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A DISCRETE-TIME ECONOMIC ORDER QUANTITY MODEL FOR AMELIORATING ITEMS WITH CONSTANT DEMAND AND LINEAR HOLDING COSTS

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ABSTRACT
This paper presents the development of a discrete time economic order quantity model with constant demand and linear holding cost for ameliorating items where shortages are not allowed and this is a generalization of GWANDA (2018) In the study, the items are assumed to undergo immediate amelioration upon arrival in stock. The purpose is to determine the optimal ordering quantity and replenishment cycle so as to minimize the total variable cost. It is a generalization of an analysis of ameliorating items under constant demand by taking time to be discrete and holding cost linear. A mathematical expression representing the instantaneous inventory level at an arbitrary time t with the cycle period T, are solved utilizing a difference equation subject to initial and boundary conditions. Numerical example has been demonstrated to validate the model and sensitivity analysis conducted to determine the impact of parameter variations.

Keywords: Economic Order Quantity, Ameliorating items, Constant Demand, Linear Holding Cost, Discrete time

INTRODUCTION

Within the scope of ameliorating inventory management, the amount of items increase by ameliorating activities within a time period which depends on the ameliorating rate of each item. Amelioration refers to the process by which the value or utility of a product improves over time. Examples of ameliorating items include fast-growing animals such as fish, chickens, ducks, cows, and sheep. These livestock are typically acquired at a young stage and reared over a period to increase their value. When kept and fed, they increase in weight and value over time. By the convention of monographs or digraphs which are plenary, time used to occupy the x-axis. But while plotting the continuous functional model, the table of values is within the discretion of discrete values. Those situated disjoint values were then joint using a (LIFO) continuous set of lines to represent the functional model of some particular realism. Thus as humans with the binary brain system, time is discrete because it is within a frame. Hwang (1997) was the first to consider the model for ameliorating items using two parameter Weibull distribution. Hwang (1999) later developed economic order quantity and partial selling price model considering issuing policies of first in first out (FIFO) and last in first out for two parameter Weibull distribution for items whose utility increases with time and those whose utility decreases with time. Biswajit et al. (2003) considered ameliorating items whose

demand is dependent on their price with instantaneous replenishment system without shortages. Hwang (2004) presented items whose quality and quantity improves with time and those whose quality and quantity decrease with time for storage among a discrete set of location sites to determine the minimum number of storage facilities so that the probability of each customer being covered is not below a critical value. Srichandan et al. (2017) considered an ordering policy with Weibull amelioration under the influence of inflation and time-value for money. Gobinda et al. (2013) first proposed a time varying second order demand rate for ameliorating items. They proposed two models: the first is an ordering policy for items whose utility improves over time following a Weibull distribution, while the second is a partial selling quantity (PSQ) model designed to manage the sale of surplus inventory generated by amelioration under linear demand conditions. In another case, Kandpal and Tinani (2013) established a stochastic ordering policy for ameliorating items under supplier's trade credit policy under authorized payment deferment. Shortages were allowed due to potential uncertainties in future supply and the model was for two suppliers. Han-Wen et al. (2017) developed an improvement for inventory policy of ameliorating items incorporating the Weibull distribution, The enhancement lies in obtaining the optimal results.

Yahaya et al. (2019) formulated an ordering policy for ameliorating inventory with linear demand rate and unconstrained retailer's capital. Fathy and Riaz (2014) Considered an inventory policy for ameliorating items with power demand, lot size dependent selling price and trade credits. The goal is to maximize the profit per unit time. It also provides insights for decision makers who need to make decisions about pricing and credits. Fathy et al. (2017) developed a deterministic inventory policy for ameliorating items with fuzzy demand under inflation and time dependent selling price. The model's goal was aiming to maximize overall profit via fuzzy technique. Fathy and Xiaolei (2020) established an optimal ordering policy for ameliorating items with higher rate of amelioration and shortages with variable rate. This also was to make more of the total profit by using Lagrangian relaxation method for the solution.

Khoumsi and Skouri (2022) considered inventory models with amelioration effect and demand patterns. The demand patterns were time dependent and ramp-type. A mathematical equation was developed for the optimal replenishment policy and numerical examples were used to examplify the model. Generally, time duration is counted in terms of complete days, months or even years. Several authors studied discrete time inventory models which includes: Dave and Jaiswal (1980) who studied a discrete in time probabilistic inventory ordering policy for deteriorating items with stationary uniform demand, constant deterioration rate with no shortages. Dave (1985) developed a discrete in time deteriorating inventory policy with demand rate as a trended time, constant deterioration, finite time horizon without shortages. Yakubu and Sani (2015) proposed a policy model for deteriorating items that exhibit non-instantaneous deterioration with discrete time.

Yazici et al. (2019) developed an EOQ model with amelioration of defective items under uncertain demand and supply with two suppliers. The article considers a multi stage supply chain in which defective items are produced by the supplier, ameliorated by the whole seller and sold to the retailer. They then developed a mathematical model to optimize the ordering and amelioration decisions under uncertainty in demand and supply. Alzzawi and Majid (2019) studied an EOQ model with amelioration of defective items under price negotiation and inflation. The authors assumed that the seller offers different prices for different order quantities and the buyer can negotiate the price, in addition they considered the inflation rate varying over time Ahmed and Vempala (2019) considered an EOQ model with amelioration of defective items under linear pricing scheme and supply uncertainty. They assumed that the whole seller can ameliorate defective items at a constant cost and the selling price is linearly related to the order quantity. Furthermore, the supply is uncertain and could be higher or lower than the agreed quantity. Khan et al. (2019) proposed an EOQ model with amelioration of defective items under uncertain demand and service level with nonlinear amelioration cost. The paper assumed that the demand is subject to variation, the buyer can negotiate the selling price while the amelioration cost is nonlinear. Luo et al. (2020) developed an EOQ model with amelioration of defective items under a multi-step delivery scheme with price negotiation. The study examines where a buyer can negotiate the selling price with the seller and the seller offering different prices depending on the delivery schedule.

In most cases during shopping trips, the statement ''we are out of stock'' seems to be a common phenomenon. This makes customers feel the stores are not managing their inventory well. This is why managing inventories is crucial for companies especially those dealing with physical products such as manufacturers, wholesalers, retailers and so on. Production of goods in large quantities leads to a large inventory and the cost associated with maintaining such goods is called holding cost. The holding cost includes that of leasing, storage, Insurance coverage for risks associated with fire, theft, or vandalism; tax obligations; and personnel expenses related to oversight and security to protect the inventory. Other authors that used linear holding cost is Gwanda (2018) considered an economic order quantity model for ameliorating items with time dependent demand and trended time dependent holding cost. Gwanda (2019) also established an inventory policy for both ameliorating and deteriorating items with exponentially increasing demand and trended time dependent holding cost. Handa et al. (2022) presented a trade credit policy in an EOQ model with stock sensitive demand and shortages for deteriorating items. Stock shortages are within time frames which is discrete. Beside being stock shortages, they are as well deteriorating. Hzang et al. (2020) considered controllable deterioration rate under stock-dependent demand rate and non-linear holding cost. Not every non-linear model is solvable, otherwise no need of studying the linear models. Adamu and Yakubu (2023) developed discrete-time economic order quantity model designed for managing items with constant demand. The objective is to identify the optimal order quantity and replenishment cycle to minimize total variable costs.

In this study, an ordering policy for items that are improved over discrete time, with constant demand and linear holding costs without shortages is presented. The objective is to remodel the work of Gwanda (2018) where continuous time was used.

MATERIALS AND METHODS

Description and Formulation of the model

The proposed model is outlined using the following notation and assumptions

Definitions and Assumptions

Notation

$I_A(t)$ *Inventory level at any given time t*

C_0 *Cost of order quantity per order*

$I_A(0)$ *The quantity to be ordered*

T *Duration of the Cycle*

C Expense associated with the item

$C_H = h_1 + h_2(t)$ *The expense associated with holding inventory per unit of time. which is assumed to be linear, where* h_1 *is the constant part of the holding cost and* $h_2(t)$ *is the variable part of the holding cost.*

D_T *Total demand in a cycle T.*

D Rate of demand for each time unit

TVC Overall variable expenses

A_m *Improved quantity*

A Amelioration rate

Premises

i. The lead time is negligible.
ii. An instantaneous ameliorating item is assumed, meaning that amelioration occurs immediately upon the item's arrival in stock.
iii. Shortages are not permitted
iv. It is assumed that the item is not retained beyond the fixed time T.

Formulation of the model

In this study, the items are sourced from an external supplier and brought to the stock. For items like livestock, they are fed, vaccinated and so on. With proper feeding and care, amelioration occurs immediately upon the arrival of the items in stock, while demand and amelioration continue until the inventory reaches zero at t = T, when the items are fully matured and sold. $I_A(0)$ is the initial order quantity. The behaviour of the inventory policy is illustrated in the figure below.

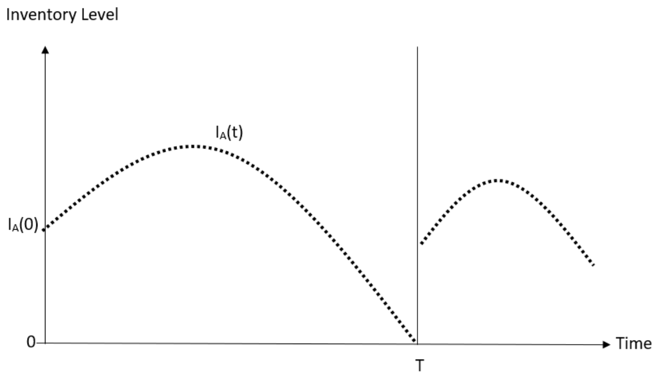

Figure 1: The graphical depiction of the inventory system's behaviour

$I_A(0)$ represents the initial inventory, $I_A(t)$ is the inventory level at any given time t. Throughout the time interval $(0 \le t \le T)$, amelioration occurs at a constant rate A, while the demand rate remains constant at D per unit of time.

Difference equation for the inventory level I(t) is given by

$$\Delta I_A(t) = AI_A(t) - D \qquad 0 \le t \le T \qquad (1)$$

With the initial and boundary conditions $I_A(0) = I(0)$, $t = 0$ and $I_A(T) = 0$, .at t = T.

This can be solved as follows:

Since $\Delta f(x) = f(x+h) - f(x)$, where h is the step length, then

$\Delta I_A(t) = I_A(t+1) - I_A(t)$ with step length of 1.

This implies

$I_A(t+1) - I_A(t) = AI_A(t) - D$ from equation (1)

$\Rightarrow I_A(t+1) = (1+A)I_A(t) - \mathrm{D}$ for t = 0, 1, 2, 3, …, (m-1), m = T

for $t = 0$

$I_A(1) = (1+A)I_A(0) - D$

for $t = 1$

$I_A(2) = (1+A)I_A(1) - \mathrm{D}$

for $t = 2$

$I_A(3) = (1+A)I_A(2) - D$

for $t = 3$

$I_A(4) = (1+A)I_A(3) - \mathrm{D}$

$= (1+A)\left[(1+A)^3 I_A(0) - \frac{D}{A}[(1+A)^3 - 1]\right] - \mathrm{D}$

$= (1+A)^4 I_A(0) - \left[\frac{(1+A)(1+A)^3}{A}\mathrm{D} - \frac{(1+A)}{A}\mathrm{D}\right] - D$

$= (1+A)^4 I_A(0) - \left[\frac{(1+A)^4}{A}\mathrm{D} - \frac{(1+A)}{A}\mathrm{D}\right] - D$

$= (1+A)^4 I_A(0) - \frac{\mathrm{D}}{A}[(1+A)^4 - (1+A)] - D$

$= (1+A)^4 I_A(0) - \frac{\mathrm{D}}{A}[(1+A)^4 - (1+A) + A]$

$= (1+A)^4 I_A(0) - \frac{\mathrm{D}}{A}[(1+A)^4 - 1]$

$\therefore \quad I_A(4) = (1+A)^4 I_A(0) - \frac{\mathrm{D}}{A}[(1+A)^4 - 1]$

So that in general we obtain

$$I_A(t) = (1+A)^t I_A(0) - \frac{\mathrm{D}}{A}[(1+A)^t - 1] \qquad (2)$$

Continuing up to T, yields

$$I_A(T) = (1+A)^T I_A(0) - \frac{\mathrm{D}}{A}[(1+A)^T - 1] \qquad (3)$$

Also for $t = (T-1)$, we have

$I_A(T-1) = (1+A)^{T-1} I_A(0) - \frac{\mathrm{D}}{A}[(1+A)^{T-1} - 1]$

For $t = (T-2)$

$I_A(T-2) = (1+A)^{T-2} I_A(0) - \frac{\mathrm{D}}{A}[(1+A)^{T-2} - 1]$

Using the boundary condition at $t = T$, $I_A(T) = 0$ from equation (3) we have

$0 = (1+A)^T I_A(0) - \frac{D}{A}[(1+A)^{\mathrm{T}} - 1]$

$\Rightarrow \quad (1+A)^T I_A(0) = \frac{\mathrm{D}}{A}[(1+A)^T - 1]$

$\Rightarrow \quad I_A(0) = \frac{\frac{\mathrm{D}}{A}[(1+A)^T - 1]}{(1+A)^T}$

$= \frac{\mathrm{D}}{A}[(1+A)^T - 1](1+A)^{-T}$

$$\therefore I_A(0) = \frac{\mathrm{D}}{A}[1 - (1+A)^{-T}] \qquad (4)$$

Substituting equation (4) into equation (2) yields

$$I_A(t) = (1+A)^t \frac{\mathrm{D}}{A}[1 - (1+A)^{-T}] - \frac{\mathrm{D}}{A}[(1+A)^t - 1] \qquad (5)$$

$= \frac{\mathrm{D}}{A}(1+A)^t - \frac{\mathrm{D}}{A}(1+A)^t(1+A)^{-T} - \frac{\mathrm{D}}{A}(1+A)^t + \frac{\mathrm{D}}{A}$

$= -\frac{\mathrm{D}}{A}(1+A)^t(1+A)^{-T} + \frac{\mathrm{D}}{A}$

$$\therefore I_A(t) = -\frac{\mathrm{D}}{A}[(1+A)^t(1+A)^{-T} - 1] \qquad (6)$$

The total demand within the interval $(0 \le t \le T)$ is given by

$D_T =$ demand rate x time period

$= DT$

The ameliorated amount A_m is given by

$A_m = D_T - I_A(0)$ i.e total demand – order quantity

$= DT - \frac{\mathrm{D}}{A}[1 - (1+A)^{-T}]$

From equation (6)

$I_A(1) = -\frac{\mathrm{D}}{A}[(1+\mathrm{A})(1+A)^{-T} - 1]$

$I_A(2) = -\frac{\mathrm{D}}{A}[(1+A)^2(1+A)^{-T} - 1]$

$I_A(3) = -\frac{\mathrm{D}}{A}[(1+A)^3(1+A)^{-T} - 1]$

$I_A(4) = -\frac{\mathrm{D}}{A}[(1+A)^4(1+A)^{-T} - 1]$

$I_A(5) = -\frac{\mathrm{D}}{A}[(1+A)^5(1+A)^{-T} - 1]$

At t = (T-1), we have

$I_A(T-1) = -\frac{\mathrm{D}}{A}[(1+A)^{T-1}(1+A)^{-T} - 1]$

and at t = (T), we get

$I_A(T) = -\frac{\mathrm{D}}{A}[(1+A)^T(1+A)^{-T} - 1]$

$\therefore \sum_{t=0}^{T} I_A(t) = -\frac{D}{A}(1+A)^{-T}[q^0 + q^1 + q^2 + q^3 + \cdots + q^{T-1} + q^T] + \frac{D}{A}(T+1)$,

Where $q = 1 + A$. However,

$\sum_{i=0}^{T} q^i = \left[\frac{(1+A)^{T+1}}{A} - \frac{1}{A}\right]$

$\therefore \sum_{t=0}^{T} I_A(t) = \frac{D}{A^2}[(1+A)^{-T} - (1+A)] + \frac{D}{A}(T+1)$

Hence Holding cost

$C_H = (h_1 + h_2(T)) \sum_{t=0}^{T} I_A(t) = (h_1 + h_2(T)) \left\{ \frac{D}{A^2}[(1+A)^{-T} - (1+A)] + \frac{D}{A}(T+1) \right\}$

The total variable cost in the cycle is given by

T V C = Ordering cost + holding cost - cost of ameliorate items

$= C_0 + C_H - CA$

$TVC = \left[C_0 + (h_1 + h_2(T)) \left\{ \frac{D}{A^2}[(1+A)^{-T} - (1+A) + A(T+1)] \right\} - C\left\{ DT - \frac{D}{A}[1-(1+A)^{-T}] \right\} \right]$

The total variable cost per unit time is then given by,

$TVC\ (T) = \frac{1}{T}\left[C_0 + (h_1 + h_2(T)) \left\{ \frac{D}{A^2}[(1+A)^{-T} - (1+A) + A(T+1)] \right\} - C\left\{ DT - \frac{D}{A}[1-(1+A)^{-T}] \right\} \right]$

$TVC(T) = \frac{C_0}{T} + \frac{(h_1+h_2(T))}{(T+1)} \left\{ \frac{D}{A^2}[(1+A)^{-T} - (1+A) + A(T+1)] \right\} - \frac{C}{(T+1)}\left\{ DT - \frac{D}{A}[1-(1+A)^{-T}] \right\}$

Therefore

$TVC(T) = \frac{C_0}{T} + \frac{(h_1)}{(T+1)} \left\{ \frac{D}{A^2}[(1+A)^{-T} - (1+A) + A(T+1)] \right\} + \frac{h_2(T))}{(T+1)} \left\{ \frac{D}{A^2}[(1+A)^{-T} - (1+A) + A(T+1)] \right\} - \frac{C}{(T+1)}\left\{ DT - \frac{D}{A}[1-(1+A)^{-T}] \right\}$

In a similar way, for TVC (T-1), let T-1 = s, so that TVC (T-1) = TVC (s)

$= \frac{C_0}{s} + \frac{(h_1)}{(T+1)} \left\{ \frac{D}{A^2}[(1+A)^{-s} - (1+A) + A(s+1)] \right\} + \frac{h_2(s))}{(T+1)} \left\{ \frac{D}{A^2}[(1+A)^{-s} - (1+A) + A(s+1)] \right\} - \frac{C}{(T+1)}\left\{ Ds - \frac{D}{A}[1-(1+A)^{-s}] \right\}$

Also, for TVC (T+1), let T+1 = e, so that TVC (T+1) = TVC (e)

$= \frac{C_0}{e} + \frac{(h_1)}{(T+1)} \left\{ \frac{D}{A^2}[(1+A)^{-e} - (1+A) + A(e+1)] \right\} + \frac{h_2(e))}{(T+1)} \left\{ \frac{D}{A^2}[(1+A)^{-e} - (1+A) + A(e+1)] \right\} - \frac{C}{(T+1)}\left\{ De - \frac{D}{A}[1-(1+A)^{-e}] \right\}$

$TVC\ (T) - TVC\ (s)$

$$= \frac{C_0(s-T)}{Ts} + \frac{h_1D[(1+A)^{-T}-(1+A)^{-s}]}{(T+1)A^2} + \frac{h_1DA[(T+1)-(s+1)]}{(T+1)A^2} + \frac{h_2D[T(1+A)^{-T}-s(1+A)^{-s}]}{(T+1)A^2} + 0 + \frac{h_2D[s(1+A)-T(1+A)]}{(T+1)A^2} + \frac{h_2DA[T(T+1)-s(s+1)]}{A^2(T+1)} + \frac{CD[s-T]}{(T+1)} + 0 + \frac{CD[(1+A)^{-s}-(1+A)^{-T}]}{(T+1)A} \quad (7)$$

Similarly,

$TVC\ (e) - TVC\ (T)$

$$= \frac{C_0[T-e]}{eT} + \frac{h_1D[(1+A)^{-e}-(1+A)^{-T}]}{(T+1)A^2} + 0 + \frac{h_1DA[(e+1)-(T+1)]}{(T+1)A^2} + \frac{h_2D[e(1+A)^{-e}-T(1+A)^{-T}]}{(T+1)A^2} + \frac{h_2D[T(1+A)-e(1+A)]}{(T+1)A^2} + \frac{h_2DA[e(e+1)-T(T+1)]}{A^2(T+1)} + \frac{CD[T-e]}{(T+1)} + 0 + \frac{CD[(1+A)^{-T}-(1+A)^{-e}]}{(T+1)A} \quad (8)$$

Condition for Optimality

An optimal solution is one that meets all problem constraints, resulting in the maximum possible objective function when maximizing (or the minimum when minimizing). The values of T for which the total variable cost TVC (T) will be minimized are

$TVC\ (T^*) \le TVC\ (\text{s})$ and $TVC(T^*) \le TVC\ (\text{e})$ $\quad (T = T^* \ge 0)$

$\Rightarrow TVC\ (T^*) - TVC\ (\text{s}) \le 0$ and $TVC\ (\text{e}) - TVC\ (T^*) \ge 0$

$\Rightarrow \Delta TVC\ (\text{s}) \le 0$ and $\Delta TVC\ (T^*) \ge 0$

Thus

$\Delta TVC\ (\text{s}) \le 0 \le \Delta TVC\ (T^*)$

Thus, for the optimal T, it is necessary to have

$$\frac{C_0(s-T)}{Ts} + \frac{h_1D[(1+A)^{-T}-(1+A)^{-s}]}{(T+1)A^2} + \frac{h_1DA[(T+1)-(s+1)]}{(T+1)A^2} + \frac{h_2D[T(1+A)^{-T}-s(1+A)^{-s}]}{(T+1)A^2} + 0 + \frac{h_2D[s(1+A)-T(1+A)]}{(T+1)A^2} + \frac{h_2DA[T(T+1)-s(s+1)]}{A^2(T+1)} + \frac{CD[s-T]}{(T+1)} + 0 + \frac{CD[(1+A)^{-s}-(1+A)^{-T}]}{(T+1)A} \le 0 \le \frac{C_0[T-e]}{eT} + \frac{h_1D[(1+A)^{-e}-(1+A)^{-T}]}{(T+1)A^2} + 0 + \frac{h_1DA[(e+1)-(T+1)]}{(T+1)A^2} + \frac{h_2D[e(1+A)^{-e}-T(1+A)^{-T}]}{(T+1)A^2} + \frac{h_2D[T(1+A)-e(1+A)]}{(T+1)A^2} + \frac{h_2DA[e(e+1)-T(T+1)]}{A^2(T+1)} + \frac{CD[T-e]}{(T+1)} + 0 + \frac{CD[(1+A)^{-T}-(1+A)^{-e}]}{(T+1)A} \quad (9)$$

Determination of the EOQ

The economic order quantity, EOQ is defined as total demand in a circle minus ameliorated amount within the circle, i.e.

EOQ = $DT - \left\{ DT - \frac{D}{A}[1-(1+A)^{-T}] \right\}$

$= DT - DT + \frac{D}{A}[1-(1+A)^{-T}]$

$= \frac{D}{A}[1-(1+A)^{-T}]$ (10)

Numerical Example

We consider an example with the following parameters: $C_0 = 5000$, C = 200, D = 3000, A = 0.41, $h_1 = 100$, *and* h_2=30. Using equations (7), (8), and (10), the optimal values of T, EOQ and the TVC are calculated as T = 125days, EOQ = 812units and TVC = N 25651.46 respectively. This results were adopted from the paper we remodel. We adopted Gwanda (2016)'s numerology as a bridge to arrive at a decision outside the sensitivity analysis or percentage error. We use any relevant software's and programming language that are well acquitted with the aforementioned languages.

Analysis of Sensitivity

Given the uncertainties in decision-making, a sensitivity analysis is conducted to determine the behavior of how the different independent variables affect a particular dependent variable under some sets of assumptions in the parameters, $C_0,\ h_1, h_{2,}\ C,\ A,\ and\ D$ on the optimal values. This is done by taking one parameter at a time while the remaining parameters are kept at their original values.

Table 1: The effect of independent variables on the dependent variables and the corresponding changes in T, TVC (T) and EOQ.

Parameter	Parameter value (%)	Corresponding results		
		T^* DAYS	$TVC(T^*)$	EOQ^*
C_0	50	147	32359.74	945.58
	25	137	29141.58	885.32
	10	130	27085.49	842.80
	5	127	26375.40	824.50
	0	**125**	**25651.46**	**812.26**
	-5	122	24912.81	793.86
	-10	120	24158.32	781.57
	-25	111	21784.87	725.97
	-50	94	17339.72	619.64
C	50	216	4999.75	1346.28
	25	159	16679.56	1017.15
	10	137	22311.01	885.32
	5	130	24017.46	842.80
	0	**125**	**25651.46**	**812.26**
	-5	120	27219.92	781.57
	-10	115	28728.44	750.74
	-25	104	32944.68	682.39
	-50	90	39160.62	594.37
A	50	221	5380.04	1228.81
	25	163	16372.57	987.57
	10	138	22101.27	873.30
	5	131	23899.69	840.30
	0	**125**	**25651.46**	**812.26**
	-5	119	27361.31	783.27
	-10	114	29033.85	759.66
	-25	101	33867.11	697.62
	-50	84	41499.10	614.75
D	50	106	30573.69	1042.30
	25	114	28244.73	930.70
	10	120	26726.24	859.73
	5	122	26195.84	833.56
	0	**125**	**25651.46**	**812.26**
	-5	127	25092.55	783.27
	-10	130	24517.31	758.52
	-25	140	22680.48	677.60
	-50	165	19106.32	526.32
H_1	50	85	41641.61	562.66
	25	100	34386.31	657.36
	10	113	29373.27	738.36
	5	119	27556.04	775.41
	0	**125**	**25651.46**	**812.26**
	-5	132	23649.23	854.98
	-10	139	21537.19	897.42
	-25	170	14392.51	1082.05
	-50	249	-1328.04	1528.90
H_2	50	114	27473.09	744.55
	25	119	26603.68	775.41
	10	122	26043.63	793.86

5	123	25849.80	799.99
0	**125**	**25651.46**	**812.26**
-5	126	25449.09	818.38
-10	128	25242.27	830.60
-25	133	24591.46	861.06
-50	143	23381.69	921.55

RESULTS AND DISCUSSION

Based on the results presented in Table I, it can be inferred that

i. "As the ordering cost C_0 increases, the economic order quantity (EOQ), total variable cost (TVC), and T all experience an upward trend. This is anticipated, as an increase in the EOQ is necessary to minimize the frequency of ordering. Additionally, since C_0 directly influences the TVC, an increase in C_0 results in a corresponding increase in the TVC. Furthermore, T also rises as a consequence of the increase in the EOQ.

ii. With an increase in the item's cost, C, the TVC decreases, the EOQ increases and T also increases. When there is high cost of the item, It is expected that both EOQ and T will decrease. However, the model is trying to reduce cost, and this probably increases both the EOQ and the T,

iii. With increase in amelioration, A, the EOQ increases, TVC decreases and T increases. This is because as the amelioration increases more items are purchased to stock so as to take the advantage of increase in amelioration which then increases the EOQ so as to get more profit. The total variable cost (TVC) decreases because amelioration leads to cost reduction. T increases due to increase in EOQ. These are all expected

iv. When the demand, D, increases, there is a corresponding rise in the EOQ and TVC but T decreases. The EOQ increases because more items are needed to cater for the large demand. The TVC increases because of the increase in EOQ. T reduces as a result of increase in demand.

v. With increase in h_1, EOQ decreases, TVC increases and T decreases. EOQ decreases as expected so as not to incur much holding cost. Since h_1 has direct cost effect on TVC, it implies increasing h_1 will increase the TVC. The cycle length T decreases since the EOQ decreases.

vi. With increase in h_2, the EOQ decreases, the TVC increases while T decreases. The EOQ decreases as expected, so as to avoid high holding cost. The TVC increases because h_2 has direct cost effect on the TVC. T decreases since EOQ decreases.

CONCLUSION

This paper presents an Economic Order Quantity (EOQ) model for ameliorating items. The demand rate is assumed to be constant, time is discrete, and the holding cost is linear. The inventory begins with items procured from external suppliers and subsequently brought into stock. Such items include fish, chicken, ducks, cows, sheep, and others. Our objective is to determine the optimal replenishment cycle that will minimize the total variable cost (TVC). A numerical example is provided to demonstrate the application of the model, and a sensitivity analysis has been conducted to assess the impact of parameter variations. The results of the sensitivity analysis indicate that changes in the parameters significantly affect the TVC, EOQ, and TTT. It shows that the decision variables are sensitive to fluctuations in all parameters. An increase in carrying cost leads to corresponding increases in the EOQ, TVC, and TTT. Conversely, as the item's cost CCC increases, the TVC decreases, while the EOQ and TTT rise. The model is trying to reduce cost and that increases both EOQ and the T. Also increase in amelioration results to decrease in TVC, but EOQ and T increase which is expected. While increase in demand results to increase in EOQ and by implication increase in TVC. However, T decreases which is expected since the demand is high. Increase in both h_1 *and* h_2 increases the TVC, but EOQ and T decrease in order not to incur much holding cost.

Some notable applications of this model includes:

i. Inventory Management in Retail: Retailers use the EOQ model to determine optimal order quantities for products, ensuring that stock levels meet customer demand without incurring excessive holding costs.

ii. Manufacturing: In manufacturing, companies apply the EOQ model to manage raw materials and component parts, balancing the costs of ordering and holding inventory to optimize production efficiency.

iii. Food and Beverage Industry: Businesses in this sector use the EOQ model to manage perishable goods, such as fruits, vegetables, and dairy products, ensuring that items are ordered in quantities that minimize spoilage while meeting demand.

iv. Pharmaceuticals: Pharmacy chains utilize EOQ models to manage medication inventories, ensuring that drugs are available when needed while minimizing storage costs and waste.

v. Automotive Industry: Auto manufacturers and suppliers apply the EOQ model to manage spare parts inventory, ensuring the availability of parts for maintenance and repairs.

vi. E-commerce: Online retailers apply the EOQ model to optimize inventory levels for various products, balancing stock availability with the costs associated with warehousing.

vii. Wholesaling: Wholesalers use the EOQ model to determine optimal order quantities for bulk purchases from manufacturers, reducing overall costs while ensuring product availability for their customers.

viii. Consumer Goods: Companies producing fast-moving consumer goods (FMCG) leverage EOQ models to manage inventory levels efficiently, ensuring products are consistently available in stores.

ix. This can be extended but not limited to: Stochastic demand with discrete time and compare results, Include partial backlogging with discrete time and analyze cost impact, Study multi item joint replenishment for ameliorating goods among others.

x. These applications demonstrate the versatility of the EOQ model in optimizing inventory management across diverse sectors.

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