

MODELING AGE-AT-DEATH IN ADAMAWA STATE, NIGERIA, USING PROBABILITY DISTRIBUTION FITS

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ABSTRACT

High mortality rates and low life expectancy remain critical public health challenges in Nigeria, yet regional variations in mortality patterns are poorly understood. This study models age at death as a proxy for life expectancy in Adamawa State, Nigeria, using probability distribution fitting approaches. Drawing on secondary data from the Specialist Hospital Yola, the study analyses age-at-death records over 61 years. Four continuous lifetime distributions—Log-normal, Log-logistic, Generalized Extreme Value (GEV), and Generalized Pareto—were evaluated for best fit using Kolmogorov-Smirnov, Anderson-Darling, and Chi-square tests. Results indicate that age-at-death is right-skewed and multimodal, with an average of 26.89 years, far below Nigeria's national average. Log-normal and GEV distributions provided the best fits among the fitted models based on multiple goodness-of-fit criteria. Regression analysis using these distributions revealed that age-at-death significantly varied by cause of death and local government area, but not by gender. The findings highlight the importance of region-specific health policy interventions and support the use of lifetime distributions in modelling mortality patterns for public health planning.

Keywords: Life expectancy, Age-at-death, Lifetime distributions, Adamawa State, Log-normal, Generalized Extreme Value, Mortality modeling

INTRODUCTION

Life expectancy in Nigeria remains among the lowest globally, with recent estimates at 53.95 years, significantly below the Sub-Saharan African average of 61.2 years (WHO, 2016; World Bank, 2018). This low life expectancy has severe socio-economic implications, including loss of productive resources and reduced GDP contributions. The World Health Organisation (WHO) defines life expectancy at birth as the average number of years a newborn is expected to live under prevailing mortality conditions (WHO, 2020). It is a key indicator of population health and socio-economic development (Bilas et al., 2014).

According to WHO, life expectancy at birth is the average number of years that a newborn could expect to live if he or she were to pass through life exposed to the sex- and age-specific death rates prevailing at the time of his or her birth, for a specific year, in a given country, territory, or geographic area. It reflects the overall mortality level of a population. It summarizes the mortality pattern across all age groups – children and adolescents, adults, and the elderly (WHO, 2020). The Organisation for Economic Cooperation and Development (OECD) defines life expectancy at birth as the length of time, on average, a newborn can expect to live if current death rates do not change (OECD, 2020). Thus, life expectancy measures the length and quality of life a person is expected to live within a geographical area, given that the factors affecting longevity do not change. Life expectancy at birth is therefore a measure of the health of a population and a reflection of the socio-economic conditions prevailing among a population in a particular area. It is the most widely used indicator of population health (Sharma, 2018). There exists a set of socio-economic indicators concerning life expectancy (Cervantes et al., 2019). Life expectancy has significant implications for individuals and the entire aggregate human behavior, affecting fertility behavior, economic growth, human capital investment, inter-generational transfers, and even incentives for pension benefits. It also implies public finance. Alluding to the

significance of life expectancy, it is crucial for developing countries to achieve socio-economic progress through significant investments in the social sectors like health, education, sanitation, environmental management and sustainability, and social safety nets. It is an important synthetic indicator for assessing a country's or region's economic and social development (Bilas, Frank, & Bosnjak, 2014).

Concerning Nigeria, as in other developing countries, variations in morbidity and mortality have been associated with a wide variety of measures of socio-economic status including per capita GDP, fertility rate, adult illiteracy rate, per capita calorie intake, health care expenditure, access to potable drinking water, urban inhabitants, unemployment rate and the nominal exchange rate (Sede & Ohemeng, 2015). Although the link between life expectancy and income, for instance, has been demonstrated in several statistical studies, it is not just the aggregate increase in income that increases life expectancy at birth, but the poverty reduction that results from the income growth (Biciunaite, 2014). Income growth reduces hunger, provides accommodation in clean environments, access to healthcare, education, and healthy, nutritious meals, and engenders good health and longevity. Life expectancy as a measure, therefore, reflects not just the overall health or mortality of a population but also provides an insight into the social and economic conditions that interplay or exist to affect longevity within a region. It is a barometer for a healthy socio-economic system. Despite improvements since 1980 (from 45.33 to 53.95 years), Nigeria's life expectancy remains critically low compared to developed regions (e.g., Europe: 77.5 years) (WHO, 2016). Factors contributing to this disparity include poor healthcare access, illiteracy, malnutrition, and inadequate sanitation (Sede & Ohemeng, 2015). While previous studies have examined socio-economic determinants of life expectancy (Muhammad & Sabo, 2018; Lin et al., 2012), few have applied probability distribution modelling to age-at-death data in Nigeria

Life expectancy at birth is the average number of years a newborn infant would be expected to live if health and living conditions at the time of birth remained the same throughout life. It reflects people's health, the quality of care they receive when ill, and social, economic, and environmental conditions that mitigate or predispose to morbidity and mortality. Furthermore, life expectancy at birth is the number of years a newborn infant of either gender may be expected to live if prevailing patterns of mortality at the time of its birth stay the same throughout its lifetime (Muhammad & Sabo, 2018).

Life Expectancy: Different authorities have advanced slightly varied definitions of life expectancy; Merriam-Webster (2016) defined it as "the average life span of an individual". According to Mela et al. (2018), life expectancy is the average number of years a person of a given age or age group may be expected to live. This implies that the estimation of life expectancy is usually done across the different levels of the age structure. Above all, life expectancy at birth is the most commonly used because it is a clear indicator of mortality conditions across the age range that is not affected by the age structure of the entire population, thereby making for comparability. This fact is borne by the definition given by the Organization for Economic Cooperation and Development (OECD) who defined life expectancy as "the average number of years that a person could expect to live if he or she experienced the age-specific mortality rates prevalent in a given country in a particular year" (OECD, 2019). This definition does not consider the effect of any future decline in age-specific mortality rates. The methodologies used in the calculation of life expectancy vary among countries. These methodological differences can affect the exact comparability of reported estimates, as different methods can slightly change a country's measure of life expectancy.

MATERIALS AND METHODS

Probability Distributions

Log-normal Distributions

Log normal distributions became popular due to Gibrat (1930) Gibrat used the log-normal distribution to explain the growth of an organism, a process known as proportionate effects. Later, the log-normal distribution was widely used in the pricing of financial securities, and one of the best-known examples is the Black Scholes model of derivative pricing. Suppose the growth of an organism is represented as (Aldahlan, 2020).

$$X_t - X_{t-1} = e_t X_{t-1} \quad (1)$$

e_t is mutually independent and at the same time independent of X_{t-1} . This process means that growth is a certain fraction of X_{t-1} independent of X_{t-1} .

Using summation over time, both sides

$$\frac{X_t - X_{t-1}}{X_{t-1}} = e_t \quad (2)$$

Assuming that each time interval is minimal

$$\sum_{t=1}^n \frac{X_t - X_{t-1}}{X_{t-1}} = \sum_{t=1}^n e_t \quad (3)$$

$$\sum_{t=1}^n \frac{X_t - X_{t-1}}{X_{t-1}} = \int_{X_0}^{X_n} \frac{dX}{X} = \log X_n - \log X_0 \quad (4)$$

The limit of the sum of independent identically distributed (i.i.d.) random variables, regardless of their distribution, is normal according to the central limit theorem, where X_i is i.i.d.

$$\sum_{i=1}^n X_i = Y \sim N(\mu, \sigma^2) \quad (5)$$

If we assume that e_t is i.i.d for $t = 1, 2, \dots, n$. If it is i.i.d., then we can say

$$\sum_{t=1}^n e_t \sim N(\mu, \sigma^2) \quad (6)$$

The above equation implies that the ratio of stock prices is usually distributed, as given by the following equations.

$$\log \left(\frac{X_n}{X_0} \right) \sim N(\mu, \sigma^2) \quad (7)$$

The same idea can be used in the case of a stock.

Consider a stock, which starts with a price. S_0 and attains a price S_t after t periods,

$$S_t = S_0(1 + r_1)(1 + r_2) + \dots (1 + r_t) \quad (8)$$

$$\frac{S_t - S_{t-1}}{S_{t-1}} = r_t \quad (9)$$

For small values of r_t The above equation can be approximated with

$$\frac{S_t - S_{t-1}}{S_{t-1}} = \log \left(\frac{S_t}{S_{t-1}} \right) \quad (10)$$

An important assumption in finance is that the returns are i.i.d. This gives us the important result that the log of the price ratio is normally distributed.

$$\log \left(\frac{S_t}{S_{t-1}} \right) \sim N(\mu_{\text{daily}}, \sigma_{\text{daily}}^2) \quad (11)$$

Log-Logistic Distribution

In recent years, the ability to propose new probability models for reliability and survival analysis has increased. Many log-logistic distribution extensions (or generalizations) have been proposed in the last two decades. In terms of applications, the log-logistic distribution and its generalizations have become the most popular models for survival and reliability data. Some recent applications have included: modeling for AIDS and Melanoma data (de-Santana, et al., 2012); used for minification process (Gui, 2013); modeling breast cancer data (Ramos et al., 2013; Tahir et al., 2015); modeling on censored survival data (Lemonte, 2014); modeling time up to first calving of cows (Louzada & Granzotto, 2016); modeling, inference, and use to a polled Tabapua Race time up to First Calving Data (Granzotto et al., 2017); modeling positive real data in many areas (Lima & Cordeiro, 2017); analysing a right-censored data (Shakhathreh, 2018); modeling lung cancer data (Alshangiti, et al. 2016); and modeling of breaking stress data (Aldahlan, 2020).

Empirical Review

In a comparative study of the risk factors that affect or cause shorter life expectancy among 15 European countries, it was discovered that income level, education, and gender among other factors are significant risk factors causing inequality in the level of life expectancy among the selected 15 European countries (Mackebach et al., 2019). The authors obtained register-based mortality and survey-based risk factor data for all these countries and examined them based on gender and education. The risk factors included a father with a manual occupation, low income, few social contacts, smoking, high alcohol consumption, high bodyweight, low physical exercise, and low fruit and vegetable consumption. They computed partial life expectancy for those between 35 and 80 years based on gender and education. They found a substantial gap in life expectancy between males and females and between the highly educated and those with low levels of education. Other factors such as smoking, low income, and heavy body weight also contributed to the inequality in life expectancy between men and women within the region. They concluded that Smoking, low income, and high body weight are critical factors to consider to reduce the inequality in life expectancy in those countries. They noted that to substantially reduce inequalities in life expectancy, decisive policy action on a broad range of health determinants is required (Bilas et al., 2014; Sede & Ohemeng, 2015; Delavari et al., 2019). Lokpriy (2013) applied a multiple regression to examine the socio-economic determinants of life expectancy in ninety lower-income countries with a per capita GNI below \$4035 in

2011. The variables of interest are improved sanitation facilities, improved water sources, secondary school enrolment, GDP per capita, and health expenditure per capita. The study finds that a higher GDP per capita, combined with access to sanitation and safe water sources, as well as secondary school education, have a positive impact on life expectancy. In contrast, the relationship between life expectancy and health expenditure per capita is found to be contradictory. It is recommended that non-medical interventions be more robust in determining life expectancy factors than medical interventions.

In an Iranian study, Agheli and Emamgholipour (2015) examined the determinants of life expectancy at birth using a Johansen-Juselius cointegration method and Error Correction Model covering 1980-2012. The findings showed a positive relationship between life expectancy, per capita income, vaccination, and education level. While the result of the Error Correction Model indicated that its coefficient is estimated at - 0.022, which shows that 2.2% of disequilibrium in life expectancy is adjusted in each period and is approached to its long-run equilibrium, using the Autoregressive Distributed Lag (ARDL) Model.

In a similar study, Sufyan (2013) examines the impacts of socio-economic determinants of life expectancy across one hundred and six countries. These countries are categorised into three categories: countries with low life expectancy as a group, countries with medium life expectancy as a group, and countries with high life expectancy as a group. The canonical discriminant analysis technique is used to discriminate between the groups. The discriminating variables are population, living in urban areas (%), currently married or in union with women of reproductive age (%), GNI purchasing power parity, population density, rural population with access to improved water supply, infant mortality rate, total fertility rate, dependent population (%), and poverty. The study shows that infant mortality is the most influential variable in discriminating among the three groups, seconded by poverty. The other important discriminating factors are total fertility rate, the percentage of currently married or in-union women of reproductive age, the percentage of rural population with access to improved water supply, population density, and the percentage of urban population. More so, infant mortality rate, poverty, and total fertility rate positively discriminate countries to belong to the group of low life expectancy at birth countries. While the percentage of population living in urban areas, currently married or in-union women of reproductive age, and rural population with access to improved water supply negatively discriminate against a country in the group of high life expectancy at birth countries.

In Nigerian studies, Sanda and Oyerinola (2014) examined the impact of life expectancy on economic growth in Nigeria throughout 1980–2012. OLS and ARDL estimation techniques were used in the analysis. The findings revealed that life expectancy has a positive impact on economic growth in Nigeria. Similarly, Ogungbenle, Olawumi, and Obasuyi (2013) analysed the relationship among life expectancy, public health spending and economic growth in Nigeria using the VAR model. The findings revealed that there is no bidirectional causality between life expectancy and public health spending, as well as life expectancy and economic growth. However, there is bidirectional causality between public health spending and economic growth. The method used is not in harmony with the findings of the study.

Ngwen and Kouty (2015) used a dynamic panel of 141 countries to determine the impact of life expectancy on economic growth in developing countries from 2000 to 2013.

The results showed that life expectancy has a positive effect on economic growth.

Lin et al. (2012) applied linear mixed models in examining the influence of four political and socio-economic factors on life expectancy at birth in one hundred and nineteen less developed countries from 1970 to 2004. The four political and socio-economic determinants are economy, educational environment, nutritional status and political regime, measured by GDP per capita at purchasing power parity, the literacy rate of the adult population aged fifteen and over, the proportion of undernourished people, and regime score, respectively. It finds that these determinants generally explain a fifty-five per cent to ninety-eight per cent increase in life expectancy given a lag period of ten years. Specifically, the political regime has the least contribution to life expectancy in LDCs, but it contributes to increasing the rate. In contrast, the other three determinants have the highest contribution, but they contribute at a decreasing rate.

Using VAR and VECM models, Sede and Ohemeng (2015) studied the socio-economic determinants of life expectancy in Nigeria. The results revealed that conventional socio-economic variables such as per capita income, education, and government expenditure on health are very effective in determining the life expectancy of developing countries yet are insignificant in Nigeria.

Bayati, Akbarian, and Kavosi (2013) explored the determinants of life expectancy in 21 Eastern Mediterranean countries from 1995 to 2007, applying a Fixed-Effect Model to estimate the parameters based on the Hausman test. The paper found that income per capita, education index, food availability, level of urbanization, and employment ratio determined health status, proxied by life expectancy at birth. Similarly, Bilas et al. (2014) investigated the determinants of life expectancy at birth in twenty-eight European countries from 2001 to 2011 using a panel data analysis approach. The variables used in the study are GDP growth rate, level of education attained, education enrollment, GDP per capita, and life expectancy. The findings revealed that GDP per capita and level of education have a positive and negative influence on life expectancy, respectively; these are the leading variables explaining between seventy-three and eighty-three per cent of differences in life expectancy. Therefore, the negativity of educational level might be due to lifestyle factor of people with higher education that incorporate more stress as a result of more complex responsibility at work, bad nutrition habits, long working hours, less physical activities, etc.

Distribution Fitting Approach to Life Expectancy Rate

The statistics literature is flooded with lifetime distributions including exponential distribution, gamma distribution, Lindley distribution, Weibull distribution and their generalizations, amongst others. The modeling and statistical analysis of lifetime data are crucial for statisticians and research workers in almost all applied sciences including behavioral sciences, engineering, medical science/biological science, insurance and finance, amongst others (Shanker et al., 2016).

There are several functions related to continuous probability distributions. The most common ones are; cumulative distribution function (cdf), probability density function (pdf), survival (reliability function), hazard (failure) rate function (HR), cumulative hazard rate (CHR) function, cumulative hazard rate average function (HRA), and the conditional survival function (CSF). The good thing about these functions is that they completely describe the distribution of lifetime,

and if you know any of these functions, it is easy to determine the others (Muse, Mwalili and Ngesa 2021).

Chikobvu and Sigauke (2020) reported the COVID-19 number of deaths in South Africa, for the period 27 March 2020 to 20 May 2020, is modeled using four statistical distributions which can be grouped under the Generalised Gamma distribution. This exploratory study also uses simple additive models to capture the underlying COVID-19 death rate. The Empirical results show that the Gamma distribution gives the best fit to the data. The hazard rate still increasing, and the peak number of deaths been reached yet despite the lockdown and other measures to try and slow down the progression of the disease. The study concluded that exploratory data analysis is simple and meant to complement the detailed and complex modelling done which is useful in informing policy and decision making.

Panagiotis and Fragkiskos (2015) in a study presented the mortality data smoothing models using a mixed version of the generalized Gompertz-Makeham distribution with Beta distribution. The proposed distribution been properly parameterized so as to produce a mathematical model with sufficient fit and robust predictive ability for other data sets. The work created mortality models that have a high value of goodness fit in different populations (other than Greek) with appropriate parameter detection method (e.g. Maximum Likelihood). The findings, revealed that the proposed BGGM (Beta Generalized Gompertz Makeham) mortality model based on mix distributions satisfactorily meets evaluation criteria (AIC, BIC).

Recently, mortality researchers have shown increased interest in studying trends in the age-at-death distribution instead of age-specific mortality rates (De Beer & Janssen 2016; Basellini et al. 2016; Bergeron-Boucher et al., 2015). One of the main reasons for analyzing the age-at-death distribution is that doing so allows researchers to distinguish between delay and compression of mortality (Bergeron-Boucher et al. 2015; de Beer & Janssen 2016; Basellini et al. 2016). Delay is defined as the shift of the age-at-death distribution to which is reflected is an increase in the modal age at death. Compression is defined as a change in the shape of the age-at-death distribution resulting from a decline in the variability of the age at dying. Until the 1970s, increases in life expectancy in low-mortality countries were largely attributable to a decline in infant and child mortality and consequently compression. But in recent years, delay of mortality has been the main cause of increases in life expectancy in these countries (De Beer & Janssen 2016).

Very few of the existing mortality projection models have made use of the two dynamics that drive the changes in the age-at-death distribution. Two recently developed projection methods that include both compression and delay can only be applied to the adult population. Terblanche (2016) projected mortality for Australian men and women aged 50–100 based on the linear extrapolation of both the modal age and the concentration of deaths around the modal age.

Basellini et al. (2016) recently proposed a methodology for modeling and forecasting adult mortality that is based entirely on the modal age and variance in the age-at-death distribution. Moreover, two mortality models have been proposed that capture mortality delay and mortality compression, i.e., the adapted Siler model by Bergeron-Boucher et al., (2015) and the compression and delay (CoDe) model by de Beer and Janssen (2016). Although the CoDe model has been used to project mortality for Japanese women (de Beer et al., 2017), the method has not yet been applied to other countries.

Several authors have proposed new approaches for forecasting age-specific central life expectancy rates using

statistical models. Instead of modelling central life expectancy rates, we consider a compositional data analysis (CoDa) approach for modelling and forecasting the age-specific numbers of deaths in period life tables. Both central life expectancy rates and life-table death counts can be derived from the other based on standard life-table relations. Using the life-table death distribution, we could model and forecast a redistribution of the density of life-table deaths, where deaths at younger ages are shifted towards older ones. Alternatively, we may consider a cohort life table which depicts the life history of a specific group of individuals but is dependent on projected life expectancy rates for those cohorts born more recently. Instead, we choose to study the period life table which represents the life expectancy conditions in a period of time (Bergeron-Boucher et al., 2017).

Methodology

This paper employed a probability distribution fitting approach to model age-at-death patterns in Adamawa State, Nigeria, using Log-normal, Log-logistic, Generalized Extreme Value (GEV), and Generalized Pareto distributions. The analysis was conducted in two phases: descriptive statistics to summarize the data distribution and parametric modeling to identify the best-fitting distribution and assess determinants of mortality.

The dataset was collected from Specialist Hospital Yola (1990–2020) and comprised secondary records of 330 age-at-death cases. The response variable was age-at-death (years), while predictors included:

- Cause of death (12 categories: cancer, HIV/AIDS, typhoid, stroke, etc.)
- Local Government Area (LGA) (21 administrative regions)
- Gender (male/female)

Probability Distribution Models

Four parametric distributions were fitted to the data:

Log-normal Distribution

A random variable X follows a Log-normal distribution if $(X) \sim N(\mu, \sigma^2)$. The probability density function (PDF) is:

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln x - \mu)^2}{2\sigma^2}\right), x > 0 \quad (12)$$

Where μ = location parameter, σ = scale parameter.

Log-logistic Distribution

The PDF of the Log-logistic distribution is:

$$f(x) = \frac{(\beta/\alpha)(x/\alpha)^{\beta-1}}{[1+(x/\alpha)^\beta]^2}, x > 0 \quad (13)$$

Where α = scale, β = shape. The hazard function is unimodal if $\beta > 1$, making it suitable for mortality data (Muse et al., 2021).

Generalized Extreme Value (GEV) Distribution

The cumulative distribution function (CDF) is:

$$G(x) = \exp\left[-\left(1 + \xi\left(\frac{x-\mu}{\sigma}\right)\right)^{-1/\xi}\right], \xi \neq 0 \quad (14)$$

Where μ = location, σ = scale, ξ = shape.

Generalized Pareto Distribution

The PDF is:

$$f(x) = \frac{1}{\alpha} \left(1 + \frac{\beta x}{\alpha}\right)^{-(1+1/\beta)}, x \geq 0 \quad (15)$$

Where α = scale, β = shape.

Model Estimation and Selection

Parameter estimation: Maximum Likelihood Estimation (MLE) was used to fit each distribution.

Goodness-of-Fit Tests

Kolmogorov-Smirnov (KS):

$$D_n = \sup_x |F_n(x) - F(x)| \quad (16)$$

Anderson-Darling (AD):

$$A^2 = -n - \sum_{i=1}^n \frac{2i-1}{n} [\ln F(X_i) + \ln(1 - F(X_{n+1-i}))] \quad (17)$$

Chi-square (χ^2):

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} \quad (18)$$

Model selection: The Akaike Information Criterion (AIC) was used:

$$AIC = 2k - 2 \ln(\hat{L}) \quad (19)$$

Where k = number of parameters, \hat{L} = likelihood value.**Regression Analysis**

To assess determinants of age-at-death, parametric survival regression was applied:

$$\text{Log}(T) = \beta_0 + \beta_1 X_1 + \dots + \beta_p X_p + \sigma W \quad (20)$$

Where;

T = survival time (age-at-death)

 X_i = predictors (cause of death, LGA, gender)

W = error term (distribution-specific, e.g., logistic for Log-logistic)

RESULTS AND DISCUSSION**Descriptive Statistics**

The descriptive summary statistics such as: mean, median, mode, skewness and kurtosis (and their standard errors), standard deviation and variances of the outcome variable, “Age-at-Death”, are presented. These summary statistics on “Age at Death”, are presented for the complete data. Other variables considered in the study include Local Government Area (LGA), Gender and Cause of death, which were considered as predictor variables in the study. The summary statistics were also structured according to these predictor variables, while box-plots were presented as graphical descriptions of the response variable and were presented according to gender, cause of death and LGA. For the inferential analysis, regression models based on suitably known lifetime distributions from literature were adopted to fit the data as well as draw inferences regarding the subject matter. The distributions considered in this study include: Log-normal, Log-logistic, Generalized Extreme Value and Generalized Pareto. These were considered and the model of best fit was selected.

Table 1: Summary Statistics on Age-at-Death

N	Mean	S.E	Median	Mode	Std. Dev	Variance	Skewness	S.E. Skewness	Kurtosis	S.E of Kurtosis	Min	Max
330	26.89	0.893	22	20 ^a	16.225	263.249	0.874	0.134	0.103	0.268	2	77

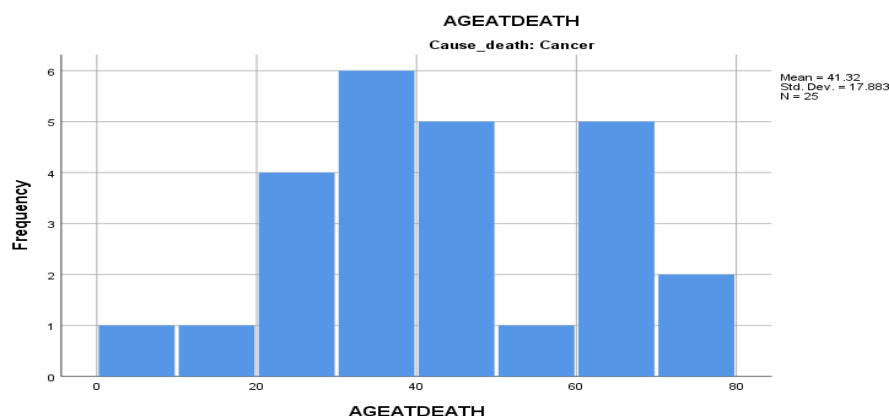


Figure 1: showing a Histogram Chart on Age-at-Death

Figure 1 presents a pictorial description of the age at death of the study population. A bar chart was used and this shows that most person died between the ages of 30 and 40 years followed by these within the ages 40 – 50 years and 60 – 70 years. The least frequency counts occurred among those aged 0 to 19 and 51 – 59.

Table 1 presents the summary statistics on the whole data. The minimum age at death 2 years while the oldest age recorded at death was 77 years. It can be observed that the age at death (which is used as a proxy to average life expectancy in this study) has an average of 26.89, approximately 27 years with a standard error of mean of 0.893 about the mean in the study population. This implies that persons in the study sample in

Adamawa State have an average life time of about 27 years based on the reported cases. As a rule of thumb, if the mean, the median and the mode can be said to be approximately equal, the data may be said to originate from a normal distribution but since this does not hold, it can be infer that other distributions may suffice. The positive value of the skewness (0.874) is indicative that age-at-death is slightly skewed to the right above the mean. The results also suggest a multimodal distribution may be appropriate, as the Table 4.1 indicates that there are more than one modal values in the distribution. This is also attested to by the high variance values which is almost eleven (11) times the size of the mean value.

Table 2: Showing Summary Statistics on Age-at-Death by Cause

Cause	N	Mean	Std.Err	Median	Mode	Skewness	Std.Err	Kurtosis	Std.Err	Min	Max
Cancer	25	41.32	3.577	40	30	0.205	0.464	-0.57	0.902	6	77
HIV/AIDS	41	23.9	1.486	21	20	0.935	0.369	0.78	0.724	4	48
Hepatitis	19	23.89	2.689	23	12	0.933	0.524	-0.288	1.014	12	48
Typhoid, Fever, Cold, Malaria	59	19.53	1.66	16	14	2.007	0.311	6.198	0.613	2	77
Kidney & Heart	14	31.86	4.673	28	13	0.338	0.597	-1.447	1.154	12	62
TB, Respiratory & Cough	33	24.61	2.571	20	20	1.38	0.409	1.585	0.798	7	70
Stroke, Aches & Pains	30	41.4	3.102	44	45	-0.306	0.427	-0.598	0.833	7	70
BP, Anxiety, Depression & Allergies	26	37.27	3.607	41	50	0.131	0.456	-1.254	0.887	12	70
Chicken Pox	8	9.38	1.438	9	8	-0.497	0.752	0.379	1.481	2	15
Chronic Ulcer	12	27	3.645	28.5	10	-0.071	0.637	-0.201	1.232	8	50
Diabetes	9	34.44	6.092	36	9	-0.1	0.717	-1.088	1.4	9	61
Delivery, Accident	44	19.18	1.615	18	18	0.449	0.357	0.133	0.702	2	48
Meningitis	10	23.2	4.541	17.5	9	0.739	0.687	-1.363	1.334	9	45

*Highlighted values have multiple Modes

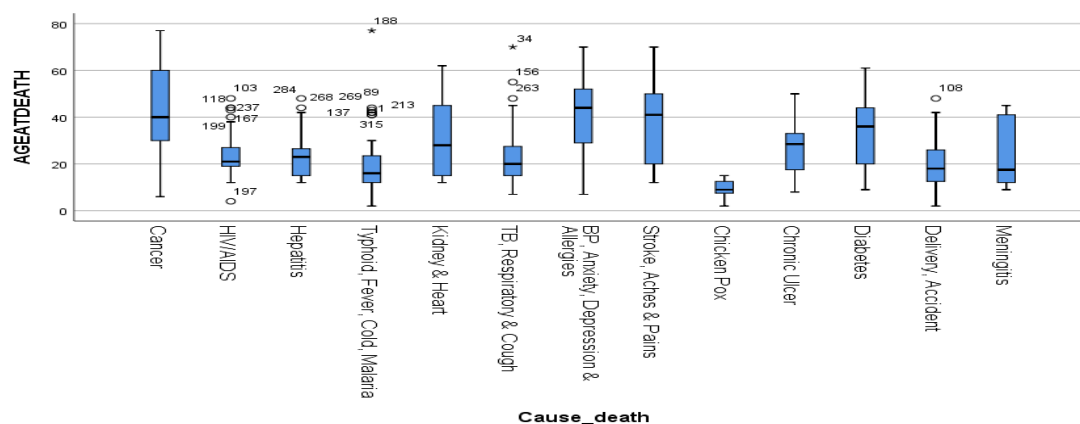
**Figure 2: Showing Summary Boxplot of Age at Death by Cause**

Table 2 presents the summary statistics on the Age-at-Death grouped according to the recorded cause of death. These causes of death include cancer, HIV/AIDS, Hepatitis, Typhoid and malaria related, Kidney and heart related problems, Tuberculosis and respiratory related diseases, diabetes etc. Table 4.2 shows that chicken pox had the least average age of 9.38 years. Deaths due to cancer have a mean age at death of 41.32 years, and those due to Stroke, depression and other stress related causes have an average age of 41.4 years. These are the highest subgroup age at death. Those due to allergies have the highest modal ages at death of 50 years. This suggests disparity in age at death could be

largely due to cause of death. Although the mean age at death of each group varies largely, it can be observed that the maximum (highest) age at death in each group across the causes of death are high except for those due to chicken pox. These statistics are presented using the box-plots on Figure 4.2, some outlier values for HIV/AIDs (197, 199, 167, 237, 118 and 103), Hepatitis (268 and 284), Typhoid, Fever, Cold and Malaria (315, 137, 213, 269, 89), TB, Respiratory and Cough (263, 156) and Delivery, Accident (108) are observed and depicted accordingly. The box-plots depict clearly the numerical summaries of Table 4.2.

Table 3: Showing Summary Statistics of Age at Death by Gender

Gender	N	Mean	Std.Err	Median	Mode	Skewness	Std.Err	Kurtosis	Std.Err	Min	Max
Female	149	25.55	1.242	21	23	1.035	0.199	0.683	0.395	2	77
Male	181	27.99	1.265	23	20	0.751	0.181	-0.231	0.359	2	77

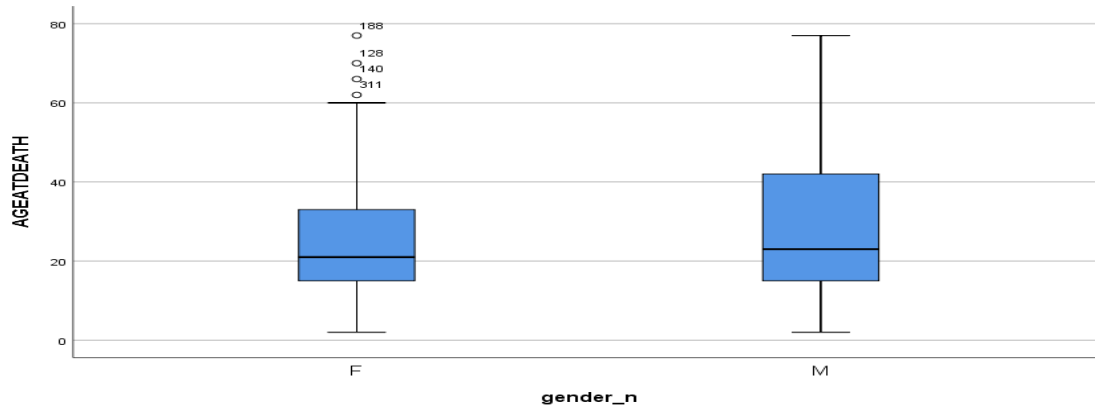


Figure 3: Showing Summary Box-Plot of Age at Death by Gender

Next, the paper examines the summary statistics on the age at death of according to gender. This is presented – Table 3 reveals that the average age at death in the study population is slightly higher for males (27.99 years) than female (25.55 years) but their youngest and oldest ages at death are the same at 2 years and 77 years respectively. It seems there is little disparity in the age at death by gender. The average life

expectancy in Nigeria is around 55.2 years according to WHO data. (www.worldlifexpectancy.com) Men live an average of 54.7 years and women live an average of 55.7 years. Life expectancy at birth in Nigeria was about 61.33 in 2022. However, Nigeria has been declared by the United Nations Population Fund (UNFPA) to be the world's third lowest life expectancy rate and the lowest in West African sub-region.

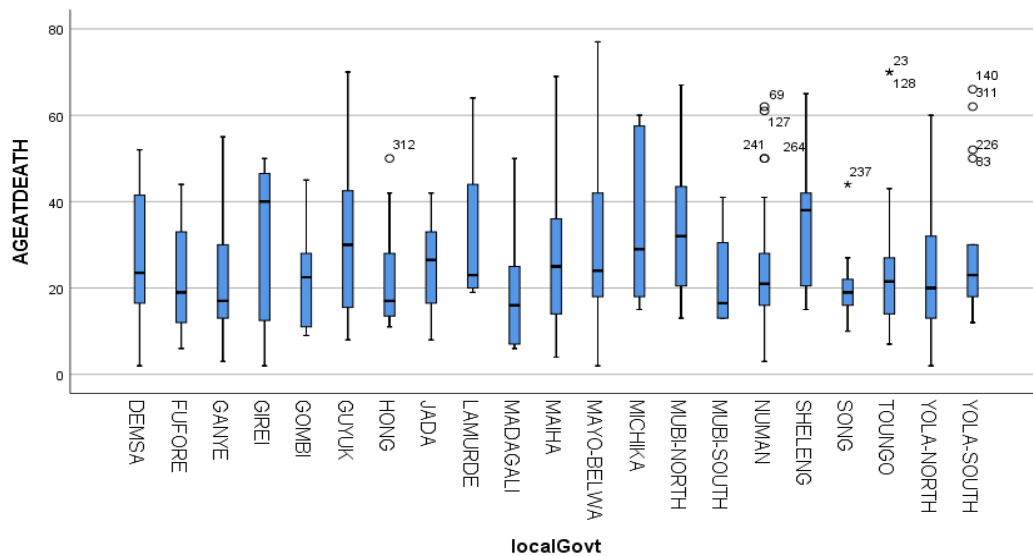


Figure 4: Showing Summary Boxplot of Age at Death by LGA

Table 4: Summary Statistics on Age at Death by Local Government Area

LGA	N	Mean	Std.Er	Median	Mode	Skewness	Std.Er	Kurtosis	Std.Er	Min	Max
Demsa	20	27.5	3.33	23.5	20	-0.012	0.512	-1.293	0.992	2	52
Fufore	14	21.93	3.56	19	6	0.525	0.597	-1.148	1.154	6	44
Ganye	21	22.43	3.318	17	12	1.254	0.501	0.493	0.972	3	55
Girei	8	31.25	6.753	40	2	-0.642	0.752	-1.656	1.481	2	50
Gombi	8	22.13	4.332	22.5	24	0.814	0.752	0.351	1.481	9	45
Guyuk	15	30.67	4.769	30	30	0.753	0.58	-0.072	1.121	8	70
Hong	11	22.64	3.95	17	12	1.269	0.661	0.543	1.279	11	50
Jada	16	24.88	2.955	26.5	30	0.052	0.564	-1.237	1.091	8	42
Lamurde	10	33.4	5.445	23	23	1.023	0.687	-0.533	1.334	19	64
Madagali	14	20.5	4.156	16	7	1.063	0.597	-0.149	1.154	6	50
Maiha	21	26.71	3.7	25	4	0.701	0.501	0.278	0.972	4	69
Mayo-Belwa	41	30.29	2.928	24	12	0.982	0.369	0.389	0.724	2	77
Michika	7	36.43	7.955	29	60	0.235	0.794	-2.552	1.587	15	60
Mubi-North	12	33.33	4.718	32	23	0.605	0.637	-0.269	1.232	13	67
Mubi-South	4	21.75	6.625	16.5	13	1.659	1.014	2.615	2.619	13	41

Numan	21	24.95	3.871	21	16	0.955	0.501	-0.006	0.972	3	62
Shelleng	15	34.33	3.861	38	18	0.386	0.58	-0.462	1.121	15	65
Song	15	20.2	2.022	19	16	2.076	0.58	6.112	1.121	10	44
Toungo	22	25.91	3.694	21.5	14	1.634	0.491	2.209	0.953	7	70
Yola-North	17	24	3.856	20	20	0.846	0.55	0.301	1.063	2	60
Yola-South	18	29.17	3.944	23	23	1.245	0.536	0.364	1.038	12	66

Examining the ages at death from the study sample across the Local Government Areas (LGAs) of Adamawa State, Table 4.4 shows the distribution of summary statistics on the age at death by LGA. Though some LGAs have small counts in terms of records of death within the study period, the details of their records reveal some interesting outcomes. Of the twenty-one (21) LGAs, thirteen (13) have records of early age at death, that is below ten (10) years, but their highest lifetime

in years is all above 40 years. The longest is from Mayo-Belwa, which is 77 years old. Some LGAs recorded under five (U5) mortalities, Demsa, Ganye, Girei, Maiha, Mayo-Belwa, Numan and Yola North. These areas require closer attention to mitigate these cases, regarded as global crises. Also, there are records of some outlier data as depicted in the box plot, which grants a pictorial representation of the age at death by LGA.

Table 5: Distribution Statistics on the Age at Death

Distribution		Kolmogorov Smirnov		Anderson Darling		Chi Squared	
		Statistic	Rank	Statistic	Rank	Statistic	Rank
1	Beta	0.09164	7	1.9897	4	36.24	8
2	Chi-Squared	0.25456	21	123.43	22	317.67	19
3	Chi-Squared (2P)	0.13565	14	6.6671	127.12	66.166	14
4	Exponential	0.2297	20	24.281	18	127.12	18
5	Exponential (2P)	0.20058	18	18.908	17	93.096	16
6	Gamma	0.08248	5	1.7963	3	33.692	5
7	Gamma (3P)	0.08023	3	1.6844	2	33.815	6
8	Gen. Extreme Value	0.08339	6	2.0448	5	24.473	1
9	Gen. Pareto	0.06712	1	50.699	19	N/A	
10	Gumbel Max	0.09759	9	2.6741	8	27.27	3
11	Laplace	0.20735	19	16.003	16	100.64	17
12	Logistic	0.17061	17	9.9971	15	63.943	13
13	Lognormal	0.07075	2	2.1386	7	26.09	2
14	Lognormal (3P)	0.08148	4	1.644	1	37.126	9
15	Normal	0.15833	16	8.5606	14	80.638	15
16	Pareto	0.39607	22	86.972	20	394.38	20
17	Rayleigh	0.12652	12	5.4975	11	52.649	11
18	Rayleigh (2P)	0.13781	15	5.0882	10	45.171	10
19	Student's t	0.95333	23	1697.3	23	29330	21
20	Triangular	0.11305	11	6.6654	12	54.078	12
21	Uniform	0.13359	13	98.728	21	N/A	
22	Weibull	0.1028	10	3.0014	9	33.12	4
23	Weibull (3P)	0.09179	8	2.0972	6	36.115	7

Table 5 presents the distribution outcome suitable to describe the deaths in age-at-death collected across the twenty-one LGAs in Adamawa State. A total of twenty-three (23) distribution are listed with their corresponding statistic estimates from the data using Easy fit version 3.0. Three goodness-of-fit statistics were used in ranking these distributions; namely, Kolmogorov-Smirnov, Anderson-Darling and the Chi-square statistic.

Summary Statistics on Goodness of Fit Test for Distribution Fitting

Table 5 presents the outcome of the suggested distributions suitable for describing the outcome variable and possible regression models for the age-at-death based on the accompanying predictor variables considered in the study. Three goodness-of-fit statistics, namely: Kolmogorov-Smirnov, Anderson-Darling, and Chi-square, were used to assess the distribution that best fits the response variable (Age at death). This was achieved using the Easy Fit software version 3.0.

Table 5 revealed that the three not unanimously ranked any particular distribution as best suited for fitting the response variable. While Kolmogorov-Smirnov ranked Generalized Pareto as best, Anderson-Darling suggested a three-parameter Lognormal as best suited, the Chi-Square test suggested the Generalized extreme Value distribution as best. However, Kolmogorov-Smirnov and the Chi-square tests ranked the Log-normal distribution as second best. Thus, since both are agreed on this, this research fits the Log-normal, the Generalized Pareto, and the Generalized extreme Value regression models to the dataset. Also, the Log-logistic lifetime distribution suggested in literature is considered in modeling the impact of predictor variables on the Age-at-death (as outcome variable).

Table 6 revealed output from regression models with Long-normal Regression, considering the impact of the levels of the independent variables on age-at-death using the log-normal regression, it was discovered that under case of death, (BP/Anxiety) has a p-value of 0.0377 (< 0.05), Delivery and Accident cases have a p-value of 0.0269 (< 0.05) compared to

the reference category (cancer), these levels of the factor are significant in predicting the age at death of the study population. Other levels of the factor “cause of death” are not significantly different from cancer impact since their p-values are greater than 0.05.

Examining Local Government of pendency as a predictor of their ages at death, this study adopted Demsa LGA as the reference category, it was observed that Guyuk (p-value = 0.0336), Jada (p-value = 0.0204), Michika (p-value = 0.0375) and Mubi-South (p-value = 0.044) were significantly different in ages at death compared to Demsa.

While considering the impact of the levels of the independent variables on age at death using the Loglogistic Regression, it was observed that under cause of death (BP/Anxiety) has a p-value of 0.047 (< 0.05), Delivery and Accident cases have a p-value of 0.015 (< 0.05) compared to the reference category (Cancer), these levels of the factor are significant in predicting the age at death of the study population.

Other levels of the factor “cause of death” are not significantly different from cancer impact since their p-values are greater than 0.05.

Examining Local Government of pendency as a predictor of their ages-at-death, thus study adopted Demsa LGA as the reference category. It was observed that Guyuk (p-value = 0.01), Jada (p-value = 0.023), Michika (p-value = 0.034) and Mubi-South (p-value = 0.025) were significantly different in ages at death compared to Demsa.

Considering the impact of the levels of the independent variables on age at death using the Generalised extreme value Regression, it was observed that under cause of death, (chronic ulcer) has a p-value 0.0191 (> 0.05), Delivery and Accident cases has a p-value of 0.8389 (> 0.05). Compared to the reference category (cancer), these levels of the factor are insignificant in predicting the age at death of the study population.

Other levels of factor “cause of death” are significantly different from cancer impact since their p-values are greater than 0.05.

Examining Local Government of pendency as a predictor of their ages-at-death, this study adopted Demsa LGA as the reference category. It was observed that Gombi (p-value = 0.0374), Jada (p-value = 0.003), Mubi-South (p-value = 0.0031) were significantly different in ages at death compared to Demsa.

Discussion

Table 1 revealed the minimum age at death 2 years old child while the oldest age recorded at death is 77 years. It was

observed that the age at death (which is used as a proxy to average life expectancy in this study). This implies that persons in the study sample in Adamawa state have an average life time of about 27 years based on the reported cases. This is different from the findings of Ramadhan, (2021) who used the model Polya Model based on the constructing result of abridged life table using data of the Taspen Mortality Table 2012, discovered that the life expectancy for the population aged 0 is 75.25 years.

Table 2 revealed that cancer, HIV/AIDS, Hepatitis, Typhoid and malaria related, Kidney and heart related problems, Tuberculosis and respiratory related diseases, diabetes etc. are the major causes of death in Adamawa State. Deaths due to cancer have a mean age at death of 41.32 years, and those due to Stroke, depression and other stress related causes have an average age of 41.4 years. These are the highest sub-group age-at-death. Those due to allergies have the highest modal ages at death of 50 years.

Table 3 revealed that the average age at death in the study population is slightly higher for males (27.99 years) than female (25.55 years) but their youngest and oldest ages at death are the same at 2 years and 77 years, respectively. There is a little disparity in the age at death by gender. The average life expectancy in Nigeria is around 54.5 years according to WHO data. Men live an average of 53.7 years and women live an average of 55.4 years.

Table 4 revealed that from the twenty-one (21) LGAs of Adamawa State, thirteen (13) have records of early age at death that is below ten (10) years, but their highest lifetime in years is all above 40 years. The longest is from Mayo-Belwa with 77 years. Some LGAs recorded under five (U5) mortalities namely: Demsa, Ganye, Girei, Maiha, Mayo-Belwa, Numan and Yola North.

Table 5 revealed that Kolmogorov-Smirnov ranked Generalized Pareto as best, Anderson-Darling suggested a three-parameter Lognormal as best suited, the Chi-Square test suggested the Generalized extreme Value distribution as best. Examining closely, we find that the Kolmogorov-Smirnov and the Chi-square tests both ranked the Log-normal distribution as second best. Thus, this research fits the Lognormal, the Generalized Pareto, and the Generalized extreme Value regression models to the dataset. This is similar to Bravo et al., (2010) who concluded that the Gompertz-Makeham functions estimated by means of generalized linear models offer a good alternative for estimating life expectancy in small population areas. The method is flexible and applicable to mortality data for a wide range of ages from any geographical conditions.

Table 6: Results of the Lognormal, Loglogistic and Gen. Extreme value Regression models on Age-at-Death

Factor	Lognormal				Loglogistic				Gen. Extreme Value			
	Estimate	Std. Error	t	p-value	Estimate	Std. Error	t	p-value	Estimate	Std. Error	t	p-value
(Intercept)	2.92004	0.205878	14.183	<2e-16***	3.0436	0.21	14.49	<2e-16	46.9182	5.8196	8.06	7.50E-16
Cause Cancer (Ref)												
HIV/AIDS	0.18579	0.168735	1.101	0.2718	0.1384	0.1734	0.8	0.425	-2.968	4.5918	-0.65	0.518
Hepatitis	0.18671	0.197902	0.943	0.3462	0.064	0.203	0.32	0.753	-2.2904	5.7026	-0.4	0.688
Typhoid_cold	0.16659	0.157182	1.06	0.2901	0.1228	0.1622	0.76	0.449	-8.3313	4.4604	-1.87	0.0618
kidneyHeart	0.16314	0.223195	0.731	0.4654	0.0534	0.2289	0.23	0.816	-5.1386	6.2522	-0.82	0.4111
TB related	0.11277	0.174704	0.646	0.5191	0.1211	0.1814	0.67	0.504	-7.5964	4.8624	-1.56	0.1182
BP Anxiety	0.37191	0.177584	2.094	0.0371	0.3534	0.1777	1.99	0.047	-5.3124	4.904	-1.08	0.2787
Stroke Aches etc	0.11317	0.185147	0.611	0.5415	0.0317	0.1891	0.17	0.867	-5.8921	5.4389	-1.08	0.2787
Chicken pox	0.21066	0.265388	0.794	0.428	0.138	0.2711	0.51	0.611	-8.3993	7.1909	-1.17	0.2428
Chronic Ulcer	0.21077	0.227692	0.926	0.3553	0.1542	0.2078	0.74	0.458	-14.3701	6.1297	-2.34	0.0191
Diabetes	0.20358	0.255637	0.796	0.4264	0.1808	0.2585	0.7	0.484	-4.0522	6.8969	-0.59	0.5568
Deliver/Accident	0.37270	0.167551	2.224	0.0269	0.4167	0.1718	2.43	0.015	-0.9779	4.8104	-0.2	0.8389
Meningitis	0.16041	0.244695	0.656	0.5126	0.091	0.2491	0.37	0.715	-6.1475	6.677	-0.92	0.3572
LGA Demsa (Ref)												
Fufore	-0.164	0.224465	-0.731	0.4656	-0.2281	0.2203	-1.04	0.301	-7.0818	6.0118	-1.18	0.2388
Ganye	-0.15868	0.201775	-0.786	0.4322	-0.2602	0.1994	-1.31	0.192	-6.0906	5.5867	-1.09	0.2756
Girei	-0.33412	0.274396	-1.218	0.2243	-0.321	0.2932	-1.09	0.274	-12.1816	7.3872	-1.65	0.0991
Gombi	0.13929	0.272316	0.512	0.6094	0.0342	0.2803	0.12	0.903	15.3388	7.3685	2.08	0.0374
Guyuk	-0.46918	0.21981	-2.134	0.0336	-0.5732	0.2237	-2.56	0.01	-2.9859	6.3919	-0.47	0.6404
Hong	-0.26483	0.241793	-1.095	0.2743	-0.351	0.2361	-1.49	0.137	-4.6638	6.4034	-0.73	0.4664
Jada	-0.50374	0.216136	-2.331	0.0204	-0.4758	0.2095	-2.27	0.023	-17.3499	5.8532	-2.96	0.003
Lamurde	0.05318	0.252424	0.211	0.8333	0.0443	0.2508	0.18	0.86	-5.7271	6.7719	-0.85	0.3977
Madagali	0.23156	0.225168	1.028	0.3046	0.1733	0.2155	0.8	0.421	2.7827	5.9975	0.46	0.6427
Maiha	0.00106	0.20236	0.005	0.9958	-0.0409	0.1909	-0.21	0.83	-4.7751	5.5685	-0.86	0.3912
Mayo-Belwa	0.06776	0.176946	0.383	0.702	-0.0228	0.1687	-0.14	0.893	-3.1297	4.9482	-0.63	0.5271
Michika	0.58965	0.28214	2.09	0.0375	0.5266	0.2477	2.13	0.034	4.079	7.508	0.54	0.5869
Mubi-North	-0.16131	0.23427	-0.689	0.4916	-0.1294	0.24	-0.54	0.59	3.3367	6.2199	0.54	0.5916
Mubi-South	-0.71368	0.352761	-2.023	0.044	-0.7526	0.3363	-2.24	0.025	-27.7877	9.3855	-2.96	0.0031
Numan	0.20136	0.200166	1.006	0.3152	0.1247	0.1875	0.67	0.506	-3.4442	5.2644	-0.65	0.513
Shelleng	0.32569	0.221729	1.469	0.1429	0.2759	0.2067	1.33	0.182	-1.5569	6.0533	-0.26	0.797
Song	0.14038	0.220845	0.636	0.5255	0.054	0.2146	0.25	0.801	-0.7058	6.0521	-0.12	0.9072
Toungo	0.15168	0.19765	0.767	0.4434	0.0845	0.1949	0.43	0.665	5.0803	5.3495	0.95	0.3423
Yola-North	0.03660	0.210987	0.173	0.8624	-0.0626	0.1949	-0.32	0.748	-7.2751	5.7752	-1.26	0.2078
Yola-South	0.13519	0.210919	-0.641	0.5221	-0.3071	0.2046	-1.5	0.133	-6.2029	5.9172	-1.05	0.2945
Gender Female (Ref)												
Male	0.01268	0.074253	-0.171	0.8645	-0.0344	0.0721	-0.48	0.634	-6.1103	2.1724	-2.81	0.0049
log(scale)					-1.053	0.0461	-22.86	<2e-16	2.8064	0.0405	69.32	<2e-16

Table 6 presented the output for the Log-normal, the Log-logistic and the Generalized Extreme Value models. The result showed that the first two identifies Blood pressure & anxiety, child delivery & accidents as critical drivers of the outcome variable and particularly in Local government areas such as Guyuk, Jada, Michika, and Mubi-South as critical locations in the State since their p-values at 5% alpha level, while the Generalized Extreme Value model identified Chronic Ulcers as reasons for death, while Gombi, Jada and Mubi South LGAs are identified as significant locations.

CONCLUSION

In conclusion, Kolmogorov-Smirnov, Anderson-Darling, Chi-Square test and Akaike Information Criteria (AIC) statistics were employed to select the best-fit model. The results show the log-logistics and log normal performed much better when compared to the other fitted distributions (AIC = Log-logistic (2743.26), Lognormal (2743.94), Generalized extreme value (2921.50) and generalized pareto (3521.50). Kolmogorov-Smirnov ranked Generalized Pareto as best, Anderson-Darling suggested a three-parameter Lognormal as best suited, the Chi-Square test suggested the Generalized extreme Value distribution as best. Examining closely, the Kolmogorov-Smirnov and the Chi-square tests both ranked the Lognormal distribution as second best. Blood pressure & anxiety, child delivery & accidents as critical drivers of the outcome variable and particularly in Local Government Areas such as Guyuk, Jada, Michika, and Mubi-South as critical locations in the State as identified by Log-normal and Log-logistics. Generalized Extreme Value model identifies Chronic Ulcers as reasons for death in Gombi, Jada and Mubi South LGAs. The outcome variable is significant between men and women.

RECOMMENDATIONS

From the findings of this research the study therefore, recommends that other probability distributions should be considered for modeling life expectancy using other proxies other than age and death. This research also recommends the application of these fitted distributions and their properties in modeling and analysis of institution specific variables in areas such as engineering, health sciences, education and economics.

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