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ON THE PRIMITIVE AND REGULAR CHARACTERISTICS OF DIHEDRAL GROUP OF DEGREE 2P AND THEIR RELEVANCE TO MUSICAL NOTE THEORY

***Samuel H. Tsok, Niri Martha Choji and Rwat Solomon Isa**

Department of Mathematics Faculty of Natural and Applied Science Plateau State University Bokkos, Plateau State Nigeria

*Corresponding authors' email: tsoksam@plasu.edu.ng

ABSTRACT

This paper investigates the primitive and regular characteristics of Dihedral Group of degree 2p, where p is an odd prime. By utilizing numerical approached, the properties of these groups were examined to shed light on their structure, behavior, and underlying algebraic characteristics. The work uses some group concept to test conditions for primitivity and regularity in these groups, with the help of Group Algorithm Programming (GAP) our results were validated. The main focus of this paper is on their applications to musical note theory. We explore the conditions under which these groups exhibit primitive and regular action on sets, highlighting their algebraic properties and symmetries. The theoretical findings are then connected to musical note arrangements, where pitch classes and transformations exhibit similar cyclic and reflective patterns. By establishing this connection, we demonstrate how group-theoretic principles can enhance the understanding of musical scales, chord structures, and symmetrical note sequences. The results presented offer new insights into the intersection of abstract algebra and music, paving the way for further interdisciplinary exploration. The work reveal that the musical note operate base on their pitch classes and musical intervals. It was discovered that the group of transpositions and inversions, denoted T_n/T_{1-n} is isomorphic to the dihedral group $D12$. Finally, Its Conjugacy classes and Character table was presented.

Keywords: Dihedral Group, Primitive, Regular, D12, Isomorphic, Musical note, GAP

INTRODUCTION

Dihedral groups whose degree is 2p, where p is prime are fundamental in abstract algebra, representing symmetries of regular polygons., Dummit and Foote, (2004). The focus here is on investigating the primitivity and regularity of such groups that are not p-groups, meaning their order is not a power of p. Using computational techniques, Holt, (2005) we address these properties and explore whether numerical analysis can offer deeper insights into their structure.

Dihedral Groups are groups that represent the symmetries of polygons and consist of rotations and reflections, Armstrong, (1997). For a group of order (n = 2p), they combine geometric interpretations with algebraic formalism, Artin, (1991).

Primitivity in a group refers, a group is primitive if it preserves no nontrivial partition of a set, Serre, (1977). In the context of dihedral groups, understanding primitivity involves analyzing their actions on various sets, Dixon, (1996).

Regularity: A group is regular if every element can be expressed uniquely as a product of generators, Burnside, (1911). Exploring this property involves understanding how elements combine in the dihedral group D_{2p}, Robinson, (1996).

Numerical Approach: In this section, numerical techniques such as group element counting, cycle structure analysis, and matrix representations of group actions are used to study the properties of D_{2p} Cannon, et al (2004).

Main computational tools include:

Cycle Decomposition: Decomposing group elements into cycles to check if the group action is primitive Butler, (2005).

Testing Regularity: Using numerical algorithms to test if every element can be uniquely written as a product of group generators, Neumann, et al, (1994).

These work-study Dihedral group of Degree 2p for small p. We present numerical simulations for small prime values of p ={3,5}. We calculate the group order, structure, and test for primitivity and regularity for each case (Suzuki, 1982, Magnus, et al, 2004). And finally, we presented an application of this work to Music.

Ben, et al (2021) work on a dihedral group of degree 3p that are not p-group he came out with a nice result that the dihedral group of degree 3p are imprimitive and soluble.

Preliminaries

The following definitions are important to this research work:

Permutation Group

A permutation group is a group G whose elements are permutations of a given set X and whose group operation is the composition of functions in G which are a bijection from the set X to itself.

Symmetric Group

The symmetric groups S_n is the group of permutations on a set with n elements. The symmetric group of degree n on a finite set is define to be the group whose elements are all bijective functions from X to X and whose group operation is that of function composition. Permutations and bijection are two the same operation meaning rearrangement.

Abelian Group

A group G is called abelian if for every $a, b \in G, ab = ba$. Otherwise G is said to be non-abelian.

Dihedral Group

A dihedral group D_n is a symmetric group for an n-sided regular polygon for $n>2$. Dihedral group are non-abelian permutation groups with group order 2n. We can mathematically write dihedral group $D_n = \{x, y | x^n = y^2 = 1, yx = x^{n-1}y = x^{-1}y\}$

Stabilizer

A kind of dual role is played by the set of elements in G which fix a specified point α. This is called the stabilizer of α in G and is denoted by $G_\alpha := \{x \in G | \alpha^x = \alpha\}$.

Transitive Group

A group G acting on a set Ω is said to be transitive on Ω if it has one orbit and so $\alpha^G = \Omega$ for all $\alpha \in \Omega$. Equivalently, G is transitive if for every pair of point $\alpha, \delta \in \Omega$ there exists $g \in \Omega$ such that $\alpha^g = \beta$. A group which is not transitive is called intransitive.

If $|\Omega| \geq 2$, we say that the action of G on Ω is doubly transitive if for any $\alpha_1, \alpha_2 \in \Omega$ such that $\alpha_1 \neq \alpha_2$ and $\beta_1, \beta_2 \in \Omega$ such that $\beta_1 \neq \beta_2$ there exist $g \in G$ such that $\alpha_1^g = \beta_1, \alpha_2^g = \beta_2$.

The group G is said to be k-transitive (or k-fold transitive) on Ω if for any sequences $\alpha_1, \alpha_2, \ldots, \alpha_k$ such that $\alpha_i \neq \alpha_j$ when $i \neq j$ and $\beta_1, \beta_2, \ldots, \beta_k$ such that $\beta_i \neq \beta_j$ when $i \neq j$ of k element on Ω, there exists $g \in G$ such that $\alpha_1^g = \beta_1\ for\ 1 \leq i \leq k$

Thus,

$G_1 = \{(1), (12), (13), (23), (123), (132)\}$ is transitive and $G_2 = \{(1), (12), (34), (12)(34)\}$ is intransitive.

Imprimitivity

A subset Δ of Ω is said to be a set of imprimitivity for the action of G on Ω, if for each $g \in G$, either $\Delta^g = \Delta$ or Δ^g and Δ are disjoint. In particular, Ω itself, the 1-element subsets of Ω and the empty set are obviously sets of imprimitivity which are called trivial set of imprimitivity.

Example

The group of symmetry $D_4 = (1), (1234), (13)(24)(1432), (13), (24), (12)(34), (14)(23)\}$, of the square with vertices 1,2,3,4 is not primitive. For take $G_1 = \{(1), (24)\}$ = reflection in the line joining vertices 1 and 3 = stabilizer of the point 1, and $H = \{(1), (24), (13)$ = reflection in m the line joining vertices 2 and 4, (13)(24) = rotation in 180^0, $H = \{(1), (24), (13), (13)(24)\}$. Thus $G_1 < H < G$.

Primitive

A permutation group G acting on a nonempty set Ω is called primitive if G acts transitively on Ω and G preserves no non trivial partition of Ω. Where non trivial partition means a partition that is not a partition into singleton set or partition into one set Ω. In other word, a group G is said to be primitive on a set Ω if the only sets of imprimitivity are trivial ones otherwise G is imprimitive on Ω, example the group (e ∗) For each $g \in G, \Delta^g = \Delta, \Delta^g \cap \Delta \neq \emptyset$. Thus $G = S_3 = \{\ldots\}$ is primitive.

p-subgroup

Let G be a group. Let H be a subgroup of G. if H is a p-group, then H is a p-subgroup of G. Thus, A p-subgroup of G is a subgroup whose order is some power of p.

Sylow p-subgroup

A Sylow p-subgroup of G is a subgroup whose order is p^k, where k is the largest natural number for which p^k divides |G|.

Normal group

A subgroup N of a group G is normal in G if the left and right cosets are the same, that is if $gH = Hg$ for every $g \in G$ and a subgroup H of G.

Semi-regular and Regular Group

A permutation group G is called semi-regular if one is the only element of G which fixes each point. In other word, G is semi-regular when $G_\alpha = 1$ for each $\alpha \in G$. A transitive semi-regular is called a regular group. Thus, the group G={(1),(12)(34),(14)(23),(13)(24)} is a regular group.

Clearly subgroups of semi-group are semi-regular; 1 is semi-regular. As we get that in a semi –regular group G, orbits have the same size, namely $|G|$, and hence, the order of G divides the degree of G. Furthermore, in a regular group G we have that $|G| = |\alpha^G| = |\Omega|$, $\alpha \in \Omega$ and so the order and the degree of G coincide.

Integers Modulo m

This is a finite group that is called the additive group of the residue class of integers modulo m. it is denoted by Z_m

MATERIALS AND METHODS

Methodology

In this work, acknowledgment of the basic facts from both the theory of abstract finite groups and the theory of permutation will be assumed throughout. Key numerical methods are employed to calculate invariants and test conditions for primitivity and regularity of these groups. (Rotman, 1999), (Humphreys, 1996).

Relevant theorems and results are given and quoted with example where necessary, in order to enhance proper understanding of the subject matter.

Theorem 1 (Cayley,1854)

Any finite group G is isomorphic to a subgroup of the symmetric group S_n of degree n, where $n = |G|$.

Proof:

Let G act on itself by right multiplication $g^h = g$h for all $g, h \in G$. If $g^h = g$ then gh $= g$ and so $h = 1$. That is, the kernel of the action is {1}. The mapping f:G → sym(G) define by $f_g \to f_g$ where $\alpha f_g = \alpha^g$ for any $\alpha \in G$ is a homomorphism. Then $G/_{Kerf} \cong$ im f but $kerf = \{1\}$ and $imf \leq sym(G) = S_n$.

Accordingly, $G \leq S_n$. In general, we have that if G acts on Ω with k kernel of the action then $G/K \leq$ sym(Ω).

Fundamental Counting Lemma or Orbit formula (Dixon and Mortimer, 1996)

Let G act on Ω and α ∈ Ω. If G is finite then $|G| = |G_\alpha||\alpha^G|$.

Proof:

We determine the length $|\alpha^G|$ of the α^G, we have that $\alpha^x = \alpha^y$ if and only if $\alpha^{xy-1} = \alpha$ if and only if $\alpha^{xy} \in G_\alpha$ if and only if $G_\alpha x = G_\alpha y$. Thus there is one to one correspondence given by the mapping $G_\alpha x \rightarrow \alpha^x$ between the set of right cosets G_α and the G-orbit α^G in Ω. Accordingly, as G is finite we have that $|G:G_\alpha| = |\alpha^G|$ and so $|G| = |G_\alpha||\alpha^G|$.

Theorem 2 (Sylow, 1872)

Let G be a finite group. If $|G| = p^r m(p, m) = 1$, then

i. There is at least one Sylow p-subgroup H of G.
ii. If B is any p-subgroup of G, then $B \subseteq x^{-1}Hx$ for some $x \in G$.
iii. If K is any Sylow p-subgroup of G, K=$g^{-1}Hg\ for\ some\ g \in G$
iv. If n is the number of Sylow p-subgroups of G, then n divides m and n ≡1modp.

Corollary (Sylow, 1872)
Let G be a finite group and H a Slow p-subgroup of G. Then H is the only Sylow p-group of G if and only if H is normal in G.
Proof:
By Sylow theorem, the Sylow p-Subgroups of G are the elements of the sets $g^{-1}Hg|g \in G$ and this reduces to a singleton set if and only if $g^{-1}Hg = H \; \forall g \in G$; that is precisely when H is normal in G;

Corollary (Thanos, 2006)
Let p be prime. If $H \leq$ G and G/H = p or p^2 then G/H is abelian. That is, every group of order p or p^2 is abelian.
Proof:
Let $|G| = p^2$. If $|Z(G)| = p^2$, then certainly G is abelian, so suppose that $|Z(G)| = p$. Then $G/Z(G)$ is a cyclic group of order p, generated say by the coset $Z(G)a$; then every element of G has the form za^i , where $z \in Z(G)$ and i = 0,1,..., p −1. By inspection, these elements commute.
Thus, G is abelian.

Lemma (Passman & Benjamin, 1968)
Let G be a dihedral group of any order, then G is transitive.
Proof:
For given α_i, α_j as any two vertices of the regular polygon with i< j, we readily see that
$(\alpha_1,\alpha_2,\ldots,\alpha_i,\ldots,\alpha_j,\ldots,\alpha_n)^{j-I}$ is the rotation about the center of the polygon through angle $2\pi^c/n$, (where n is the number of edges of the polygon) which take α_i to α_j. As such G is transitive.

Theorem 3 (Passman & Benjamin, 1968)
Let G be a non-trivial transitive permutation group on Ω. Then G is primitive iff G_α, (α∈Ω) is a maximal subgroup of G or equivalently, G is imprimitivity if and only if there is a subgroup H of G properly lying between G_α, (α∈Ω) and G.
Proof::
Suppose G is imprimitive and ψ a non-trivial subset of imprimitivity of G.
Let $H = \{g \in G|\psi^g = \psi\}$.
Clearly H is a subgroup of G and a proper subgroup of G because $\psi \subset \Omega$ and G is transitive.
Now choose $\alpha \in \psi$. If $g \in G$ then $\alpha^g = \alpha$, showing that $\alpha \in \psi \cap \psi^g$ and so $\psi = \psi^g$.
Hence $G \leq H$.
Hence, $G_\alpha \leq H \leq G$.
Since $|\psi| = 1$, choose $\beta \in \psi \; such \; that \; \beta \neq \alpha$. By transitivity of G, there exist some $h \in G$ with $\alpha^h = \beta \; so \; that \; h \in G_\alpha$. Now $\beta \in \psi \cap \psi^h$ so $\psi = \psi^g$ and $h \in H - G_h$. Thus, $H \neq G_\alpha$ Hence G_α is not a maximal subgroup.
Conversely, suppose that $G_\alpha \leq H \leq G$ for some subgroup H.
Let $\psi = \alpha^H$.
Since $H > G_\alpha$, $|\psi| \neq 1$.
Now If $\psi = \Omega$, then H is transitive on Ω and hence $\Omega = |G: G_\alpha| = |H: G_\alpha|$ showing that H = G, a contradiction. Hence, $\psi = \Omega\psi = \Omega$. Now we shall show that ψ is a subset of imprimitivity of G.
Let $h \in G$ and $\beta \in \psi \cap \psi^g$ then $\beta = a^h = a^{hg}$ for some h, $h' \in H$.
Hence $\alpha_{hgh^{-1}} = \alpha$. So $hgh^{-1} \in G_\alpha < H$.
Thus $\psi = \psi^g$. Hence ψ is a non-trivial subset of imprimitivity. So G is imprimitive.

Theorem 4 (Passman & Benjamin, 1968)
Let G be a transitive permutation group of prime degree on Ω. Then G is primitive.
Proof:
Now since G is transitive, it permutes the sets of imprimitivity bodily and all the sets have the same size. But $\Omega = \cup |\Omega_i|$, Ω_i being the sets of imprimitivity. As $|\Omega|$ is prime we Have that either each $|\Omega_i|$=1 or Ω is the set of imprimitivity. So G is primitive.

Theorem 5 (Audu, et al, 2000)
Let G be a transitive abelian group. Then, G is regular.
Proof:
Fix $\alpha \in \Omega$. If $\beta \in \Omega$ such that $\exists \, g \in G$ with $\alpha^G = \beta$. Now $G_\alpha = G_\alpha^g = (G_\alpha)^g = g^{-1}(G_\alpha)g = G_\alpha$ (since G is abelian). As α, β are arbitrary, we get that G_α =1 since G is transitive, it is regular.

Proposition (Neumann, 1980)
A transitive group is regular if and only if its order and degree are equal
Proof:
Let G be a regular on Ω. of degree n since $|\alpha^G| = |G|$ and G is transitive Hence $|G| = n$, conversely, by transitivity of G it follows that, $n|G_\alpha| = |G|$. Hence $G_\alpha = 1$, since $|G| = n$ by assumption Hence G is semi-regular, but G is transitive so G is regular

Proposition (Neumann, 1980)
An intransitive group is irregular if and only if its order and degree are not equal
Proof:
Let G be an irregular group on Ω. of degree n, since $|\alpha^G| \neq |G|$ and G is intransitive Hence $|G| = n$. Conversely by transitivity of G it follows that, $n|G_\alpha| = |G|$. Hence $|G_\alpha| \neq 1$, since $|G| = n$ by assumption. Hence G is Semi-regular, but G is intransitive so G is irregular.

Regularity Property (Dedekind, 1879)
A group G is called regular if every subgroup of G is normal.
Proof:
The wreath product $H \wr G$ is defined as the semi-direct product $HX \rtimes G$, where HX is the direct product of X copies of H. The elements of $H \wr G$ can be represented as pairs (f, g), where f:$X \to H$ is a function and $g \in G$. Multiplication in $H \wr G$ is defined component-wise as $(f1,g1)\cdot(f2,g2)=(f1f2,g1g2)$, where $f1f2$ is the point-wise product of functions. Consider a subgroup K of $H \wr G$. Since HX is a normal subgroup of $H \wr G$, the projection of K onto HX is also a normal subgroup. The projection map π:$H \wr G \to HX$ is defined as $(f, g) \mapsto f$. This is a group homomorphism, and its kernel is the set of elements of the form $(1,g)$ for all $g \in G$, which is isomorphic to G. Therefore, HX is a normal subgroup of $H \wr G$. Let K be a subgroup of $H \wr G$, and let $a \in H \wr G$. We need to show that $aKa-1=K$ for all $a \in H \wr G$. Consider an element $a = (fa, ga)\in H \wr G$. The conjugate of K by a is given by $aKa^{-1}=\{(fakfa^{-1},ga)| \; k \in K\}$. Since HX is a normal subgroup, the conjugation $fakfa^{-1}$ lies in HX. Therefore, $aKa^{-1} \subseteq HX \times G$. The subgroup HX is normal in $H \wr G$, and $K \cap HX$ is normal in HX. As HX is normal in $H \wr G$ and $K \cap HX$ is normal in HX, the subgroup K is normal in $H \wr G$.
Thus, the wreath product $H \wr G$ is regular, as every subgroup is normal.

Theorem 6
Let $H \leq G$ be groups and $g \in G$. Then: (i) $g \in gH$ (ii) Two left cosets of H in G are either identical or disjoint. (iii) The number of elements in gH is $|H|$

Proof::

(i) Since, $1 \in H$, we have that $g^{-1} \in gH$ (ii) Take the left coset aH of H in G. By (i) above, $a \in aH$. Suppose that $a \in bH$ for some $b \in G$. Then we have to show that $aH = bH$ since Since $a \in bH$. We have that, $a = bh_1$ for some $h_1 \in H$, so that for any $h \in H$,. $ah = (bh_1)h = b(h_1h) \in bH$ That is, $aH \subseteq bH$: and, Thus $bh = (ah_1^{-1})h = a(h_1^{-1}h) \in aH$ That is, $bH \subseteq aH$. Thus $aH = bH$. It follows that if $aH \cap bH \neq \emptyset$, then $aH = bH$ and as such distinct left cosets are disjoint (iii) The map $H \to gH$ defined by $h \to gh$ is bijective. Thus, $|H| = |gH|$

Theorem 7 (Langrange's Theorem)

The order of a subgroup of a finite group is a factor of the order of the group.

Proof:

Let $|G| = n < \infty$ $let\ H \leq G$ and let $|H| = m$ Now, G is the union of pairwise disjoint cosets of H. Let there be j distinct cosets of H in G. We know that for any $a \in G$ $|aH| = |H| = m$. Therefore, the total number of elements in G is m_j So $n=m_j$, that is, m divides n and the result follow $|G| = |G: H||H|$.

Theorem 8

Every subgroup of a cyclic group is cyclic.

Proof:

Let $H \leq G = \langle g \rangle$ If $H = \{1\}$, $then\ H = \{g^0\}$ is trivially cyclic. Then $H \neq \{1\}$ and choose $h \in H$. Then $h = g^s$ for some $s \in Z$. And $h^{-1} = g^{-s}$. Thus there are positive integers $t \ni g^t \in H$. Take the least of such positive integers and call it I. By the well-ordering principle of natural number, any set of positive integers contains a smallest number. By division algorithm we may write $S = ql + r$, $0 \leq r < l$. $Then\ h = q^s = g^{ql+r} = (g^l)^q g^r$ so that $g^r = (g^l)^{-q} h \in H$ If $r \neq 0$,$then\ r < l$ which contradicts the choice of l. thus, $r = 0$ $and\ so\ h = (g^l)^q$ Hence $H \subseteq \langle g^l \rangle$. $now\ g^l \in H\ and\ so\ \langle g^l \rangle \in H$. Accordingly, $H = \langle g^l \rangle$ and the theorem follows.

Theorem 9 (First Sylow's Theorem)

Let G be a finite group, p a prime and $p\ r$ the highest power of P diving the order of G. Then there is a subgroup of G of order of G. Then there is a subgroup of G of order $p\ r$.

Proof:**;**

We will prove the theorem by induction on the order n of G. For $|G| = 1$ the theorem is trivial. Assume $n > 1$ and the theorem is true for groups of order $< n$. Suppose $|Z(G)| = c$ We have two possibilities; (i) $c|p$ or (ii) $p \dagger c$,

i. Suppose $c|p$ $Z(G)$ is an abelian group. Therefore, Z(G) has an element of order p. Let N be a cyclic subgroup of Z(G) is normal in G consider G/N. Then $|G/N| = n/p$ by theorem 3.1 Hence by our induction assumption, G/N has a subgroup H of order p^{r-1} therefore $\exists$ a subgroup H of $G\exists$ $H/N = \bar{H}$ as $p^{r-1} = |\bar{H}| = |H|/|N| = |H|/p$, We conclude that $|H| = q^r$. Thus, in this case, G has a subgroup of order p^r.
ii. Suppose $p|c$. The class equation for G is of the form $|G| = |Z(G)| + \sum [G: C(R)]$ $R \in \Re$. Since $p||G|$ $and\ p \dagger c$, we have $p \dagger \sum [G: C(R)]$ $R \in \Re$.

Therefore, for at least one $R \in \Re*$, $p \dagger |G: C(R)|$ But $|G| = |G: C(R)|/|C(R)|$ by Theorem 3.1 Hence $p^r ||C(R)|$, $Since\ p^r ||G|$. Now $|C(R)| \neq |G|$; $for\ if$ $|C(R)| = |G|$, Then C(R) = G and $R \cap Z(G) = \phi$. Thus, by the induction assumption, C(R) has a subgroup H of order P^r. Consequently, so does G. In either case we have found a subgroup H of order P^r. AS require

RESULTS AND DISCUSSION

In this section, we Generate Dihedral groups of degree 2p. We stated a proposition and provided its proofs using the concepts of Group Theory. Also is the introduction and test of primitive and regular nature of the groups generated, GAP 4.11.1 was use to validate our claim. Application of this work was presented in Musical note.

Throughout the letter p is an odd prime number

Primitivity and Regularity of Dihedral Group of Degree 2p

Our main result on the dihedral group of degree 2p is as below.

Proposition 1

Suppose G is a dihedral group of degree 2p. Then G is imprimitive and irregular.

Proof:

Let $|G| = 2\times2p$ and $\Omega = \{1, 2, 3, \ldots\ldots., 2p\}$. That G is transitive follows easily from a lemma by Passman Now, name the vertices of G as 1,2,3,…, 2p and let l be the line of symmetry joining the middle of the vertex 1 and with the middle of the vertices $p\ and\ p + 1\ so\ that$

$\alpha = (2, 2p)(3, 2p - 1) \ldots (p, p + 1)$, is the reflection in l.

Then $G_1 = \{(1), (2, 2p)(3, 2p - 1) \ldots (p, p + 1)\}$ is the stabilizer of the point 1. We readily see that G_1 is a non-identity proper subgroup of G which has

$H = \{(1), (2, 2p), (3, 2p - 1), (4, 2p - 2), \ldots, (p, p + 1), \alpha\}\ldots$

as a subgroup properly lying between G_1 and G. i.e, $G_1 < H < G$. It follows by virtue of Theorem 3, G is imprimitive proved. Now $|G| = 2\times2p = 2^2p$. Therefore, G has Sylow 2-subgroups ($Syl_2(G)$) of order 4 and Sylow p-subgroups ($Syl_p(G)$) of order p.

Let n_p denote the number of Sylow p-subgroups in G of order p. Then, by Sylow Therefore $n_p \equiv 1$modulo p and n_p divides 4 $\Rightarrow n_p = 1$ (for $p > 3$)

This clearly implies that $Syl_p(G)$ say H is unique and hence normal in G from a Corollary by Thanos. H has order p implying $|G:H| = 1$, therefore the factor group G/H is of order p and hence abelian. However, there exist a subgroup of G say $K = Syl_2(G)$ which is not normal in G since $n_2 = \{1, p\}$. Thus, G is irregular by Dedekind. Also, from (Orbit-formula) $|\alpha^G||G_\alpha| = |G|$ we have that, $|G_\alpha| = 2 \neq 1$ and again, $|G| = 4p \neq 2p = |\Omega|$ by theorems 5 and a Proposition by Neuman, that G is irregular as required

Comparison of Result 1 with a Standard Programme Groups, Algorithms, and Programming

We shall now generate a dihedral groups of degree 2p where (p = 3) regular polygon and discuss whether they are transitive, primitive and regular using the concepts of Sylow p-subgroups and group actions. We use GAP to generate the group.

```
gap> G :=DihedralGroup(IsGroup,12);
Group([ (1,2,3,4,5,6), (2,6)(3,5) ])
gap> Order(G);
12
gap> Elements(G);
[ (), (2,6)(3,5), (1,2)(3,6)(4,5), (1,2,3,4,5,6), (1,3)(4,6), (1,3,5)(2,4,6), (1,4)(2,3)(5,6), (1,4)(2,5)(3,6), (1,5)(2,4), (1,5,3)(2,6,4),
(1,6,5,4,3,2), (1,6)(2,5)(3,4) ]
gap> NG :=NormalSubgroups(G);
[Group([(1,2,3,4,5,6),(2,6)(3,5)]),Group([(1,2,3,4,5,6),(1,3,5)(2,4,6)]),Group([(1,6)(2,5)(3,4),(1,3,5)(2,4,6)]),Group([(2,6)(3,5),(1,3,5)(
2,4,6)]),Group([(1,4)(2,5)(3,6)]),Group([(1,3,5)(2,4,6)]),Group(())]
gap> SG :=AllSubgroups(G);
[Group(()),Group([(2,6)(3,5)]),Group([(1,3)(4,6)]),Group([(1,5)(2,4)]),Group([(1,4)(2,5)(3,6)]),Group([(1,2)(3,6)(4,5)]),Group([(1,4)(
2,3)(5,6)]),Group([(1,6)(2,5)(3,4) ]),
Group([(1,3,5)(2,4,6)]),Group([(2,6)(3,5),(1,4)(2,5)(3,6)]),Group([(1,5)(2,4),(1,4)(2,5)(3,6)]),Group([(1,3)(4,6),(1,4)(2,5)(3,6)]),Group
([ (1,3,5)(2,4,6),(2,6)(3,5)]), Group([(1,3,5)(2,4,6),(1,2,3,4,5,6)]),Group([(1,3,5)(2,4,6),(1,2)(3,6)(4,5)]),Group([
(1,3,5)(2,4,6),(2,6)(3,5),(1,2,3,4,5,6)]) ]
gap> Size(NG);
7
gap> IsTransitive(G);
true
gap> IsPrimitive(G);
false
gap> IsAbelian(G);
false
gap> IsRegular(G);
false
gap> OrbitsDomain(G);
[ [ 1, 2, 3, 6, 4, 5 ] ]
gap> S2 :=SylowSubgroup(G,2);
Group([ (2,6)(3,5), (1,4)(2,5)(3,6) ])
gap> Order(S2);
4
gap> List(S2);
[ (), (2,6)(3,5), (1,4)(2,5)(3,6), (1,4)(2,3)(5,6) ]
gap> S3 :=SylowSubgroup(G,5);
Group(())
gap> S3 :=SylowSubgroup(G,3);
Group([ (1,3,5)(2,4,6) ])
gap> Order(S3);
3
gap> List(S3);
[ (), (1,5,3)(2,6,4), (1,3,5)(2,4,6) ]
gap> IsNormal(G,S3);
true
gap> Stb1 :=Stabilizer(G,1);
Group([ (2,6)(3,5) ])
gap> Order(Stb1);
2
gap> Elements(Stb1);
[ (), (2,6)(3,5) ]
gap> CC :=ConjugacyClasses(G);
[ ()^G, (2,6)(3,5)^G, (1,2)(3,6)(4,5)^G, (1,2,3,4,5,6)^G, (1,3,5)(2,4,6)^G, (1,4)(2,5)(3,6)^G ]
gap> Size(CC);
6
gap> List(CC,X->Order(Representative(X)));
[ 1, 2, 2, 6, 3, 2 ]
gap> Display (CharacterTable(G));
CT1

 2 2 2 2 1 1 2
 3 1 . . 1 1 1

 1a 2a 2b 6a 3a 2c
 2P 1a 1a 1a 3a 3a 1a
 3P 1a 2a 2b 2c 1a 2c
 5P 1a 2a 2b 6a 3a 2c

X.1 1 1 1 1 1 1
X.2 1 -1 -1 1 1 1
X.3 1 -1 1 -1 1 -1
X.4 1 1 -1 -1 1 -1
X.5 2 . . 1 -1 -2
X.6 2 . . -1 -1 2
gap>
```

From the Gap Result above the group of symmetry $G = D_{12}$ of the regular polygon viz:

Now $|D_{12}| = 12$ and $\Omega = \{1,2,3,4,5,6\}$ is the set of points of G. It follows from a Lemma by Passman that G is transitive as we can see from the result above $\alpha^G = \Omega \ \forall \alpha \in \Omega$. Also the stabilizer of the point 1 in G is given by G_1= {(),(2,6)(3,5)} which is obviously a non-identity proper subgroup of G. we readily see from the subgroups of G that G has a subgroup. H= {[(),(2,6)(3,5),(1,4)(2,5)(3,6),(1,4)(2,3)(5,6)]}

H=Syl_2(G) properly lying between G_1 and G that is $G_1 < H < G$ hence G is imprimitive by theorem 3

D_{12} = {(),(2,6)(3,5),(1,2)(3,6)(4,5),(1,2,3,4,5,6),(1,3)(4,6),(1,3,5)(2,4,6),(1,4)(2,3)(5,6),(1,4)(2,5)(3,6),(1,5)(2,4),(1,5,3)(2,6,4),(1,6,5,4,3,2),(1,6)(2,5)(3,4)}

Now $|D_{12}| = 12 = 2^2 \cdot 3$

i. The Sylow 2-subgroups of D_{12} have order 4. The number is congruent to 1 modulo 2 and it divides 3, n_2 = {1, 3,9,15,21,27, . . . } hence, not normal in D_{12}.
ii. The Sylow 3-subgroups of D_{12} have order 3 and it divides 4. We readily see that D_{12} has a normal Sylow 3-subgroup given by K = {(),(1,5,3)(2,6,4),(1,3,5)(2,4,6)}

Hence n_3 = {1}

This implies by Sylow's, K is normal in D_{12} . K has order 3 and $|D_{12}:K| = 4$, therefore the factor group D_{12}/K is of order 4 which is a p-Group and hence abelian as in a Corollary by Thanos . Now the stabilizer of the point 1 in D_{12} is given by $D_{12(1)}$ = {(1),(2,10)(3,9)(4,8)(5,7)}. We can see that $|D_{12}| \neq \Omega$ Thus $|D_{12(1)}| = 2$, $\forall \alpha \in \Omega$ where $\Omega = \{1, 2,...,6\}$, Since $|D\alpha_{(1)}| = 2$ therefore, D_{12} is transitive and not semi regular, then as $|D_{12(1)}| \neq 1$ it implies that D_{12} is irregular by theorems 5 and Proposition by Neuman.

Comparison of Result 1 With A Standard Programme Groups, Algorithms, and Programming

We shall now generate dihedral groups of degree 2p regular polygon where (p = 5) and discuss whether they are transitive, primitive and regular using the concepts of Sylow p-subgroups and group actions. We use GAP to generate the group.

```
GAP 4.11.1 of 2021-03-02
GAP │ https://www.gap-system.org
└──────────┘ Architecture: x86_64-pc-cygwin-default64-kv7
Configuration: gmp 6.2.0, GASMAN, readline
Loading the library and packages ...
Packages: AClib 1.3.2, Alnuth 3.1.2, AtlasRep 2.1.0, AutoDoc 2020.08.11, AutPGrp 1.10.2, Browse 1.8.11,

gap> # Dihedral group of Degree 2p (where p = 5) #
gap> G:=DihedralGroup(IsGroup,20);
Group([ (1,2,3,4,5,6,7,8,9,10),(2,10)(3,9)(4,8)(5,7) ])
gap> Order(G);
20
gap> Elements(G);
[(),(2,10)(3,9)(4,8)(5,7),(1,2)(3,10)(4,9)(5,8)(6,7),(1,2,3,4,5,6,7,8,9,10),(1,3)(4,10)(5,9)(6,8),(1,3,5,7,9)(2,4,6,8,10),(1,4)(2,3)(5
,10)(6,9)(7,8),(1,4,7,10,3,6,9,2,5,8),(1,5)(2,4)(6,10)(7,9),(1,5,9,3,7)(2,6,10,4,8),(1,6)(2,5)(3,4)(7,10)(8,9),(1,6)(2,7)(3,8)(4,9)(5
,10),(1,7)(2,6)(3,5)(8,10),(1,7,3,9,5)(2,8,4,10,6),(1,8)(2,7)(3,6)(4,5)(9,10),(1,8,5,2,9,6,3,10,7,4),(1,9)(2,8)(3,7)(4,6),(1,9,7,5,3)(
2,10,8,6,4),(1,10,9,8,7,6,5,4,3,2),(1,10)(2,9)(3,8)(4,7)(5,6)]
gap> SG :=AllSubgroups(G);
[Group(()),Group([(2,10)(3,9)(4,8)(5,7)]),Group([(1,3)(4,10)(5,9)(6,8)]),Group([(1,5)(2,4)(6,10)(7,9)]),Group([(1,7)(2,6)(3,5)(
8,10)]),Group([(1,9)(2,8)(3,7)(4,6)]),Group([(1,6)(2,7)(3,8)(4,9)(5,10)]),Group([(1,2)(3,10)(4,9)(5,8)(6,7)]),Group([(1,4)(2,3)(
5,10)(6,9)(7,8)]),Group([(1,6)(2,5)(3,4)(7,10)(8,9)]),Group([(1,8)(2,7)(3,6)(4,5)(9,10)]),Group([(1,10)(2,9)(3,8)(4,7)(5,6)]),Gr
oup([(2,10)(3,9)(4,8)(5,7),(1,6)(2,7)(3,8)(4,9)(5,10)]),Group([(1,7)(2,6)(3,5)(8,10),(1,6)(2,7)(3,8)(4,9)(5,10)]),Group([(1,3)(4,
10)(5,9)(6,8),(1,6)(2,7)(3,8)(4,9)(5,10)]),Group([(1,9)(2,8)(3,7)(4,6),(1,6)(2,7)(3,8)(4,9)(5,10)]),Group([(1,5)(2,4)(6,10)(7,9),(
1,6)(2,7)(3,8)(4,9)(5,10)]),Group([(1,3,5,7,9)(2,4,6,8,10)]),Group([(1,3,5,7,9)(2,4,6,8,10),(2,10)(3,9)(4,8)(5,7)]),Group([(1,3,5,
7,9)(2,4,6,8,10),(1,2,3,4,5,6,7,8,9,10)]),Group([(1,3,5,7,9)(2,4,6,8,10),(1,2)(3,10)(4,9)(5,8)(6,7)]),Group([(1,3,5,7,9)(2,4,6,8,10
),(2,10)(3,9)(4,8)(5,7),(1,2,3,4,5,6,7,8,9,10)])]
gap> Size(SG);
22
gap> NG :=NormalSubgroups(G);
[Group([(1,2,3,4,5,6,7,8,9,10),(2,10)(3,9)(4,8)(5,7)]),Group([(1,2,3,4,5,6,7,8,9,10),(1,3,5,7,9)(2,4,6,8,10)]),Group([(1,10)(2,9)(
3,8)(4,7)(5,6),(1,3,5,7,9)(2,4,6,8,10)]),Group([(2,10)(3,9)(4,8)(5,7),(1,3,5,7,9)(2,4,6,8,10)]),Group([(1,6)(2,7)(3,8)(4,9)(5,10)])
,Group([(1,3,5,7,9)(2,4,6,8,10)]),Group(())]
gap> Size(NG);
7
gap> IsTransitive(G);
true
gap> IsPrimitive(G);
false
gap> IsAbelian(G);
false
gap> IsRegular(G);
false
gap> OrbitsDomain(G);
[ [ 1, 2, 3, 10, 4, 9, 5, 8, 6, 7 ] ]
gap> S2 :=SylowSubgroup(G,2);
Group([(2,10)(3,9)(4,8)(5,7),(1,6)(2,7)(3,8)(4,9)(5,10)])
gap> Order(S2);
4
```

```
gap> List(S2);
[(),(2,10)(3,9)(4,8)(5,7),(1,6)(2,7)(3,8)(4,9)(5,10),(1,6)(2,5)(3,4)(7,10)(8,9)]
gap> S5 :=SylowSubgroup(G,5);
Group([ (1,7,3,9,5)(2,8,4,10,6) ])
gap> Order(S5);
5
gap> List(S5);
[(), (1,5,9,3,7)(2,6,10,4,8),(1,9,7,5,3)(2,10,8,6,4),(1,3,5,7,9)(2,4,6,8,10), (1,7,3,9,5)(2,8,4,10,6)]
gap> IsNormal(G,S2);
false
gap> IsNormal(G,S5);
true
gap> Sb1 :=Stabilizer(G,1);
Group([(2,10)(3,9)(4,8)(5,7)])
gap> Order(Sb1);
2
gap> Elements(Sb1);
[(), (2,10)(3,9)(4,8)(5,7)]
gap> Sb2 :=Stabilizer(G,2);
Group([(1,3)(4,10)(5,9)(6,8)])
gap> Order(Sb2);
2
gap> Sb5 :=Stabilizer(G,5);
Group([(1,9)(2,8)(3,7)(4,6)])
gap> Order(Sb5);
2
gap>
gap> Sb2 :=Stabilizer(G,2);
Group([(1,3)(4,10)(5,9)(6,8)])
gap> Order(Sb2);
2
gap> Sb5 :=Stabilizer(G,5);
Group([(1,9)(2,8)(3,7)(4,6)])
gap> Order(Sb5);
2
gap>
```

Example 2
From the Gap Result above the group of symmetry G = D_{20} of the regular polygon viz:
Now $|D_{20}|$ = 20 and Ω = {1,2,3,4,5,6,7,8,9,10} is the set of points of G. It follows from a Lemma by Passman that G is transitive as we can see from the result above $\alpha^G = \Omega \; \forall \alpha \in \Omega$. Also the stabilizer of the point 1 in G is given by G_1={(1),(2,10)(3,9)(4,8)(5,7)} which is obviously a non-identity proper subgroup of G. we readily see from the subgroups of G that G has a subgroup. H={[(),(2,10)(3,9)(4,8)(5,7),(1,6)(2,7)(3,8)(4,9)(5,10),(1,6)(2,5)(3,4)(7,10)(8,9)]}
H=$Syl_2(G)$ properly lying between G_1 and G that is $G_1 < H < G$ hence G is imprimitive by theorem 3
D_{20}={(),(2,10)(3,9)(4,8)(5,7),(1,2)(3,10)(4,9)(5,8)(6,7),(1,2,3,4,5,6,7,8,9,10),(1,3)(4,10)(5,9)(6,8),(1,3,5,7,9)(2,4,6,8,10),(1,4)(2,3)(5,10)(6,9)(7,8),(1,4,7,10,3,6,9,2,5,8),(1,5)(2,4)(6,10)(7,9),(1,5,9,3,7)(2,6,10,4,8),(1,6)(2,5)(3,4)(7,10)(8,9),(1,6)(2,7)(3,8)(4,9)(5,10),(1,7)(2,6)(3,5)(8,10),(1,7,3,9,5)(2,8,4,10,6),(1,8)(2,7)(3,6)(4,5)(9,10),(1,8,5,2,9,6,3,10,7,4),(1,9)(2,8)(3,7)(4,6),(1,9,7,5,3)(2,10,8,6,4),(1,10,9,8,7,6,5,4,3,2),(1,10)(2,9)(3,8)(4,7)(5,6)}
Now $|D_{20}| = 20 = 2^2 \cdot 5$

i. The Sylow 2-subgroups of D_{20} have order 4. The number is congruent to 1 modulo 2 and it divides 5, n_2 = {1, 5} hence, not normal in D_{20}.
ii. The Sylow 5-subgroups of D_{20} have order 5 and it divides 4. We readily see that D_{20} has a unique Sylow 5-subgroup given by K = {(1),(1,5,9,3,7)(2,6,10,4,8),(1,9,7,5,3)(2,10,8,6,4),(1,3,5,7,9)(2,4,6,8,10),(1,7,3,9,5)(2,8,4,10,6)}

Hence $n_5 = 1$

This implies by Sylow's, K is normal in D_{20} . K has order 5 and $|D_{20}:K| = 4$, therefore the factor group D_{20}/K is of order 4 which is a p-Group and hence abelian by Thanos. Now the stabilizer of the point 1 in D_{20} is given by $D_{20(1)}$ = {(1),(2,10)(3,9)(4,8)(5,7)}. We can see that $|D_{20}| \neq \Omega$ Thus $|D_{20(1)}| = 2$, ∀ α ∈ Ω where Ω = {1, 2,...,10}, Since $|D\alpha_{(1)}| = 2$ therefore, D_{20} is transitive and not semi regular, then as $|D_{20(1)}| \neq 1$ it implies that D_{20} is irregular by theorems 5 and Proposition by Neumann.

Musical Application of Permutation Groups of degree 2p Using Dihedral group
We decided to bring the application of our work through the means of musical note which reveal that the musical note operate base on their pitch classes and musical intervals.
In fact, Group theory helps in understanding the structure and symmetry within musical compositions, providing insights into harmony, chord progressions, and more. Every finite group has a sylow p-subgroup which is in line with the first sylow's theorem. Group theory as a Structure for Atonal Music Theory the numbering of the pitch classes reveals their isomorphism to Z_{12}. More interestingly, the group of transpositions and inversions, denoted T_n/T_{1-n} is isomorphic to the dihedral group D_{12}.
Thomas M. Flore (1993). He referred to C, C#, D, D#, E, F, F#, G, G#, A, A#, B. As the Z_{12} Model of pitch class.
He constructed a musical clock as below:

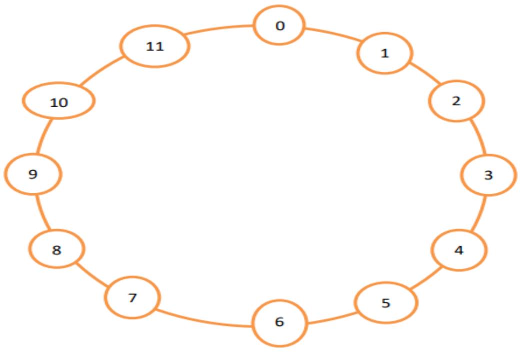

Figure 1: Musical Clock

Figure 1: Musical Clock He also said, we have a bijection between the set of pitch classes and he defined transposition as: $Tn : Z_{12} \rightarrow Z_{12} \ni T_n\ (X){:}X + n$ and invertion was also defined as $In : Z_{12} \rightarrow Z_{12} \ni In\ (X) : -x + n$ where n is in mod12 Ada Zhang, (2009).

Considered possibly musical notes with their corresponding integers as:

C C# D D# E F F# G G# A A# 0 1 2 3 4 5 6 7 8 9 10 He defined transposition, Tn as that which moves a pitch-class or pitch-class set up by n (mod12). And inversion was also defined here as TnI as the pitch (A) about C(0) and then transposes it by n. that is , $TnI(a) = -a + n(mod12)$. Then further, laid out all the pitches in a circular pattern on a 12-sided polygon. That is, consider the transposition $T11$. It sends C to B, C# to C, Alissa (2009). Assert that the musical actions of the dihedral groups. This paper considers two ways in which the dihedral groups act on the set of major and minor triads. 0 1 11 11 10 2 9 3 6 5 4 7 8 to Emma, (2011), referred to the musical notes with their corresponding integers as in Ada Zhang, (2009) as M_{12}, that is the Mathieu group. He asserts that this can be generated by just two permutations Expressed below in both two-line notation and cycle notation. We denote these generating permutations as P_1 and P_0

$$P_1 = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 5 & 6 & 4 & 4 & 7 & 8 & 2 & 9 & 1 & 10 & 0 & 11 \end{pmatrix}$$

= (0 5 8 1 6 2 4 7 9 10)(11)

$$P_0 = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 6 & 5 & 7 & 4 & 8 & 3 & 9 & 2 & 10 & 1 & 11 & 0 \end{pmatrix}$$

= (0 6 9 1 5 3 4 8 10 11)(2 7)

Adam, (2011) defined transposition and inversion as: Transposition is defining as $Tn : Z_{12} \rightarrow Z_{12} \ni T_n\ (X){:}\ x + n$ mod12 and he also define Inversion as $In : Z_{12} \rightarrow Z_{12} \ni In\ (X) : -x + n$ where n is in mod12. This operation was composite function.

Numbering of the Musical Notes

C C# D D# E F F# G G# A A# B

0 1 2 3 4 5 6 7 8 9 10 11

Note that $B\# = C$. It shows that the musical notes form a group of integers of Modulo 12.

That is $Z_{12} = \{C, C\#, D, D\#, E, F, F\#, G, G\#, A, A\#, B\}$. Let the operation be $* = \# = +$ Result of the behavior of the musical note on Groups Musical Notes as it related to groups axiom. Without loss of generality

i. Closureness , $F \in Z_{12}$, hence $E * F = A \in Z_{12}$
ii. Associativity : $E, F\ and\ F\# \in Z_{12},$

hence, $(E * F) * F\# = E * (F * F\#)$

$= A * F\# = E * B$

$= D\# = D\#$

iii. Identity $\in Z_{12} \ni C \in Z_{12}$, hence $F * C = C * F = F$
iv. Inverse $F \in Z_{12}\ \exists G \in Z_{12}$, hence $F * G = G * F = C \in Z_{12}$

Therefore, musical note behavior satisfied all the mathematical group axioms.

v. Furthermore, $\forall F, G \in Z_{12}\ F * G = G * F = C \in Z_{12}$ This shows that it is not just a group, but also an abelian group.

With the behavior of the musical notes we have just seen, we personally suggest for the root note of musical scales (notes) to be algebraically named as the identity note.

Table 1: List of Musical Notes and their inverse Note

Note	Inverse
C	C
C#	B
D	A#
D #	A
E	G #
F	G
F#	F#

The behavior of the musical note as related to Table 1. Gave us insight to formulate this Proposition 2.

Proposition 2

If G is cyclic, then there is at least an element which is unique with its inverse

Proof:

Suppose G is cyclic $\Longrightarrow \forall x \in G$, each $x \in G$ can be written in the form $x = g^m$ for some $g \in G$, Where $m \in Z \ni some\ y \in G \ni x * y = e \in G \Longrightarrow x = y$ where e is the identity element $y = x^{-1} \Longrightarrow x = x^{-1}$

This completes the proof. Remark table 1 give better understanding of the proposition above The Result of theorem 3.2.3 $Z_{12} = \{C, C\#, D, D\#, E, F, F\#, G, G\#, A, A\#, B\}$. $H = \{C, C\#, B\} \Longrightarrow H \leq Z_{12}$

$DH = \{D * C, D * C\#, D * B\}$ $DH = \{D, \#, C\#\}$ Clearly, $D \in DH$. And again, $|H| = |DH| = 3$. Furthermore, for some $A, F \in Z_{12}$ $(A * F)\ H = DH \Longrightarrow$ two left cosets are identical in this

case for some $A, G \in Z_{12}$ $A * G \in Z_{12}$ but $A * G = E \neq D$, $EH \neq DH$
Two left cosets are disjoint in this case.
The Result of [Theorem 3.2.6 previous results] $|Z_{12}| = 12$ Since $|Z_{12}| = 12$ and $|H| = 3$

$\Rightarrow {|Z_{12}|}/{|H|} = \frac{12}{3} = 4$

It is true that the order of a subgroup divides the order of a group. The Result of theorem 3..2.5 From
$Z_{12} = \{C, C\#, D, D\#, E, F, F\#, G, G\#, A, A\#, B\}$
For $C \in Z_{12}$, $C^0 = C$, $C^1 = C\#$, $C^2 = D$, $C^3 = D\#$, $C^4 = E$, $C^5 = F$, $C^6 = F\#$, $C^7 = G$, $C^8 = G\#$, $C^9 = A$, $C^{10} = A\#$, $C^{11} = B$, $C^{12} = C$
Musical notes are cyclic. That is $Z_{12} = \langle C \rangle$.
Consider the subgroup $H = \{C , C\#, B\}$, $B \in H \leq Z_{12}$ $B^0 = B$ $B^1 = C$ $B^2 = C\#$ Clearly, H is cyclic.
Using musical notes, we are satisfied with the theorem which states that "every cyclic group has a subgroup which is also cyclic.

Proposition 3: Every musical note is a generator of Z_{12}
Proof:
$C^0 = C$, $C^1 = C$, $C^2 = D$, $C^3 = D\#$, $C^4 = E$, $C^5 = F$, $C^6 = F\#$, $C^7 = G$, $C^8 = G\#$, $C^9 = A$, $C^{10} = A\#$, $C^{11} = B$, $C^{12} = C$ Therefore, C has generated Z_{12}. Similarly, every other note can behave as such. The Result from first Sylow's Theorem Recall that the $|Z_{12}| = 12$, since $12 = 2x2x3 = 2^2x3$ $\exists H \leq Z_{12} \ni |H| = 2^2$ which is sylow-2 subgroup of Z_{12}
It is true that every finite group has a sylow p-subgroup which is in line with the first sylow's theorem. Group theory is a Structure for Atonal Music Theory The numbering of the pitch classes reveals their isomorphism to Z_{12}.
Tsok S. (2018), More interestingly, the group of transpositions and inversions, denoted T_n/T_{1-n} is isomorphic to the dihedral group D_{12}.
The result of these findings shows those musicals notes behaviors satisfied all group axioms and are related to group theory. Since music is food for the soul and mind, we suggest that a good understanding of group theory to musician can help in composing best musical composition that will give satisfaction to Audience and as well bring healings to their minds.

We used GAP package to get the follows information on the Group of the Musical Note Modulo 12 which is isomorphic to D_{12} :

```
gap> # Dihedral group of Degree 2p (where p = 3) #
gap>
gap> G := DihedralGroup(IsGroup,12);
Group([ (1,2,3,4,5,6), (2,6)(3,5) ])
gap> Order(Z12);
12
gap> Elements(Z12);
[ (), (2,6)(3,5), (1,2)(3,6)(4,5), (1,2,3,4,5,6), (1,3)(4,6), (1,3,5)(2,4,6), (1,4)(2,3)(5,6), (1,4) (2,5)(3,6), (1,5)(2,4), (1,5,3)(2,6,4),
(1,6,5,4,3,2), (1,6)(2,5)(3,4) ]
gap> CC :=ConjugacyClasses(Z12);
[ ()^G, (2,6)(3,5)^G, (1,2)(3,6)(4,5)^G, (1,2,3,4,5,6)^G, (1,3,5)(2,4,6)^G, (1,4)(2,5)(3,6)^G ]
gap> List(CC,x->Order(Representative(x)));
[ 1, 2, 2, 6, 3, 2 ]
gap> Display(CharacterTable(Z12));
CT1
```

Table 2: The Representatives of Conjugacy Classes of Musical Note Modulo 12

S/N	Representative	Size	Name
1		1	1a
2	(2,6)(3,5)	2	2a
3	(1,2)(3,6)(4,5)	2	2b
4	(1,2,3,4,5,6)	6	6a
5	(1,3,5)(2,4,6)	3	3a
6	(1,4)(2,5)(3,6)	2	2c

Table 3: Character Table for Musical note Modulo12

	1a	**2a**	**2b**	**6a**	**3a**	**2c**
2P	1a	1a	1a	3a	3a	1a
3P	1a	2a	2b	2c	1a	2c
5P	1a	2a	2b	6a	3a	2c
χ_1	1	1	1	1	1	1
χ_2	1	1	1	1	1	1
χ_3	1	1	1	1	1	1
χ_4	1	1	1	1	1	1
χ_5	2	0	0	1	1	2
χ_6	2	0	0	1	1	2

Table 2 display the Representation of Conjugacy Classes of Musical Note Modulo 12 which have 6 conjugacy classes of different sizes with their names as seen in the table above.

Table 3 Represent the Character of the Musical Note Modulo 12. This table shows 6 characters of the Musical Notes.

CONCLUSION

The results of the numerical experiments were discussed, particularly focusing on which values of p lead to non-primitive actions and the conditions under which (D2p}) fails to be regular. This work shows that the Dihedral Group of degree 2p (p=3) is found to be isomorphic to musical notes

In Conclusion the findings as summarize and highlighted proof numerical approaches provide useful insights into the primitive and regular nature of Dihedral groups of degree 2p. These methods, computationally intensive, offer a clear pathway to understanding these fundamental group-theoretic properties as clearly shown by our results that the Dihedral groups of degree 2p where p is odd prime number are Imprimitive and irregular

Contributions to Knowledge

i. We generated a new group of degrees 2p using dihedral method procedures, which shade more light to the Primitivity and Regularity nature of the groups generated as our contribution.
ii. We proved that dihedral Group of degree 2p that are not p-groups are imprimitive and irregular. A proposition was formulated and proved to back up our claims while a standard program namely Groups, Algorithms and Programming (GAP) version 4.11.1, 2021), was employed to compare the results. These findings cannot be found in any text of higher learning.
iii. We add to the research space the novel discovery of an Application to our generated dihedral Group of degree 2p (where p = 3) in Musical note.

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