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CATALAN COLLOCATIONS TAU METHOD FOR SOLUTION OF FRACTIONAL INTEGRO-DIFFERENTIAL EQUATIONS

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ABSTRACT

This article is concern with the numerical solution of Fractional order Fredholm Integro-differential Equations using Catalan tau collocation method. The concept of Catalan tau collocation method was implemented on some of fractional Fredholm integro-differential equations to illustrate the efficienciency and practicability of the method. Fractional derivatives in Riemann-Liouvilles sense were adopted throughout this article.. The results showed the reliable, efficacy and accuracy that the method exhibited for this kind of problems when compared to the analytic solutions.

Keywords: Collocation Tau Method, Fractional Fredholm Integro-differential Equations, Fractional derivatives

INTRODUCTION

Fractional integro-differential equations were studied recently by many scholars in the field of physical science, technology and engineering due to their ability and appropriateness in the description of various natural and physical phenomenon (Ali et.al, 2020)). The mathematical models used in the area of banking and finance science and technology, mechanics, to describe the concept and notion of fractional integro-differential equations have been useful and important in the area of numerical methods and analysis (Ullah et.al, 2024). The challenges arising from solving modelling equations of complex phenomenon are very difficult to handle analytically and this is mainly because most of the problems of integro-differential equations do not have analytic solutions in the canonical or closed forms. Therefore, numerical methods are adopted to attempt to solve them (Oyedepo *et. al,* 2023).

Catalau Tau collocation method has emerged as a pivotal numerical technique for addressing the computational challenges inherent in these intricate mathematical models (Odekunle, 2006). Numerical methods for solving fractional-order equations have been a critical area of research. Investigations demonstrated the efficacy of tau methods in approximating solutions to fractional differential equations, showcasing improved computational accuracy (Shahmorad, 2005; Mamadu, 2016). The comprehensive review emphasized the significance of collocation methods in handling complex mathematical models (Jiang & Gao, 2024). A detailed analysis provided insights into the method's computational capabilities, highlighting its potential for solving complex mathematical problems with enhanced precision. Empirical studies have consistently demonstrated the method's superior performance (Çevik et,at, 2025). Many scholars developed and implemented several numerical methods which includes, and not limited to, Iterative Decomposition Method (Wohlmuth, 2012)), Adomian Decomposition Method (Turkyilmazoglu, 2019), Collocation Method (Herrera et al, 2007)). Fractional spectral collocation method (Zayernouri, & Karniadakis, 2014). Variational Iteration Method (Tomar et al, 2023), , Laplace Transform Method (Abou-Hayt., & Dahl, 2023), Successive Substitution Method (García-Vidal, 2019), Galerkin Method (Bozyigit, 2021), Homotopy Analysis Method (Guled et al, 2023) etc. Catalan tau collocation method is a method used in the field of numerical analysis that simplifies the problem by adding a perturbation term and using Catalan number which makes it easier to be solved. The system of equations derived as a result of using this method are presented in terms of arbitrary constant coefficients which will be solved and substituted into the assumed solution to obtain the approximate solution. Chebychev Galerkin method was applied by Mostafa et al, (2021) to solve integro-differential equations of the second kind, their method proves to be efficient and reliable for solving many kinds of integro-differential equations.

A generalized Jacobi polynomials as basis functions for the approximate solution was applied, using Discrete Galerkin method to solve fractional integro-differential equations (Mokhtary, ,2016). Convergence analysis to approximate solutions on exact solution was carried out, the results produced using the method were in good agreement with the analytical solution. Pseudo-spectral Legendre Galerkin method was applied in (Fakhar-Izadi & Dehghan, 2013) to solve problem in population dynamics that involved nonlinear partial integro-differential equations. Perturbed collocation method of Lane-Emden type was used to solve singular multi-order fractional differential equations (Uwaheren et al, 2020). The scheme is to transform the differential equations to a system of linear algebraic equations using collocation method. From the numerical results, the method produced good estimate for the differential equations considered. Mahdy & Mohamed, (2016) and Mahdy et al, (2016) solved integro-differential equations of fractional order, using shifted Chebyshev basis functions by least squares method and the results obtained converge to the analytical solution. Fathy, (2021) obtained numerical solution of second kind nonlinear integral equation of Fredholm type using Chebyshev wavelets with a very good approximation result. Iweobodo et al. (2023) applied Homotopy Perturbation method combined with Wavelet-Galerkin method to solve some classes of integro-differential equations. In this research, we present the Catalan tau collocation method for solving linear fractional order Fredholm integro-differential equations.

The advantage of our proposed method is that it can solve fractional integro-differential equations without discretization of the equations.

MATERIALS AND METHODS

Collocation Tau Method

Let $y(x)$ be an unknown function that satisfies the equation

$$D^{\alpha}y(x) = f(x) + \lambda\int_a^b K(x,t)y(t)dt \qquad a \le x \le b \quad (1)$$

where $D^{\alpha}y(x)$ represents the fractional derivative of order α of $y(x)$, $K(x,t)$ is a kernel of the function. Approximating the (1) by the power series solution of the form

$$y(x) = \sum_{i=0}^{n} a_i x^i \quad (2)$$

so that equation (1) can be perturbed to become

$$D^{\alpha}y(x) = f(x) + \lambda\int_a^b K(x,t)y(t)dt + H(x) \quad a \le x \le b \quad (3)$$

where $H(x)$ is the perturbation term.

Catalan Polynomial

Among the known mathematical constants, Catalan polynomials have attracted attention in recent years. Catalan constants can be written in the form (Kim & Kim, 2017)

$$C_n = \sum_{i=0}^{\infty} \frac{(-1)^k}{(2n+1)^2} \quad (4)$$

It status is not known, whether Catalan constants C_n is rational or irrational. Previously, Catalan constants in differential equations involves rewriting in the form such as Hurwitz zeta function, Dirichlet beta function, Legendrry chi-function. In this work, we attempt to apply the constants in solving fractional integro-differential equation directly without rewriting them in terms of any other functions.

Definition of Terms

Fractional Derivative

Fractional derivative is a non-integer type of derivative of a function. Its has a lot of benefits and importance. Riemann-Liouvilles differential operator of a noninteger/fractional order β is given as

$$(D_a^{\beta}f)(x) = \frac{1}{\Gamma(n-\beta)}\frac{d^n}{dx^n}\int_0^x (x-t)^{n-\beta-1}f(t)dt. \quad (5)$$

The differential operator in Caputo sense D_{β}^{*} is defined by

$$(D_{\beta}^{*}f)(x) = \frac{1}{\Gamma(n-\beta)}\int_0^x (x-t)^{n-\beta-1}\frac{d^n}{dt^n}f(t)dt \quad (6)$$

for $0 \le x \le 1$.

Fractional Integro-Differential Equations

An integro-differential equation is a differential equation that contains integrals. if the unknown function $u(x)$ is both inside and outside of the integral, the is called an integro-differential equation that is either the Fredholm (with costants limits of integratin) or Volterra (with variable limits of integratin) type. Where the integro-differential equation contains a fractional derivative D^{β} It is known as fractional integro-differential.The general form of fractional integro-differential equation is:

$$D^{\alpha}y(x) = f(x) + \lambda\int_{l(x)}^{p(x)} K(x,t)y(t)dt \quad (7)$$

subject to the initial conditions: $D^{\alpha}y_k(0) = \phi_k$, and $K(x,t)$ is a given smooth function.

Riemann-Liouvilles Fractional Integral

A continous function $u\colon(0,\infty) \to R$ has a Riemann-Liouvilles fractional integral of order $\alpha > 0$ is defined as

$$I^{\alpha}u(x) = \frac{1}{\Gamma(\alpha)}\int_0^x (x_i - t)^{\alpha-1}u(t)\,dt \quad (8)$$

Integral of n^{th} Derivative

It is defined as

$$I^{\alpha}D^{\alpha}u(x) = f(t) - \sum_{k=0}^{\alpha-1}\frac{c_k t^k}{k!} \quad (9)$$

Fredholm Fractional Integro-Differential Equation

A fractional integro-differential equation is called a Fredholm fractional integro-differential equation if the upper and the lower limits of the equation are both constants. For instance:

$$D^{\alpha}y(x) = f(x) + \lambda\int_0^1 K(x,t)y(t)dt \quad (10)$$

subject to the conditions: $y^{(\alpha)}(0) = \phi_k$. (11)

Volterra Fractional Integro-Differential Equation

When the limits of such equation are not both constants but one a constant and the other a variable, then the equation is called Volterra fractional integro-differential equation:

$$D^{\alpha}y(x) = f(x) + \lambda\int_0^x K(x,t)y(t)dt \quad (12)$$

Catalan number: A series of numbers given by the equation

$$C_n = \frac{1}{n+1}\binom{2n}{n}, \quad n = 0,1,2,\cdots \quad (13)$$

where $\binom{n}{m} = \frac{n!}{m!(n-m)!}$, $n,m = 0,1,2,\cdots$ is known as the Catalan numbers. Using (12), it is therefore

$$C_0 = 1,\quad C_1 = 1,\quad C_2 = 2,\quad \mathbf{C_3} = 5,\quad C_4 = 14,\quad C_5 = 42,\quad C_6 = 132, \quad (14)$$

Catalan polynomial: The Catalan polynomial $C_n(x)$ can be defined as

$$C_n(x) = \sum_{n=0}^{\infty} C_n x^n = \sum_{n=0}^{\infty}\left[\frac{1}{n+1}\binom{2n}{n}\right]x^n, \; n = 0,1,2, \quad (15)$$

Main Tools

Let the perturbation term be defined as

$$H_i(x) = \tau C_i(x) \quad (16)$$

where $C_i(x)$ is the Catalan polynomial of order i.

A polynomial $y_n(x)$ of degree n can be expressed as

$$y_n(x) = \sum_{n=0}^{\infty} a_n x^n = A^T X \quad (17)$$

Mathematical Background

From the Fredholm integral differential equation of fractional order, using (7) into perturbed equation (3) subject to the initial condition given in (10), to obtain

$$I^{\alpha}D^{\alpha}y(x) = I^{\alpha}f(x) + I^{\alpha}H(x) + I^{\alpha}\left(\lambda\int_0^1 K(x,t)y(t)dt\right) \quad (18)$$

Using (8), equation 17 becomes

$$y(x) - \sum_{k=0}^{\alpha-1}\frac{y^j(0)x^j}{j!} = I^{\alpha}f(x) + I^{\alpha}H(x) + I^{\alpha}\left(\lambda\int_0^1 K(x,t)y(t)dt\right) \quad (19)$$

Substituting the initial condition equation (10) into equation (18), to have

$$y(x) = \sum_{j=0}^{n}\frac{\phi_j x^j}{j!} + I^{\alpha}f(x) + +I^{\alpha}H(x) + I^{\alpha}\left(\lambda\int_0^1 K(x,t)y(t)dt\right) \quad (20)$$

RESULTS AND DISCUSSION

Method of Solution

Substituting (16) into equation (19), we obtain

$$\sum_{n=0}^{\infty} a_n x^n = \sum_{j=0}^{n}\frac{\phi_j x^j}{j!} + I^{\alpha}f(x) + I^{\alpha}H(x) + I^{\alpha}\left(\lambda\int_0^1 K(x,t)\left(\sum_{n=0}^{\infty} a_n t^n\right)dt\right) \quad (21)$$

or

$$\sum_{n=0}^{\infty} a_n\left(x^n - I^{\alpha}\left(\lambda\int_0^1 K(x,t)t^n dt\right)\right) = \sum_{j=0}^{n}\frac{\phi_j x^j}{j!} + I^{\alpha}f(x) + I^{\alpha}H(x) \quad (22)$$

Using (7) into (22), we obtain

$$\sum_{n=0}^{\infty} a_n\left(x^n - \frac{1}{\Gamma(\alpha)}\int_0^x (x-s)^{\alpha-1}\left(\lambda\int_0^1 K(x,s)t^n dt\right)ds\right) = \sum_{j=0}^{n}\frac{\phi_j x^j}{j!} + \frac{1}{\Gamma(\alpha)}\int_0^x (x-t)^{\alpha-1}(f(t) + H_n(t))\,dt \quad (23)$$

Substituting the perturbation term and collocating the resulting equation at

$x_j = \frac{b-a}{i+1} j \quad j = 1,2,3, \cdots\ i+1 \qquad (24)$

to obtain.

$\sum_{n=0}^{\infty} a_n \left(x_i^n - \frac{1}{\Gamma(\alpha)} \int_0^x (x_i - s)^{\alpha-1} \left(\lambda \int_0^1 K(x,s) t^n dt \right) ds \right) = \sum_{j=0}^{n} \frac{\phi_j x_i^j}{j!} + \frac{1}{\Gamma(\alpha)} \int_0^x (x_i - t)^{\alpha-1} \left(f(t) + \tau(1 + t + 2t^2 + 5t^3 + \cdots) \right) dt \qquad (25)$

After evaluating the integral in (25). The resulting equation is then collocated at equally spaced point to give

$a_0 \left(1 - \frac{1}{\Gamma(\alpha)} \int_0^x (x_i - s)^{\alpha-1} \lambda \int_0^1 K(s,t) dt\, ds \right) + a_1 \left(x_i - \frac{1}{\Gamma(\alpha)} \int_0^x (x_i - s)^{\alpha-1} \lambda \int_0^1 K(s,t)(t) dt\, ds \right) + a_2 \left(x_i^2 - \frac{1}{\Gamma(\alpha)} \int_0^x (x_i - s)^{\alpha-1} \lambda \int_0^1 K(s,t)(t^2) dt\, ds \right) \cdots + a_j \left(x_i^j - \frac{1}{\Gamma(\alpha)} \int_0^x (x_i - s)^{\alpha-1} \lambda \int_0^1 K(s,t)(t^j) dt\, ds \right) = \phi_0 + \phi_1 x_i + \frac{1}{2} \phi_2 x_i^2 + \frac{1}{6} \phi_3 x_i^3 + \frac{1}{j!} \phi_j x_i^j + \frac{1}{\Gamma(\alpha)} \int_0^x (x_i - t)^{\alpha-1} \left(f(t) + \tau(1 + t + 2t^2 + 5t^3) \right) dt \qquad (26)$

Solving equation (26) subject to the given initial conditions will result in (i+2)
system of equation in (i+2) unknown $a_0, a_1, a_1, \cdot$ and τ which can be determined later

Numerical Examples

We present three different numerical examples which are solved by Catalan tau collocation method to show efficiently and accuracy of our proposed method. All the numerical computations are carried out with Maple 18.

Example One

Consider the fractional integro-differential equation

$D^{\frac{1}{2}} y(x) = \frac{x}{12} + \frac{\frac{8}{3} x^{3\backslash 2} - 2\sqrt{x}}{\sqrt{\pi}} + \int_0^1 xty(t)$

with an initial condition $y(0) = 0$. The exact solution of this problem is $y(x) = x^2 - x$.

Solution

Using (24), with $\lambda = 1$, $f(x) = \frac{x}{12} + \frac{\frac{8}{3} x^{3\backslash 2} - 2\sqrt{x}}{\sqrt{\pi}}$, $H(t) = \tau(14x^4 + 5x^3 + 2x^2 + x + 1)$

$y_4(x) = a_0 \left(1 - \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} \left(\int_0^1 K(s,t) ds \right) dt \right) + a_1 \left(x - \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} \left(\int_0^1 K(s,t) t ds \right) dt \right) + a_2 \left(x^2 - \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} \left(\int_0^1 K(s,t) t^2 ds \right) dt \right) + a_3 \left(x^3 - \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} \left(\int_0^1 K(s,t) t^3 ds \right) dt \right) + a_4 \left(x^4 - \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} \left(\int_0^1 K(s,t) t^4 ds \right) dt \right) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} \left(f(t) + H(t) \right) dt \qquad (27)$

Which gives

$y_4(x) = a_0 \left(1 - \frac{2}{3} \frac{x^{\frac{3}{2}}}{\sqrt{\pi}} \right) + a_1 \left(x - \frac{4}{9} \frac{x^{\frac{3}{2}}}{\sqrt{\pi}} \right) + a_2 \left(x^2 - \frac{1}{3} \frac{x^{\frac{3}{2}}}{\sqrt{\pi}} \right) + a_3 \left(x^3 - \frac{4}{15} \frac{x^{\frac{3}{2}}}{\sqrt{\pi}} \right) + a_4 \left(x^4 - \frac{2}{9} \frac{x^{\frac{3}{2}}}{\sqrt{\pi}} \right) = \frac{1}{\Gamma(\alpha)} \int_0^x \left(\frac{1}{\sqrt{x-1}} + \frac{t}{12} + \frac{\frac{8}{3} t^{3\backslash 2} - 2\sqrt{t}}{\sqrt{\pi}} + \tau(14t^4 + 5t^3 + 2t^2 + t + 1) \right) dt \qquad (28)$

Collocating (28) at a point in (24) with $i = 4$, couple with the initial condition, such that $a_0 = 0$, we obtain the following of equations

$0.9663582330 a_0 + 0.1775721554 a_1 + 0.2317911652e - 1 a_2 - 0.5456706790e - 2 a_3 - 0.9613922320e - 2 a_4 = -0.1543930389 + 0.6072614474\tau$

$0.9048467139 a_0 + 0.3365644759 a_1 + 0.1124233569 a_2 + 0.2593868555e - 1 a_3 - 0.611776205e - 2 a_4 = -0.2241411189 + 1.234081936\tau$

$.8251922511 a_0 + .4834615008 a_1 + .2725961256 a_2 + .1460769004 a_3 + 0.7133075037e - 1 a_4 = -.2108653752 + 2.635221819\tau$

$.7308658643 a_0 + .6205772430 a_1 + .5054329322 a_2 + .4043463458 a_3 + .3198886215 a_4 = -.1151443107 + 5.769349010\tau$

$.6238736110 a_0 + .7492490740 a_1 + .8119368055 a_2 + .8495494444 a_3 + .8746245370 a_4 = 12.08261248\tau + 0.6268773149e - 1 \quad a_0 = 0$

The six unknowns $\{a_0, a_1, a_2, a_3, a_4, \tau\}$ can be obtained using Gaussian elimination method.

Table 1: Exact and Approximate Solutions and Absolute Error for (n = 4&5) Example 1

x	$y(x)$	$y_4(x)$	$\lvert y(x) - y_4(x)\rvert$	$y_5(x)$	$\lvert y(x) - y_5(x)\rvert$
0.1	-0.9e-1	-0.8999999744e-1	$2.56 * 10^{-9}$	-0.8999998017e-1	$1.983 * 10^{-8}$
0.2	-.16	-.1599999949	$5.1 * 10^{-9}$	-.1599999603	$3.97 * 10^{-8}$
0.3	-.21	-.2099999920	$8.0 * 10^{-9}$	-.2099999362	$6.38 * 10^{-8}$
0.4	-.24	-.2399999884	$1.16 * 10^{-8}$	-.2399999029	$9.71 * 10^{-8}$
0.5	-.25	-.2499999834	$1.66 * 10^{-8}$	-.2499998528	$1.472 * 10^{-7}$
0.6	-.24	-.2399999764	$2.36 * 10^{-8}$	-.2399997740	$2.260 * 10^{-7}$
0.7	-.21	-.2099999665	$3.35 * 10^{-8}$	-.2099996483	$3.517 * 10^{-7}$
0.8	-.16	-.1599999529	$4.71 * 10^{-8}$	-.1599994486	$5.514 * 10^{-7}$
0.9	-0.9e-1	-0.8999993439e-1	$6.561 * 10^{-8}$	-0.8999913915e-1	$8.6085 * 10^{-7}$
1	0	$9.11 * 10^{-8}$	$9.11 * 10^{-8}$	0.0000013300	0.0000013300

Table 1: shows the exact solutions and approximation solutions obtained by the Catalan tau collocation method and the absolute error in $x \in [0.1,\ 1]$ are compared in Table 1. From Table 1 above, it shows that the approximate solutions ($y_4(x)$ and $y_5(x)$) are quite near to the exact solution for all $x \in [0.1,\ 1]$. $y_4(x)$ errors vary from 2.56×10^{-9} to 9.11×10^{-8}, whereas $y_5(x)$ mistakes are significantly greater but still on the order of 10^{-8} to 10^{-7}. The error tends to increase slightly as x approaches one. The two approximations yield extremely good results, but $y_4(x)$ is better than $y_5(x)$ in terms of error magnitude.

Example 2

Consider the fractional integro-differential equation

$D^{\frac{5}{3}} y(x) = \frac{3\sqrt{(3)}\Gamma\left(\frac{2}{3}\right) x^{\frac{1}{3}}}{\Pi} - \left(\frac{1}{5}\right) x^2 - \left(\frac{1}{4}\right) x + \int_0^1 (xt + x^2 t^2) y(t)$

with an initial condition $y(0) = 0$. The exact solution of this problem is $y(x) = x^2$.

Solution

$y_4(x) = -3.653100213 * 10^{-8} x^4 + 3.171432180 * 10^{-8} x^3 + .9999999805 x^2 + 5.993272047 * 10^{-10} x$

$y_5(x) = 1.224332709 * 10^{-7} x^5 - 1.309968256 * 10^{-7} x^4 + 6.871680473 * 10^{-8} x^3 + .9999999972 x^2 + 1.988575130 * 10^{-9} x$

Table 2: Exact and Approximate Solutions and Absolute Error for (n = 4&5) Example 2

x	$y(x)$	$y_4(x)$	$\|y(x)-y_4(x)\|$	$y_5(x)$	$\|y(x)-y_5(x)\|$
0.1	0.01	0.009999999893	$1.07*10^{-10}$	0.01000000023	$2.3*10^{-10}$
0.2	0.04	0.03999999954	$4.6*10^{-10}$	0.04000000068	$6.8*10^{-10}$
0.3	0.09	0.08999999898	$1.02*10^{-9}$	0.09000000141	$1.41*10^{-9}$
0.4	0.16	0.1599999982	$1.8*10^{-9}$	0.1600000027	$2.7*10^{-9}$
0.5	0.25	0.2499999971	$2.9*10^{-9}$	0.2500000045	$4.5*10^{-9}$
0.6	0.36	0.3599999954	$4.6*10^{-9}$	0.3600000077	$7.7*10^{-9}$
0.7	0.49	0.4899999930	$7.0*10^{-9}$	0.4900000127	$1.27*10^{-8}$
0.8	0.64	0.6399999893	$1.07*10^{-8}$	0.6400000214	$2.14*10^{-8}$
0.9	0.81	0.8099999840	$1.60*10^{-8}$	0.8100000358	$3.58*10^{-8}$
1	1	0.9999999763	$2.37*10^{-8}$	1.000000059	$5.9*10^{-8}$

Table 2: shows the exact solutions and approximation solutions obtained by the Catalan tau collocation method and the absolute error in $x \in [0.1,\ 1]$ are compared in Table 2. Table 2 shows that the approximate solutions; $y_4(x)$ and $y_5(x)$ are very near to the exact solution for all $x \in [0.1,\ 1]$. $y_4(x)$ errors vary from $1.07*10^{-10}$ to $2.37*10^{-8}$, whereas $y_5(x)$ mistakes are significantly greater but still on the order of 10^{-10} to 10^{-8}. The error tends to increase slightly as x approaches one. The two approximations yield extremely good results,

***Example* 3**

Consider the Fredholm integro-differential equation

$D^{\alpha}y(x) = 2x - 3x^2 + \frac{1}{30} - \int_0^1 y(t)dt$

Subject to the initial condition $y(0) = y'(0) = 0$. For $\alpha = 2$, the exact solution is $y(x) = \frac{1}{3}x^3 - \frac{1}{4}x^4$.

$y_5(x) = 3.547603876*10^{-8}x^5 - .2500000447x^4 + .3333333585x^3 - 4.064677763*10^{-9}x^2 + 4.481541350*10^{-10}x$

$y_4(x) = -.2500000005x^4 + .3333333337x^3 - 1.923396909*10^{-10}x^2 + 8.257575665*10^{-12}x$

Table 3: Exact and Approximate Solutions and Absolute error for (n = 4&5) Example 3

x	$y(x)$	$y_4(x)$	$\|y(x)-y_4(x)\|$	$y_5(x)$	$\|y(x)-y_5(x)\|$
0.1	0.0003083333333	0.0003083333325	$8.0*10^{-13}$	0.0003083333586	$2.53*10^{-11}$
0.2	0.002266666667	0.002266666664	$3.0*10^{-12}$	0.002266666736	$6.9*10^{-11}$
0.3	0.006975000000	0.006974999994	$6.0*10^{-12}$	0.006975000174	$1.74*10^{-10}$
0.4	0.01493333333	0.01493333332	$1.0*10^{-11}$	0.01493333369	$3.6*10^{-10}$
0.5	0.02604166667	0.02604166666	$1.0*10^{-11}$	0.02604166732	$6.5*10^{-10}$
0.6	0.03960000000	0.03959999993	$7.0*10^{-11}$	0.03960000121	$1.21*10^{-9}$
0.7	0.05430833330	0.05430833323	$7.0*10^{-11}$	0.05430833549	$2.19*10^{-9}$
0.8	0.0682666667	0.06826666650	$2.0*10^{-10}$	0.06826667063	$3.93*10^{-9}$
0.9	0.0789750000	0.07897499983	$1.7*10^{-10}$	0.07897500708	$7.08*10^{-9}$
1	0.08333333333	0.08333333302	$3.1*10^{-10}$	0.08333334569	$1.236*10^{-8}$

Table 3 shows the exact solutions, approximation solutions and the absolute error obtained in $x \in [0.1,\ 1]$. Table 3 shows that the approximate solutions; $y_4(x)$ and $y_5(x)$ are very near to the exact solution for all $x \in [0.1,\ 1]$. $y_4(x)$ errors vary from $8.0*10^{-13}$ to $3.1*10^{-10}$, whereas $y_5(x)$ mistakes are significantly greater but still on the order of 10^{-11} to 10^{-8}. The error tends to increase slightly as x approaches one.

CONCLUSION

In this paper, a numerical method to solve the linear fractional type Fredholm integro-differential equation using the Catalan tau collocation method was proposed. For the approximate solution, we used the Riemann-Liouvilles differential operator and power series and Catalan polynomial. The fractional derivatives of Riemann-Liouvilles and power series transformed the equation into an algebraic system. Using Gaussian elimination method, Maple 18 software; gave the solution of the power series coefficients and the perturbation term, which allowed us to describe the approximate solution. Three numerical examples were used to illustrate the approximate results. As a result, Therefore, it is observed that Catalan tau collocation method is resilient, computationally viable and effective for variety of fractional order integro-differential equation of Fredholm types with high accuracy.

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