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APPLICATION OF CURVE FITTING METHODS TO MAPPING OF TUBERCULOSIS IN EASTERN CAPE PROVINCE, SOUTH AFRICA

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ABSTRACT

There are accurate and imprecise ways for interpolating data that is regularly distributed or scattered. Nonetheless, some techniques may be used to irregular grids and others to regular grids for data interpolation. Examining the spatial distribution of illness prevalence rates and their relationships within a specific distance and direction is crucial for spatial epidemiology. This study's goal is to use 3-D curve fitting techniques to create a graphical disease map for TB prevalent patterns. In this work, the distribution patterns of tuberculosis (TB) in the Eastern Cape Province were identified, localized, and compared for smoothing using linear and biharmonic spline methods implemented in MATLAB for the geographic and graphic depiction of the disease prevalence. The datasets are typically displayed as 3D or XYZ triplets, where Z is the variable of interest—in this case, the province's TB counts—and X and Y are the spatial coordinates. For the years 2012–2015, surface and contour maps were created to show the prevalence of tuberculosis at the province level. Biharmonic interpolations demonstrated smooth surfaces with lower sum of squares errors and regular patterns in the distribution of tuberculosis cases in the province, according to the overall aspect of all the fittings. These innovative interpolation techniques are infrequently employed in disease mapping applications, and they offer the advantage of being evaluated at subjective places as opposed to just on a rectangular grid, as is the case with the majority of conventional GIS techniques for geospatial analysis.

Keywords: Linear, Biharmonic, Splines, Tuberculosis, South Africa

INTRODUCTION

The majority of earth science data are spatially dispersed and can be either raster (gridded topography) or vector (points, lines, and polygons). Digitizing map objects, like drainage networks or lithologic unit frameworks, produces vector data. Although raster data can be obtained straight from the output of a satellite sensor, grid data is typically interpolated from field samples that are irregularly distributed (a process known as gridding) (Trauth, 2007). Cubic splines are commonly used to find the smoothest curve (Ahlberg et al., 1967) or surface (De Boor, 1962; Bhattacharyya, 1969) that passes through a set of irregularly spaced data appoints. This technique corresponds physically to forcing an elastic beam (or elastic sheet) to match the data points. The interpolating curve (or surface) satisfies the biharmonic equation and therefore has minimum curvature (Briggs, 1974).Because of its great precision, ease of use, and adaptability, Sandwell's (1987) biharmonic spline interpolation approach—which is based on Green's function—has become the standard method. When filling in a sparse grid, spline smoothing works best at removing angular curves or surfaces. In the physical sciences, interpolation and gridding of data are natural processes that are carried out on an equidistant grid using an averaging or finite difference scheme. Because of their smooth look, cubic splines are frequently used; nonetheless, these functions may exhibit undesired oscillations between data points (Wessel et al., 1998).

In linear methods, the new surface is restricted to the area that contains the disease points and by default, it does not extrapolate beyond its regions. Additionally, compared to biharmonic approaches, which smooth the contour shapes, the linear method's outlines are more angular. The technique also offers a quick interpolation algorithm, but it has the drawback of only using the interpolation function within the region enclosed by a convex random set of data points, which leaves the surface unsmoothed. However, the main benefit of the biharmonic technique is that slope measurements can be used as data, which is useful for some remote sensing applications where slopes are more accurately measured than heights. The technique is also easily applicable to interpolation problems in three or more dimensions, which is equivalent to multiquadric interpolation in three dimensions (Hardy, 1971; Hardy et al., 1986).

Although not directly applied to "diseases" in a medical context, biharmonic spline interpolation is a mathematical technique that fits a smooth surface through a set of data points; for instance, it can be used to analyze patterns in disease progression or to create smooth representations of data from medical imaging. Biharmonic interpolation is a more sophisticated technique that yields smoother, more accurate results, particularly when dealing with complex surfaces or when smoothness is a priority (Liping et al., 2018). A good number of statistical reviews on disease mapping have been conducted (Clayton et al., 1992; Smans et al., 1992; Wakefield et al., 2000; Manda, et al., 2011). Disease mapping is typically carried out to investigate the geographical distribution of disease burden. By estimating the disease burden in specific areas, area specific estimates of risk may inform public health resource allocation, and the informal comparison of risk maps with exposure maps may generate hypotheses or provide clues to the etiology (Wakefield et al., 2007).As a component of the traditional triad of person, place, and time, disease mapping has a long history in epidemiology (Walter, 2000). There are several statistical reviews available, such as Mollie (1996). Examining the spatial distribution of illness prevalence rates and their relationships within a specified distance and direction is crucial for disease mapping. Given that some regions' socioeconomic and demographic characteristics may transcend physical borders, it is generally expected that nearby regions or places will have comparable rates to those that are farther away. Biharmonic spline interpolation can help create smooth illustrations of disease advancement or spatial distribution patterns, such as when interpolating data from satellite imagery or other sources to model the spread of a disease (Feliciano-Cruz, et al., 2012). This spatial correlation is often measured as a function of distance between pairs of locations, with measurement enabled by Geographic Information Systems (GIS) technology.

For three years in a row (2007, 2008, and 2009), tuberculosis was the top cause of mortality among the top ten underlying natural causes of death in South Africa, which has the third-highest global burden of the disease (Mollie, 1996). In the Eastern Cape Province, tuberculosis primarily affects those in the working age range. For 2003, 2004, and 2005, the percentage distribution of reported TB cases in the 25–34 age range was 15.9%, 0.7%, and 23.1%, respectively. TB was the second highest cause of death for women and the third major cause of death for men aged 15 to 44, according to a 2000 MRC report on the South African National Burden of Disease study for the Eastern Cape Province. After KwaZulu Natal, the Eastern Cape Province has the second-highest provincial TB burden (WHO, 2010). HIV weakens the immune system, increasing the lifetime risk of developing from latent TB infection to active TB disease by 10% in people with normal immune systems. The prevalence of TB, TB/HIV co-infection, and MDR-TB is extremely high in the area. In 2008, the province saw over 60,000 new cases of tuberculosis.

One of the data analysis processes, curve fitting, is useful for prediction analysis by graphically illustrating the relationship between the data points, whether in a linear or non-linear model. The curve fit typically finds the peaks along the curve or simply smoothes the data and improves the plot's visibility. The goal of curve fitting is to describe a well-fitting model by examining the connection between independent and dependent variables. The mathematical equation that best fits the provided data is found using curve fitting (Vidyullatha et al., 2016). Biharmonic splines are a type of curve fitting method that are used in medical imaging to reconstruct multivariate functions from sampled data, offering advantages over traditional methods. They are especially useful in medical applications where the need to create smooth and accurate models of complex data is crucial (Deng et al., 2011). Biharmonic splines can also be used to interpolate data between data points, creating a smooth representation of the underlying surface, which can be useful for mapping disease distribution or visualizing disease patterns. A study on the geospatial mapping of drug-resistant tuberculosis prevalence in Africa at national and sub-national levels assembled a geolocated dataset from 173 sources across 31 African countries, comprising drug susceptibility test results and covariate data from publicly available databases. Bayesian model-based geostatistical framework with multivariate Bayesian logistic regression model was used to estimate DR-TB prevalence at lower administrative levels and it showed significant variation by country (Alemneh Mekuriaw Liyew, et al., 2025). It is notably seen that there is a dearth of research which employs curve fitting models to an infectious disease like TB. Therefore, in order to obtain surface and contour plots of reported TB and to study and compare the prevalence dynamics between districts or regions, this paper presents simple methods based on curve fitting methods, using the Linear and Biharmonic interpolants with smoothness as a priority. The methods are also used to identify and localize areas of the observed TB counts from the twenty-four (24) health subdistricts of the Eastern Cape province in order to visually represent the disease incidence.

MATERIALS AND METHODS

The datasets are typically represented as 3D or XYZ triplets, where X and Y are the spatial coordinates and Z is the variable of interest, in this case the province's TB counts. The entire method is implemented in MATLAB, and produces the surface and contour plots. For comparison, we performed the spatial analysis using two curve fitting methods, surface and contour plots, respectively, using the linear and biharmonic spline methods for the year (2012-2015) on the same geographical scales.

Study Area and Population

The Eastern Cape Province of South Africa is where this study was conducted. Located on South Africa's east coast, between KwaZulu-Natal and Western Cape provinces, is the Province of the Eastern Cape. It is bordered by Lesotho, the Free State Province, and the Northern Cape Province. Stretching from the semi-arid Great Karoo to the forests of the Wild Coast, the Eastern Cape Province is home to an incredible array of natural features. It also encompasses the rich Langkloof, the mountainous southern Drakensberg region, and the Keiskamma valley. The stunning coastline of the Eastern Cape, which borders the Indian Ocean, is its primary feature. The province covers an area of 168 966km² and with a population of 6 562 053 (Statistics South Africa, Censuses). The province is situated at 32.2968° S and 26.4194°E of the country.

With its capital at Bhisho, the Eastern Cape is the third most populous province in South Africa. The province's major towns and cities include Port Elizabeth, East London, Grahamstown, Mthatha (formerly Umtata), Graaf Reinet, Cradock, and Port St Johns. The province is comprised of two metropolitan municipalities: Buffalo City Metropolitan Municipality and Nelson Mandela Bay Metropolitan Municipality. It has six district municipalities, each of which is further subdivided into 37 local municipalities. The Eastern Cape is considered one of the poorest provinces in South Africa, primarily due to the poverty prevalent in the former homelands, where subsistence farming is the main source of income.

Epidemiological Data Sources

Based on TB notification and survey data from the Eastern Cape Province, this is a retrospective secondary data source. The Eastern Cape Department of Health provided the data for this study. In 2014, twenty-four health sub-districts in the province, including the two metropolitan municipalities, had electronic tuberculosis register (ETR) records from which 37,365 TB cases were reported. Figure 1 is an Eastern Cape Province map that displays the 24 health sub-districts where data was gathered.

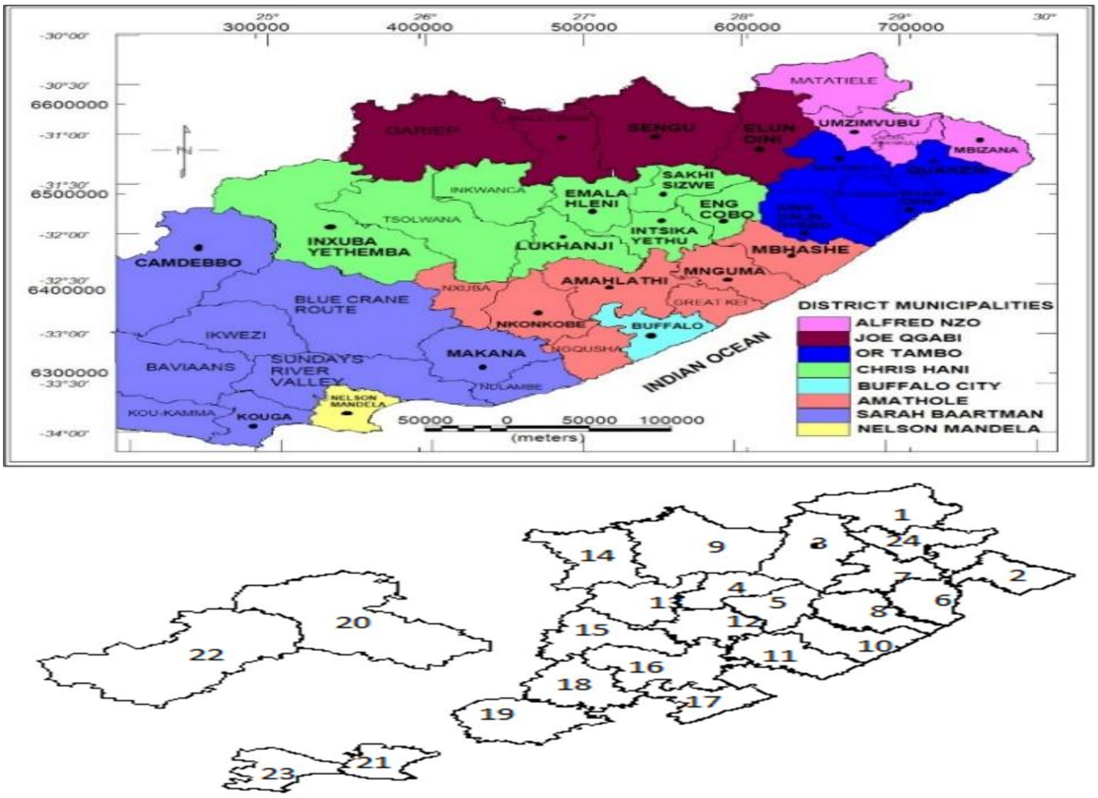


Figure 1: First Map. Eastern Cape Province showing 37 district municipalities and 2 metropolitans and Second Map. 24 health sub-districts for TB dataset extracted by R software

Biharmonic Spline Interpolation: Curve fitting

We have decided to fit an exponential curve, a shape that is frequently seen in mechanical part design and can, for instance, represent the shape of ultrasonic waveguides in practice:

$$y = e^{-x}, x \in [0, 1] \quad (1)$$

Splines

The spline functions may be defined using a set:

$$S = \pi_{b,k}^{m} \quad (2)$$

of linear piecewise polynomials of order k with breaks b1 <... < bp + 1 and no jump in any lower-order derivative than the mth derivative, which naturally includes a basis and a sequence f1,...,fn, each of which can be expressed uniquely in the form:

$$f(x) = \sum_{i=1}^{n} a_i \, f_i(x) \quad (3)$$

where n is the dimension of the linear space S and the coefficients are ai. The coordinates of f with regard to this basis are the coefficients ai. B-splines make up the basis of the space S. The third-degree polynomial that defines the interpolant function for a cubic spline is

$$p_i(x) = ai\,(x - x_i)^3 + bi(x - x_i)^2 + ci(x - x_i) + d \quad (4)$$

where for pi the variable xi takes values on intervals $x_{i-1} \leq x \leq x_i$, $i = 1, \ldots, n-1$

A piecewise-polynomial function is a function built from polynomials.

Interpolation

Building a function f that corresponds to a set of data values is a process(x_i, y_i): $f(x_i) = y_i$, all i.

The function f is the interpolant, developed as a unique function of the form (3).

Polynomial interpolation may be selected because for n data points (xi, yi) there is one polynomial of order n–1 that matches these data:

$$f_{i(x)} = \prod_{i \neq j}(x - x_j) \quad (5)$$

In spline interpolation, one chooses the fj to be the n consecutive B-splines, using Schoenberg-Whitney theorem.

RESULTS AND DISCUSSION

Table 1: Descriptive Statistics for TB Counts from 2012-2015

Variable	Observation	Mean	Std. Dev.	Min	Max
TB 2012	24	1373.792	869.462	34	3409
TB 2013	24	1588.542	1198.957	435	5409
TB 2014	24	1556.875	1217.089	388	5147
TB 2015	24	1537.167	1167.017	336	5147

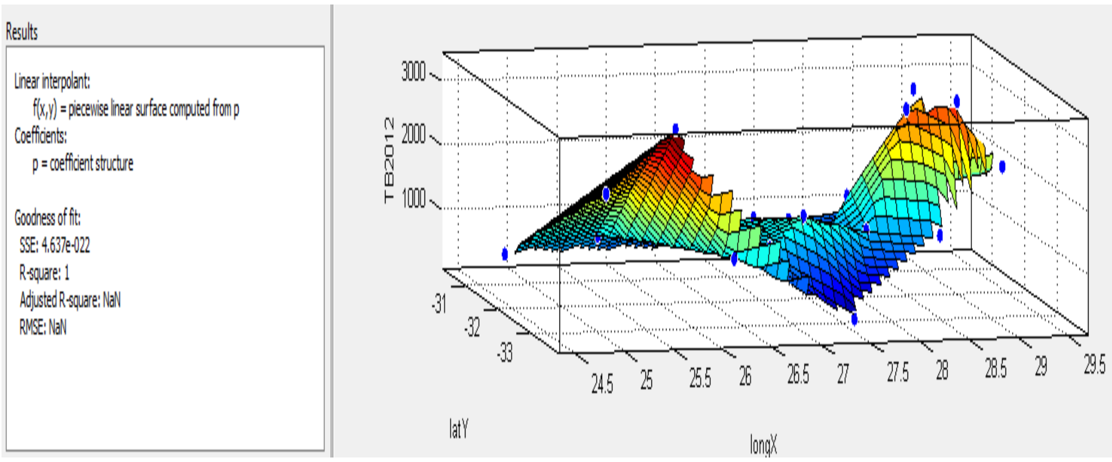
TB data analysis of cases collected between 2012 to 2015 showed that the highest cases were recorded in 2013 (mean = 1588.54), while the least was observed in 2012 (mean = 869.42) (Table 1).Skewness and Kurtosis tests for normality assessment of the variables showed that only TB data for 2012 is normally distributed (P-value > 0.05), while TB data for 2013, 2014 and 2015 are not distibuted normal (P-values < 0.05) (Table 2).

Table 2: Test for Normality for the TB Data (2012 – 2015)

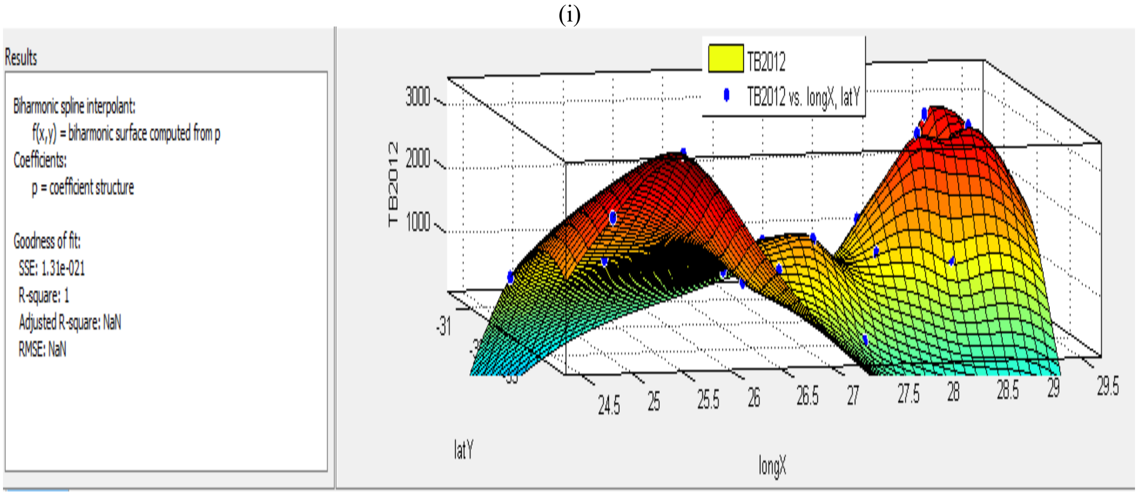
Variable	Observation	Pr (Skewness)	Pr (Kurtosis)	adj chi2(2)	Prob>chi2
TB 2012	24	0.0415	0.6588	4.47	0.1071
TB 2013	24	0.0007	0.0113	13.59	0.0011
TB 2014	24	0.0004	0.0063	14.84	0.0006
TB 2015	24	0.0011	0.0216	12.27	0.0022

Curve fittings with linear and biharmonic (Spline) methods for 3-Dpatterns of TB cases

The plottings of all the figures shown below displayed the spatial patterns of TB cases by the surface and contour plots using the linear and the biharmonic spline methods for the year (2012-2015). The datasets are represented generally as a 3D or XYZ triplets, where X and Y are the spatial coordinates and Z is the variable of interest,the TB counts in the province.

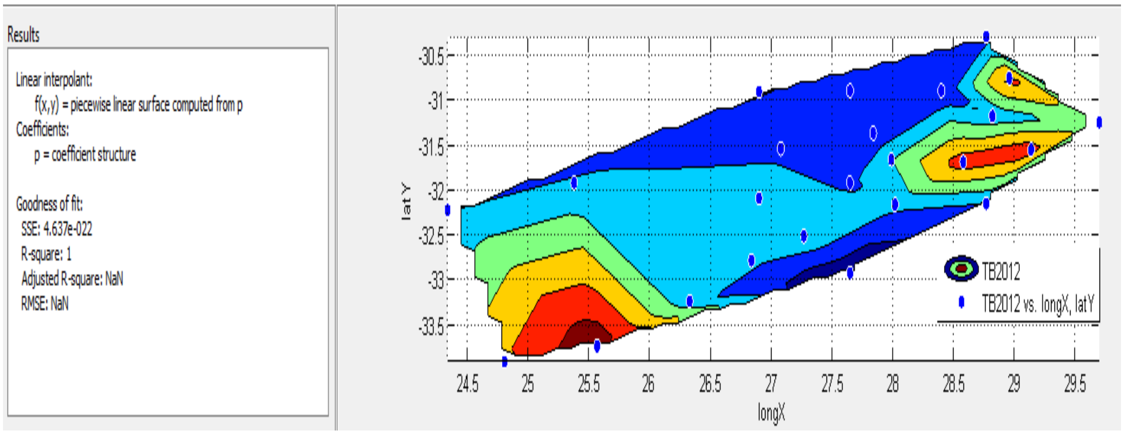


(i)



(ii)

Figure 2: Surface plots for (i) Linear interpolant and (ii) Biharmonic interpolant for TB 2012 data



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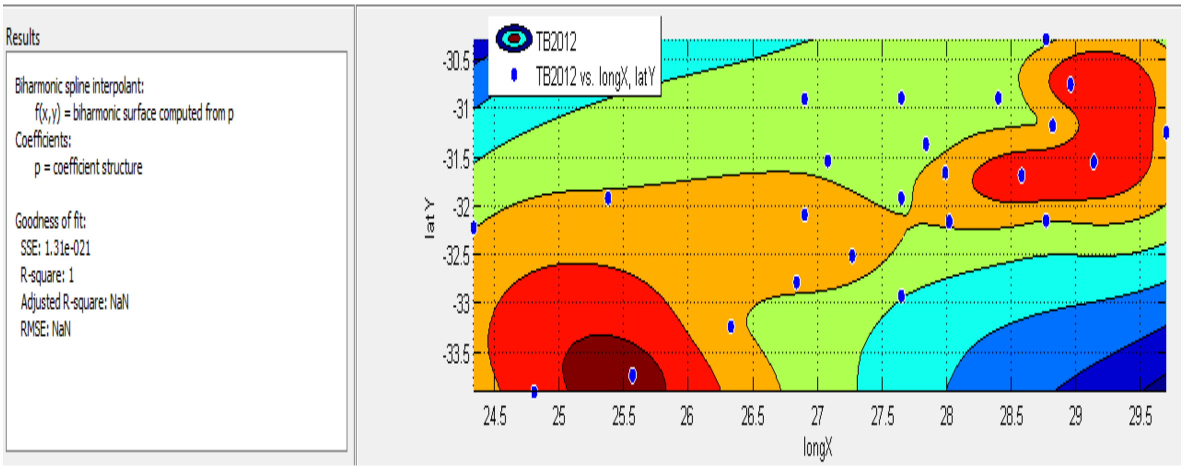
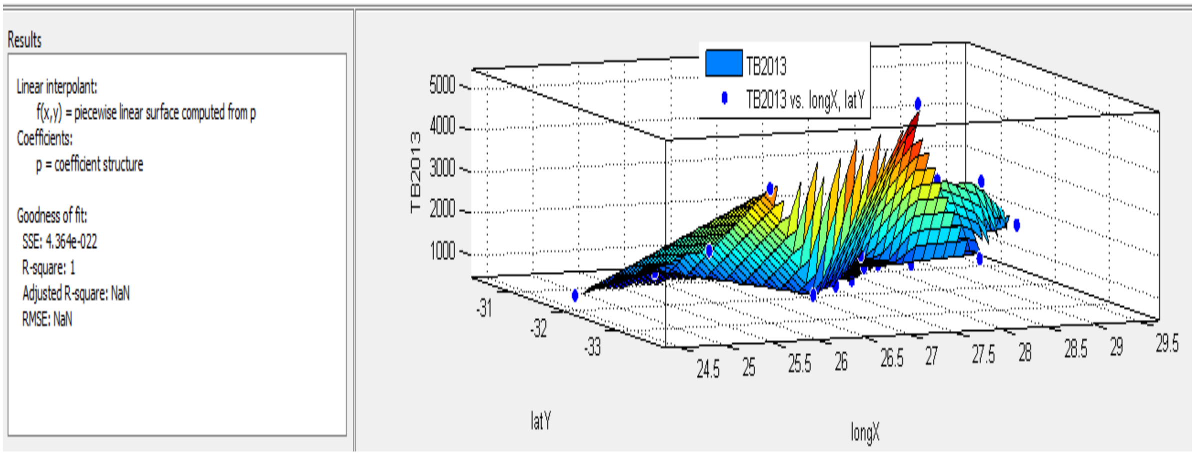


(ii)

Figure 3: Contour plots for (i). Linear interpolant and (ii). Biharmonic interpolant for TB 2012 data

The surface and contour biharmonic interpolant plots for TB 2012 data showed lower SSE values (SSE = 1.32e-021) respectively, indicating a better model fit, meaning the model's predictions are closer to the actual data points (Fig. 2 and 3).



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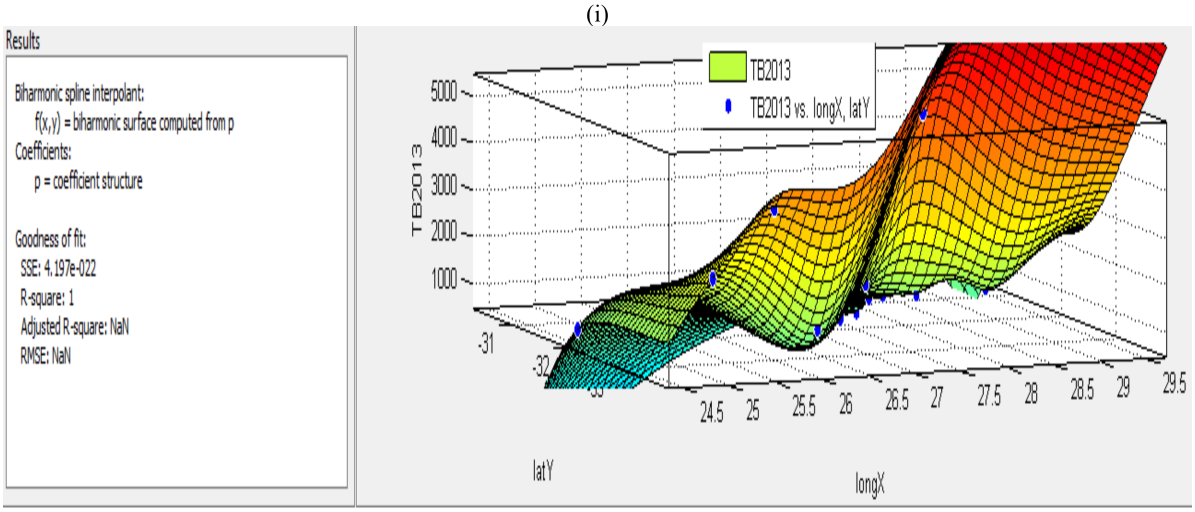
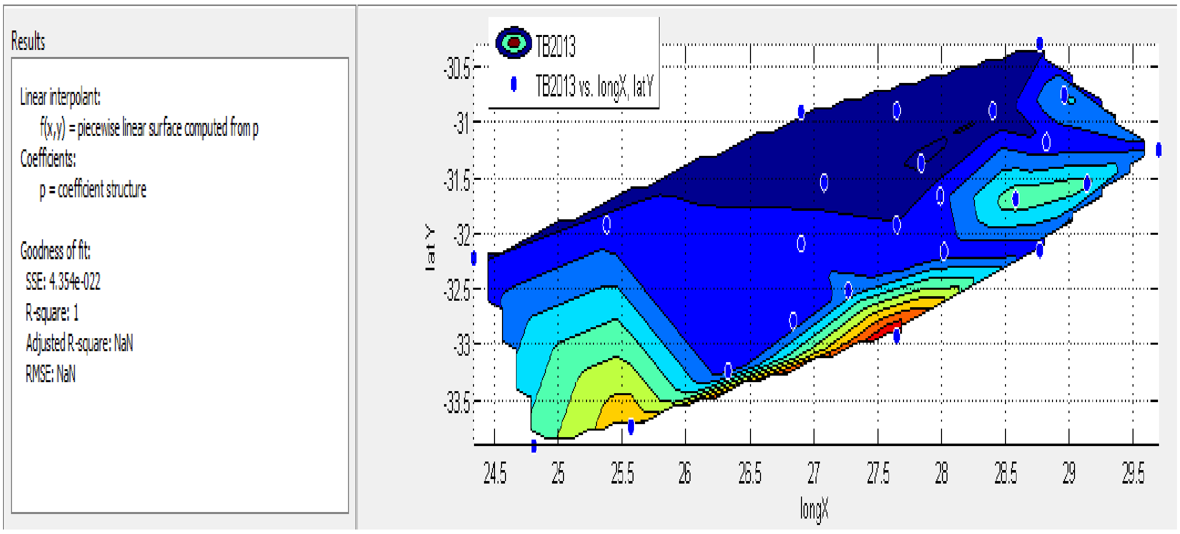


(ii)

Figure 4: Surface plots for (i) Linear interpolant and (ii) Biharmonic interpolant for TB 2013 data



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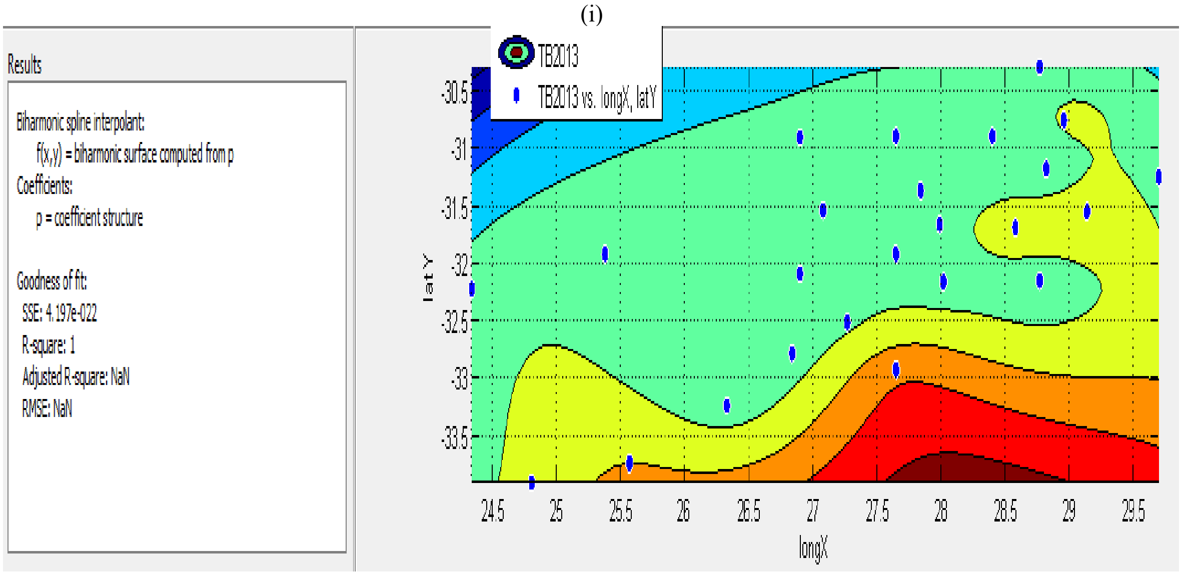
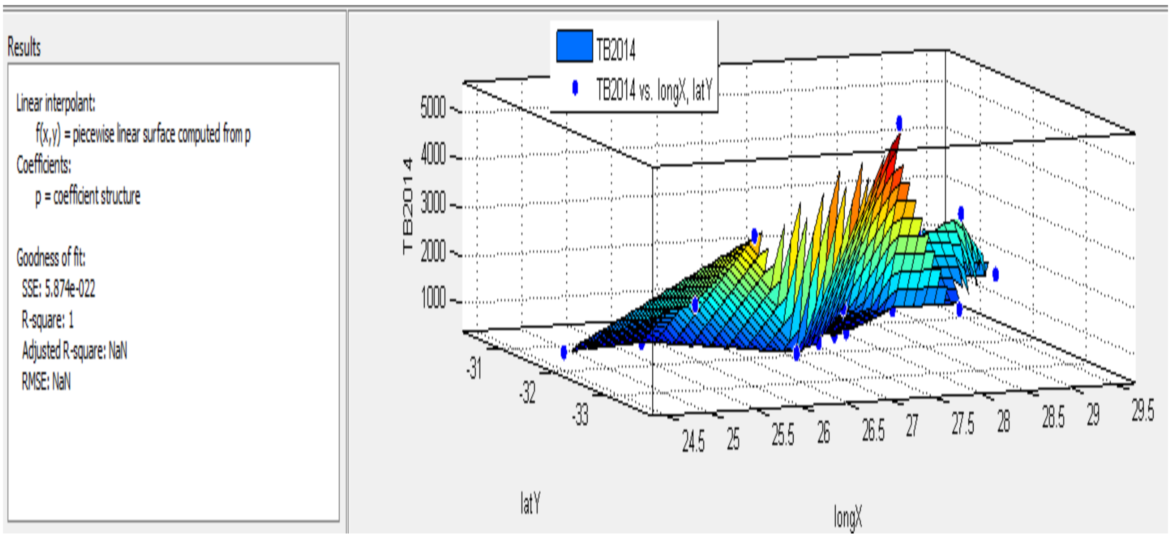


(ii)

Figure 5: Contour plots for (i). Linear interpolant and (ii). Biharmonic interpolant for TB 2013 data

The surface and contour plots for TB 2013 data also showed lower SSE values in the biharmonic interpolants (SSE = 4.197e-022) respectively, indicating a better model fit, meaning the model's predictions are closer to the actual data points (Fig. 4 and 5).



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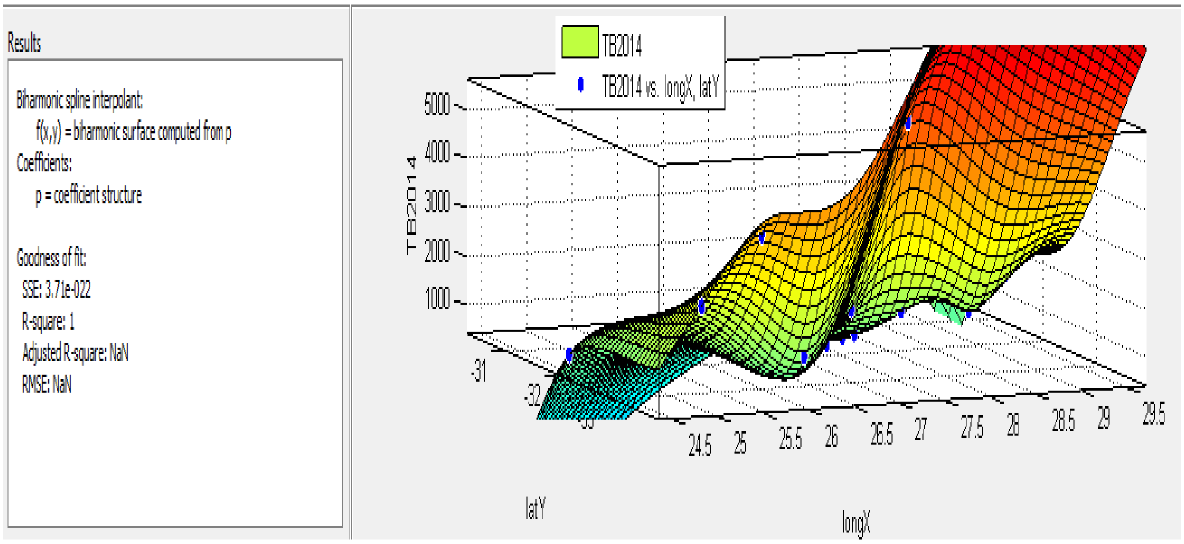
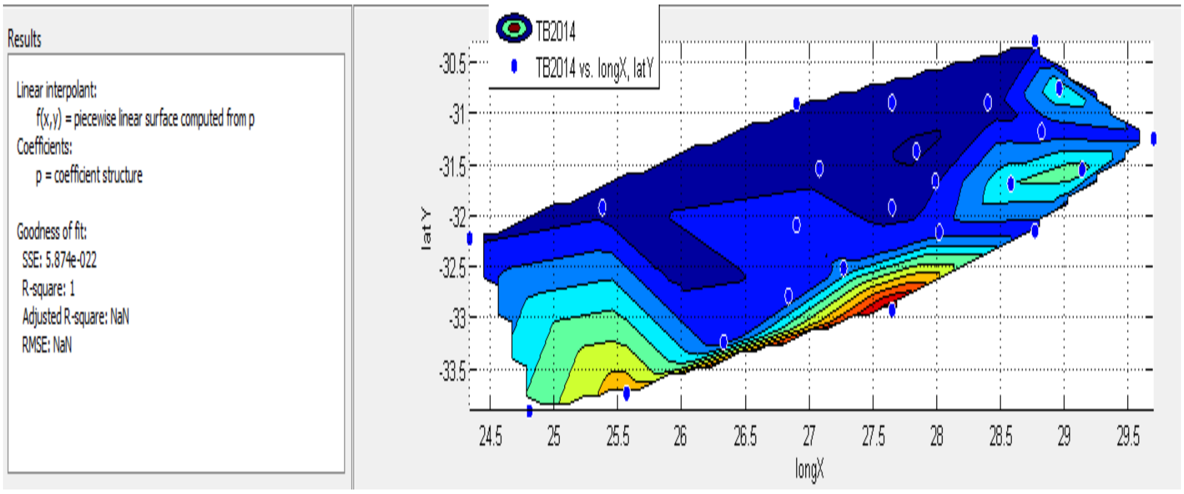


(ii)

Figure 6: Surface plots for (i) Linear interpolant and (ii) Biharmonic interpolant for TB 2014 data



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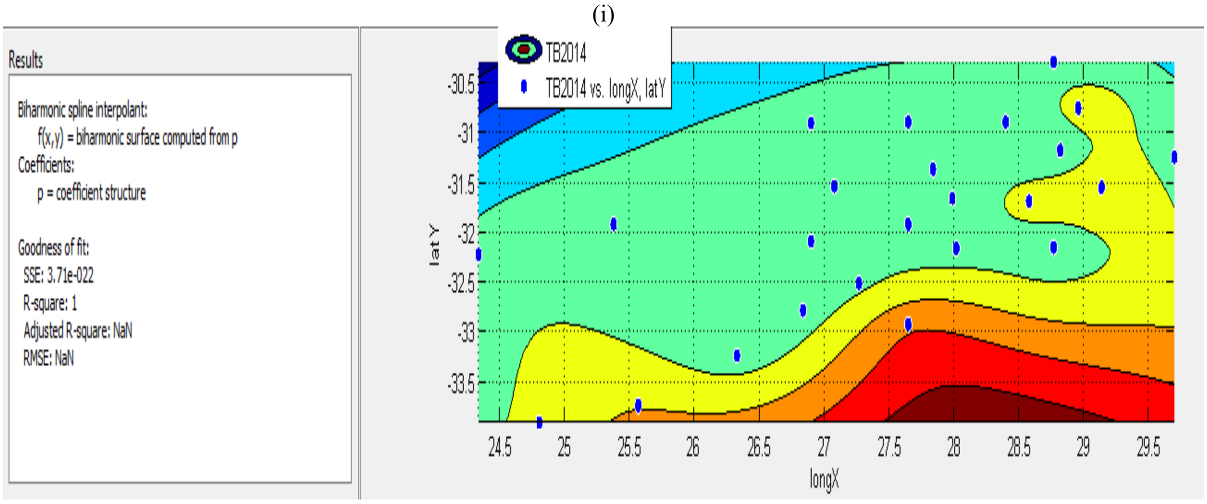
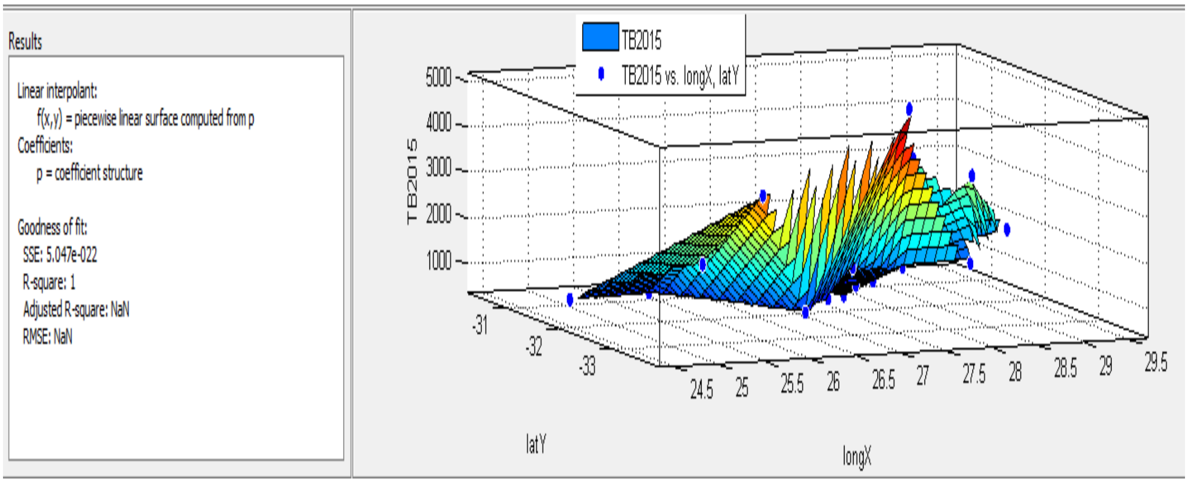


(ii)

Figure 7: Contour plots for (i). Linear interpolant and (ii) Biharmonic interpolant for TB 2014 data

The surface and contour plots for TB 2014 data also showed lower SSE values in the biharmonic interpolants with SSE = 3.71e-022 as against the linear interpolnat with SSE = 5.874e-022, indicating a better model fit, meaning the model's predictions are closer to the actual data points (Fig. 6 and 7).



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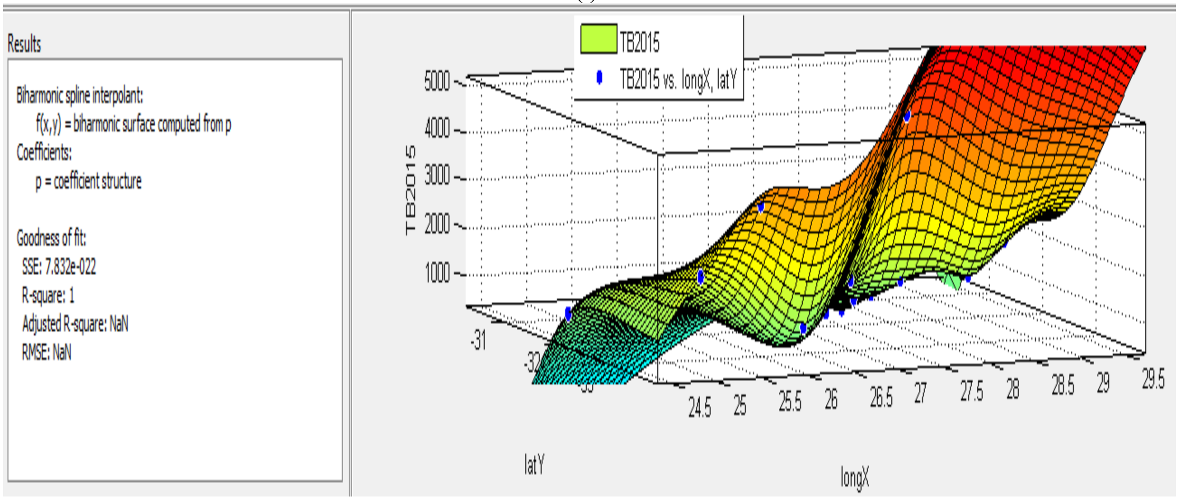
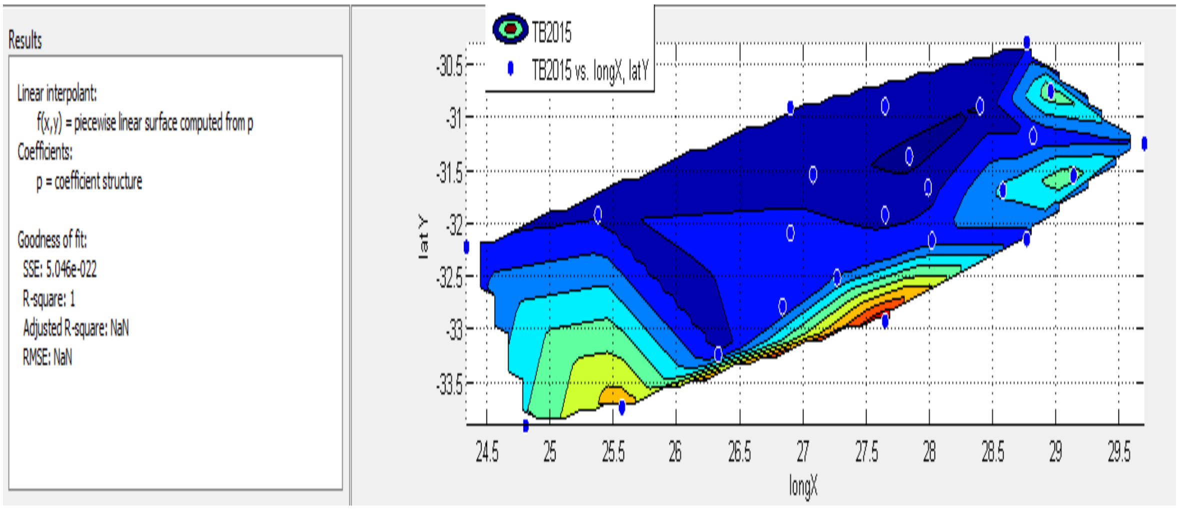
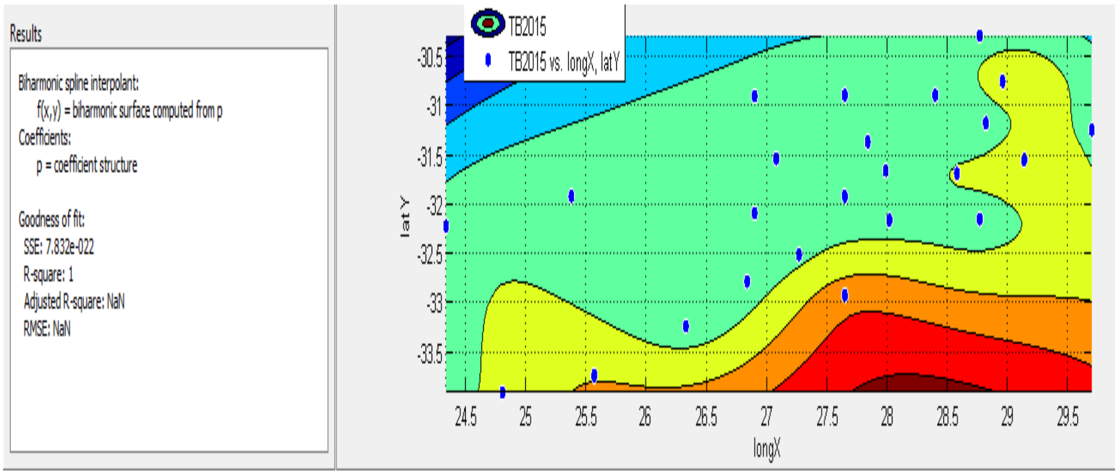


(ii)

Figure 8: Surface plots for (i). Linear interpolant and (ii) Biharmonic interpolant for TB 2015 data



(i)



(ii)

Figure 9: Contour plots for (i). Linear interpolant and (ii). Biharmonic interpolant for TB 2015 data

The surface and contour plots for TB 2015 data conversely showed lower SSE values in the linear interpolants with SSE = 5.046e-022 as against the biharmin interpolant with SSE = 7.832-022, indicating a better model fit, meaning the model's predictions are closer to the actual data points (Fig. 8 and 9).

Discussion

Estimating values between known data points can be done using both linear and biharmonic interpolation techniques, however their methods and levels of complexity vary. When used to surfaces, biharmonic interpolation uses more sophisticated mathematical functions to produce smoother, more curved surfaces than linear interpolation, which uses basic linear polynomials to produce straight lines between points. The more sophisticated method of biharmonic interpolation yields smoother, more accurate results, particularly when working with complicated surfaces or when smoothness is important.

Surface modeling is the process of determining a surface, whether natural or artificial, using one or more mathematical formulas. In order to model the three-dimensional surface in space, a function must be foundz = f (x, y) that represents the entire surface of the values z = f (x, y) associated with the point P (x, y) arranged irregularly. In addition, this function can predict the values z = f (x, y) and for other positions regularly arranged. Data gridding and interpolation are physical science procedures that are naturally carried out on an equidistant grid utilizing an averaging or finite difference scheme.

An interpolating curve or surface that passes through each data point is fitted by the interpolant fit category in curve fitting applications. Overall, all of the fittings' results indicated a systematic pattern in the Eastern Cape province's TB case distribution, indicating that the disease's spread is not random. It has also been noted that the province's TB distribution tends to concentrate more in the southern region, and that the disease incidence increases as one moves closer to the Indian Ocean's coast.

In Kouga (Sarah Baartman Municipality), Nelson Mandela, King Dalingyebo, and Nyandeni (OR Tambo), as well as Umzimvubu in Alfred Nzo municipality, there were only two high incidence hotspots of the disease in 2012. Similar clusters and hotspots of illness incidence were seen in 2013, 2014, and 2015.

The municipalities of Amathole, OR Tambo, and the coastal portion of Sarah Baartman Districts have a particularly high concentration of TB cases. Once more, they all showed a concerning trend of occurrences that were closer to the Indian Ocean. As TB occurrences either decrease away from the hotspot locations or rise as they get closer to them, the disease's effects on the neighborhood may also be seen. Although not completely devoid of the disease, the center and northern regions of the Eastern Cape province had a very low prevalence of tuberculosis.

Since curve smoothness is important in this situation, the biharmonic interpolant plots of all TB cases fit better with fewer sum of squares errors for every year except 2015. According to Tarpey (2000), a low sum of squares owing to error means that the model fits the data well, but a high sum of squares due to error implies that the model might not be correctly capturing the relationships between the variables. As a result, the results of this investigation demonstrated that nearly all biharmonic interpolants had lower SSE values than linear interpolants. This edge produced smoother plots that, even when there were some underlying transmitting factors present, accurately depicted the links between TB cases and their locations.

CONCLUSION

The MATlab data representation techniques have been explained in this research study. By default, linear algorithms do not extrapolate beyond their regions; instead, the new surface is limited to the region containing the illness points. Additionally, the linear technique produces more angular outlines than the biharmonic method, which smoothes out the contours. The method's drawback is that it only applies the interpolation function to the region enclosed by a convex random set of data points, which results in an unsmoothed surface. However, it does offer a quick interpolation process. Since its inception, MATLAB has included a biharmonic spline interpolation. Sandwell created this interpolation technique (Sandwell, 1987). Smooth surfaces produced by this particular gridding technique are best suited for noisy data sets with irregular control point distributions.

The study employed the linear and biharmonic spline interpolation techniques to determine the disease's geographical distribution based on the observed TB counts from the Eastern Cape province's twenty-four (24) health subdistricts. Both the surface and contour plottings were compared using these two approaches; the latter (biharmonic) was found to provide a better spatial pattern of TB prevalence for the regions and a well-defined disease map and fit from the values of their respective sum of squares errors (SSE). The irregularly spaced dataset is interpolated at grid points using the

spline interpolation technique on the biharmonic smoothing method.The biharmonic spline interpolation method used in this study provides a solution to most gridding problems. The biharmonic spline interpolations are in many ways, said to be the other extreme, because they can be used for very irregular-spaced and noisy data, hence the contours suggest an extremely smooth surface.
Green's function or the biharmonic technique is better or higher than the traditional finite-difference methods for reasonable amounts of data because (1) the model surface can be constrained by both data values and directional gradients, (2) noise and disturbances can be easily curbed by seeking a least-squares fit instead of exact interpolation, and (3) the model can be evaluated at random locations instead of only on a rectangular grid.
These novel approaches are infrequently employed in disease mapping applications, but they have the advantage of being able to be evaluated at subjective places as opposed to just on a rectangular grid, which is what most conventional GIS methods of geospatial studies do. In conclusion, the biharmonic function approach for splines has several valuable advantages, one of which is the significant simplification of the method's computer implementation and interpretation.

ACKNOWLEDGMENTS
We are grateful to data and research department, administrators, staff and members of Eastern Cape Department of Health for releasing the data of the province.

Availability of Data and Materials
This is a retrospective secondary data source from Eastern Cape Province TB notification and survey data. The data used in this study was obtained from Eastern Cape Department of Health. A total of 37,365 TB cases was reported and extracted from the electronic tuberculosis register (ETR) records from the twenty-four health sub-districts of the province, including the two metropolitan municipalities for the year 2012 – 2015.

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