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ADOMIAN DECOMPOSITION METHOD FOR STEADY FREE CONVECTIVE COUETTE FLOW IN A VERTICAL CHANNEL WITH NON-LINEAR THERMAL RADIATION, DYNAMIC VISCOSITY AND DYNAMIC THERMAL CONDUCTIVITY EFFECTS

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ABSTRACT

In this paper, we investigate steady free convective Couette flow in a vertical channel with nonlinear thermal radiation, dynamic viscosity and dynamic thermal conductivity effects. The investigation is motivated by the studies of some researchers which assumed linear thermal radiation and constant fluid properties. However, this is uncalled for; as these assumptions do not reflect true behavior of the flow. For instance; increase in temperature affects fluid viscosity, thermal conductivity thereby changing the transport phenomenon. Here; the investigation considers both the fluid viscosity and thermal conductivity to be dependent on temperature with the thermal radiation adopting nonlinear form. Due to this reasons, the associated flow equations are highly nonlinear and exhibit no analytical solution and therefore require the use of Adomian decomposition method (ADM) of solution. The attained ADM solution is then coded into computer algebra package of mathematica where results under the parameters of interest are presented and discussed. Results of the investigation show that raising the thermal radiation leads to corresponding rise in both the velocity and temperature of the fluid in the channel. Furthermore; lessening the viscosity and thermal conduction of the fluid were identified to escalate both velocity and temperature of the fluid.

Keywords: Natural convection; Couette flow; Steady flow; Variable Fluid Properties; Nonlinear Thermal Radiation

INTRODUCTION

Flow of fluid induced by density difference occurring between the fluid particles due to temperature gradients is referred to as free convection flow. This type of flow has fundamental importance in many technological and industrial applications such as nuclear reactor, radiators, furnaces, rapid cooling process, sewage disposal and many more. Natural convection flows due to the movement of bounding surface surrounding the fluid is termed as "Couette flow". This type of flow occurs in fluid machineries involving moving parts; especially in hydrodynamics lubrications. Couette flow has been used as a fundamental method for measurement of viscosity and as a means of estimating drag force in many wall driven applications (Yasutomi (1984)). A situation in flow formation which is not time dependent is called steady flow. This type of flow has applications in many engineering devices like boiler, turbine, condenser and water pump that run nonstop for many months before they are shut down for maintenance. Several scholars considered steady natural convection flow through channels due to its significance in engineering technology; especially; in cooling/heating applications. For example; it is used in computer engineering where electronic cabinets containing circuits are design in channel forms so as to enhance cooling of the computer system; in civil engineering, channels are used for irrigation purposes, measuring discharge of water in a river, studying the spread of pollutants and so on. In relation to this, Ostrach (1952) investigated steady laminar natural convection flow of viscous incompressible fluid between two vertical walls while Ostrach (1954) and Sparrow *et al*. (1952) studied combined effects of steady free and forced convective flow and heat transfer between vertical walls. The study of Miyatake and Fuzii (1972) presented results for steady natural convection between vertical walls on considering different physical situations of the flow process. Transient flow between two vertical walls heated/cooled asymmetrically was investigated by Singh and Paul (2006) and revealed that formation of upward flow occurs near the heated wall with down ward flow achieved near the cooled wall. Couette flow of heat generating/absorbing fluid was investigated by Jha and Ajibade (2010) and their result shows that reverse flow of the fluid is achieved with external heating of the moving plate. The study of Miyatake *et al.* (1973) pointed that the rate of heat transfer near the hotter wall is enhanced by the buoyancy force with the reversal flow attained near the cooler wall. Other connected studies can be witnessed in Mandal *et al.* (2014), Jha and Ajibade (2011). Nelson and Wood (1989a,b) and Jha *et al.* (2012).

Studies related to viscous fluid with temperature-dependent viscosity are of paramount importance; especially in petroleum industries for purification and filtration processes; it is also used in food processing and coating of metals. The ancient expression of temperature-dependent viscosity was first given by Reynold (1984). With the advent of this; several scholars have modeled the expression for temperature-dependent fluid viscosity in different forms; all of which revolved around the ancient Reynold's expression. These can be viewed in Elbashbashy *et al.* (2000), Mukhapdyay *et al.* (2009) and Vanden Berg *et al.* (2005); just a few to mention among others. In a related article, Carey and Mollendorf (1978) affirmed that when the viscosity of water is raised from

$10^0C(\overline{\mu} = 0.0033g/cms)$ to $50^0C(\overline{\mu} = 0.00548g/cms)$; its viscosity is decreased by 240% while that of Grey *et al.* (1982) conveyed that; when the viscosity of a fluid is temperature-dependent, the flow mechanism of the fluid changes significantly compared to the assumption of constant viscosity. Mehta and Sood (1992) disclosed that; the usual assumption of constant viscosity of fluids evaluated at some reference temperature is not sufficient to describe a correct situation in the transport characteristics of viscous fluids. Temperature-dependent viscosity on free convective laminar boundary layer flow past a vertical isothermal flat plate was studied by Kafousius and Williams (1995) while the effect of temperature-dependent viscosity on mixed convection flow past a vertical flat plate in the region near a leading edge was investigated by Kafousius and Rees (1998). In the afore mentioned studies the latter researcher disclosed that when viscosity of fluid is sensitive to temperature change, the effect of temperature-dependent viscosity has to be taken into cognizance or else significant errors may occur in the flow mechanisms. Furthermore, Makinde and Ogulu (2011) concluded that a reduction in fluid viscosity amounts to the rise in its velocity. Interrelated scholarly articles can be seen in Costa and Macedonio (2003), Seddeek and Salem (2006), and Hossain *et al.* (2001).

Temperature-dependent thermal conductivity in the study of flow of viscous fluids has been considered by scholars due to its solicitations in technological innovations like in the extrusion of plastic sheets, polymer processing, spinning of fibers, cooling of elastic sheets etc. For instance, in heat sink/source applications; materials of high thermal conductivity are used while those of low conductivity are usedin designing insulators. Similarly, metals in liquid form with small Prandtl number in the interval of 0.01 – 0.1 are commonly used for cooling purposes because of their high thermal conductivity. Numerous scholars investigated flow of viscous fluids on the assumption of constant thermal conductivity. However this is uncalled for; as variation in temperature affects the thermal conduction of the fluid. This is evidently observed in the study of Adrian *et al*. (1997) which disclosed that; when dry air is heated to 1000^0C its thermal conductivity is $31.39\times10^{-3}W/mK$ while at 2000^0C the thermal conductivity is $37.95\times10^{-3}W/mK$ Similarly; Van den Berg *et al.* (2001) divulged that the use of variable thermal conductivity to study flow of molten magma can delay secular cooling of the mantle with constant viscosity model. Sharma and Aisha (2014) submitted that thermal conduction of fluid increases with decrease in Prandl number. Other associated studies can be referred to the articles of Rihab *et al.* (2017), Dubuffet *et al.*(1999), Starlin (2000), Blas (2019) and Hofmeister (1999).

Release of energy in the form of electromagnetic waves by hot objects is termed thermal radiation. This has fundamental importance in cooling/heating processes; especially in the aspect of engineering applications for human survival on the earth. For instance thermal radiation is used in sterilization of medical instruments, toasting of bread, treatment of cancer and tumor, air conditioners and heaters. Due to this, Rosseland (1931), first gave the expression for thermal radiation and this expression was further simplified by Sparrow and Cess (1962). The simplified form is being used by scholars to study flow of fluids with thermal radiation; refer to Makinde *et al.* (2007), Makinde and Ibrahim (2017), Ganji e*t al.* (2015), Sheikholesmi (2015), Makinde (2008) and Abel and Mashesha (2007). In view of this novelty, some researchers mentioned above have discussed the effect of thermal in their flow formation using linearized temperature in in their flow formation. The usage of this was queried by Magyari and Patokratoras (2011) arguing that the flow behavior is not accurately predicted via this procedure. They therefore proposed alternative method which adopted the use of nonlinear temperature in the expression for thermal radiation. In recognition to this, connected studies can be witnessed in Yusuf and Ajibade (2018a, 2020), Ajibade and Yusuf (2019), Yabo *et al.* (2016) and Jha *et al.* (2017).

There exists different method of solving differential equations arising from fluid flows. These include the method of undetermined coefficients, Runge-Kuta method, finite difference method, Laplace transform, Adomian decomposition method (ADM) and many others.

The present article investigates steady free convective Couette flow in a vertical channel on adopting nonlinear thermal radiation, dynamic viscosity, dynamic thermal conductivity and ADM method of solution (Adomian (1994)). This investigation is motivated by the works of some authors which failed to adopt the above parameters upon which the flow behaviors are either under-determined or over-determined. The choice of ADM is due to the following reasons: the technique avoids perturbation, it gives efficient, accurate and approximate solution, it does not require discretization of the solution, does not results to large equations. Furthermore; the method is not affected by computational round off errors, consumes less time and less amount of computer memory (Makinde *et al.* (2007).

MATHEMATICAL FORMULATION OF THE PROBLEM

Figure 1 consists of an infinite vertical channel formed by two parallel plates kept h distance apart. The channel is filled with an optically thick viscous incompressible fluid at the expense of radiative heat flux of intensity q_r ; which is absorbed by the plates and transferred to the fluid. Neglecting the effect of viscous dissipation and assuming all the fluid's physical properties are constant except for its viscosity and thermal conduction which are assumed to be temperature-dependent. The x'-axis coordinate is taken along the channel in the vertically upward direction, being the direction of the flow while the y'-axis is taken normal to it. Also; assuming effect of radiative heat flux in the x'- direction to be negligible compared to that in the y'- direction with the temperature of the plate kept at $y' = 0$ rise to T_w and thereafter maintained constant while the other plate at $y' = h$ remains at T_0. Furthermore, the plate at $y' = 0$ moves on its own plate impulsively at uniform velocity $u' = Mu_0$ while the other plate remains at rest.

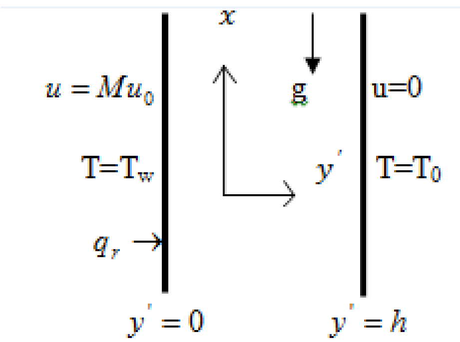

Figure 1: Diagram of the problem

The appropriate governing equations under these assumptions together with the assumption of Boussinesq approximation are:

$$\frac{1}{\rho}\frac{\partial}{\partial y'}\left(\mu\frac{\partial u'}{\partial y'}\right)+g\beta\left(T-T_0\right)=0 \quad (1)$$

$$\frac{1}{\rho c_p}\frac{\partial}{\partial y'}\left(k\frac{\partial T'}{\partial y'}\right)-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y'}=0 \quad (2)$$

with the radiative heat flux of Sparrow and Cess.(1962) given as:

$$q_r = \frac{-4\sigma\,\partial T'^4}{3\delta\,\partial y} \quad (3)$$

Following Carey and Mollendorf (1978); the dynamic viscosity and dynamic thermal conductivity of the fluid are respectively expressed in the form:

$$\mu = \mu_0\left(1-\lambda\left(\frac{T'-T_0}{T_w-T_0}\right)\right), \quad k = k_0\left(1-\varepsilon\left(\frac{T'-T_0}{T_w-T_0}\right)\right), \quad \lambda, \varepsilon \in \Re \quad (4)$$

with the boundary conditions for the velocity and temperature fields as:

$$u' = Mu_0, \quad T' = T_w \quad at \quad y' = 0 \quad (5)$$

$$u' = 0, \quad T' = T_0 \quad at \quad y' = h \quad (6)$$

Non-dimensional of the Problem

The problem under consideration involves quantities in different dimensions and so equations (1-6) are therefore required to be transformed into non-dimensional form using the quantities:

$$u = \frac{u'}{u_0}, \quad y = \frac{y'}{h}, \quad \theta(y) = \frac{T'-T_0}{T_w-T_0} \quad (7)$$

Using equation (4) and (7); the momentum equation (1) is transformed into dimensionless form and the following equation is obtained:

$$u''(y) = \lambda\left(1+\lambda\theta(y)\right)\theta'(y)u'(y) - Gr\theta(y)\left(1+2\lambda\theta(y)\right) \quad (8)$$

The radiative heat flux in equation (3) is expanded nonlinearly on adopting Magyari and Pantokratoras (2011) and the following equation is grasped:

$$\frac{\partial q_r}{\partial y'} = -\frac{4\sigma}{3h^2\delta}\left(12(T_w - T_0)^4\left[\theta(y)+\phi\right]^2 \frac{\partial}{\partial y}(\theta(y)).\frac{\partial}{\partial y}(\theta(y))\right)$$
$$-\frac{4\sigma}{3h^2\delta}\left(4(T_w - T_0)^4\left[\theta(y)+\phi\right]^3 \frac{\partial^2}{\partial y^2}(\theta(y))\right) \quad (9)$$

Now substituting equation (4), (7) and (9) into equation (2) gives the equation:

$$\theta''(y) = \varepsilon\theta'^2(y)\left(1+\varepsilon\theta(y)\right)\left[1-\frac{4R_T}{3}\left(1+\varepsilon\theta(y)\right)\left[\theta(y)+\phi\right]^3\right]$$
$$-4R_T\left(1+\varepsilon\theta(y)\right)\left[\theta(y)+\phi\right]^2\theta'^2(y)\left[1-\frac{4R_T}{3}\left(1+\varepsilon\theta(y)\right)\left[\theta(y)+\phi\right]^3\right] \quad (10)$$

Again, using equation (7) in equation (5) and (6), the boundary conditions are:

$$u = M, \quad \theta = 1 \quad at \quad y = 0 \quad (11)$$
$$u = 0, \quad \theta = 0 \quad at \quad y = 1 \quad (12)$$

where $R_T = \frac{4\sigma(T_w - T_0)^3}{3k_0\delta}$, $\phi = \frac{T_0}{T_w - T_0}$, $Gr = \frac{g\beta(T_w - T_0)}{4\upsilon U_0}$ (13)

Meaning of the parameters involved in equations (1-13) see the table of nomenclature.

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Mathematical Description of ADM

Consider the differential equation in Adomian form:

$$Lu + Su + Nu = g \quad (14)$$

where u is unknown function which is to be determined by a recursive relation, L is the highest order derivative which is also invertible, S is the remainder of the linear operator whose order is less than L, Nu represents the nonlinear terms and g is the system input.

Operating L^{-1} to both sides of equation (14) and using the given initial boundary conditions, the following differential equation is obtained:

$$u = w - L^{-1}(Su) - L^{-1}(Nu) \quad (15)$$

where w represents the term arising from integrating g and the auxiliary conditions.

According to ADM the solution u is defined by the series:

$$u = \sum_{n=0}^{\infty} u_n \quad (16)$$

and Nu comprises the series of the Adomian polynomials:

$$Nu = \sum_{n=0}^{\infty} A_n \quad (17)$$

where A_n are Adomian polynomials generated from the equation:

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{\infty}\lambda_i u_i\right)\right]\right]_{\lambda=0} \quad (18)$$

The solution components $u_0, u_1, u_2, \ldots\ldots$ are determined recursively as:

$$u_0 = w \quad (19)$$

. . . . .

$$u_{j+1} = -L^{-1}(Su_j) - L^{-1}(Nu_j), \; j \geq 0 \quad (20)$$

where w is referred to as the zeroth-order component.

ADM Solution of the Problem

The differential equations (8) and (10) subject to equations (11) and (12) are solved using ADM as follow:

Let $Lu(y) = u''(y)$, $L\theta(y) = \theta''(y)$ where $L^{-1}\chi(\bullet) = \int\int \chi(\bullet)dydy$ (21)

Putting equation (15); equations (8) and (11) can now be written as:

$$Lu(y)=\lambda\left(1+\lambda\theta(y)\right)\theta'(y)u'(y)-Gr\theta(y)\left(1+2\lambda\theta(y)\right) \tag{22}$$

$$L\theta(y)=\varepsilon\theta'^{2}(y)\left(1+\varepsilon\theta(y)\right)\left[1-\frac{4R_T}{3}\left(1+\varepsilon\theta(y)\right)\left[\theta(y)+\phi\right]^3\right]$$
$$-4R_T\left(1+\varepsilon\theta(y)\right)\left[\theta(y)+\phi\right]^2\theta'^{2}(y)\left[1-\frac{4R_T}{3}\left(1+\varepsilon\theta(y)\right)\left[\theta(y)+\phi\right]^3\right] \tag{23}$$

Operating L^{-1} to equations (22) and (23) we obtain:

$$L^{-1}Lu(y)=\lambda L^{-1}\left\{\left(1+\lambda\theta(y)\right)\theta'(y)u'(y)\right\}-GrL^{-1}\left\{\theta(y)\left(1+2\lambda\theta(y)\right)\right\} \tag{24}$$

$$L^{-1}L\theta(y)=\varepsilon L^{-1}\left\{\theta'^{2}(y)\left(1+\varepsilon\theta(y)\right)\left[1-\frac{4R_T}{3}\left(1+\varepsilon\theta(y)\right)\left[\theta(y)+\phi\right]^3\right]\right\}$$
$$-4R_TL^{-1}\left\{\left(1+\varepsilon\theta(y)\right)\left[\theta(y)+\phi\right]^2\theta'^{2}(y)\left[1-\frac{4R_T}{3}\left(1+\varepsilon\theta(y)\right)\left[\theta(y)+\phi\right]^3\right]\right\} \tag{25}$$

But $L^{-1}Lu(y)=u(y)-u(0)-yu'(0)$ and $L^{-1}L\theta(y)=\theta(y)-\theta(0)-y\theta'(0)$ (26)

Using equations (11) in equations (24) and (25) we have:

$$u(y)=M+yA+\lambda L^{-1}\left\{\left(1+\lambda\theta(y)\right)\theta'(y)u'(y)\right\}-GrL^{-1}\left\{\theta(y)\left(1+2\lambda\theta(y)\right)\right\} \tag{27}$$

$$\theta(y)=1+By+\varepsilon L^{-1}\left\{\theta'^{2}(y)\left(1+\varepsilon\theta(y)\right)\left[1-\frac{4R_T}{3}\left(1+\varepsilon\theta(y)\right)\left[\theta(y)+\phi\right]^3\right]\right\}$$
$$-4R_TL^{-1}\left\{\left(1+\varepsilon\theta(y)\right)\left[\theta(y)+\phi\right]^2\theta'^{2}(y)\left[1-\frac{4R_T}{3}\left(1+\varepsilon\theta(y)\right)\left[\theta(y)+\phi\right]^3\right]\right\} \tag{28}$$

Where $A=f'(0)$ and $B=\theta'(0)$ are values to be obtained using (12).

The ADM defines $u(y)$ and $\theta(y)$ in the forms:

$$u(y)=\sum_{n=0}^{\infty}u_n(y) \quad \text{and} \quad \theta(y)=\sum_{n=0}^{\infty}\theta_n(y) \tag{29}$$

Substituting equation (29) into equation (27) and (28), we have:

$$\sum_{n=0}^{\infty}u_n(y)=M+yA+\lambda L^{-1}\left\{\left(1+\lambda\sum_{n=0}^{\infty}u_n(y)\right)\frac{d}{dy}\sum_{n=0}^{\infty}\theta_n(y)\frac{d}{dy}\sum_{n=0}^{\infty}u_n(y)\right\}$$
$$-GrL^{-1}\left\{\sum_{n=0}^{\infty}\theta_n(y)\left(1+2\lambda\sum_{n=0}^{\infty}\theta_n(y)\right)\right\} \tag{30}$$

$$\sum_{n=0}^{\infty}\theta_n(y)=1+yB$$
$$+\varepsilon L^{-1}\left\{\frac{d}{dy}\left(\sum_{n=0}^{\infty}\theta_n(y)\right)\frac{d}{dy}\left(\sum_{n=0}^{\infty}\theta_n(y)\right)\left(1+\varepsilon\sum_{n=0}^{\infty}\theta_n(y)\right)\left[1-\frac{4R_T}{3}\left(1+\varepsilon\sum_{n=0}^{\infty}\theta_n(y)\right)\left[\sum_{n=0}^{\infty}\theta_n(y)+\phi\right]^3\right]\right\}$$
$$-4R_TL^{-1}\left\{\left(1+\varepsilon\sum_{n=0}^{\infty}\theta_n(y)\right)\left[\sum_{n=0}^{\infty}\theta_n(y)+\phi\right]^2\frac{d}{dy}\left(\sum_{n=0}^{\infty}\theta_n(y)\right)\frac{d}{dy}\left(\sum_{n=0}^{\infty}\theta_n(y)\right)\left[1-\frac{4R_T}{3}\left(1+\varepsilon\sum_{n=0}^{\infty}\theta_n(y)\right)\left[\sum_{n=0}^{\infty}\theta_n(y)+\phi\right]^3\right]\right\} \tag{31}$$

Setting $\theta_0(y)=1+By$ and $u_0(y)=M+yA-GrL^{-1}\left\{\left(1+2\lambda\theta(y)\right)\theta(y)\right\}$ (32)

$u_{n+1}(y)$ and $\theta_{n+1}(y)$ for $n\geq 0$ are determined using the generating relations:

$$u_{n+1}(y)=\lambda L^{-1}\left\{(1+\lambda\theta_n(y))\frac{d}{dy}(\theta_n(y))\frac{d}{dy}(u_n(y))\right\} \quad \text{and} \tag{31}$$

$$\theta_{n+1}(y)=\varepsilon L^{-1}\left\{\frac{d}{dy}(\theta_n(y))\frac{d}{dy}(\theta_n(y))(1+\varepsilon\theta_n(y))\left[1-\frac{4R_T}{3}(1+\varepsilon\theta_n(y))\left[\theta_n(y)+\phi\right]^3\right]\right\}$$
$$-4R_T L^{-1}\left\{(1+\varepsilon\theta_n(y))\left[\theta_n(y)+\phi\right]^2\frac{d}{dy}(\theta_n(y))\frac{d}{dy}(\theta_n(y))\left[1-\frac{4R_T}{3}(1+\varepsilon\theta_n(y))\left[\theta_n(y)+\phi\right]^3\right]\right\} \tag{33}$$

For more details on ADM, see Adomian (1994).

Convergence/Termination Criteria of the ADM Solution
It has been proven in Adomian (1994) and Cherruault (1990) that convergence of ADM solution always exists and is rapidly. Based on this the convergence of the solution is not tested here. For the termination criteria; the ADM solutions for u and θ are all paused after the 3rd terms as subsequent terms after these contribute insignificantly to the final solution. The final solutions are not presented here due to their cumbersomeness but are used for discussing the results.

Nusselt Number and Skin Friction on the Channel Plates
The Nusselt number on the channel plates are evaluated on adopting Kay (2017) via:

$$Nu_0=(1-\varepsilon\theta)\frac{d\theta}{dy}\Big|_{y=0} \quad \text{and} \quad Nu_1=(1-\varepsilon\theta)\frac{d\theta}{dy}\Big|_{y=1} \tag{33}$$

with the skin friction o calculated using:

$$\tau_0=(1-\lambda\theta)\frac{du}{dy}\Big|_{y=0} \quad \text{and} \quad \tau_1=(1-\lambda\theta)\frac{du}{dy}\Big|_{y=1} \tag{34}$$

RESULTS AND DISCUSSION
Steady free convective Couette flow in a vertical channel with dynamic viscosity, dynamic thermal conductivity and nonlinear thermal radiation has been investigated in this article. Influences of the physical parameters involved are examined with the results presented in figure 2 – 10 and on tables 1-3. For the aim of discussion, the values of R_T and ε taken in the range $0\le\varepsilon,R_T\le 1$ as terms associated with them behave as strong heat source/sink and their large values lead to finite time temperature blow up (Makinde and Chinyoka, 2010). Similarly, the values of ϕ,λ,M are arbitrarily chosen between 0.1 – 3 while that of Gr are selected between 10, 12 and 14 which represent cooling of the plates by free convection current.

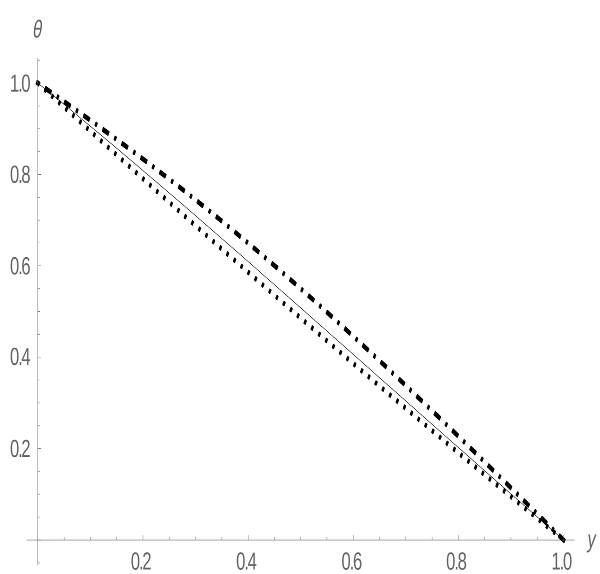

Fig 2: Temperature profiles for different R_T
($\phi=0.1,\epsilon=0.1 \ldots R_T=0.001$, ____ $R_T=0.1$,

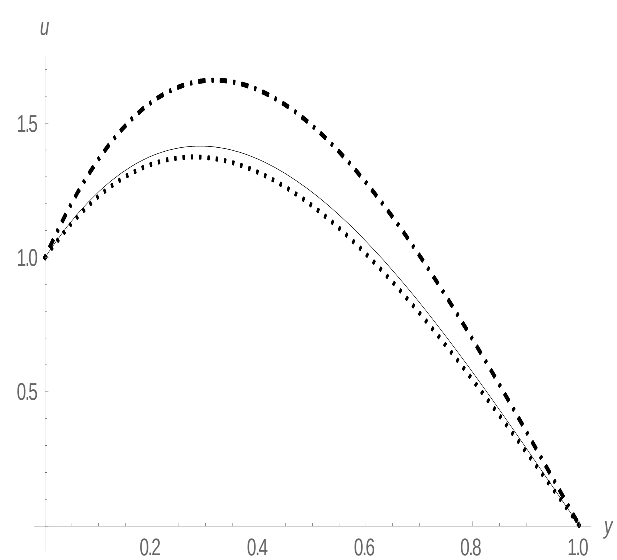

Fig 3: Temperature profiles for for different R_T
($\phi=0.1,\lambda=0.1,\mathrm{M}=1,\mathrm{Gr}=10\ \epsilon=0.1$

-.-.- R_T = 0.5)

… R_T =0.001, ___ R_T = 0.1,-.-.- R_T = 0.5)

Figure 2 demonstrates the effect of thermal radiation on the fluid temperature within the channel. The figure shows that an increase in R_T contributes to the corresponding increase to the fluid temperature. The consequential effect of this on the fluid velocity is reflected on figure 3 where the fluid velocity is also seen to rise with increase in R_T, these ehaviors are the attributes of the decrease in thermal conductiom of the fluid.

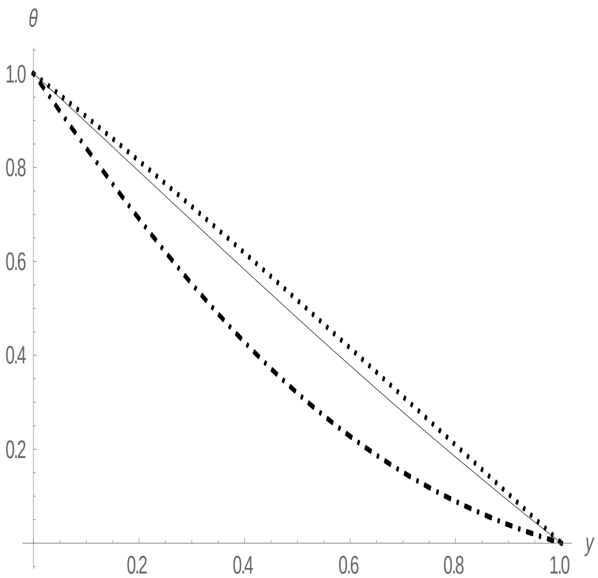

Fig 4: Temperature profiles for different ϵ: ($\phi = 0.1$, $R_T = 0.1$,…ϵ = 0.001,___ϵ =0.3, . - . - ϵ = 0.6)

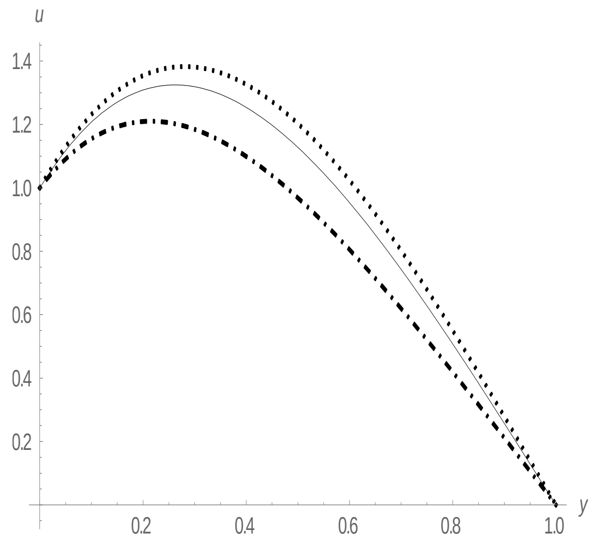

Fig 5: Velocity profile for different ϵ: ($\phi = 0.1, \lambda = 0.1$, M $= 1, R_T = 0.1$, …ϵ =0.001,___ϵ =0.3,. - . - ϵ = 0.6)

Figure 4 and 5 illustrate the effect of changing thermal conductivity on the temperature and velocity of the fluid within the channel when other parameters are fixed. From these figures the temperature is seen to decrease with decrease in thermal conduction of the fluid. Similarly; on decreasing thermal conduction of the fluid the velocity in the channel is also witness to decrease. These fashions are the consequence of the decrease in thermal conduction of the fluid. This physically reveals the effect of decrease in thermal diffusivity with growing ϵ which act to diminish the influence of the applied boundary temperature, thus causing a decrease in the thermodynamics and consequent decrease of fluid velocity in the channel.

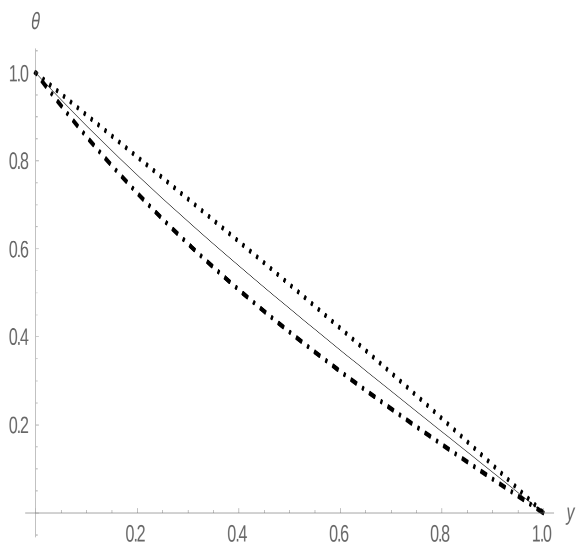

Fig 6: Temperature profiles for different ϕ: ($\epsilon = 0.1, R_T = 0.1, -.-.\phi = 1$, __$\phi = 2$, …$\phi = 3$)

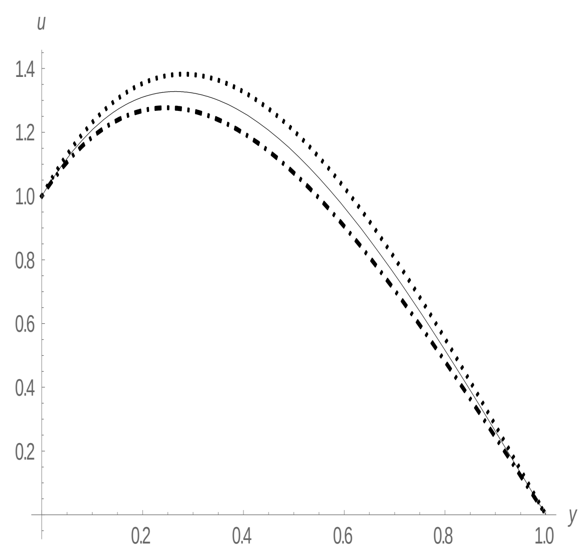

Fig 7: Velocity profiles for different ϕ: ($M = 1$, $\lambda = 0.1, Gr = 10$, $\epsilon = 0.1, R_T = 0.1$, $-.-.\phi = 1$, ___$\phi = 2$, . …$\phi = 3$)

Figure 6 displayed the influence of temperature difference on the fluid temperature where the figure illustrates that the temperture in the channel increases with increase in ϕ. This trend is attributed to the icrease in the initial temperature of the fluid. The reflective effect of this on fluid velocity within the channel is graphed in figure 7 where the fluid velocity also increases with increase in ϕ.

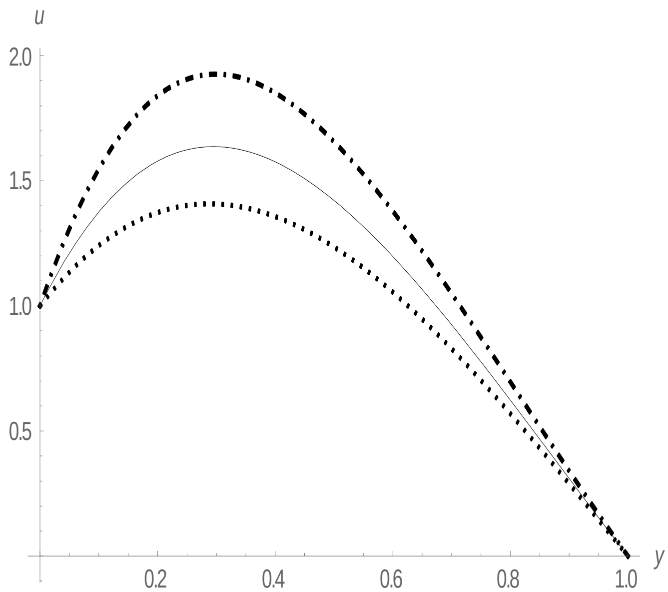

Fig 8: Velocity profile for different λ

($\phi = 0.1, \mathrm{M} = 1, \mathrm{G}r = 10, \epsilon = 0.1, R_T = 0.1, \ldots$ $\lambda = 0.1$, ___ $\lambda = 0.4$, -.-. $\lambda = 0.8$)

Effect of varying fluid viscosity on the fluid velocity is portrayed in figure 8. The figure demonstrates that the fluid velocity within the channel increase with decrease in viscosity (increase in λ) of the fluid. This behavior is accredited to the loosing of cohesive force between the fluid molecules.

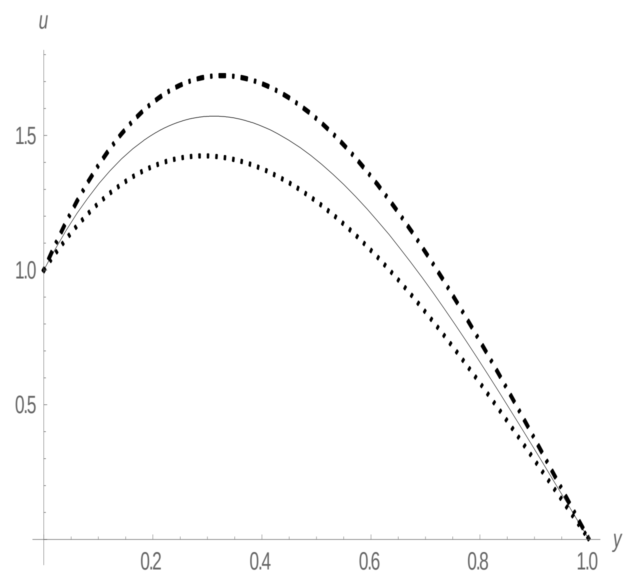

Figure 9: Velocity profile for different Gr

($\phi = 0.1, \mathrm{M} = 1,$ $\lambda = 0.1, \epsilon = 0.1, R_T = 0.1, \ldots.$ Gr = 10, ___Gr=12, −. −. Gr=14)

The effect of varying Gr on the fluid velocity is depicted in figure 9 where the figure shows that the fluid velocity increases with increase in Gr. This is the consequence of the increase in the buoyancy force of the fluid molecules within the channel.

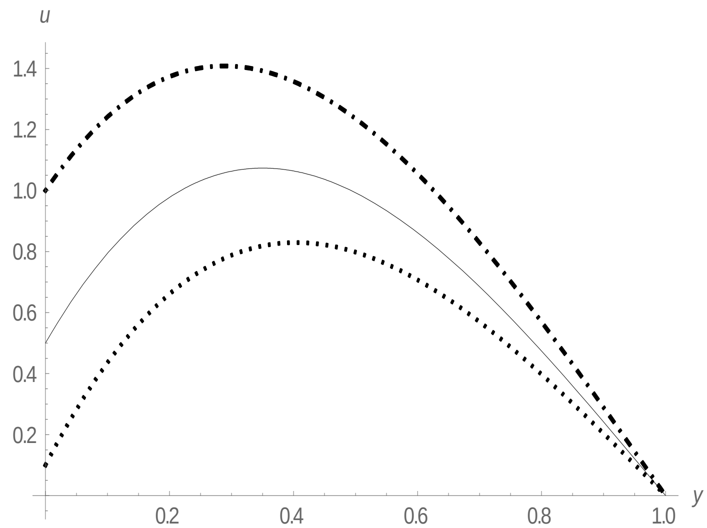

Fig 10: Velocity profile for different values of M
($\phi = 0.1$, $\lambda = 0.1$, Gr $= 10$, $\epsilon = 0.1$, $R_T = 0.01$, ... M = 0.1, ___M = 0.5, .-.-. M = 1)

Figure 10 reflects the effect of varying the velocity of the moving boundary. The figure shows that the fluid velocity increases with increase in M. This is due to the physical fact that in a nonslip regime, the thin film of fluid adjacent to the moving plate moves with the velocity of the moving plate.

Table 1: Nusselt number on the channel plates.

R_T	$\varepsilon = 0.01$, $\phi = 0.1$		$\varepsilon = 0.04$, $\phi = 0.1$		$\varepsilon = 0.04$, $\phi = 0.4$	
	Nu_0	Nu_1	Nu_0	Nu_1	Nu_0	Nu_1
0.02	0.980063	1.00437	0.994207	0.989897	0.973672	1.00434
0.04	0.957673	1.01452	0.969960	1.000520	0.935962	1.02708
0.06	0.937537	1.02385	0.943387	1.010740	0.905951	1.04742
0.08	0.919534	1.03289	0.929055	1.020560	0.832045	1.06566
0.10	0.903285	1.04161	0.911729	1.029990	0.863127	1.08208

The effect of varying parameters on the rate of heat transfer (Nusselt number) between the working fluid and the channel plates is presented on Table1. The table displays that with growing R_T ; Nu_0 decreases while Nu_1 increases. This clearly reveals the increase in temperature difference between the channel plates. The table further shows that with small increase in ε ; Nu_1 increases whereas Nu_0 increases. Again; with small increase in ϕ and growing R_T . Nu_0 decreases while Nu_1 increases.

Table 2: Numerical values for skin friction on the channel plates

λ	ε = 0.01, R_T = 0.01, ϕ =0.1, Gr =10, M=1		ε = 0.04, R_T = 0.01, ϕ =0.1, Gr =10, M=1		ε = 0.04, R_T = 0.04, ϕ =0.1, Gr = 10, M=1		ε = 0.04, R_T = 0.04, ϕ =0.4, Gr = 10, M=1	
	τ_0	τ_1	τ_0	τ_1	τ_0	τ_1	τ_0	τ_1
0.1	2.55095	2.73325	2.53689	2.71957	2.56191	2.74020	2.59111	2.76723
0.3	2.71718	2.79358	2.70290	2.77980	2.72773	2.80043	2.75712	2.82752
0.5	2.54614	2.75695	2.53506	2.74646	2.55277	2.76074	2.57488	2.78055
0.7	2.17884	2.95170	2.18980	2.98933	2.16251	2.90715	2.14527	2.84532

Effect of varying physical parameters on skin friction is presented on table 2. The table reflects that both τ_0 and τ_1 increases with initial increase in λ and later they decreases with more increase in λ. As the thermal conduction of the fluid decreases; both τ_0 and τ_1 decreases with initial increase in λ and later they decreases with further increase in λ. On escalating thermal radiation and temperature difference, τ_0 and τ_1 are view to increase with initial increase in λ and they later decreases with further increase in λ.

VALIDATION OF THE RESULTS

This section validates the accuracy of the results realized in this investigation. In order to do this, the parameters R_T , λ and ε are suppressed and the parameter M is set to one (all in the present study). The resulting equations are then compared with those obtained in the published work of Jha and Ajibade (2010) on silencing heat generating/ absorbing parameter (i.e. S = 0). The results of the comparison are displayed in table 3 below:

Table 3: Numerical values on table 3 shows that the two studies agree excellently with each other.

Jha and Ajibade (2010) when S = 0			Present study when $R_T = \varepsilon = 0$ and M = 1	
y	$\theta(y)$	$u(y)$	$\theta(y)$	$u(y)$
0.1	0.9000	1.1850	0.9000	0.1850
0.3	0.7000	1.3000	0.7000	1.3000
0.5	0.5000	1,1250	0.5000	1.1250
0.7	0.3000	0.7600	0.3000	0.7600
0.9	0.1000	0.2650	0.1000	0.2650

CONCLUSION

In this paper, Adomian decomposition method has been successfully applied to study and investigate the steady free convective Couette flow through a vertical channel with nonlinear thermal radiation, dynamic viscosity and dynamic thermal conductivity. The following results are deduced:

i. The fluid velocity and temperature within the channel are both found to increase with increase in thermal conduction of the fluid.
ii. Decrease in viscosity of the fluid in the channel has been identified to increase the fluid velocity during flow.
iii. Increase in thermal radiation has been acknowledged to increase both the fluid velocity and temperature in the channel.
iv. Decreasing the fluid viscosity has been recognized to cause initial increase in the skin friction between the plates and the fluid.
v. With fixed parameters and with decrease in thermal conduction, the skin frictions on the plates are found to decrease with decrease in viscosity of the fluid.

vi. On decreasing the fluid thermal conduction, the Nusselt number on the heated plate is grasped to increase while it decrease on the cold plate.

Nomenclature and Greek symbols:

Symbols	Interpretation	Unit
y'	Dimensional length	m
y	Dimensionless length	
g	Acceleration due to gravity	ms^{-2}
k	Thermal conductivity	W/mK
T	Dimensional temperature	K
h	Dimensional channel width	m
T_w	Wall temperature	K
T_0	Ambient temperature	K
u'	Dimensional velocity	ms^{-1}
u	Dimensionless velocity	
υ	Kinematic viscosity of the fluid	m^2s^{-1}
α	Thermal diffusivity	m^2s^{-1}
δ	Absorption coefficient	
β	Volumetric expansion coefficient	K^{-1}
μ	Variable fluid viscosity	$kgm^{-1}s^{-1}$
μ_0	Dynamic fluid viscosity	$kgm^{-1}s^{-1}$
Gr	Grasshop number	
R_T	Thermal radiation parameter	
S	Heat generating/absorbing parameter	
q_r	Radiative heat flux	Wm^{-2}
ϕ	Temperature difference parameter	K
θ	Dimensionless temperature	
σ	Stefan-Boltzman constant	JK^{-1}
ε	Thermal conductivity variation parameter	
λ	Viscosity variation parameter	
$\Re$	Set of real numbers	
Nu_0	Nusselt number on the plate at $y=0$	
Nu_1	Nusselt number on the plate at $y=1$	
Nu	Nusselt number	
τ_0	Skin friction on the plate at $y=0$	
τ_1	Skin friction on the plate at $y=1$	
τ	Skin friction	

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