left[(1-\lambda)\cos\left(\frac{\pi}{2}\left(1-e^{-(x_i/\beta)^{\alpha}}\right)\right) + \lambda\cos\left(\frac{3\pi}{2}\left(1-e^{-(x_i/\beta)^{\alpha}}\right)\right)\right] \quad (22)$$

For the Maximum Likelihood Estimation (MLE), we derive the Log-Likelihood by taking the natural logarithm of the likelihood function as follows:

Let the log-likelihood function$\ell(x;\alpha,\beta,\lambda) = log\, L\,(x;\alpha,\beta,\lambda)$, therefore;

$$LL(x;\alpha,\beta,\lambda) = n\,log\left(\frac{\pi}{2}\right) + n\,log\,\alpha - n\alpha\,log\,\beta + (\alpha-1)\sum_{i=1}^{n} log\,x_i - \sum_{i=1}^{n}\left(\frac{x_i}{\beta}\right)^{\alpha} + \sum_{i=1}^{n} log\left[(1-\lambda)\cos\left(\frac{\pi}{2}\left(1-e^{-(x_i/\beta)^{\alpha}}\right)\right) + \lambda\cos\left(\frac{3\pi}{2}\left(1-e^{-(x_i/\beta)^{\alpha}}\right)\right)\right] \quad (23)$$

Differentiating ℓpartially with respect toλ,α andβrespectively gives;

$$\frac{\partial\ell}{\partial\lambda} = \sum_{i=1}^{n}\frac{-\cos\left(\frac{\pi}{2}\left(1-e^{-(x_i/\beta)^{\alpha}}\right)\right)+\cos\left(\frac{3\pi}{2}\left(1-e^{-(x_i/\beta)^{\alpha}}\right)\right)}{1+\lambda\cos\left(\frac{\pi}{2}\left(1-e^{-(x_i/\beta)^{\alpha}}\right)\right)+\lambda\cos\left(\frac{3\pi}{2}\left(1-e^{-(x_i/\beta)^{\alpha}}\right)\right)} \quad (24)$$

$$\frac{\partial\ell}{\partial\alpha} = \frac{n}{\alpha} - n\,log\,\beta + \sum_{i=1}^{n} log\,x_i - \sum_{i=1}^{n}\left(\frac{x_i}{\beta}\right)^{\alpha} log\left(\frac{x_i}{\beta}\right) + T_1 \quad (25)$$

$$\frac{\partial\ell}{\partial\beta} = \frac{-n\alpha}{\beta} + \alpha\sum_{i=1}^{n}\left(\frac{x_i}{\beta}\right)^{\alpha}\frac{1}{\beta} + T_2 \quad (26)$$

Where T_1 and T_2 are the derivatives of the cosine-related terms. Equating equations (22) and (23) to zero and solving for the solution of the non-linear system of equations will give us the maximum likelihood estimates of parameters αandβrespectively. However, the solution cannot be obtained analytically except numerically with the aid of suitable statistical software like Python, R, SAS, etc., when data sets are given.

RESULTS AND DISCUSSION

This section presents two datasets, their summary of descriptive statistics and applications to some selected extensions of the Weibull distribution. We have compared the performance of the Transmuted New Class Sine Weibull distribution (*TNCS-W*) to those of the New Weighted Weibull distribution (*NWWD*) and the classical Weibull distribution (W*D*).

We evaluated the performance of the above-listed models using the *AIC* (Akaike Information Criterion). It is considered that the model with the smallest value of AIC was chosen as the best model to fit the data.

Data set I: The first data set (Chen et al., 2010) corresponds to fifty-two ordered annual maximum antecedent rainfall measurements in mm from Maple Ridge in British Columbia, Canada. The data are: 264.9, 314.1, 364.6, 379.8, 419.3, 457.4, 459.4, 460, 490.3, 490.6, 502.2, 525.2, 526.8, 528.6, 528.6, 537.7, 539.6, 540.8, 551.0, 573.5, 579.2, 588.2, 588.7, 589.7, 592.1, 592.8, 600.8, 604.4, 608.4, 609.8, 619.2, 626.4, 629.4, 636.4, 645.2, 657.6, 663.5, 664.9, 671.7 ,673.0, 682.6, 689.8, 698, 698.6, 698.8, 703.2, 755.9, 786, 787.2, 798.6, 850.4, 895.1. It's summarized as follows:

Table 1: Summary Statistics for Dataset I

Parameters	n	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis
Values	52	264.9	528.1	596.8	672.0	595	895.1	16158	-0.1895	0.3454

Table 2: The performance of the selected models using the *AIC* value of the models evaluated at the *MLEs* based on Dataset I

Distributions	Parameter estimates (standard errors)	-ll=(-log-likelihood value)	*AIC*	Ranks
TNCS-W	α =839.5006 (36.5343) β =5.3606 (0.6094) λ =0.99 (0.3814)	-301.7842	609.5685	**1**
NWWD	α =599.9739 (73.8817) β =4.3035 (1.3871) θ =1.4858 (0.8949)	-325.1424	656.2848	**3**
WD	α =644.8914 (17.8587) β =5.2805 (0.5540)	-325.3783	654.7565	**2**

Using table 2 and comparing the value of the *AIC* for each model shows that the *TNCS-W* distribution has better performance compared to the *NWW* and Weibull distributions. This is due to the decision rule, which states that the model with the lowest AIC is considered the most adequate or efficient. The positive value of λ indicates right-skewness, while the shape parameter β > 1 suggests an increasing hazard rate.

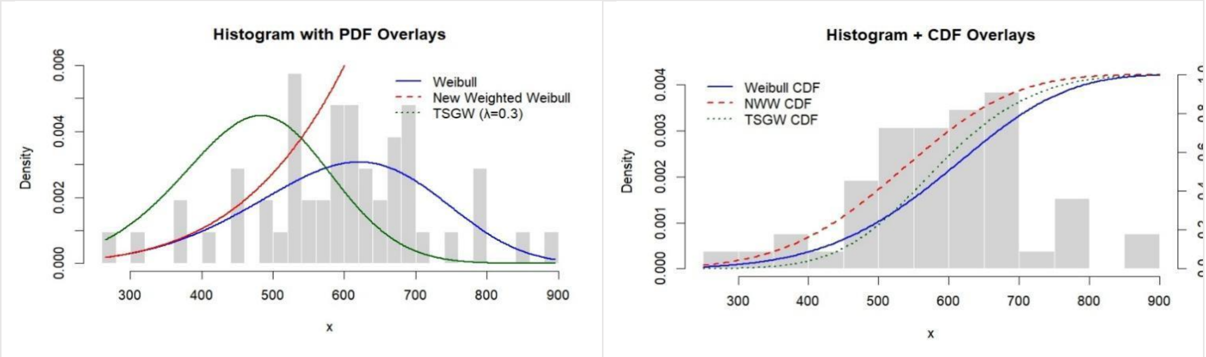

Figure 3: Histogram and plots of the estimated densities *pdf* and *cdf* of the TNCS-W, NWWD and WD fitted to the dataset I

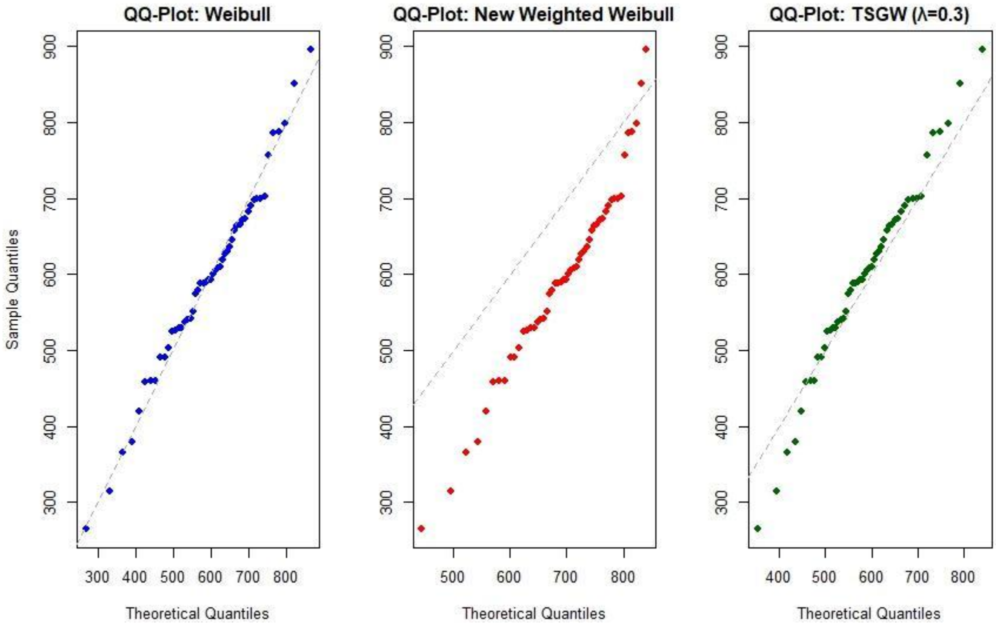

Figure 4: Probability plots for the fit of the TNCS-W, NWWD and WD based on dataset I

Data set II: This second dataset represents the survival times of 121 patients with breast cancer obtained from a large hospital in the period from 1929-1938 from Lee (1986), Ramos *et al*. (2013) and Oguntunde *et al.* (2015). The observations are as follows: 0.3, 0.3, 4.0, 5.0, 5.6, 6.2, 6.3, 6.6, 6.8, 7.4, 7.5, 8.4, 8.4, 10.3, 11.0, 11.8, 12.2, 12.3, 13.5, 14.4, 14.4, 14.8, 15.5, 15.7, 16.2, 16.3, 16.5, 16.8, 17.2, 17.3, 17.5, 17.9, 19.8, 20.4, 20.9, 21.0, 21.0, 21.1, 23.0, 23.4, 23.6, 24.0, 24.0, 27.9, 28.2, 29.1, 30.0, 31.0, 31.0, 32.0, 35.0, 35.0, 37.0, 37.0, 37.0, 38.0, 38.0, 38.0, 39.0, 39.0, 40.0, 40.0, 40.0, 41.0, 41.0, 41.0, 42.0, 43.0, 43.0, 43.0, 44.0, 45.0, 45.0, 46.0, 46.0,47.0, 48.0, 49.0, 51.0, 51.0, 51.0, 52.0, 54.0, 55.0, 56.0, 57.0, 58.0, 59.0, 60.0, 60.0, 60.0, 61.0, 62.0, 65.0, 65.0, 67.0, 67.0, 68.0, 69.0, 78.0, 80.0, 83.0, 88.0, 89.0, 90.0, 93.0, 96.0, 103.0, 105.0, 109.0, 109.0, 111.0, 115.0, 117.0, 125.0, 126.0, 127.0, 129.0, 129.0, 139.0, 154.0.

Table 3: Summary Statistics for the dataset II

Parameter	n	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis
Values	121	0.30	17.5	40.00	60.0	46.33	154.00	1244.464	1.04318	0.40214

Table 4: The performance of the selected models using the *AIC* value of the models evaluated at the *MLEs* based on Dataset II

Distributions	Parameter estimates (standard errors)	-ll=(-log-likelihood value)	*AIC*	Ranks
TNCS-W	α =148.93 (16.532) β =1.31 (0.097) λ =-0.99 (0.26)	-525.115	1056.23	**1**
NWWD	α =57.93 (15.132) β =1.49 (0.393) θ =0.80 (0.333)	-578.880	1163.76	**3**
WD	α =50.138 (3.672) β =1.306 (0.093)	-579.024	1162.05	**2**

Now, comparing the value of the *AIC* for all the distributions, it is very clear that the proposed *T-NCS-W* performs best, outperforming both the *NWWD* and the classical Weibull distribution (W*D)*. This conclusion is again due to the decision rule, which says that the distribution or model with a smaller value of the test statistic (AIC) should be considered as the most fitted or efficient model. This decision also agrees with the fact that extending or generalising any continuous distribution always provides a compound distribution with a better fit than the classical distribution. The negative λ reflects left-skewness, aligning with the long-tailed survival data. These interpretations confirm that the transmutation parameter allows the TNCS-W to flexibly adjust for skewness direction."

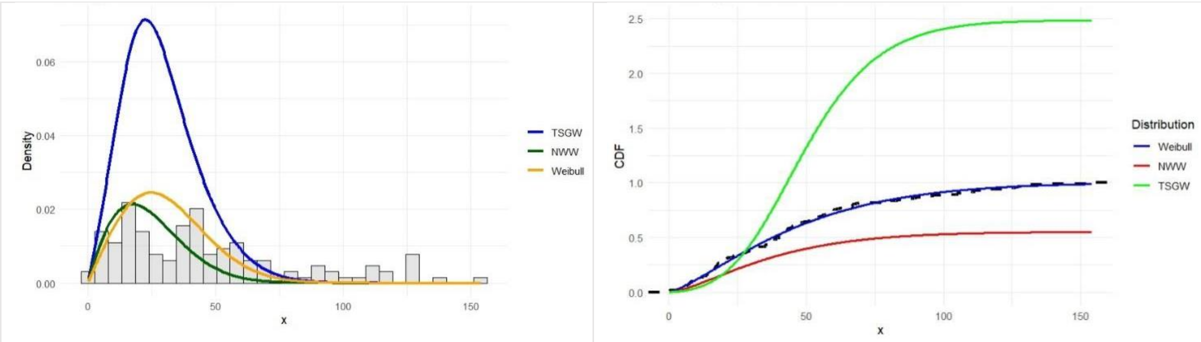

Figure 5: Histogram and plots of the estimated *pdf* and *cdf* of the TNCS-W, NWW and WD fitted to the dataset II

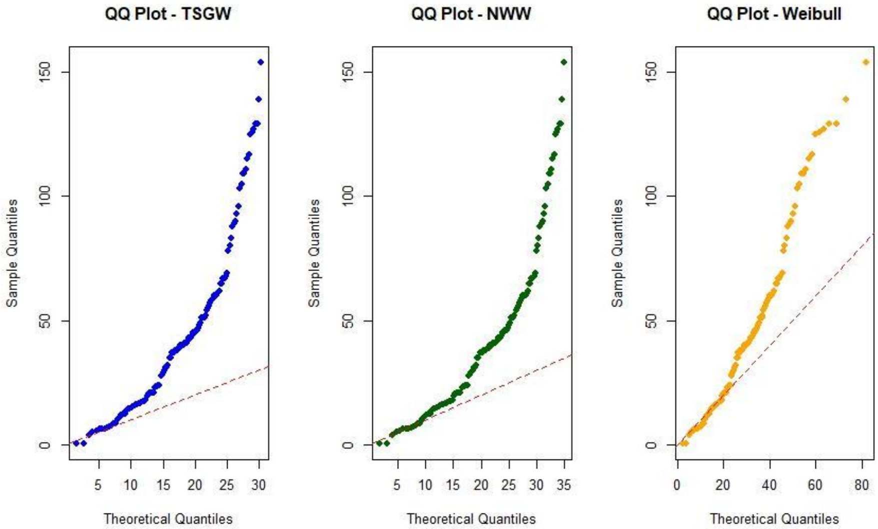

Figure 6: Probability plots for the fit of the TNCS-W, NWW and WD (dataset II)

Figures 3 and 5 show that the fitted TNCS-W curves closely follow the empirical histograms and probability plots, particularly in the tails, where the NWWD and WD deviate. This highlights the practical advantage of the additional transmutation and sine components in achieving better alignment with observed data.

The consistent improvement in AIC values across both datasets connects directly to the study's aim: demonstrating the TNCS-W distribution's capacity to provide more accurate fits compared to existing alternatives.

CONCLUSION

This paper introduced the Transmuted New class Sine G (TNCS-G) family of distribution and also proposed and studied the Transmuted New Class Sine Weibull (TNCS-W) distribution as it's sub-model, motivated by the limitations of existing Weibull-based models in handling skewness and heavy-tailed data. Key properties, including moments and the moment-generating function, were derived, and parameters were estimated via maximum likelihood. Application to rainfall and survival data demonstrated that the TNCS-W consistently outperforms both the New Weighted Weibull and the standard Weibull distributions in terms of AIC and log-likelihood values. The findings contribute both theoretically, by expanding the family of sine–transmuted distributions, and practically, by offering a more flexible tool for modelling real-life data in fields such as survival analysis, reliability engineering, and environmental science.

One limitation of this study is that only maximum likelihood estimation was considered; alternative estimation methods (e.g., Bayesian or L-moments) could be explored in future work. Additionally, future research could investigate regression extensions of the TNCS-W for covariate-dependent lifetime data or explore its performance in censored datasets.

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